Comments and Addenda

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Comment on the decay of $\psi(3.1)$ into even-G-parity states*

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We discuss what can be learned about the dynamics of the new particles by studying the decays of $\psi(3.1)$ into even-G-parity states.

In this note I will discuss what we can learn about the dynamics of the new particles[1,2] by studying the decays of $\psi(3.1)$ into even-G-parity final states. Experimental indications that $G=+1$ decays are rare are consistent with the hypothesis that $\psi(3.1)$ is a hadron with $G=-1$. As a hadron, it is created by a virtual photon in e^+e^- annihilation, and so it may decay via a virtual photon into hadrons, as in Fig. 1(A). Because the purely hadronic decays of $\psi(3.1)$ are suppressed by $\sim 10^3$, the process indicated in Fig. 1(A) (hereafter referred to as the "type-A" process) has a much larger branching ratio (by $\sim 10^3$) than the 10^{-4} we would have guessed by just counting powers of α. Squaring this amplitude, we obtain a contribution to the hadronic width,

$$\Gamma_A(\psi(3.1)\to\gamma\to\text{hadrons}) = R|_{3\text{ GeV}} \times \Gamma(\psi(3.1)\to\mu^+\mu^-) \cong 12\text{ keV}, \quad (1)$$

where we have used[3]

$$R|_{3\text{ GeV}} \equiv \sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)|_{3\text{ GeV}} = 2.5$$

and[4] $\Gamma(\psi(3.1)\to\mu^+\mu^-) = 5$ keV. Since[4] $\Gamma(\psi(3.1)\to\text{hadrons}) \cong 60$ keV, about 20% of the hadronic width of $\psi(3.1)$ is due to the type-A process.

Similarly for exclusive $G=+1$ channels such as the decay into $2n$ pions, we have

$$\Gamma_A(\psi(3.1)\to(2n)\pi) = \left[\frac{\sigma(e^+e^-\to(2n)\pi)}{\sigma(e^+e^-\to\mu^+\mu^-)}\right]_{\sqrt{s}=3\text{ GeV}} \times \Gamma(\psi(3.1)\to\mu^+\mu^-). \quad (2)$$

Using experimental values,[3,4] we find that the branching ratios for $\psi(3.1)\to 2\pi^+2\pi^-$ and $\psi(3.1)\to 3\pi^+3\pi^-$ are each $\sim\frac{1}{2}\%$. Given the enormous cross section for $e^+e^-\to\psi(3.1)$ at the peak of the resonance, it should be possible to measure these decay modes if they are indeed present near the 1% level.

The above remark has often been made in recent months. Here I will discuss two related issues:

(1) By determining whether there are other important mechanisms by which $\psi(3.1)$ decays into $G=+1$ states, we can learn about the nature of the dynamics which underlies the new particles.

(2) How large are the dynamically uninteresting

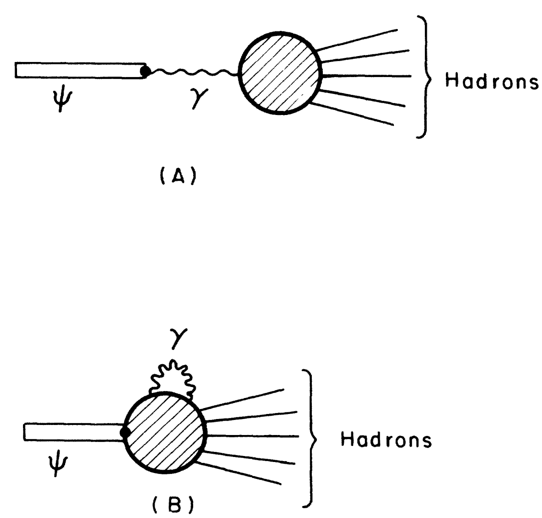

FIG. 1. Type-A and Type-B decays of $\psi(3.1)$ into hadrons.

sources of $G=+1$ states in e^+e^- annihilation at $\sqrt{s}=3.1$ GeV which determine how well we can detect the dynamically interesting mechanisms referred to in (1) above?

The alternative mechanism (hereafter referred to as "type B") for the decay of $\psi(3.1)$ into a $G=+1$ state is illustrated in Fig. 1(B), which is single-photon irreducible as opposed to Fig. 1(A). The simplest examples of type-B mechanisms are processes in which ψ decays by strong interactions into a $G=-1$ state which then undergoes electromagnetic final-state scattering or electromagnetic mixing into a $G=+1$ state. A specific example is illustrated in Fig. 2: ψ decays into $\omega\pi\pi$, which mixes electromagnetically into $\rho\pi\pi$.

The important point is that from our knowledge of the electromagnetic properties of ordinary hadrons, we can be confident that such final-state interaction amplitudes are much smaller than the type-A amplitudes. *Therefore, if there is a large discrepancy between the observed rate of $G=+1$ decays and that which we would expect due to the type-A mechanism, it would mean that*

(1) *either the hypothesis that $\psi(3.1)$ is a $G=-1$ hadronic state is false, or*

(2) *the type-B mechanism is involved in a way which illuminates the internal dynamics of the new particles.*

In the remainder of this note I will make the preceding statement more precise and will illustrate how the presence or absence of important type-B amplitudes can help us to understand the dynamics of the new particles.

First consider what is meant by a "large discrepancy" between the observed $G=+1$ decay rate and the rate which is expected from the type-A mechanism. In addition to the type-A mechanism, there are two trivial sources of $G=+1$ states in e^+e^- annihilation at $\sqrt{s}=3100$ MeV—the nonresonant background and type-B decays of $\psi(3.1)$ in which the electromagnetic interaction involves just the final-state hadrons (e.g., Fig. 2). These two amplitudes may interfere with the type-A amplitudes, and since the relative phases are unknown we cannot calculate the sum but can only estimate the range of possible values. Therefore, the presence of these two trivial sources of $G=+1$

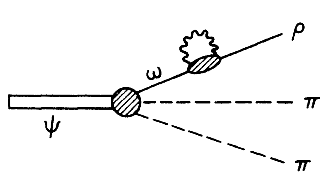

FIG. 2. Example of a Type-B decay involving electromagnetic mixing of the hadrons in the final state.

states determines a "noise level" beneath which we cannot hope to detect the more interesting type-B processes which probe the internal dynamics. This "noise level" provides the scale which determines what is meant by a "large discrepancy."

Consider first the nonresonant background. The background cross sections are[3]

$$\sigma(e^+e^-\to 2\pi^+2\pi^-)|_{3.1\text{ GeV}}\cong\sigma(e^+e^-\to 3\pi^+3\pi^-)|_{3.1\text{ GeV}}\cong 0.7\text{ nb}.$$

Taking the "true" cross section at the peak of the resonance[5,6] as $\sigma(e^+e^-\to\psi(3.1)\to\text{hadrons})\cong 1.0\times10^5$ nb, we compute "true" cross sections of

$$\sigma_A(e^+e^-\to\psi(3.1)\to 2\pi^+2\pi^-)\cong\sigma_A(e^+e^-\to\psi(3.1)\to 3\pi^+3\pi^-)\cong 500\text{ nb}.$$

(With the SPEAR resolution, "true" cross sections of 500 nb correspond to observable cross sections of 12 nb.) Therefore, the interference between the nonresonant background and type-A amplitudes gives rise to cross sections which may differ from 500 nb by a "noise" factor of $1\pm2(0.7/500)^{1/2}=1\pm0.07$.

Consider next type-B processes involving electromagnetic interactions among the final-state hadrons. These amplitudes are typically of order α compared to the nonelectromagnetic decay amplitudes, which are in turn only a factor of

$$\left[\frac{\Gamma_{\text{tot}}(\psi(3.1)\to\text{hadrons})}{R|_{3\text{ GeV}}\times\Gamma(\psi(3.1)\to\mu^+\mu^-)}-1\right]^{1/2}\cong 2$$

larger than the type-A amplitude. Thus these type-B amplitudes added to the type-A amplitudes will create interference effects of order $1\pm2(\frac{2}{137})\cong1\pm0.03$.

To improve on the above order-of-magnitude estimate of electromagnetic final-state effects, we would have to do a detailed study of each hadronic final state. Such an analysis would require more experimental information about the final states than we are likely to have in the near future. As an especially simple and experimentally accessible example, consider the process illustrated in Fig. 2. The probability for ρ-ω mixing is $\Gamma(\omega\to\pi^+\pi^-)/\Gamma(\rho^0\to\pi^+\pi^-)=0.85\times10^{-3}$. If x is the fraction of nonresonant $e^+e^-\to 2\pi^+2\pi^-$ events at $\sqrt{s}=3$ GeV which are in the $\rho^0\pi^+\pi^-$ mode and y is the branching ratio $\Gamma(\psi(3.1)\to\omega\pi^+\pi^-)/\Gamma(\psi(3.1)\to\text{hadrons})$, then the type-A mechanism contributes a branching ratio for $\psi(3.1)\to\rho^0\pi^+\pi^-$ of $\frac{1}{2}10^{-2}x$, and the B process of Fig. 2 causes this to be multiplied by an interference "noise" factor of $1\pm2[(0.85)10^{-3}y/\frac{1}{2}10^{-2}x]^{1/2}$. With[7] $y\sim\frac{1}{4}\times10^{-2}$

and $x \sim \frac{1}{3}$ we get 1 ± 0.07, which is consistent with the above order-of-magnitude estimate. If x were much smaller than $\frac{1}{3}$, the noise factor would be larger, but this would not have important consequences since $\rho\pi\pi$ would then be only a very small fraction of the $2\pi^+2\pi^-$ events (the number of $\rho\pi\pi$ events decreases as x while the noise only increases as $1/\sqrt{x}$).

Since we cannot now hope to carry out such an analysis for each important final state, we will have to use an order-of-magnitude estimate. To be conservative, we allow an extra factor of 3 beyond the initial estimate of 1 ± 0.03, so that we estimate interference effects due to final-state interaction at 1 ± 0.1. Combining this with the above estimate of interference with the nonresonant background (1 ± 0.07) we obtain a generously large estimate of 1 ± 0.2 for the total noise level due to "uninteresting" interference effects. As a practical matter, the uncertainty in this estimate is not at the moment very important, because the experimental errors in the measurement of the nonresonant $2\pi^+2\pi^-$ and $3\pi^+3\pi^-$ cross sections are even larger.[3] These errors, which are $\gtrsim 20\%$, introduce greater uncertainty in the estimate that $\sigma_A(2\pi^+2\pi^-) \sim \sigma_A(3\pi^+3\pi^-) \sim 500$ nb than the uncertainties in the estimate of the "uninteresting" interference effects.

We now briefly illustrate how the dynamics underlying the new particles determines whether there are additional large type-B amplitudes for $\psi(3.1)$ to decay into $G=+1$ final states. As a first example, consider the hypothesis that $\psi(3.1)$ has the Han-Nambu[8] assignment $(1,8)$, i.e., a singlet under ordinary SU(3) and an octet under the color SU(3). Since the electromagnetic current is $J_{em} = J(8,1) + J(1,8)$, there are type-B amplitudes

$$\langle \psi(1,8) | J(8,1)(x) J(1,8)(0) | (8,1) \rangle$$

which transform $\psi(3.1)$ into an ordinary $(8,1)$ hadronic state which may then decay with a typical hadronic width $\gtrsim 100$ MeV. Thus we estimate the partial width for such decays to be $\Gamma_B \gtrsim \alpha^2 \times 100 \text{ MeV} \sim 10$ keV, which is of the same order of magnitude as the contribution of the type-A process, $\Gamma_A(\psi(3.1) \to \gamma \to \text{hadrons}) \cong 12$ keV. Therefore, with Han-Nambu dynamics, we expect $G=+1$ decays of $\psi(3.1)$ at a rate which differs from the type-A rate by much more than the 20% "noise level" discussed above.

As a second example, we consider a model which is unattractive from a theoretical point of view but which, given the depth of our ignorance, nonetheless deserves consideration from a phenomenological point of view. Suppose, in partial analogy with charmonium models, that $\psi(3.1)$ is a bound state of new quarks, $\bar{q}'q'$, which carry a new quantum number. The new quantum number is conserved by strong interactions but, unlike charm, is violated by first-order electromagnetic interactions. Exchange of a single photon then allows the transition of the new quarks to an ordinary $\bar{q}q$ quark pair which has an uninhibited transition into hadrons. As in the preceding estimate we then expect $\Gamma_B \gtrsim 10$ keV, and there should be substantial deviations from the rates expected for type-A processes alone.

Consider next those versions of the charmonium model[9] in which the narrow width of $\psi(3.1)$ is viewed as analogous to the poorly understood suppression of $\phi \to \pi\pi\pi$ (often referred to as "Zweig's rule"). Experimental information from ϕ decays suggests that the suppression mechanism continues to operate in decays in which both strong and electromagnetic interactions occur. For instance, $\Gamma(\phi \to \pi^0\gamma)$ as measured at Orsay is consistent with the rate to be expected using vector-meson dominance and the hypothesis that $\phi \to \pi^+\pi^-\pi^0$ proceeds primarily via $\phi \to \rho\pi^0 \to \pi^+\pi^-\pi^0$.[10] Therefore, if the ψ-ϕ analogy is correct, all type-B decay amplitudes of $\psi(3.1)$ are of order α smaller than the (suppressed) strong decay amplitudes. All type-B amplitudes are then of the same order of magnitude as the previously discussed type-B amplitudes involving electromagnetic interactions of the final-state hadrons. Therefore, in these models we expect $\psi(3.1)$ to decay into $G=+1$ states at the rate to be expected from type-A amplitudes, with corrections in the rate of no more than $\sim 30\%$, which includes the $\sim 20\%$ "noise level" discussed above. (A caveat: It is possible, though it seems far-fetched, that second-order electromagnetic processes could violate Zweig's rule although first-order processes do not. In this case, type-B amplitudes could be much larger.)

A more quantitative evaluation of the type-B amplitude is possible in the charmonium model of Appelquist and Politzer,[11] since in that model the underlying dynamics is precisely specified to be an asymptotically free, non-Abelian gauge theory. The strong-interaction decay of $\psi(3.1)$ into hadrons is calculated from the amplitude of Fig. 3, as if the three gluons were actually produced on their mass shell. It is assumed that the confinement mechanism which is conjectured to operate at large distances (~ 1 fm) causes the gluons to have a transition into ordinary hadrons with unit probability. The result of their calculation is

$$\Gamma(\psi(3.1) \to 3 \text{ gluons} \to \text{hadrons}) = \tfrac{5}{18} \times \tfrac{2}{9} \times \frac{\pi^2 - 9}{\pi} \alpha_s^{\,3} (\tfrac{4}{3}\alpha_s)^3 m_c, \quad (3)$$

where $\alpha_s \cong 0.19$ is determined[6] from

$$\Gamma(\psi(3.1) \to e^+e^-)/\Gamma(\psi(3.1) \to 3 \text{ gluons} \to \text{hadrons}) \cong \tfrac{5}{48}$$

and m_c is the mass of the charmed quark.

The leading type-B amplitude is obtained by replacing one of the three gluons in Fig. 3 with a photon. For this process I have obtained[12]

$$\Gamma(\psi(3.1) \to \gamma + 2 \text{ gluons}) = \frac{8}{9} \times \frac{2}{9} \times \frac{\pi^2 - 9}{\pi} \alpha \alpha_s^{\,2} (\tfrac{4}{3}\alpha_s)^3 m_c. \quad (4)$$

However, at distances of order 1 fm, the effective coupling of the photon is still $e = (4\pi\alpha)^{1/2} = (4\pi/137)^{1/2}$, so that the transition $\gamma + 2$ gluons $\to$ hadrons occurs with a rate P of order α and not of order unity. Therefore, we obtain

$$\frac{\Gamma_B(\psi(3.1) \to \gamma + 2 \text{ gluons} \to \text{hadrons})}{\Gamma_{\text{tot}}(\psi(3.1) \to 3 \text{ gluons} \to \text{hadrons})} = \frac{16}{5} \frac{\alpha}{\alpha_s} P. \quad (5)$$

Since

$$\frac{\Gamma_A(\psi(3.1) \to \gamma \to \text{hadrons})}{\Gamma_{\text{tot}}(\psi(3.1) \to 3 \text{ gluons} \to \text{hadrons})} \cong \tfrac{1}{4},$$

and taking, for example, $P \cong \alpha$ we have

$$\frac{\Gamma_B(\psi(3.1) \to \gamma + 2 \text{ gluons} \to \text{hadrons})}{\Gamma_A(\psi(3.1) \to \gamma \to \text{gluons})} \cong 3.6 \times 10^{-3}. \quad (6)$$

Interference between the A and B amplitudes can therefore modify the rate due to the type-A process alone by a factor $1 \pm 2(3.6 \times 10^{-3})^{1/2} = 1 \pm 0.12$. Taking account of the previously discussed "noise level," the observed rate for $\psi(3.1)$ to decay into $G = +1$ states should not differ by more than ~30% from the rate due to the type-A process alone.

Equation (5) is in the spirit of the original charmonium calculation[11] of $\Gamma(\psi \to 3 \text{ gluons} \to \text{hadrons})$ since we treat the photon and the two gluons as though they were real and ignore (except for the factor P) the details of the final-state interaction $\gamma + 2$ gluons $\to$ hadrons. This hypothesis is only a conjecture both in its original version and in the extension to the virtual photon which has been introduced here. It is a necessary hypothesis if we wish to make quantitative predictions, since we cannot hope to calculate in detail how the gluons (or, here, gluons and photon) become the composite quark states, which are the hadrons of the model. The hypothesis could be totally incorrect or it could be qualitatively correct yet fail quantitatively (the latter possibility seems more likely in the extension to the virtual photon than in the original model). If, for instance, we had $P \cong 10\alpha$, then for Eq. (6) we would have 3.6×10^{-2} and the interference effect could be 1 ± 0.4, or 1 ± 0.6 when the noise level is added.

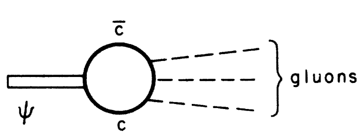

FIG. 3. The strong-interaction decay of $\psi(3.1)$ in the charmonium model of Appelquist and Politzer.

We should also keep in mind that there is no model-independent quantitative connection between the type-A and type-B processes and the production of $G = +1$ final states [except for certain type-A processes, as in Eq. (2)]. However, in the models discussed here, we would expect these processes to give rise to $G = +1$ and $G = -1$ states in comparable proportions. Whereas the SU(3) electromagnetic current of Gell-Mann and Nishijima is $\frac{3}{4}$ part $G = +1$ and $\frac{1}{4}$ part $G = -1$, in the extension to SU(4) the current is $\frac{9}{20}$ part $G = +1$ and $\frac{11}{20}$ part $G = -1$, and in the Han-Nambu model (i.e., the SUB version[8]) it is $\frac{3}{8}$ part $G = +1$ and $\frac{5}{8}$ part $G = -1$. For type-A decays in both models and for the type-B decays of charmonium which proceed by $\gamma + 2$ gluons, the virtual photon couples to ordinary hadrons in the final state, so we expect $G = +1$ states to predominate over $G = -1$ states by approximately 3:1. For type-B decays in the Han-Nambu model the situation is more complicated, but again we expect $G = +1$ and $G = -1$ states to occur with rates which are equal within an order of magnitude.

The analysis which has been presented here is possible because of the enormous cross section for $e^+e^- \to \psi(3.1)$, which makes the "signal-to-noise" ratio extremely favorable, that is, the ratio of the type-A amplitude to the uninteresting sources of $G = +1$ states. It would not be profitable to pursue such an analysis of the decays of ϕ and $\psi(3.7)$. For instance, $\sigma_A(e^+e^- \to \phi \to \gamma \to \pi^+\pi^-)$ is only ~2% of the nonresonant background for $e^+e^- \to \pi^+\pi^-$ at $\sqrt{s} = 1$ GeV, and $\sigma_A(e^+e^- \to \psi(3.7) \to \gamma \to 2\pi^+2\pi^-)$ is probably a few times smaller than the nonresonant cross section $\sigma(e^+e^- \to 2\pi^+2\pi^-)$ at $\sqrt{s} = 3.8$ GeV.[13]

To conclude: The hypothesis that $\psi(3.1)$ is a $G = -1$ hadron implies that ~20% of the hadronic decays proceed through a virtual photon—the type-A mechanism of Fig. 1(A). Regardless of dynamics, interference effects may cause decay rates into $G = +1$ states to differ by as much as 20% from the rate due to the type-A mechanism alone. Additional large type-B interference effects may or may not be present, depending on the internal dynamics of the new particles. For example, in the charm model, additional interference effects are probably small, but in the Han-Nambu color model type-B interference effects would be as large as the type-A amplitudes.

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[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[2]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974).

[3]B. Richter, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. IV-37.

[4]The decay parameters used here are preliminary values culled from a variety of sources. See, for instance, R. Pearson *et al.*, SLAC Report No. SLAC-PUB-1515, 1974 (unpublished).

[5]R. Schwitters, presented at the Annual Meeting of the American Physical Society, Anaheim, California, 1975 (unpublished).

[6]According to Ref. 5, the measured cross section for $e^+e^- \to \psi(3.1) \to$ hadrons is ~ 2500 nb. Taking $\Gamma(\psi(3.1) \to \text{hadrons}) \sim 60$ keV and 2.5 MeV as the SPEAR resolution (FWHM), we compute the true cross section.

[7]These values for x and y are based on my own very rough analysis of Fig. 18 of Ref. 3 and of a figure presented in Ref. 5. In addition I have used a ~2% branching ratio for $\psi(3.1) \to 2\pi^+ 2\pi^- \pi^0$.

[8]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965). Actually the model used here is the so-called SUB version considered by N. Cabibbo, L. Maiani, and G. Preparata, Phys. Lett. 25B, 132 (1967). In the original proposal of Han and Nambu, ordinary SU(3) is identified with the diagonal subgroup of $SU(3)' \times SU(3)''$, while in the SUB version it is identified with $SU(3)'$.

[9]For instance, see S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975); A. De Rújula and S. L. Glashow, *ibid.* 34, 46 (1975); C. G. Callan, R. L. Kingsley, S. B. Treiman, F. Wilczek, and A. Zee, *ibid.* 34, 52 (1975).

[10]J. Lefrançois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, New York, 1972).

[11]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[12]This result is larger by a factor 3 than that obtained by T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, Phys. Rev. Lett. 34, 365 (1975) (see their footnote 12). The discrepancy is due to a statistical factor 3!/2! in going from three to two identical particles, which was omitted by the above authors. I thank Tom Appelquist for a discussion.

[13]These estimates are based on data from Refs. 3, 5, and 10.