Coherent states with definite charge and isospin and cluster production

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A solvable unitary model of Auerbach *et al*. is generalized to take into account the isospin structure of the pion and the leading-particle system. The coherent-state representation is used to discuss the production of isovector objects, such as pions and resonances (or clusters). Particular attention is paid to productions when the isospin state of the ingoing and outgoing leading-particle system is fixed. The isospin analysis of the pion-pion correlation is performed by assuming both the direct-pion-emission mechanism and the cluster-emission mechanism. The slope of $\langle n_0 \rangle_{n_-}$ is predicted to increase with energy and is positive in the π-ρ and ρ-B regions (about 3/4 and about 14/17, respectively).

I. INTRODUCTION

It is generally accepted today that a realistic model of particle production in hadron collisions at high energy should include quantum numbers, such as charge, isospin, baryon number, strangeness, etc., while at the same time satisfying the constraints imposed by unitarity. Recently, several solvable unitary models have been proposed in the approximation of small transverse momenta and small center-of-mass energy of secondaries. They all belong to a class of models first proposed by Auerbach, Aviv, Blankenbecler, and Sugar[1] (AABS model).

In these models, incident hadrons are pictured as propagating through the interaction region, without making significant fractional changes in their c.m. energies. A generalization of the AABS model[1] which includes internal symmetries,[2] such as charge and isospin, enables one to study the correlation between pions in accord with isospin and charge conservation. The isospin structure of the underlying dynamics can be studied by measuring correlations between definite charge combinations, such as $+-$, $--$, -0, etc. For example, from charge conservation alone the correlation parameter f_2^{--} is negative[3] and fails to fit the data if the independent emission of charged pions is a dominant production mechanism.

On the other hand, if isospin conservation is included, it alone gives strong positive correlations[4] for pairs of like pions ($f_2^{--} \geq 0$) and for pairs of charged pions. It fails to give a positive correlation between a neutral and a charged pion ($f_2^{0-} < 0$). Experimentally, however, one finds a strong positive correlation between π^0 and π^{ch} pions.

The correlation between charged and neutral pions has also been considered within the framework of other theoretical models[5]: (a) the independent emission of resonances (or clusters) constrained by charge conservation and (b) the statistical-like approach using a suitable functional averaging procedure[6] for the pion field.

In this paper we use the coherent-state representation[6,7] to discuss the production of isovector clusters in nucleon-nucleon scattering within the AABS model. Clusters emitted in such a process decay subsequently into pions outside the region of interaction. The isospin-invariant coupling of isovector clusters to a nucleon system is constructed. In Sec. II, we generalize the AABS model[1] to take into account the isotopic spin of pions and construct coherent states with definite charge and isotopic spin. These coherent states are then coupled to definite isospin states of ingoing and outgoing leading-particle systems. In Sec. III, we confront the problem of dynamics. The simple form of the input function W, which we consider, corresponds to scattering from a gray disk with a constant radius and opacity that grows logarithmically with energy. In Sec. IV, we illustrate our technique by an example of $pp \to pp$ + pions scattering. Results for other cases are tabulated. In Sec. V, we discuss a general formalism for the independent emission of isovector clusters and clusters which have more complicated isospin structure. Effects on the pion-pion correlation are indicated. In Sec. VI, we briefly summarize and discuss our results.

II. AABS MODEL WITH ISOSPIN

In the AABS model, the scattering operator (the S matrix) is diagonal in the rapidity difference $Y \sim \ln s$, and the relative impact parameter $\vec{B}$ of the two incident hadrons. It can be written in the explicitly unitary form

$$S(Y, \vec{B}) = \exp[i\chi(Y, \vec{B})] , \tag{2.1}$$

where the Hermitian operator $\chi(Y, \vec{B})$ determines the amplitude for emitting or absorbing a given number of pions with each interaction. The scattering operator $S(Y,\vec{B})$ acts only in the space spanned by pions. The only reference to the nucleon system that remains is contained in the diagonal variables Y and $\vec{B}$. On the other hand, inclusion of isospin requires $S(Y,\vec{B})$ to be not only an operator in the space spanned by pions but also a matrix in the isospace of nucleons.

The creation and annihilation operators of physical pions are written in the spherical bases and are normalized so that

$$[a_\sigma(q),a^\dagger_{\sigma'}(q')]=\delta_{\sigma\sigma'}4\pi(2\pi)^2\delta(y-y')\delta^{(2)}(q_T-q'_T)\,,\quad \sigma,\sigma'=0,+,-\,. \tag{2.2}$$

The four-momenta of pions are expressed in terms of their rapidity y and transverse momenta $\vec{q}_T$.

The scattering operator $S(Y,\vec{B})$ is best analyzed in the so-called coherent-state representation,[6,7] which provides a useful basis for parametrizing the scattering operator. The coherent states $|\vec{W};Y,\vec{B}\rangle$ are defined as eigenstates of $a_\sigma(q)$

$$a_\sigma(q)|\vec{W};Y,\vec{B}\rangle=W_\sigma(Y,\vec{B};q)|\vec{W};Y,\vec{B}\rangle\,, \tag{2.3}$$

where $W_\sigma(Y,\vec{B};q)$ denote arbitrary complex functions of $Y,\vec{B}$, and q. $|\vec{W};Y,\vec{B}\rangle$ can also be written in the form

$$|\vec{W};Y,\vec{B}\rangle=D(\vec{W};Y,\vec{B})|0\rangle\,. \tag{2.4}$$

Here $D(\vec{W};Y,\vec{B})$ is the unitary coherent-state displacement operator

$$D(\vec{W};Y,\vec{B})=\exp\left[\sum_\sigma\int dq\,W_\sigma(Y,\vec{B};q)a^\dagger_\sigma(q)-\text{H.c.}\right], \tag{2.5}$$

where $dq\equiv[4\pi(2\pi)^2]^{-1}dy\,d^2q_T$. $D(\vec{W};Y,\vec{B})$ is unitary in the sense that

$$D^\dagger(\vec{W};Y,\vec{B})=D^{-1}(\vec{W};Y,\vec{B})=D(-\vec{W};Y,\vec{B})\,. \tag{2.6}$$

In the uncorrelated jet model,[4] one expects the isospin of all pions, regardless of their momenta, to be coupled to form the total isospin. This is achieved by considering coherent states with $\vec{W}(Y,\vec{B};q)$ of the form

$$\vec{W}(Y,\vec{B};q)=W(Y,\vec{B};q)\hat{n}\,, \tag{2.7}$$

where $\hat{n}$ is a unit vector independent of q giving the direction of $\hat{W}$ in the isotopic-spin space.

Let us define

$$D^I_{I_3}(W;Y,\vec{B})=\frac{1}{(4\pi)^{1/2}}\int d^2\hat{n}\,Y^*_{II_3}(\hat{n})D(W\hat{n};Y,\vec{B})\,,$$
$$\sum_{II_3}D^I_{I_3}(W;Y,\vec{B})[D^I_{I_3}(W;Y,\vec{B})]^\dagger=1\,. \tag{2.8}$$

The coherent state with the total isotopic spin I and the z component I_3 is then defined as

$$|W;Y,\vec{B};II_3\rangle=D^I_{I_3}(W;Y,\vec{B})|0\rangle\,. \tag{2.9}$$

Normalization is such that

$$\langle 0|D^I_{I_3}(W;Y,\vec{B})|0\rangle=\delta_{I0}\delta_{I_30}\exp[-\tfrac{1}{2}A(Y,\vec{B})]\,, \tag{2.10}$$

where

$$A(Y,\vec{B})=\int dq\,|W(Y,\vec{B};q)|^2\,. \tag{2.11}$$

The scattering operator is now written as an integral

$$S(Y,\vec{B})=\int d^2\hat{n}|\hat{n}\rangle D(W\hat{n};Y,\vec{B})\langle\hat{n}|\,, \tag{2.12}$$

where $|\hat{n}\rangle$ represents the isospin-state vector of the leading-two-particle system. It has the property that

$$\langle\hat{n}|\hat{n}'\rangle=\delta^{(2)}(\hat{n}-\hat{n}')\,,$$
$$\int|\hat{n}\rangle d^2\hat{n}\langle\hat{n}|=1\,, \tag{2.13}$$
$$\langle\hat{n}|TT_3\rangle=Y_{TT_3}(\hat{n})\,,$$

where $Y_{TT_3}(\hat{n})$ are the spherical harmonics. The probability distribution of producing n_+ π^+, n_- π^-, and n_0 π^0 pions is

$$\int d^2B\,dq_1dq_2\cdots dq_n|\langle T'T'_3n_+n_-n_0|S(Y,\vec{B})|TT_3\rangle|^2$$
$$=N(T'T'_3;TT_3)P(n_+n_-n_0,T'T'_3;TT_3)\,,\quad n=n_++n_-+n_0\,, \tag{2.14}$$

where TT'_3 and TT_3 label the isospin states of the outgoing and ingoing leading-two-particle system, respectively. $N(T'T'_3;TT_3)$ is the corresponding normalization factor.

III. GRAY-DISK DYNAMICS

In order to extract some simple properties of the model described in Sec. II and to obtain some detailed results for charged-particle multiplicity, multiplicity distributions, and correlations, we must choose an explicit form for the dynamical function $W(Y,\vec{B};q)$.[1] It must fall off rapidly with $\vec{q}_T^{\,2}$ and be a smooth function of y in the central region $|y|<Y/2$, going to zero for y outside this interval. As a result, $A(Y,\vec{B})$ [Eq. (2.11)] will grow linearly with Y. For simplicity, we assume $A(Y,\vec{B})$ to have the form

$$A(Y,\vec{B})=\Delta\Theta(R-B)\,, \tag{3.1}$$

where

$$\Delta=aY+b\,, \tag{3.2}$$

and a and b go to a constant value at high energies.

R could, in principle, also depend on Y. This form of $A(Y,\vec{B})$ corresponds to scattering from a gray disk with a constant radius. The pion-emission strength parameter Δ is linear in Y, which is expected from a multiperipheral model, for example. It is evident that in this model the total and elastic cross section go to constant values at high energies.

The multiplicity distribution simplifies considerably and is now of the form

$$P(n_+n_-n_0, T'T_3'; TT_3)N(T'T_3'; TT_3) = e^{-\Delta}\frac{\Delta^n}{n_+!n_-!n_0!}C^{(0)}_{\sigma_1\sigma_2\cdots\sigma_n}(T'T_3'; TT_3), \tag{3.3}$$

where $n=n_0+n_++n_-$ and

$$N(T'T_3'; TT_3)=e^{-\Delta}\sum_{l=0} G_l(\Delta)C^{(l)}(T'T_3'; TT_3) \tag{3.4}$$

is the normalization factor.

The numerical coefficients $C^{(l)}_{\sigma_1\cdots\sigma_n}$ are defined as

$$C^{(l)}_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3)=4\pi\left|\int d^2\hat{n}\,Y^*_{T'T_3'}(\hat{n})Y_{lm}(\hat{n})Y_{TT_3}(\hat{n}) \times D^{1*}_{\sigma_1 0}(\hat{n})\cdots D^{1*}_{\sigma_n 0}(\hat{n})\right|^2, \tag{3.5}$$

where D^l denote the familiar Wigner rotation matrices. Note that

$$m=T_3'-T_3+\sigma_1+\sigma_2+\cdots+\sigma_n,$$
$$|m|\leq l\leq |T+T'+n|. \tag{3.6}$$

Among $\sigma_1\cdots\sigma_n$, there are n_+ indices with $\sigma=+1$, n_- with $\sigma=-1$, and n_0 with $\sigma=0$. The relation connecting $C^{(l)}$ and G_l functions has the form

$$\sum_{n_+n_-n_0}\frac{\Delta^n}{n_+!n_-!n_0!}C^{(0)}_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3) = \sum_l G_l(\Delta)C^{(l)}(T'T_3'; TT_3). \tag{3.7}$$

Here

$$G_l(\Delta)=\left(\frac{\pi}{2\Delta}\right)^{1/2} I_{l+1/2}(\Delta) \tag{3.8}$$

is a modified spherical Bessel function of the first kind.[8]

The average number of emitted pions of a given type is defined by

$$N(T'T_3'; TT_3)\langle n_\sigma(T'T_3'; TT_3)\rangle = \Delta e^{-\Delta}\sum_l G_l(\Delta)C^{(l)}_\sigma(T'T_3'; TT_3). \tag{3.9}$$

The two-body factorial moments are obtained from

$$N(T'T_3'; TT_3)\langle n_\sigma n_{\sigma'}-\delta_{\sigma\sigma'}n_\sigma\rangle = \Delta^2 e^{-\Delta}\sum_l G_l(\Delta)C^{(l)}_{\sigma\sigma'}(T'T_3'; TT_3). \tag{3.10}$$

The total number of produced pions is simply related to the normalization factor $N(T'T_3'; TT_3)$ as

$$\langle n(T'T_3'; TT_3)\rangle = \Delta\partial_\Delta \ln[e^\Delta N(T'T_3'; TT_3)]. \tag{3.11}$$

According to Eq. (3.2), Δ in this model plays the role of the energy parameter. At very high energies, we can use the asymptotic expansion $(\Delta\to\infty)$ of $G_l(\Delta)$,[8] which is

$$e^{-\Delta}G_l(\Delta)\sim x[1-l(l+1)x+\cdots];\quad x=\frac{1}{2\Delta}. \tag{3.12}$$

From Eq. (3.12), the following asymptotic expressions are found for $\langle n_\sigma\rangle$ and $f_2^{\sigma\sigma'}$ functions:

$$\langle n_\sigma(T'T_3'; TT_3)\rangle_{\text{asym}} \sim \Delta A_\sigma(T'T_3'; TT_3)+\tfrac{1}{2}B_\sigma(T'T_3'; TT_3)+\cdots,$$
$$f_2^{\sigma\sigma'}(T'T_3'; TT_3)_{\text{asym}} \sim \Delta^2(A_{\sigma\sigma'}-A_\sigma A_{\sigma'})+\tfrac{1}{2}\Delta(B_{\sigma\sigma'}-A_\sigma B_{\sigma'}-B_\sigma A_{\sigma'})+\cdots, \tag{3.13}$$

where

$$A_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3)\sum_l C^{(l)}(T'T_3'; TT_3)=\sum_l C^{(l)}_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3),$$
$$B_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3)\sum_l C^{(l)}(T'T_3'; TT_3)=-\sum_l l(l+1)C^{(l)}_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3) +A_{\sigma_1\cdots\sigma_n}(T'T_3'; TT_3)\sum l(l+1)C^{(l)}(T'T_3'; TT_3). \tag{3.14}$$

IV. CASE $T=T_3=T'=T'_3=1$

In order to illustrate the general properties of the model, we shall confine our consideration to the process $pp \to pp +$pions. Other cases are listed in Tables I and II.

The case $T=T_3=T'=T'_3=0$ is a particularly simple one. It has already been treated by other authors.[4]

A straightforward calculation yields

$$C^{(0)}_{\sigma_1 \cdots \sigma_n}(11;11) = 9\delta_{n_+ n_-}\left[\frac{1+(-1)^{n_0}}{2}\right]\left[\frac{(n_0-1)!!\,(n_-+1)!}{(2n_-+n_0+3)!!}\right]^2 . \tag{4.1}$$

From this expression and Eq. (3.13), we obtain the following asymptotic expressions for $\langle n_\sigma(11;11)\rangle$:

$$\begin{aligned}
&\langle n_0\rangle \sim \tfrac{1}{7}\Delta - \tfrac{2}{7} + \cdots ,\\
&\langle n_+\rangle = \langle n_-\rangle \sim \tfrac{3}{7}\Delta - \tfrac{5}{14} + \cdots , \qquad (4.2)\\
&\langle n\rangle = \sum_\sigma \langle n_\sigma\rangle \sim \Delta - 1 .
\end{aligned}$$

Notice that $\langle n_0\rangle$, $\langle n_+\rangle$, and $\langle n_-\rangle$ do not satisfy the relation

$$\tfrac{1}{2}[\langle n_+\rangle + \langle n_-\rangle] = \langle n_0\rangle = \tfrac{1}{3}\langle n\rangle , \tag{4.3}$$

which is often quoted in the literature. This result is encountered in all cases when the isospin of the initial and final leading-particle system is different from zero. Relation (4.3) holds exactly only in the case $T=T_3=T'=T'_3=0$.

From Table II, we see that the charged-pion–neutral-pion correlation function $f_2^{0-}(T'T'_3;TT_3)$ is negative in all cases, regardless of the value of the isospin $(T'T'_3)$ of the outgoing leading-particle system, if the independent emission of pions dominates the production mechanism.

Many authors have pointed out that the study of correlations among charged and neutral pions offers information on the underlying isospin structure. Quantities on which experimental data exist are the charged-pion–neutral-pion correlation function f_2^{0-} and the associated average neutral multiplicity $\langle n_0\rangle_{n_-}$. Experimentally, f_2^{0-} and $(\partial\langle n_0\rangle_{n_-}/\partial n_-)|_{n_-=0} = \alpha$ are both positive.[9] It is expected that the dependence of $\langle n_0\rangle_{n_-}$ on n_- will be approximately linear, i.e.,

$$\langle n_0\rangle_{n_-} = \alpha n_- + \beta \tag{4.4}$$

if multibody correlations are small.[10] α and β in Eq. (4.4) are energy- and isospin-dependent parameters. Data indicate that the correlation between π^0 and π^- increases monotonically with incident c.m. energy. The parameter α is an increasing function and will probably approach a constant limit at high energies.

To the leading order in Δ, α may be related to f_2^{0-} and f_2^{--} functions in the following way:

$$\alpha(T'T'_3;TT_3) \sim \frac{f_2^{0-}(T'T'_3;TT_3)}{f_2^{--}(T'T'_3;TT_3)+\langle n_-(T'T'_3;TT_3)\rangle} + \cdots \tag{4.5}$$

The pion-independent-emission model thus gives

$$\alpha_\pi(11;11)_{\text{asym}} \sim -2 + \cdots . \tag{4.6}$$

In order to show the relationship between (4.5) and (4.6), let us consider $P(n_-)$ and $\langle n_0\rangle_{n_-}$. After a slight amount of algebra using Eq. (4.1), we find that

$$P(n_-) = (Ne\Delta)^{-1}\frac{9}{4}\left(\frac{\Delta}{2}\right)^{2n_-}\left(\frac{1}{n_-!}\right)^2 F_{n_-}(\Delta)$$

and

$$\langle n_0\rangle_{n_-} = \Delta\partial_\Delta \ln F_{n_-}(\Delta) , \tag{4.7}$$

where

$$F_{n_-}(\Delta) = \frac{\pi}{4}\left[\frac{(n_-+1)!}{\Gamma(n_-+\frac{5}{2})}\right]^2 {}_1F_2(\tfrac{1}{2}; n_-+\tfrac{5}{2}, n_-+\tfrac{5}{2}; \tfrac{1}{4}\Delta^2) \tag{4.8}$$

is a generalized hypergeometric function.[11] In the Appendix, we show that $F_{n_-}(\Delta)$ can be well represented by the modified spherical Bessel function as

$$F_{n_-}(\Delta) \sim \tfrac{1}{2}[(n_-+1)!]^2\left(\frac{2}{\Delta}\right)^{2n_-+3} G_{2n_-+3}(\Delta) . \tag{4.9}$$

Using (3.12) and the results given in the Appendix, a correct asymptotic expansion $(\Delta\to\infty)$ of $F_{n_-}(\Delta)$ is obtained in the form

$$F_{n_-}(\Delta)_{\text{asym}} \sim \tfrac{1}{2}[(n_-+1)!]^2\left(\frac{2}{\Delta}\right)^{2n_-+3}\frac{e^\Delta}{2\Delta} \times\left[1+\frac{n_-+2}{\Delta}+\frac{(n_-+2)(3n_-+7)}{2\Delta^2}+\cdots\right] . \tag{4.10}$$

TABLE I. Asymptotic expressions for average pion multiplicities.

σ	$7\langle n_\sigma(11;11)\rangle$	$7\langle n_\sigma(10;11)\rangle$	$7\langle n_\sigma(1-1;11)\rangle$	$5\langle n_\sigma(00;11)\rangle$	$3\langle n_\sigma(00;00)\rangle$
0	$\Delta-2$	$3\Delta-2$	$\Delta-3$	$\Delta-2$	$\Delta-1$
+	$3\Delta-\frac{5}{2}$	$2\Delta+1$	$3\Delta+5$	$2\Delta+1$	$\Delta-1$
−	$3\Delta-\frac{5}{2}$	$2\Delta-6$	$3\Delta-9$	$2\Delta-4$	$\Delta-1$

TABLE II. Leading asymptotic terms of two-particle correlation moments.

$\sigma\sigma'$	$147f_2^{\sigma\sigma'}(11;11)$	$147f_2^{\sigma\sigma'}(10;11)$	$147f_2^{\sigma\sigma'}(1-1;11)$	$175f_2^{\sigma\sigma'}(00;11)$	$45f_2^{\sigma\sigma'}(00;00)$
00	$4\Delta^2-2\Delta$	$8\Delta^2-20\Delta$	$4\Delta^2-10\Delta$	$8\Delta^2-12\Delta$	$4\Delta^2-2\Delta$
0−	$-2\Delta^2+\Delta$	$-4\Delta^2+10\Delta$	$-2\Delta^2+5\Delta$	$-4\Delta^2+6\Delta$	$-2\Delta^2+\Delta$
−−	$\Delta^2-32\Delta$	$2\Delta^2-26\Delta$	$\Delta^2-34\Delta$	$2\Delta^2-38\Delta$	$\Delta^2-8\Delta$
+−	$\Delta^2+31\Delta$	$2\Delta^2+16\Delta$	$\Delta^2+29\Delta$	$2\Delta^2+32\Delta$	$\Delta^2+7\Delta$

From (4.7) and (4.10), it follows that

$$\langle n_0\rangle_{n_-} \sim \Delta - 2n_- - 4 + \cdots , \tag{4.11}$$

which together with

$$f_2^{0-} \sim -\frac{1}{147}\,\Delta^2\left(2-\frac{1}{\Delta}\right)+\cdots$$

and (4.12)

$$f_2^{--} \sim \frac{1}{147}\,\Delta^2\left(1-\frac{32}{\Delta}\right)+\cdots$$

shows the validity of the asymptotic relation (4.6). It is evident that the behavior indicated by Eq. (4.11) is in disagreement with present trends of data.[9]

Another interesting quantity is the associated two-body correlation function $(f_2^{00})_{n_-}$, which has been advocated by many authors[5, 10] as a possible sensitive test of the underlying isospin structure. It is believed that measurements of $(f_2^{\sigma\sigma'})_{n_-}$ moments would allow better discrimination between different isospin models for pion production.

In our pion-independent-emission model, the factorial moments of the π^0 distribution for a given n_- are defined as

$$\langle n_0(n_0-1)\cdots(n_0-k+1)\rangle_{n_-} = \Delta^k\,\frac{\partial_\Delta{}^k F_{n_-}(\Delta)}{F_{n_-}(\Delta)}\,. \tag{4.13}$$

From Eq. (4.10), we see that we can write asymptotically

$$\langle n_0(n_0-1)\rangle_{n_-} \sim \Delta^2 - 4(n_-+2)\Delta + 3(n_-+2)^2 + \cdots ,$$

so that

$$(f_2^{00})_{n_-} \sim -(n_-+2)^2 + \cdots \tag{4.14}$$

approaches a constant value at asymptotic energies.

Using the Mueller-Regge analysis, Weber[12] derived a sum rule for $f_2^{\sigma\sigma'}$ functions in pp collisions, which to $O(s^{-1/4})$ reads

$$R_2 = (f_2^{00}+f_2^{-0})/(f_2^{+-}+f_2^{--}) \to 1\,.$$

We find the same result valid to $O(1/\Delta^2)$, as can be seen from Table II.

V. ISOVECTOR-CLUSTER EMISSION

Analysis of the preceding sections shows that the independent emission of pions is not sufficient to explain most of the experimental findings. Various suggestions to remedy this situation have been offered.[5, 13] One of them is the independent emission of clusters which decay into two or more pions. The more pions in a cluster the larger the correlation effect expected. The independent emission of charged particles and clusters has been discussed by different authors[5] using either complete charge conservation or the Cerulus[14] statistical approach. Here we shall discuss the independent emission of an arbitrary number of various types of isovector clusters from an excited leading-particle system having the total isospin T and the third component T_3. We can straightforwardly perform a generalization to more complicated clusters, such as isotensor ones. Clusters might be resonances. The known prominent nonstrange isovector resonances up to $M_{\rm res} \sim 1.3$ GeV are π, ρ, B, A_2, etc.[15] To each "cluster type" we associate the corresponding coherent-state displacement operator $D(W_c\hat{n};Y,\vec{B})$ [see Eq. (2.5)]. The extension to include isotensor clusters can be performed by replacing $\sum_\sigma n_\sigma c_\sigma^\dagger(q)$ in Eq. (2.5) by

$$[D^I(\hat{n})\cdot c^{(I)\dagger}(q)] = \left(\frac{2I+1}{4\pi}\right)^{1/2} \sum_{I_3} (-)^{I_3} Y_{II_3}(\hat{n}) \times c^{(I)}_{-I_3}(q)^\dagger\,. \tag{5.1}$$

Here $c^I_{I_3}(q)^\dagger$ is the creation operator of a cluster having the total isospin I and the z component I_3.

The independence of clusters of different type means that $D(W_c\cdots)$ commute for different clusters, that is,

$$[D(W_c,\cdots),\, D(W_{c'},\cdots)] = 0 \quad (c \neq c')\,. \tag{5.2}$$

Therefore, the independent emission of an arbitrary number of various types of clusters can be described by the following coherent-state displacement operator

$$D(W\hat{n};Y,\vec{B})\to\prod_c D(W_c\hat{n};Y,\vec{B})=D\Big(\sum_c W_c\hat{n};Y,\vec{B}\Big),\qquad(5.3)$$

where the product and the sum run over all different types of clusters.

For definiteness, we shall consider only the isovector clusters π, ρ, B, and A_2. It is quite simple to include the simultaneous emission of either isoscalar clusters or more complicated isospin clusters. The parameter specifying the strength for the emission of a cluster c is defined by

$$A_c(Y,\vec{B})=\int dq\,|W_c(Y,\vec{B};q)|^2=\Delta_c\Theta(R-B).\qquad(5.4)$$

A relatively laborious calculation using the techniques developed in the preceding sections gives the asymptotic values for pion multiplicity $\langle n_\sigma\rangle$ and pion two-body correlation functions $f_2^{\sigma\sigma'}$ in the process $pp\to pp+$pions:

$$\begin{aligned}\langle n_0\rangle&\simeq\frac{1}{7}\Big[\Delta_\pi+6\Delta_\rho+8\Delta_B+13\Delta_a+\Delta_b-\frac{1}{\Delta}(2\Delta_\pi+5\Delta_\rho+9\Delta_B+12\Delta_a+2\Delta_b)\Big]+\cdots,\\ \langle n_+\rangle&=\langle n_-\rangle\simeq\frac{1}{7}\Big\{3\Delta_\pi+4(\Delta_\rho+\Delta_a)+10(\Delta_B+\Delta_b)-\frac{1}{2\Delta}[5\Delta_\pi+9(\Delta_\rho+\Delta_a)+19(\Delta_B+\Delta_b)]\Big\}+\cdots\end{aligned}\qquad(5.5)$$

and

$$\begin{aligned}f_2^{00}&\simeq\frac{2}{147}\Big(2-\frac{1}{\Delta}\Big)(\Delta_\pi-\Delta_\rho+\Delta_B-\Delta_a+\Delta_b)^2+\frac{2}{7}\Big[\Delta_B\Big(1-\frac{2}{\Delta}\Big)+2\Delta_a\Big(3-\frac{5}{2\Delta}\Big)\Big]+\cdots,\\ f_2^{--}&=f_2^{++}\simeq\frac{1}{147}\Big\{(\Delta_\pi-\Delta_\rho+\Delta_B-\Delta_a+\Delta_b)^2-\frac{32}{\Delta}\Big[\Delta^2-\frac{1}{16}(\Delta_\pi\Delta_\rho+\Delta_\pi\Delta_a+\Delta_\rho\Delta_B+\Delta_\rho\Delta_b+\Delta_a\Delta_b)\Big]\Big\}\\ &\quad+\frac{2}{7}\Big(3-\frac{5}{2\Delta}\Big)(\Delta_B+\Delta_b)+\cdots,\\ f_2^{0-}&=f_2^{0+}\simeq-\frac{1}{147}\Big(2-\frac{1}{\Delta}\Big)(\Delta_\pi-\Delta_\rho+\Delta_B-\Delta_a+\Delta_b)^2\\ &\quad+\frac{1}{7}\Big[\Delta_\rho\Big(3-\frac{5}{2\Delta}\Big)+\Delta_B\Big(11-\frac{23}{2\Delta}\Big)+7\Delta_a\Big(1-\frac{1}{\Delta}\Big)+\Delta_b\Big(1-\frac{2}{\Delta}\Big)\Big]+\cdots,\\ f_2^{+-}&\simeq\frac{1}{147}\Big\{(\Delta_\pi-\Delta_\rho+\Delta_B-\Delta_a+\Delta_b)^2+\frac{31}{\Delta}\Big[\Delta^2+\frac{2}{31}(\Delta_\pi\Delta_\rho+\Delta_\pi\Delta_a+\Delta_\rho\Delta_B+\Delta_\rho\Delta_b+\Delta_a\Delta_b)\Big]\Big\}\\ &\quad-\frac{1}{7}\Big[\Big(1-\frac{2}{\Delta}\Big)(\Delta_\rho+\Delta_a)+\Big(13-\frac{12}{\Delta}\Big)(\Delta_B+\Delta_b)\Big]+\cdots,\end{aligned}\qquad(5.6)$$

where $\Delta=\Delta_\pi+\Delta_\rho+\Delta_B+\Delta_a+\Delta_b$ and $\Delta_a\Delta_b=\frac{1}{2}\Delta_A$ are strength parameters of A_2-decay modes.

It should be noticed that

$$\langle n\rangle=\sum_\sigma\langle n_\sigma\rangle\simeq(\Delta_\pi+2\Delta_\rho+3\Delta_A+4\Delta_B)\Big(1-\frac{1}{\Delta}\Big)+\cdots\qquad(5.7)$$

Thus, if there is an average of $\langle k\rangle$ pions per cluster, the net mean number $\langle n\rangle$ of pions produced in the final state is

$$\langle n\rangle=\sum_c\langle k_c\rangle\langle N_c\rangle,\qquad(5.7')$$

where $\langle N_c\rangle$ is the average number of clusters of type c.

The results of these simple calculations imply that clustering phenomena in the presence of a direct-pion emission should become dominant mechanisms at higher energies. We suggest the following isovector cluster-emission picture[5]: At low energies, $E_{\rm c.m.}\lesssim 7$ GeV, the independent emission of pions should dominate. At higher energies, the emission of ρ, B, A_2, etc., should become a dominant mechanism.

Let us calculate the maximal value of the slope parameter α of the $\langle n_0\rangle_{n_-}$ vs. n_- plot according to the relation given by Eq. (4.5) for the π-ρ and ρ-B cluster-combination mechanisms. We find

$$(\alpha_{\pi\rho})_{\max} \sim \tfrac{3}{4} + \cdots ,$$
$$(\alpha_{\rho B})_{\max} \sim \tfrac{14}{17} + \cdots . \quad (5.8)$$

These maximal values are obtained in the limits $\Delta_\pi \sim \Delta_\rho$, $\Delta_B \sim 0$, $\Delta_A \sim 0$, and $\Delta_\rho \sim \Delta_B$, $\Delta_\pi \sim 0$, $\Delta_A \sim 0$, respectively. The predictions in Eq. (5.8) will be in reasonable agreement with data (Fig. 1) if the $\Delta_\pi \sim \Delta_\rho$ regime starts at $E_{\text{c.m.}} \sim 25$ GeV and the $\Delta_\rho \sim \Delta_B$ regime at $E_{\text{c.m.}} \sim 30$ GeV. Above an incident c.m. energy of 7 GeV, the energy constraints become less dominant, and any increase in α reflects the effect of dynamical correlations.

It is also of interest to notice that in the $\Delta_\pi \sim \Delta_\rho$ and $\Delta_\rho \sim \Delta_B$ regions we find an approximate equality between

$$\langle n_+ \rangle = \langle n_- \rangle \simeq \langle n_0 \rangle \simeq \tfrac{1}{3} \langle n \rangle . \quad (5.9)$$

Analysis of this section shows that the cluster-emission mechanism seems necessary to give positive values of f_2^{0-} and the positive slope of $\langle n_0 \rangle_{n_-}$, thus indicating that the underlying dynamics should contain attractive couplings between the charged and neutral components of the pion field.

VI. DISCUSSION

In this work we have extended the AABS model, for which the scattering operator satisfies exact s-channel unitarity, to include internal symmetries, such as isospin. Isospin is included via the coherent-state representation. Special attention is paid to the production when the isospin state of the ingoing and outgoing leading-particle system is fixed.

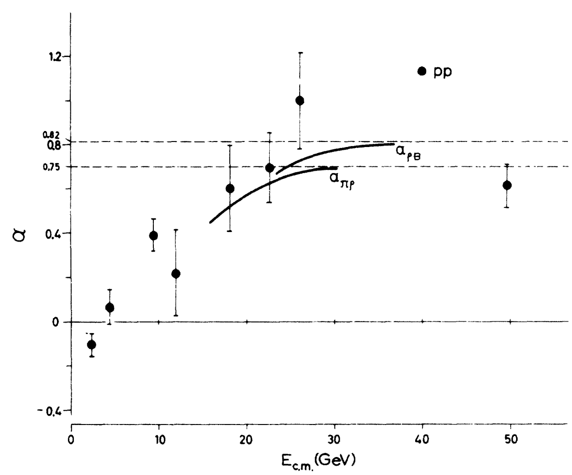

FIG. 1. α **as a function of the available c.m. energy,** $E_{\text{c.m.}}$.[9] **The solid curves are schematic pictures of the dependence of** α **on** $E_{\text{c.m.}}$ **if** $|\Delta_\pi - \Delta_\rho| \ll (\Delta_\pi + \Delta_\rho)$ **and** $|\Delta_\rho - \Delta_B| \ll (\Delta_\rho + \Delta_B)$, **respectively, as seems to be indicated by the data.**

The main points of the direct-pion emission are the following.

(i) The total number of pions produced in such a process has a Poisson-like distribution

$$P(n) = (Ne^{\Delta})^{-1} \frac{\Delta^n}{n!} [\partial_\Delta^n (Ne^{\Delta})|_{\Delta=0}] .$$

This behavior is typical of any model with uncorrelated-particle emission.

(ii) An average number of charged and neutral pions does not satisfy the relation

$$\tfrac{1}{2}(\langle n_+ \rangle + \langle n_- \rangle) = \langle n_0 \rangle = \tfrac{1}{3}\langle n \rangle$$

if the leading-particle system has isospin different from zero. This relation holds exactly only in the case when $T = T_3 = T' = T_3' = 0$.

(iii) A strong positive correlation for pairs of like pions and for pairs of charged pions is found (Table II).

(iv) The correlation among charged and neutral pions is found negative. This negative value of f_2^{0-} seems to be responsible for the negative slope of $\langle n_0 \rangle_{n_-}$ as a function of n_-

$$\langle n_0 \rangle_{n_-} \sim \Delta - 2n_- - 4 + \cdots .$$

The result is in disagreement with existing experimental data.

(v) The associated moment $(f_2^{00})_{n_-}$ in the process $pp \to pp + \text{pions}$ approaches a constant value

$$(f_2^{00})_{n_-} \sim -(n_- + 2)^2 + \cdots$$

at high energy.

The conclusion is that qualitative agreement with data cannot be reached by combining the AABS model and an isospin-conserving hypothesis if the direct-pion emission dominates the production mechanism.

To circumvent these difficulties, we have extended the AABS model in such a way as to allow the independent emission of resonances (or clusters) as well. The cluster-emission picture we follow is such that the direct-pion emission dominates the low-energy region ($E_{\text{c.m.}} \lesssim 7$ GeV). With increasing energy, the π-ρ regime becomes a dominant production mechanism (at $E_{\text{c.m.}} \sim 25$ GeV). At still higher energies, the ρ-B regime starts to dominate the production mechanism.

Interesting points to note here are the following.

(a) Average multiplicities of charged and neutral pions satisfy the approximate asymptotic equality

$$\langle n_+ \rangle = \langle n_- \rangle \simeq \langle n_0 \rangle \simeq \tfrac{1}{3}\langle n \rangle$$

in the π-ρ and ρ-B regions.

(b) The slope parameter α of the $\langle n_0 \rangle_{n_-}$ vs n_- plot is positive and increases with energy,

$$(\alpha_{\pi\rho})_{\max} \sim \tfrac{3}{4} + \cdots ,$$
$$(\alpha_{\rho B})_{\max} \sim \tfrac{14}{17} + \cdots .$$

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APPENDIX

In this Appendix, we consider a suitable transformation of the series

$$f(z)=\sum_{k=0}^{\infty} \frac{\Gamma(a+k)}{[\Gamma(b+k)]^2}\frac{z^k}{k!} \quad (a,b>0), \tag{A1}$$

in order to obtain a form convenient for asymptotic expansion. We use Kummer's method for transforming a convergent series into another one. Let

$$\sum_{k=0}^{\infty} a_k = f \tag{A2}$$

be a given convergent series and

$$\sum_{k=0}^{\infty} c_k = c \tag{A3}$$

be a given convergent series with a known sum c such that

$$\lim_{k\to\infty} \frac{a_k}{c_k} = \lambda \neq 0 . \tag{A4}$$

Then

$$f = \lambda c + \sum_{k=0}^{\infty}\left(1-\lambda\frac{c_k}{a_k}\right)a_k . \tag{A5}$$

For our case, we choose

$$a_k = \frac{\Gamma(a+k)}{[\Gamma(b+k)]^2}\frac{z^k}{k!}, \qquad c_k = \frac{1}{\Gamma(2b-a+k)}\frac{z^k}{k!}, \tag{A6}$$

so that

$$\frac{a_k}{c_k} \sim 1 + \frac{(a-b)^2}{k} + \cdots , \quad k \text{ being very large}, \tag{A7}$$

and

$$\lim_{k\to\infty}\frac{a_k}{c_k} = \lambda = 1 . \tag{A8}$$

Then

$$c = \sum_{k=0}^{\infty} c_k = (\sqrt{z}\,)^{-2b+a+1} I_{2b-a-1}(2\sqrt{z}\,). \tag{A9}$$

The correction term is obtained from

$$c_{\text{corr}} = \sum_k c_k\left(\frac{a_k}{c_k} - \lambda\right). \tag{A10}$$

The values of the parameters a, b, and z used in this paper are

$$a=\tfrac{1}{2}, \quad b = n_- + \tfrac{5}{2}, \quad z = (\tfrac{1}{2}\Delta)^2 .$$

A close examination of Eqs. (A9) and (A10) yields the following asymptotic expansion of $F_{n_-}(\Delta)$ used in the text

$$F_{n_-}(\Delta) \sim \tfrac{1}{2}[(n_-+1)!]^2\left(\frac{2}{\Delta}\right)^{2n_-+3} \times \frac{e^{\Delta}}{2\Delta}\left(d_0 + \frac{d_1}{\Delta} + \frac{d_2}{\Delta^2} + \cdots\right), \tag{A11}$$

where

$$d_k = \sum_{i=0}^{N} \frac{(2i+k)!}{(i!)^2 2^{2i}}\,\frac{(2N-2i-1)!!}{(2N-2i)!!} \quad (N = n_- + 1). \tag{A12}$$

We have evaluated the first few d_k coefficients. They are as follows:

$$d_0 = 1, \qquad d_1 = N+1, \qquad d_2 = \tfrac{1}{2}(N+1)(3N+4). \tag{A13}$$

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