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# Local quantum-number compensation in multiple production

C. Quigg*
*Fermi National Accelerator Laboratory,*† *Batavia, Illinois 60510*
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Experimental distributions are defined which are sensitive to the existence and nature of short-range phenomena. Clustering of produced hadrons is established directly. The possibility that clusters responsible for strange particle or baryon production differ from those responsible for pion production is entertained. Similarities and distinctions between transverse momentum and other additive quantum numbers are discussed.

## I. INTRODUCTION

Short-range correlations in rapidity appear to be one of the prominent features of multiple production at high energies. It is also generally accepted that long-range correlations arise from interference between diffractive and nondiffractive components of the production cross section.[1] Whether intrinsic long-range correlations are present within the nondiffractive component is uncertain. However, whatever other effects might be present, considerable indirect evidence does exist for the dominance of short-range phenomena in the nondiffractive component. This paper deals with the existence and the origins of such phenomena.

It is now widely held that a useful, if somewhat ingenuous, description of the origin of short-range correlations is provided by cluster emission models.[2,3] Despite successes of cluster models and the circumstantial evidence for clustering, some efforts persist to show, by means of Monte Carlo simulations, that experimental features adduced in support of the short-range correlation hypothesis do not depend upon short-range correlation dynamics. In my opinion, these efforts need not be taken seriously in themselves,[4] but they raise a valid challenge which should be met: to provide unambiguous evidence for short-range phenomena. I see, therefore, two immediate objectives in the study of nondiffractive multiple production. First, it is important to verify the existence of short-range phenomena and to quantify their properties. It will then be obligatory to ask whether the cluster description of the short-range correlation dynamics is a necessity of merely a useful fiction.[5] In other words, it will be necessary to find the clusters.

In this paper two topics are studied in pursuit of these objectives. In Sec. II I discuss tests of the hypothesis that internal quantum numbers are conserved locally in rapidity. This hypothesis follows naturally from any (factorizable) $t$-channel exchange picture, and is an immediate corollary of cluster or short-range correlation dynamics. Data are presented which give a direct demonstration of the local compensation of electric charge, and the extension to other internal quantum numbers is treated. In Sec. III I investigate the consequences of local compensation of transverse momentum. The information contained in a number of new experimental distributions is developed in detail. If the transverse momenta of all secondaries (including neutrals) can be measured, a technique exists for probing the transverse momentum distribution of clusters. A summary is given in Sec. IV.

## II. LOCAL COMPENSATION OF INTERNAL QUANTUM NUMBERS

In this section I shall develop direct experimental evidence for the local compensation of electric charge and provide a direct measure of the mobility[6] of electric charge. My immediate interest is to deduce characteristics of multiple production and to integrate them with existing information. However, as a number of authors have emphasized recently,[7] the unitarity equation connects the locality of quantum-number compensation with the energy dependence of two-body to two-body quantum-number exchange reactions. An accurate determination of, for example, charge mobility in collisions at different primary energies bears not only on the tenability of the cluster description, but also on the origin of the energy dependence of charge exchange cross sections.[8]

### A. Local charge compensation

The analysis of charge transfer observables[9] has verified in detail[10] the predictions of the independent cluster emission picture.[11-14] This success was a psychological prerequisite for the present investigation, in which I rely upon a specific, idealized short-range-order model to anticipate the data. It will, however, soon become apparent that the qualitative theoretical expectations do not rest on details of the model. The basic assumption to be tested is that charge compensation is a short-range phenomenon. In the specific language of cluster models, the corresponding assumption is that observed hadrons emanate from independently emitted clusters, are characterized by a mobility in rapidity from their parent cluster, and experience no final-state interactions. In this picture all short-range correlations are intracluster effects. I shall use a simplified one-dimensional model to explore the consequences of this assumption.[15] In the model neutral three-pion clusters ($\pi^+\pi^-\pi^0$) have an equal chance to be produced anywhere in the available rapidity interval $[-Y/2, Y/2]$. A cluster produced at rapidity $\hat{y}$ will yield pions at rapidities $\hat{y}-\Delta$, $\hat{y}$, and $\hat{y}+\Delta$, where $\Delta$ will be called the mobility. The transfer of charge from one c.m. hemisphere to the other is therefore a consequence of decays of clusters in the active region $(-\Delta, \Delta)$. For every cluster the correspondence between rapidities $(\hat{y}-\Delta, \hat{y}, \hat{y}+\Delta)$ and pion charges $(-1, 0, +1)$ can be made in six equally probable ways. Each cluster in the active region has a $\frac{1}{3}$ probability to contribute $(-1, 0, +1)$ to the *net charge transfer,*

$u=\frac{1}{2}$ (total charge in the forward hemisphere minus beam charge)

$-\frac{1}{2}$ (total charge in the backward hemisphere minus target charge),

independent of the behavior of other clusters.

The consequences of this model in the usual experimental situation were dealt with in Ref. 11. Let us now turn our attention to charge transfer across a gap centered at zero c.m. rapidity. Clearly, when the gap width exceeds the cluster mobility, no charge is actually exchanged between the forward and backward regions. The tedious calculations to be described lead to the following important qualitative result. As the gap width $G$ is increased from 0 to $2\Delta$, the mean-squared charge fluctuation at fixed topology, $\langle u^2\rangle_n$, decreases. For $G>2\Delta$, $\langle u^2\rangle_n$ remains constant, independent of $G$, until forced by kinematical end effects to decrease to zero. The experimental observation of this behavior provides direct evidence for a short-range, or clustering, effect and yields a direct measure of the mobility $\Delta$.

It is convenient to employ the generating function technique introduced in Ref. 11. Let us define

$$p \equiv 2\Delta/Y, \qquad g \equiv G/Y, \tag{1}$$

and construct the generating function $P_N(x)$ corresponding to the emission of $N$ clusters, in terms of which[16]

$$\langle u^k\rangle_N = (\tfrac{1}{2}x\,\partial_x)^k P_N(1). \tag{2}$$

Three cases must be distinguished: (i) $G<\Delta$; (ii) $\Delta<G<2\Delta$; (iii) $G>2\Delta$. In each case, pions which are products of clusters emitted in certain intervals may go unobserved in the gap. The various possibilities are identified in Fig. 1. It is then a straightforward counting exercise to determine, for each interval marked in Fig. 1, the probability that a specific charge transfer will occur and so to construct the appropriate generating function. The results are

$$P_N(x)=\left[(1-p-g)+\frac{p-2g}{3}(x^2+1+x^{-2}) + \frac{g}{6}(x^2+2x+2/x+x^{-2})+\frac{2g}{3}(x+1+1/x)\right]^N, \tag{3}$$

$$\langle u^2\rangle_N = N\left[\frac{2p}{3}-\frac{g}{2}\right], \tag{4}$$

if $0<G<\Delta$;

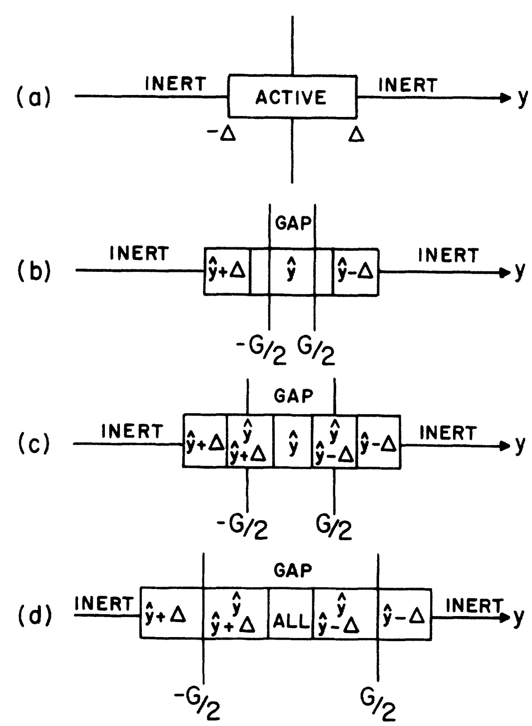


FIG. 1. Partitions used in the discussion of quantum-number transfer across a gap. The abscissa is c.m. rapidity. Notations (e.g., $\hat{y}+\Delta$) in an indicated region specify the lost pions originating from clusters in that region. (a) No gap; (b) $G<\Delta$; (c) $\Delta<G<2\Delta$; (d) $G>2\Delta$.

$$P_N(x) = \left[ (1 - p - g) + \frac{2g}{3}(x + 1 + 1/x) + \frac{p - g}{6}(x^2 + 2x + 2/x + x^{-2}) \right]^N, \quad (5)$$

$$\langle u^2 \rangle_N = N\left[\frac{p}{2} - \frac{g}{6}\right], \quad (6)$$

if $\Delta < G < 2\Delta$; and

$$P_N(x) = \left[ (1 - 2p) + \frac{2p}{3}(x + 1 + 1/x) \right]^N, \quad (7)$$

$$\langle u^2 \rangle_N = Np/3, \quad (8)$$

if $G > 2\Delta$.

The expected gap dependence of the charge transfer fluctuation is sketched in Fig. 2. This prediction is to be compared with the experimental results[17] from the Fermilab 205-GeV/$c$ bubble-chamber exposure, which are shown in Fig. 3. The two-prong and four-prong events are largely quasi-two-body in character, so they should not display the features expected in the cluster picture. Indeed, they do not. For the 6-, 8-, and 10-prongs, there is clear evidence of a rapid decrease in the charge transfer fluctuation as $G$ increases from 0 to 1.5, followed by an interval in which $\langle u^2 \rangle_N$ is essentially independent of $G$. The absence of any plateau in the 12-prongs is consistent with the observed shape of $d\sigma_{12}/dy$. For such a high multiplicity, the region (in $G$) in which $\langle u^2 \rangle_N$ decreases because of local charge compensation overlaps with the region in which the kinematically imposed decrease occurs. To emphasize the importance of these results, I remark that if individual pions were independently emitted, $\langle u^2 \rangle_N = (N/4)(1 - G/Y)$, so no plateau would be seen.

Several important conclusions may be drawn from these data.[18]

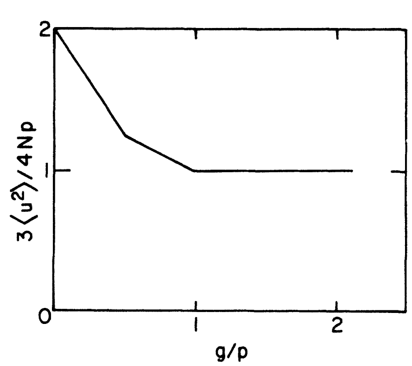


FIG. 2. Prediction of Eqs. (4), (6), and (8) for the fluctuation in charge transferred across a gap in events of fixed topology.

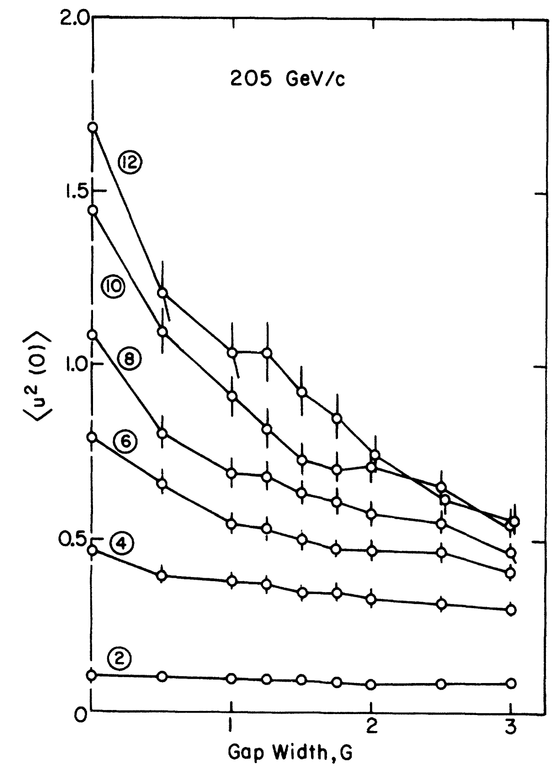


FIG. 3. Fluctuations in charge transfer across a gap in fixed-topology events in 205-GeV/$c$ $pp$ collisions (from Ref. 18).

(1) Charge is compensated locally in rapidity in multiparticle production. For this to be so in the sense of Refs. 7, it must further be shown that the mobility of charge is independent of energy.

(2) The mobility of electric charge, which is $\Delta \approx 0.75$ in 205-GeV/$c$ $pp$ collisions, appears not to depend strongly on event topology.

(3) The measured charge mobility is quite consistent with the expectations of the isotropic cluster decay model and with the range parameter deduced from fits to the two-particle correlation function in the central region.[19] It also agrees with the value inferred from data on forward-backward multiplicity correlations across a gap at this energy.[2,17]

It is of great interest to compare these results with measurements of charge mobility at other energies, and in collisions initiated by other beams. Unfortunately the existing statistics at 102 and 405 GeV/$c$ are inadequate for this purpose.[20]

### B. Other internal quantum numbers

In the context of explicit exchange models it is natural to suppose that rare quantum numbers such as strangeness and baryon number might be conserved more locally than charge, because the corresponding Regge trajectories are lower-lying than those which carry charge. Equivalently, arguments based on $Q$ values of known resonances[21] (e.g., $\phi, A_2 \to K\bar{K}$ *versus* $\rho, f \to \pi\pi$) lead to the same expectations. According to the Mueller-Regge analysis of two-particle correlations in the central region, for $K\bar{K}$ correlations there will be a correlation length $=1$ term arising from $\phi$ and $f^*$ exchange, in addition to the usual correlation length $=2$ term. The same considerations applied to the $N\bar{N}$ case are less enlightening.[22] It is highly desirable to subject these preconceptions to experimental tests. Given the difficulty of identifying particles in a large-acceptance detector, the task is not an easy one, but in the following discussion I will assume that all such technical difficulties can be overcome.

The transfer of any additive quantum number across a gap in rapidity can be treated in the same manner as electric charge, so that the results of Sec. II A are immediately applicable. A closely related distribution also is useful for studying the mobility of quantum numbers explicitly absent from the initial state (e.g., strangeness in $pp$ collisions). Consider a bin of extent $B$ in rapidity which lies fully within the central region. The dependence upon bin size of the mean-squared fluctuation in the amount of an additive quantum number confined within the bin is governed by the locality of compensation of that quantum number. To be specific, let us consider strangeness compensation in a model with "$\phi$" $\to K\bar{K}$ clusters. A cluster produced at $\hat{y}$ gives rise to kaons at $\hat{y} \pm \Delta$. By performing the same kind of counting as occurred in the charge transfer example, we can construct a generating function $R_N(x)$ appropriate for the emission of $N$ "$\phi$" clusters, in terms of which

$$\langle S^2 \rangle_N = (x\partial_x)^2 R_N(1), \tag{9}$$

where $S$ represents the total strangeness contained in the bin.

For $B < 2\Delta$, the generating function is

$$R_N(x) = [(1-2b) + b(x + 1/x)]^N, \tag{10}$$

where $b \equiv B/Y$, and

$$\langle S^2 \rangle_N = 2Nb = 2NB/Y. \tag{11}$$

For $B > 2\Delta$, the corresponding results are

$$R_N(x) = [(1-2p) + p(x+1/x)]^N \tag{12}$$

and

$$\langle S^2 \rangle_N = 2Np = 4N\Delta/Y. \tag{13}$$

The average over cluster multiplicities yields

$$\langle\langle S^2 \rangle\rangle = \begin{cases} 2\langle N \rangle B/Y, & B < 2\Delta \\ 4\langle N \rangle \Delta/Y, & B > 2\Delta. \end{cases} \tag{14}$$

The strangeness fluctuation increases with bin size, attaining a saturation value at $B = 2\Delta$, the point at which entire clusters may be confined within the bin. This behavior is sketched in Fig. 4.[23] It is completely analogous to the case of quantum-number transfer across a gap sketched in Fig. 2. The onset of $B$ independence both establishes the locality of strangeness compensation and measures the mobility of strangeness. Evidently the same expectations apply to any additive quantum number.

## III. LOCAL COMPENSATION OF TRANSVERSE MOMENTUM

Knowledge of the mobility of transverse momentum will be especially useful for making inferences about the underlying (exchange?) mechanism of particle production. In many respects, transverse momentum is simply another additive quantum number[24] (although a continuous, rather than a discrete one), and can be treated on the same footing as the others. The important practical distinction is that, unlike visible charge, the visible transverse momentum is not balanced event-by-event in bubble-chamber pictures. If all produced particles including neutrals could be measured, the techniques of Sec. II would be useful for measuring transverse momentum mobility as well. If neutrals go undetected, the theoretical expectations are modified to the extent that no effects as striking as those already discussed will appear.

A very simple model suffices to explore the possibilities. I assume that transverse momenta $\gamma, 0, -\gamma$ are carried by the pion products of an "$\omega$" cluster. For each cluster, the correspondence between rapidities $(\hat{y}-\Delta, \hat{y}, \hat{y}+\Delta)$, pion charges $(-1, 0, 1)$, and transverse momenta $(-\gamma, 0, \gamma)$ can be made in 36 equally probable ways. It is

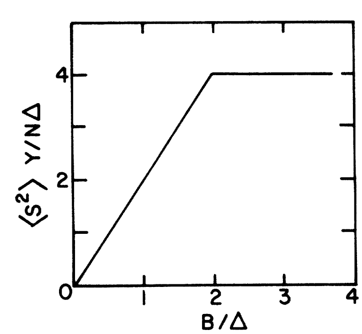


FIG. 4. Prediction for quantum-number deposit in a bin.

convenient to define, in analogy with charge transfer, the net transverse momentum transfer

$$v \equiv \tfrac{1}{2}\,(\text{visible transverse momentum in the forward hemisphere}) - \tfrac{1}{2}\,(\text{visible transverse momentum in the backward hemisphere}). \quad (15)$$

Because neutrals go undetected, each inactive cluster will make a contribution to $v$ of $(-\gamma/2, 0, \gamma/2)$ with equal probability. Each cluster in the active region will contribute $(-\gamma, -\gamma/2, 0, \gamma/2, \gamma)$ with probability $(\frac{1}{9}, \frac{1}{3}, \frac{1}{9}, \frac{1}{3}, \frac{1}{9})$. Hence

$$T_N(x) = \left[\frac{(1-p)}{3}(x+1+1/x) + \frac{p}{9}(x^2+3x+1+3/x+x^{-2})\right]^N$$

is a generating function in terms of which

$$\langle v^k \rangle_N = (\tfrac{1}{2}\gamma x \partial_x)^k\, T_N(1). \quad (17)$$

The mean-squared fluctuation of $p_\perp$ transfer in $N$-cluster events is

$$\langle v^2 \rangle_N = \frac{4\Delta\gamma^2}{9}\,\frac{N}{Y} + \frac{\gamma^2 N}{6}, \quad (18)$$

which should be compared with the charge transfer fluctuation

$$\langle u^2 \rangle_N = \frac{4\Delta}{3}\,\frac{N}{Y}. \quad (19)$$

The first term in (18) has the expected form for the fluctuation of $p_\perp$ transfer arising from the decay of active clusters. The second term, which is proportional to the number of clusters instead of the cluster density, represents the event-to-event imbalance in visible transverse momentum. The origin of this term in the nonobservation of neutrals is made more explicit if $N$ is replaced by the associated multiplicity of neutrals, so that

$$\langle v^2 \rangle_N = \frac{4\Delta\gamma^2}{9}\,\frac{N}{Y} + \frac{\gamma^2}{6}\langle n_0(N)\rangle. \quad (20)$$

The quantity $N/Y$ represents the mean density of clusters in the active region, $\langle dN(\bar{y})/dy\rangle$, where the brackets imply an average over the region $(\bar{y}-\Delta, \bar{y}+\Delta)$. This in turn is related to the observed density of charged pions by

$$\left\langle \frac{dN}{dy}(\bar{y})\right\rangle \simeq \frac{1}{2}\left\langle \frac{1}{\sigma_n}\frac{d\sigma_n}{dy}(\bar{y})\right\rangle, \quad (21)$$

where the factor $\frac{1}{2}$ occurs because each cluster produces two charged particles, and $n = 2N+2$ in $pp$ collisions. Therefore, the fluctuation in charge transferred across an arbitrary boundary at $\bar{y}$ in $n$-prong events will be

$$[D_n^{(\text{charge})}(\bar{y})]^2 \simeq \frac{2\Delta}{3\sigma_n}\left\langle \frac{d\sigma_n}{dy}(\bar{y})\right\rangle, \quad (22)$$

whereas the corresponding fluctuation in $p_\perp$ transfer will be

$$[D_n^{(p_\perp)}(\bar{y})]^2 \simeq \frac{2\Delta\gamma^2}{9\sigma_n}\left\langle \frac{d\sigma_n}{dy}(\bar{y})\right\rangle + \frac{\gamma^2}{6}\langle n_0(n)\rangle. \quad (23)$$

The structure of Eqs. (22) and (23) is characteristic of the cluster picture, but the numerical factors depend on the explicit parameters of the model. The forms of both (22) and (23) have been verified in the 205-GeV/$c$ data,[25] for events with at least six charged prongs. However, the extraction of charge mobility or transverse momentum mobility from these distributions would be rather model-dependent.

The transfer of transverse momentum across a gap can be studied in the same fashion as that of other additive quantum numbers. Without neutral detection, it is not possible to give direct evidence for short-range effects. The remaining results, therefore, are presented not because they are intrinsically interesting, but to indicate how much experiments are compromised by the inability to measure neutrals. What follows may be regarded as case for highly efficient detection of *all* produced particles. The results are

$$T_N(x) = \left[\frac{1-p-g}{3}(x+1+1/x) + \frac{p-2g}{9}(x^2+3x+1+3/x+x^{-2}) + \frac{g}{18}(x^2+6x+4+6/x+x^{-2})\right]^N, \quad (24)$$

$$\langle v^2 \rangle_N = \frac{N\gamma^2}{18}[3+4p-6g], \quad (25)$$

if $0 < G < \Delta$;

$$T_N(x) = \left[\frac{1-g}{3}(x+1+1/x) + \frac{p-g}{18}(x^2+6x+4+6/x+x^{-2}) + \frac{2g-p}{9}(2x+5+2/x)\right]^N, \quad (26)$$

$$\langle v^2 \rangle_N = \frac{N\gamma^2}{18}[3+3p-4g], \quad (27)$$

if $\Delta < G < 2\Delta$;

$$T_N(x) = \left[(g-p) + \frac{1-g}{3}(x+1+1/x) + \frac{p}{9}(2x+5+2/x)\right]^N, \quad (28)$$

$$\langle v^2 \rangle_N = \frac{N\gamma^2}{18}[3+2p-3g], \quad (29)$$

if $G > 2\Delta$ (but $G+2\Delta < Y$). The dependence of $\langle v^2\rangle_N$ upon the gap size is sketched in Fig. 5. In contrast to the behavior indicated in Fig. 2, which would apply to $p_\perp$ if neutrals were also observed,

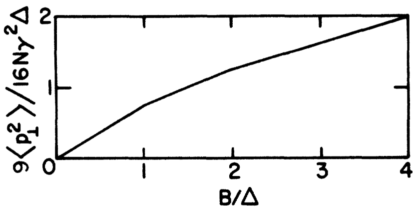


FIG. 5. Prediction of Eqs. (25), (27), and (29) for the fluctuation in transverse momentum transferred across a gap if neutrals are not observed.

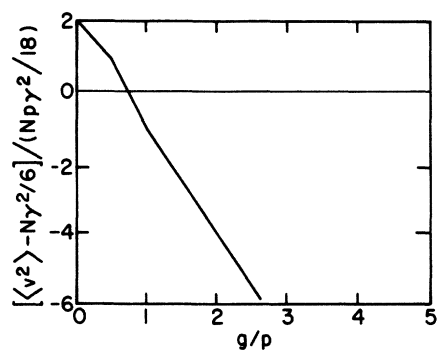


FIG. 6. Prediction for transverse momentum deposit in a bin if neutrals are not observed.

there is no striking change in the behavior of $\langle v^2\rangle_N$ at the point in $G=2\Delta$. Basically this is because the larger the gap, the smaller are the event-to-event fluctuations in visible transverse momentum transfer.

The same shortcoming applies to the dependence upon bin size of the mean-squared transverse momentum confined in a bin within the central region. In the schematic model considered here, one expects

$$\langle p_\perp{}^2\rangle_N = \frac{4\gamma^2\Delta}{9}\frac{N}{Y}\times\begin{cases} 3B/\Delta, & 0\leq B<\Delta \\ 1+2B/\Delta, & \Delta\leq B<2\Delta \\ 2+3B/2\Delta, & B\geq 2\Delta. \end{cases} \quad (30)$$

This behavior, which is sketched in Fig. 6, is to be contrasted with what would be found if neutrals were measured also. The latter situation corresponds to Fig. 4.

## IV. SUMMARY

Charge is balanced locally in rapidity, as expected in short-range correlation models. In such models it is required that the mobility of charge be independent of the incident beam energy. This prediction has not been tested. The measured mobility of charge is compatible with the observed range of two-particle rapidity correlations. Measurement of the mobility of other internal quantum numbers will provide important insights into the nature of the particle production mechanism. Although it is attractive to suppose that transverse momentum is balanced locally, and existing experimental results are consistent with this hypothesis, no direct proof is possible unless the momenta of all produced particles can be measured. Verification of local transverse momentum compensation is an important goal.

## ACKNOWLEDGMENTS

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*Alfred P. Sloan Foundation Fellow. Also at Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637.

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[1]C. Quigg, in *Proceedings of the Canadian Institute of Particle Physics Summer School, McGill University, 1973,* edited by R. Henzi and B. Margolis (Institute of Particle Physics, Montreal, 1974), p. 517; H. Harari, in *Phenomenology of Particles at High Energies,* edited by R. L. Crawford and R. Jennings (Academic, New York, 1974), p. 297. These lecture notes provide extensive references to the primary literature.

[2]A. W. Chao and C. Quigg, Phys. Rev. D 9, 2016 (1973); S. Pokorski and L. Van Hove, Acta. Phys. Polon. B5, 229 (1974).

[3]The most extensive development since the summaries contained in the introductory sections of Refs. 2 has been in the area of semi-inclusive correlations. See, for example, E. L. Berger, Phys. Lett. 49B, 369 (1974); Nucl. Phys. B85, 61 (1975); J. Ranft and G. Ranft, Phys. Lett. 49B, 286 (1974); A. Morel and G. Plaut, Nucl. Phys. B78, 541 (1974); R. Arnold and G. H. Thomas, Phys. Rev. D 9, 3121 (1974); F. Hayot and M. Le Bellac, Nucl. Phys. B86, 333 (1975).

[4]When such programs succeed, it invariably develops that dynamics have been imposed, either implicitly or unwittingly.

[5]For some recent evidence in favor of the cluster interpretation, see C. Quigg, P. Pirilä, and G. H. Thomas, Phys. Rev. Lett. 34, 290 (1975).

[6]Although I shall consistently use this nomenclature, the reader who is more comfortable with terms such as

range, correlation length, cluster mass, $Q$ value, etc. is welcome to substitute them at this point.

[7]Chan Hong-Mo and J. E. Paton, Phys. Letters 46B, 228 (1973); A. Krzywicki and D. Weingarten, *ibid.* 50B, 265 (1974); P. Grassberger, C. Michael, and H. I. Miettinen, *ibid.* 52B, 60 (1974); A. Krzywicki, Nucl. Phys. B86, 296 (1975); D. Weingarten, Phys. Rev. D 11, 1924 (1974).

[8]In exchange models these questions are not separate; charge mobility and charge-exchange energy dependence both are prescribed by (or prescribe) the hadron spectrum. If the only theoretical constraint imposed is unitarity (which is the spirit of Refs. 7), it is not disingenuous to seek an explanation of the one in terms of the other.

[9]T. T. Chou and C. N. Yang, Phys. Rev. D 7, 1425 (1973).

[10]U. Idschok *et al.* (Bonn-Hamburg-Munich Collaboration), Nucl. Phys. B67, 93 (1973); T. Ferbel, in *Particles and Fields—1973,* proceedings of the Conference on Particles and Fields, Berkeley, California, 1973, edited by H. H. Bingham, M. Davier, and G. Lynch (A.I.P., New York, 1973), p. 400; J. Whitmore, in *Experiments on High Energy Particle Collisions—1973*, proceedings of the International Conference on New Results from Experiments on High Energy Particle Collisions, Vanderbilt University, 1973, edited by Robert S. Panvini (A.I.P., New York, 1973), p. 14; T. Kafka *et al.*, Phys. Rev. Lett. 34, 687 (1975); C. Bromberg *et al.*, Rochester Report No. UR-522 (unpublished); E. Malamud *et al.*, Bull. Am. Phys. Soc. 20, 591 (1975).

[11]C. Quigg and G. H. Thomas, Phys. Rev. D 7, 2752 (1973).

[12]A. Białas, in *Proceedings of the IVth International Symposium on Multiparticle Hadrodynamics, Pavia, Italy, 1973,* edited by F. Duimio, A. Giovannini, and S. Ratti (Instituto Nazionale di Fisica Nucleare, Pavia, 1974), p. 93.

[13]P. Bosetti *et al.* (Aachen-Berlin-CERN-London-Vienna Collaboration), Nucl. Phys. B62, 46 (1973); Argonne–Fermilab–Stony Brook Collaboration, quoted by Chao and Quigg, Ref. 2.

[14]A. Białas, K. Fiałkowski, M. Jezabek, and M. Zieliński, Acta Phys. Polon. B6, 59 (1975).

[15]It will be obvious, and has also been shown explicitly in Ref. 14, that no important features of the cluster picture are compromised by the idealization, for the observables discussed here.

[16]That is, the coefficient of $x^{2q}$ in a power-series expansion of $P_N(x)$ is the probability that an $N$-cluster event will lead to charge transfer $q$.

[17]ANL–Fermilab–Stony Brook Collaboration (unpublished).

[18]The presence of intrinsic (but limited) charge exchange between clusters would not affect these conclusions. Whether clusters are neutral or only nearly neutral (carrying limited charge) is immaterial except for numerical details I shall not discuss.

[19]For a review, see G. H. Thomas, in *Proceedings of the XVII International Conference on High Energy Physics*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I-83.

[20]I thank C. Bromberg and T. Ferbel for sending me the Michigan-Rochester data at these energies.

[21]From all indications, clusters closely resemble the prominent resonances. See Ref. 5, and P. Pirilä, G. H. Thomas, and C. Quigg, Phys. Rev. D 12, 92 (1975).

[22]I suspect this is because of the familiar problems of duality diagrams for two-body $N\bar{N}$ scattering.

[23]Although the boundaries of the allowed phase space have been excluded from my considerations, it is obvious that as $b \to 1$ (so that all produced particles are contained in the bin) the fluctuations $\langle\langle S^2\rangle\rangle \to 0$.

[24]A. Białas emphasized this similarity in a parallel session of the XVII International Conference on High Energy Physics, London, 1974. His remark stimulated the investigation summarized here, in which I have tried to specify what is to be learned from a study of $p_\perp$ transfer. Since this paper was submitted for publication, A. Białas *et al.*, Nucl. Phys. B86, 365 (1975), have reported a study of $p_\perp$ transfer in fitted events in 12- and 24-GeV/$c$ $pp$ collisions. More recently, Krzywicki, Ref. 7, has attempted to use local $p_\perp$ compensation as a starting point for overlap function calculations.

[25]T. Kafka (private communication).