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Muon charge ratio prediction from hadronic scaling*

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In view of a number of contrasting results for the cosmic-ray muon ratio, a careful calculation is carried out and extended down to $E_\mu = 50$ GeV. It is found that even with the large uncertainties involved the calculated μ^+/μ^- ratio is significantly higher than the experimental values over the energy range considered. Nuclear corrections have only a small effect if a coherent production model is used and cannot remove the discrepancy, which we conclude requires some new features of high-energy interactions, a higher n/p ratio in the primary flux, or both.

I. INTRODUCTION

Theoretical predictions of the charge ratio of cosmic-ray muons at sea level are based on the assumption of scaling of accelerator data at very high energies. These data are used to compute the fluxes of nucleons, pions, and kaons produced in the atmosphere in hadronic cascades. The differential intensity of muons produced by pion and kaon decay can then be derived from the meson fluxes as a function of energy and zenith angle. Such calculations have been performed by several groups[1-6] over the past few years, with results for the μ^+/μ^- ratio ranging all the way from about 1.26 to 1.58. The experimental data, on the other hand, fluctuate between 1.26 and 1.33 at mean muon energy ranging from a few GeV up to about 3 TeV (see Refs. 6 and 7) with good statistical accuracy in the low- and medium-energy range ($E_\mu < 1$ TeV). It appears difficult to state to what degree the theoretical predictions agree with experiment, since most previous calculations used a number of approximations such as neglecting kaon contribution, energy dependence, or nuclear effects. It thus seems justified to perform yet another calculation attempting to include all major effects, or at least estimating their relative impact on the final result. Furthermore, a reliable over-all error estimate must be established so that statements about the degree of agreement with experiment can be more conclusive, particularly since this reflects on the acceptable models of high-energy hadronic interactions as well as on cosmic-ray composition.

Since this paper is intended to serve as a critical evaluation of theoretical calculations, it might be useful to recapitulate some of the results previously derived. This is done in Sec. II, where hadronic fractional moments are derived in the framework of scaling, and in Sec. III, where diffusion equations are used to determine the meson intensities which then lead to the muon flux and charge ratio. Throughout the calculation, a flat isothermal atmosphere is assumed with its density decreasing exponentially. A flat atmosphere is a good approximation when the earth's curvature can be neglected, i.e., for zenith angles smaller than 80°, and the atmospheric temperature is essentially independent of depth within the top 200 g/cm^2 of the atmosphere (which is the region of most importance for our purposes since most unstable particles are produced at about 100 g/cm^2 atmospheric depth). In addition, we neglect lateral distributions in the laboratory system of particles produced in collisions with air nuclei and by decay, so that one-dimensional diffusion equations can be written down. This approximation is justified because of the small transverse momenta involved at high particle energies. Furthermore, deflection of charged particles by the earth's magnetic field can be neglected. Other approximations inherent in the calculations are discussed where relevant. Section IV then contains a presentation of numerical results including errors. Section V is a discussion of the effect of different rates of approach to scaling for π^+ and π^-, while in Sec. VI we describe the effect of intranuclear interactions in the framework of a coherent-production model. Section VII is a discussion and comparison with previous results.

II. KINEMATICS AND THE SCALING HYPOTHESIS

Measurements of the cosmic-ray primary flux extending up to about 1 TeV indicate that it consists mainly of protons and α particles with smaller amounts of heavier nuclei. The energy spectrum obeys a power-law behavior with the flux decreasing rapidly as the energy increases. According to balloon-flight measurements performed by Ryan *et al.*,[8] the α/p ratio in the primary flux is about 1:26 and stays approximately constant up to 400 GeV/nucleon. This corresponds to a p/n ratio of about 9:1 when 6% contribution from heavier nuclei is included.[3] We therefore write for the differential intensity of the incident cosmic radiation at the top of the atmosphere

$$p(E, h=0)=\frac{dN_p(E,0)}{dE}=p_0E^{-(\gamma+1)}\ ,$$
$$n(E, h=0)=\frac{dN_n(E,0)}{dE}=n_0E^{-(\gamma+1)}\ , \tag{1}$$

where $\gamma=1.75\pm0.03$ is taken as a constant independent of energy.[9]

The quantity of interest which is generally extracted from accelerator data is the single-particle inclusive distribution for the process $a+b\to c+X$:

$$f^c_{ab}(E,E')=\int d^{(2)}q_\perp \mathfrak{f}^c_{ab}(E,E',q_\perp)\ , \tag{2}$$

where

$$\mathfrak{f}^c_{ab}(E,E',q_\perp)=\frac{1}{\sigma^{\rm incl}_{ab}}\ \frac{d^3\sigma_{a\to c}}{d^3q/E}\quad ,$$

E' is the energy of the incident particle, and E is the energy of the secondary particle c in the laboratory.

A consequence of the exponential form of the primary spectrum is that the flux of the produced particle c will typically contain terms of the form

$$n_c(E)=\frac{dN_c(E)}{dE}$$
$$=\text{const}\times\int_E^\infty\frac{dE'}{E}E'^{-(\gamma+1)}f^c_{ab}(E,E')\ . \tag{3}$$

The scaling hypothesis now requires that at high energies

$$\mathfrak{f}^c_{ab}(E,E',q_\perp)_{E,E'\to\infty}\ \mathfrak{f}^c_b(\tilde{x},q_\perp)\ , \tag{4}$$

where $\tilde{x}=E/E'$. Since we also assume factorization, the target subscript b in (4) can be dropped. A change of variable in (3) leads to

$$n_c(E)=\text{const}\times E^{-(\gamma+1)}Z_{ac}\ , \tag{5}$$

where Z_{ac} is the fractional moment defined as

$$Z_{ac}=\int_0^1 d\tilde{x}\ \tilde{x}^{(\gamma-1)}f^c_a(\tilde{x})\ . \tag{6}$$

The variable $\tilde{x}$ can be related to the usual Feynman variable x through

$$\tilde{x}=x+\frac{\mu^2}{2m_aE}\ \text{ with } \mu=(m_c{}^2+q_\perp{}^2)^{1/2}\ , \tag{7}$$

assuming only $\mu^2\ll E^2$, which is a very reasonable approximation in view of the very high energies and generally low transverse momenta of the secondary particles. This yields an energy dependence in (6) when rewritten in terms of x:

$$Z_{ac}(E)=\int_{-\mu^2/2m_aE}^{1-\mu^2/2m_aE}dx\int d^{(2)}q_\perp\mathfrak{f}^c_a(x,q_\perp)$$
$$\times\left(x+\frac{\mu}{2m_aE}\right)^{\gamma-1}\ . \tag{8}$$

Taking $\mu^2\ll 2m_aE$ (or alternatively $x\gg2\mu/\sqrt{s}$), one is led to the kinematic approximation $\tilde{x}=x$. No such approximation is needed when the rapidity variable is used instead of x. If we define $y^{\rm proj}=y^{\rm beam}-y_c$ (where y_c is the c.m. rapidity of the produced particles and $y^{\rm beam}$ is the c.m. rapidity of the incoming particle), the Z's assume the following general form:

$$Z_{ac}=\frac{1}{m_a{}^\gamma}\int_{\ln\mu/m_a}^\infty dy^{\rm proj}\int d^{(2)}q_\perp e^{-\gamma y^{\rm proj}}$$
$$\times\mathfrak{f}^c_a(y^{\rm proj},q_\perp)\mu^\gamma\ . \tag{9}$$

III. THE MUON ENERGY SPECTRUM AND CHARGE RATIO

Following the procedure outlined in Refs. 1 and 2, we describe the propagation of nucleons and mesons through the atmosphere by diffusion equations, which can then be solved for the appropriate fluxes using the scaling hypothesis, the definition of fractional moments, and charge-symmetry relations. Neglecting production of nucleons by mesons, the nucleon fluxes obtained are[1-2]

$$N(E,h)=p(E,h)+n(E,h)$$
$$=N_0e^{-h/\Lambda_N}E^{-(\gamma+1)}\ ,$$
$$\Delta(E,h)=p(E,h)-n(E,h) \tag{10}$$
$$=\Delta_0e^{-h/\Lambda'_N}E^{-(\gamma+1)}\ ,$$

where

$$N_0=p_0+n_0\ ,\quad \Lambda_N=\lambda_N(1-Z_{pp}-Z_{pn})^{-1}\ ,$$
$$\Delta_0=p_0-n_0,\quad \Lambda'_N=\lambda_N(1-Z_{pp}+Z_{pn})^{-1}\ ,$$

λ_N is the nucleon-interaction mean free path for protons and neutrons assumed independent of energy,[10] and Λ_N represents the attenuation length of nucleons in the atmosphere.

Assuming that pions retain the same direction as the parent particles and taking into account pion fractional loss and production through nucleon-nucleon and nucleon-pion interaction as well as π-meson decay, the diffusion equation one finds for the pions is

$$\frac{d\Pi(E_\pi,h)}{dh}=-\Pi(E_\pi,h)\left(\frac{1}{\Lambda_\pi}+\frac{B_\pi}{hE_\pi\cos\theta}\right)$$
$$+\frac{N_0Z^+_{p\pi}}{\lambda_N}e^{-h/\Lambda_N}E_\pi{}^{-(\gamma+1)}\ , \tag{11}$$

where $\Pi(E_\pi,h)=n_{\pi^+}(E_\pi,h)+n_{\pi^-}(E_\pi,h)$, $Z^+_{p\pi}=Z_{p\pi^+}$

$+Z_{p\pi^-}$, $\Lambda_\pi = \lambda_\pi(1 - Z_{\pi^+\pi^+} - Z_{\pi^-\pi^-})^{-1}$, and λ_π denotes the pion mean free path in the air. To get Eq. (11) into this form, charge-symmetry relations were used (namely, $f_p^{\pi^+} = f_n^{\pi^-}$, $f_n^{\pi^+} = f_p^{\pi^-}$, $f_{\pi^+}^{\pi^+} = f_{\pi^-}^{\pi^-}$, and $f_{\pi^+}^{\pi^-} = f_{\pi^-}^{\pi^+}$). The decay constant for the π meson, appearing in Eq. (11), is defined as

$$B_\pi = \frac{Hm_\pi c}{\tau_0} \simeq 115 \text{ GeV},$$

where τ_0 is the mean lifetime of pions at rest and H is the atmospheric scale height assumed independent of depth. We take H to be ~6.4 km, which corresponds to a weighted average over the atmosphere.[11] The zenith angle θ is related to H through $H\rho = h\cos\theta$, ρ being the atmospheric density. Contributions from neutral pions and kaons to the over-all pion flux were neglected because of the lower intensity of these components in the atmosphere. A similar equation can be written for the pion flux difference: $\Delta_\pi(E_\pi, h) = n_{\pi^+}(E_\pi, h) - n_{\pi^-}(E_\pi, h)$, but with Λ_N' and $Z_{p\pi}^- = Z_{p\pi^+} - Z_{p\pi^-}$ instead of Λ_N and $Z_{p\pi}^+$, and $\Lambda_\pi' = \lambda_\pi(1 - Z_{\pi^+\pi^+} + Z_{\pi^+\pi^-})^{-1}$ replacing Λ_π. Since $\Pi(E_\pi, h)$ and $\Delta_\pi(E_\pi, h)$ are factorizable, solutions for the pion spectrum can be written down immediately:

$$\Pi(E_\pi, h) = \frac{N_0 Z_{p\pi}^+}{\lambda_N} E_\pi^{-(\gamma+1)} e^{-h/\Lambda_\pi} h^{-B_\pi/E_\pi\cos\theta} \times \int_0^h e^{-h'/l_\pi} h'^{B_\pi/E_\pi\cos\theta} dh' ,$$

$$\Delta_\pi(E_\pi, h) = \frac{\Delta_0 Z_{p\pi}^-}{\lambda_N} E_\pi^{-(\gamma+1)} e^{-h/\Lambda_\pi'} h^{-B_\pi/E_\pi\cos\theta} \times \int_0^h e^{-h'/l_\pi'} h'^{B_\pi/E_\pi\cos\theta} dh' , \tag{12}$$

where

$$\frac{1}{l_\pi} = \frac{1}{\Lambda_N} - \frac{1}{\Lambda_\pi} , \quad \frac{1}{l_\pi'} = \frac{1}{\Lambda_N'} - \frac{1}{\Lambda_\pi'} .$$

Equations (12) yield particularly simple expressions in the high- and low-energy limits:

(i) If $E_\pi\cos\theta \gg B_\pi$,

$$\Pi(E_\pi, h) = \frac{N_0 Z_{p\pi}^+}{\lambda_N} E_\pi^{-(\gamma+1)} \frac{e^{-h/\Lambda_\pi} - e^{-h/\Lambda_N}}{1/l_\pi} ,$$

$$\Delta_\pi(E_\pi, h) = \frac{\Delta_0 Z_{p\pi}^-}{\lambda_N} E_\pi^{-(\gamma+1)} \frac{e^{-h/\Lambda_\pi'} - e^{-h/\Lambda_N'}}{1/l_\pi'} , \tag{13}$$

in agreement with previous results.[1,2]

(ii) If $E_\pi\cos\theta \ll B_\pi$, we can expand the integral in Eqs. (12) and take the low-energy limit. The pion flux then reduces to

$$\Pi(E_\pi, h) = \frac{N_0 Z_{p\pi}^+}{\lambda_N} E_\pi^{-(\gamma+1)} e^{-h/\Lambda_N} \frac{hE_\pi\cos\theta}{B_\pi} ,$$

$$\Delta_\pi(E_\pi, h) = \frac{\Delta_0 Z_{p\pi}^-}{\lambda_N} E_\pi^{-(\gamma+1)} e^{-h/\Lambda_N'} \frac{hE_\pi\cos\theta}{B_\pi} . \tag{14}$$

Thus, we may conclude that the pion flux should be isotropic at high energies but go like $\cos\theta$ at low energies. In both limits the flux decreases with increasing energy but more sharply so in the high-energy limit.

Entirely analogous expressions hold for kaons, the main difference being that B_π is replaced by B_K, which is considerably larger because of the kaon's larger mass. We calculate $B_K \simeq 854$ GeV. A source of uncertainty in the kaon flux, however, arises from our lack of data for kaon production by neutrons. To circumvent this problem, the calculations are carried out comparing several extreme possibilities, such as setting $f_n^{K^\pm} = 0$, $f_n^{K^\pm} = f_p^{K^\pm}$, or $f_n^{K^\pm} = f_p^{K^\mp}$.

We can now proceed to calculate the muon flux from

$$\mu^\pi(E_\mu, h) = n_{\mu^+}^\pi(E_\mu, h) + n_{\mu^-}^\pi(E_\mu, h) = \frac{B_\pi}{(1 - m_\mu^2/m_\pi^2)\cos\theta} \times \int_0^h \frac{dh'}{h'} \int_{E_\mu}^{E_\mu m_\pi^2/m_\mu^2} \frac{dE_\pi}{E_\pi^2} \Pi(E_\pi, h) , \tag{15}$$

where muon decay was neglected since it should not influence the muon ratio. An analogous expression again can be written for muons produced by kaon decay. The upper limit in the integral over h' can usually be approximated by infinity since we are concerned with sea-level measurements ($h \sim 1000$ g/cm^2). At the high-energy limit $E_\mu\cos\theta \gg B_\pi, B_K$ this leads to the familiar result for the muon ratio

$$\frac{\mu^+}{\mu^-} = \frac{\mu^\pi + \mu^K + \Delta_\mu^\pi + \Delta_\mu^K}{\mu^\pi + \mu^K - \Delta_\mu^\pi - \Delta_\mu^K} = \frac{A_\pi^+ + A_\pi^- + k(C_K^+ + C_K^-)}{A_\pi^+ - A_\pi^- + k(C_K^+ - C_K^-)} , \tag{16}$$

where the following notation was used:

$$A_\pi^+ = \frac{N_0}{\Delta_0} Z_{p\pi}^+ L_\pi , \quad A_\pi^- = Z_{p\pi}^- L_\pi' ,$$

$$C_K^+ = \frac{N_0}{\Delta_0} \frac{Z_{pK}^+ + Z_{nK}^+}{2} L_K + \frac{Z_{pK}^+ - Z_{nK}^+}{2} \overline{L}_K ,$$

$$C_K^- = \frac{Z_{pK}^- - Z_{nK}^-}{2} L_K' + \frac{N_0}{\Delta_0} \frac{Z_{pK}^- + Z_{nK}^-}{2} \overline{L}_K' ,$$

$$L_{\pi,K} = \frac{\ln(\Lambda_{\pi,K}/\Lambda_N)}{1/l_{\pi,K}} , \quad L_{\pi,K}' = \frac{\ln(\Lambda_{\pi,K}'/\Lambda_N')}{1/l_{\pi,K}'} ,$$

$$\overline{L}_K = \frac{\ln(\Lambda_K/\Lambda'_N)}{1/\Lambda'_N - 1/\Lambda_K}, \quad \overline{L}'_K = \frac{\ln(\Lambda'_K/\Lambda_N)}{1/\Lambda_N - 1/\Lambda'_K},$$

$$k = b_{K\mu}\frac{B_K G_K}{B_\pi G_\pi},$$

$$G_{\pi,K} = \frac{1}{1 - m_\mu{}^2/m_{\pi,K}{}^2}\,\frac{1-(m_\mu{}^2/m_{\pi,K}{}^2)^{\gamma+2}}{\gamma+2},$$

and $b_{K\mu} = 0.64$ is the branching ratio for kaon decay into muons. Note that if we were to set $f_p^{K^\pm} = f_n^{K^\mp}$ in analogy with pions, the expressions involving kaons would simplify considerably, i.e., we would have

$$C_K^+ \to A_K^+ = \frac{N_0}{\Delta_0} Z_{pK}^+ L_K$$

and

$$C_K^- \to A_K^- = Z_{pK}^- L'_K.$$

For simplicity we will present the succeeding formulas for this case only, although in discussing numerical results (Sec. IV), all the possibilities mentioned above will be considered.

At medium and high energies ($E_\mu > 0.2$ TeV), the muon ratio will exhibit an energy dependence attributable to the kaons with their larger energy spread in the laboratory system. If we start from Eq. (15) and use Eq. (14) but with kaons interchanged with pions, we arrive at

$$\mu^K(E_\mu, \cos\theta) = \frac{b_{K\mu}B_K N_0 Z_{pK}^+}{(1-m_\mu{}^2/m_K{}^2)\cos\theta}\int_{E_\mu}^{E_\mu m_K{}^2/m_\mu{}^2}\frac{dE_K}{E_K^2}E_K{}^{-(\gamma+1)}\left(\frac{1}{\eta+1} - \frac{\Lambda_K/l_K}{\eta+2} + \frac{(\Lambda_K/l_K)^2}{\eta+3} - \cdots\right), \tag{17}$$

where we denote $\eta = B_K/E_\mu\cos\theta$ and assume $Z_{pK^\pm} = Z_{nK^\mp}$. In the high-energy limit, the above reduces to the usual muon intensity, which goes like $1/\cos\theta$, and leads to a charge ratio independent of E_μ. For $\eta \gg 1$, on the other hand, the muon spectrum is isotropic and the charge ratio is energy dependent. Thus for a medium-energy range $B_\pi < E_\mu < B_K$, we would expect the muons resulting from pions to go approximately like $1/\cos\theta$, whereas those from kaons should be isotropic. A fairly good approximation to the series in (17) at all energies was given by Barrett *et al.*[11]:

$$\int_{E_\mu}^{\infty}\frac{E_K{}^{-(\gamma+1)}}{a+E_K}\,dE_K \simeq \frac{E_\mu{}^{-(\gamma+1)}}{(\gamma+2)E_\mu + (\gamma+1)a + (aE_\mu)^{1/2}/(2\gamma+3)}, \tag{18}$$

where we made the approximation $E_\mu m_K{}^2/m_\mu{}^2 \to \infty$ and set $a \equiv B_K L_K/\Lambda_N\cos\theta$. In order to extend the calculations to still lower energies (down to $E_\mu = 50$ GeV), the energy dependence resulting from pion decay has to be taken into account also. This can be done in a manner similar to what was done for kaons except that Eq. (17) now has to be integrated numerically, the approximation (18) rendered inapplicable by the pion's smaller energy range.

IV. NUMERICAL RESULTS

We use for the proton excess and the primary spectrum exponent γ the values calculated by Yekutieli and Rotter[3] from measurements performed by Ryan *et al.*[8] on cosmic-ray protons and α particles (see Table I). The fractional moments can be calculated from the recent high-energy CERN ISR data (between 31 and 63 GeV total c.m. energy),[12] if we assume that scaling has been reached at these energies. (The effect of relaxing this assumption is discussed in Sec. V.) Inclusive distributions for protons, pions, and kaons produced in pp collisions are presently available in terms of both x and y^{proj} and we computed the Z values for each case using a simple parametrization of the transverse-momenta dependence as a pure exponential in $q_\perp$ with a varying coefficient. In this manner, the invariant cross section $[\sigma_{\text{inv}} \equiv E(d^3\sigma/d^3q)]$ can be written as

$$\sigma_{\text{inv}}(q_\perp, x) = g(x)|_{q_\perp=0.4}e^{-b(x)q_\perp},$$
$$\sigma_{\text{inv}}(q_\perp, y^{\text{proj}}) = g(y^{\text{proj}})|_{q_\perp=0.4}e^{-b(y^{\text{proj}})q_\perp}, \tag{19}$$

where $q_\perp = 0.4$ GeV/c was chosen since this is where the scaling behavior manifests itself best. From the data we note that an exponential dependence on $q_\perp$ is a more reasonable fit when y^{proj} is held fixed for $\pi^\pm$ and $K^\pm$ production. Accordingly, the fractional moments we use are the ones calculated in terms of rapidity for pions and kaons but in terms of x for protons—a procedure which

TABLE I. Input data used in the calculations.

Constant	Value	Reference
λ_N	81 ± 4 g/cm^2	3 and 16
Λ_N	120 ± 10 g/cm^2	1
λ_π	117 ± 6 g/cm^2	3 and 16
λ_k	123 ± 6 g/cm^2	1
N_0/Δ_0	1.245 ± 0.186	3 and 7
γ	1.75 ± 0.03	7

minimizes the parametrization errors. In either case, it turns out, the Z's are not too sensitive to the choice of parameter b, which was determined by a least-squares fit over the range of y^{proj} (or x) used. The over-all error on the Z's is still relatively large (~10–15% with only 2–4% due to parametrization). This can be attributed mostly to measurement errors present in the data as well as a smearing out of the points over the energy range considered. In addition, the only data available for high x ($x > 0.4$) and low y^{proj} ($y^{\text{proj}} < 0.1$, 0.6 for pions and kaons, respectively) are of low energy (Allaby *et al.* at 6.8 GeV).[13] Since these regions still contribute about 15–20% to the Z's, a fairly substantial error is introduced by either the need to extrapolate the high-energy data or the use of the low-energy ones, and so we chose, as a compromise, a curve which lies about halfway in between. Our calculated Z values are presented in Table II alongside previously calculated values for comparison's sake.

We have also checked the validity of the kinematic approximation, which underestimates contributions near $x \approx 0$. However, it becomes obvious if we expand Eq. (8) in terms of E_π that, considering the high energies involved, energy-dependent terms (which go as E_π^{-1}) will have only a small effect. In fact, going from $E \approx 50$ GeV to $E \approx 2000$ GeV induces a change in $Z_{p\pi^+}$ which is only slightly more than 1%. This is to be expected since the steep cosmic-ray primary spectrum gives extra weight to particles with large values of x and thus reduces the effect of uncertainties introduced at low x.[14] The major contribution to the Z's comes in fact from the region around $x = 0.2$.

Meson-proton collision data at 25 and 16 GeV/c (see Ref. 15) were used to calculate the pion fractional moments. We find

$$Z_{\pi^+\pi^+} \approx Z_{\pi^-\pi^-} = 0.242 \pm 0.03 ,$$

$$Z_{\pi^+\pi^-} \approx Z_{\pi^-\pi^+} = 0.074 \pm 0.009 .$$

Unfortunately, since no adequate data on high-energy kaon-proton collisions are available, we had to use the following approximations (which hold to within 10–20% in the low-energy range):

$$Z_{K^+K^+} \approx Z_{K^-K^-} \approx Z_{\pi^+\pi^+} ,$$

$$Z_{K^+K^-} \approx Z_{K^-K^+} \approx Z_{\pi^+\pi^-} .$$

The same situation exists for pion production by kaons and vice versa and so pion $\leftrightarrow$ kaon exchange was neglected. The effect of this approximation will be discussed later. With these input data, those from Table I, and assuming $Z_{pK^\pm} = Z_{nK^\mp}$, Eq. (16) yields for the muon ratio at the high-energy limit

$$\mu^+/\mu^- = 1.52 \pm 0.17 . \tag{20}$$

This ratio reduces to 1.39 ± 0.14 when kaons are not taken into account. Setting $Z_{nK^\pm} = 0$ would increase the ratio in (20) by 0.07, whereas using an opposite extreme assumption $Z_{pK^\pm} = Z_{nK^\pm}$ would cause an increase of nearly 0.1 or approximately 6%. Apart from these uncertainties, the over-all error quoted above results in most part from uncertainties in $Z_{p\pi^\pm}$ (where a 15% error gives 7% in the muon ratio), and the primary spectrum proton excess which contributes about 5% to the over-all error. Variations in the power γ, however, have a relatively small effect on the charge ratio (going from $\gamma = 1.75$ to $\gamma = 1.7$ reduces the charge ratio by only 0.01). Uncertainties in the attenuation length have only about 3% effect on μ^+/μ^- and variations in the mean free paths have an even smaller impact. Setting $Z_{\pi^+\pi^-} = 0$, as was done in some previous calculations, has the effect of increasing the

TABLE II. The fractional moments calculated from the high-energy ISR data. Values found in terms of both x and y^{proj} variables are given as well as some of the previously calculated values for comparison purposes.

	This calculation y^{proj} variable used	This calculation x variable used	Garraffo[a] *et al.*, Ref. 2	Yekutieli and Rotter, Ref. 3	Ashley[b] *et al.*, Ref. 4
$Z_{p\pi^+}$	0.0392 ± 0.0065	0.0426 ± 0.0054	0.053 78	0.0334 ± 0.015	0.0564
$Z_{p\pi^-}$	0.0239 ± 0.0033	0.0252 ± 0.0025	0.031 56	0.0226 ± 0.001	0.0386
Z_{pk^+}	0.0061 ± 0.0009	0.0063 ± 0.0001	...	...	0.0062
Z_{pk^-}	0.002 ± 0.0002	$0.002\,03 \pm 0.000\,36$	...	...	0.0022
Z_{pp}	0.3132 ± 0.0481	$0.265\,09 \pm 0.05$	0.1821	...	0.2

[a] These are values corrected to include nuclear effects using the cascade model.
[b] Values derived from fits to interpolations between 19.2-GeV p-Be and p-Al data.

ratio by about 0.06.

We have also investigated the dependence of the charge ratio on muon energy and zenith angle, extending the calculations down to $E_\mu = 50$ GeV and over a wide range of angles—from $\theta = 0°$ up to $\theta = 60°$. Figure 1 shows the resulting μ^+/μ^- ratio at sea level and at $\theta = 0°$ as compared to the measured values. Our calculations predict a nearly constant muon ratio up to about 200 GeV (~1.44) which then increases slowly with energy reaching a value of 1.54 at 10 TeV, i.e., less than 7% increase. The effect of including pion production by kaons is to reduce the π^+/π^- ratio (and hence μ^+/μ^-) some, but the kaons' lower flux in the atmosphere renders this correction rather small. On the other hand, kaon production by pions is expected to reduce the K^+/K^- ratio more noticeably, especially at higher energies where kaon contribution to the muon flux is appreciable. Nevertheless, we estimate that the muon ratio will not decrease by more than a few percent at higher energies (above 1 TeV/nucleon), and the calculated curve will still be significantly higher than the experimental points (which average to about 1.28–1.33 for zenith angles near 0°). Our ratio decreases as the zenith angle increases but only slightly so. Statistical and other uncertainties in the Z's and uncertainties in the primary spectrum and the mean free paths could shift the curve up or down by about 0.15. An additional shift of 0.02–0.03 is caused by assigning different values to $Z_{nK^\pm}$ in accordance with what was discussed before.

As a further check, we have also calculated the n/p ratio at various depths and compared it to the data in Fig. 2.[16] While the experimental points fluctuate between 0.55 and 0.75 for h between approximately 500 and 800 g/cm^2,[17] indicating some rise, our calculations yield an n/p ratio which increases from about 0.45 to 0.6 for the same range of depths. It is doubtful, however, whether this can be taken as a serious disagreement between theory and experiment in view of the sensitivity of the neutron-to-proton ratio to uncertainties in the input data. E.g., increasing Λ_N by only 10 g/cm^2 will increase n/p by nearly 40% at $h = 500$ g/cm^2, with comparable error resulting from the uncertainty inherent in Z_{pp}. A much smaller error results from the uncertainty in the primary cosmic-ray spectrum (~12%) and an even smaller one (~3%) from the nucleon interaction mean free path.

V. APPROACH TO SCALING FOR π^+ AND π^-

We now turn our attention to the fact that the ISR data do not quite scale in the central region. This may affect our calculations some since plots of the invariant cross section against the energy, when either $s^{-1/2}$ or $s^{-1/4}$ scales are being used, indicate a different rate of approach to scaling for π^+ and π^-. Since reliable plots were available to us only on an $s^{-1/2}$ scale, we will base our estimate on parameters derived from it (while an $s^{-1/4}$ scale is more plausible, being the one theoretically derived from Mueller analysis, the qualitative features exhibited on both scales are essentially the same). Thus, assuming π^+ and π^- reach the same asymptotic value at infinite energy, we write $\sigma_{\text{inv}}|^{\pm}_{x=0;q_\perp=0.4} = a + b^\pm/\sqrt{s}$ with $a \sim 14$ mb/(GeV$^2/c^3$), $b^+ \sim 20$ mb/(GeV$/c^3$) and $b^- \sim 45$ mb/(GeV$/c^3$).[18] The contribution to the ratio $Z_{p\pi^+}/Z_{p\pi^-}$ from the central region is hence reduced (by about 20%), π^- approaching its scaling limit faster than π^+ (only the magnitude of de-

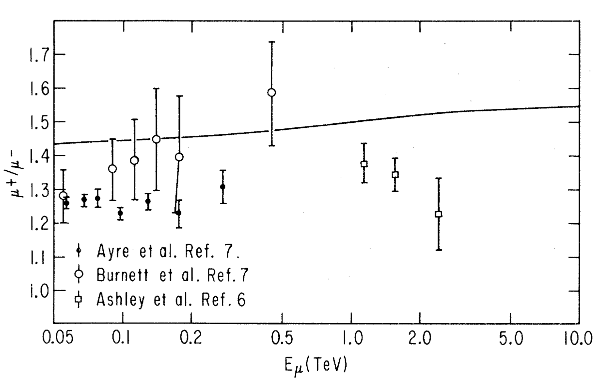

FIG. 1. The muon ratio plotted as a function of muon energy at mean zenith angle 0°. The data points shown are compiled from Refs. 6 and 7. The curve represents our predicted values.

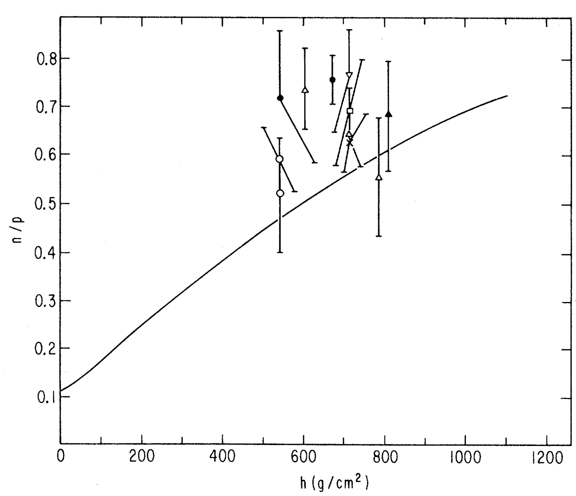

FIG. 2. Neutron/proton ratio as a function of atmospheric depth at high energy. The data used are those of Ref. 17.

crease would be changed were we to use an $s^{-1/4}$ parametrization instead). As it turns out, the central plateau contributes no more than 2% to the fractional moments (this contribution being evidently suppressed owing to the exponential rapidity dependence) and hence even if the central region fraction of $Z_{p\pi^+}/Z_{p\pi^-}$ is considerably reduced the effect on the over-all pion excess is very small.

In this connection it may be useful to note that the data indicate only a weak energy dependence in the fragmentation region ($x > 0$), the ratio π^+/π^- decreasing slightly between accelerator and ISR energies. The error introduced thereby is relatively negligible. We may add here that the kaon ratio exhibits a more pronounced effect in the fragmentation region, but as mentioned in the previous section, much larger sources of error are present for the kaons, relative to which the effect of a different approach to scaling for K^+ and K^- is again negligible.

VI. NUCLEAR EFFECTS

Our calculations so far were based on the assumption that the incident primary nucleons interact with other free nucleons. Since the atmosphere is actually composed of light nuclei, the spectra of the particles produced in atmospheric collisions will be modified to some extent because of intranuclear multiple scatterings. Nuclear effects have been considered previously, but the complicated processes involved render the results rather model dependent. Garraffo *et al.*[2] provided, for example, one calculation of intranuclear cascade effects by assuming independent inelastic collisions in the nucleus, each occurring with a certain probability taken to be a simple Poisson distribution. Such a model belongs to a class of incoherent-production models (IPM) in which the final state for an intranuclear collision appears immediately and is capable of generating a cascade in the nucleus (i.e., the time scale for hadronic collisions is assumed to be smaller than the nuclear radius). However, IPM appears to be ruled out on the basis of experimental evidence. According to recent emulsion experiments performed by Gibbs *et al.*,[19] incoherent-production models, although capable of giving a reasonable fit to accelerator data, appear to predict too many particles at cosmic-ray energies and yield a dependence of the average charge multiplicities on the mass number A which is too flat. For the same reasons, any model in which many particles propagate through the nucleus (e.g., the Glauber theory or mean-free-path approximations) cannot give an adequate description of the high-energy interactions that take place in the nucleus. It would thus appear that the hadronic state traversing the nucleus bears little resemblance to what is finally observed. Two recent models proposed along these lines—Gottfried's energy flux model (EFM)[20] and Fishbane and Trefil's coherent-production model (CPM),[21] are probably more realistic in that they give a reasonable fit to cosmic-ray data and are compatible with the observed charged-pion multiplicity dependence on A. In the CPM, which we now use in estimating the magnitude of nuclear corrections, a long-lived intermediate projectile excitation (e.g., a "fireball") propagates through the nucleus producing target excitations characteristic of a target nucleon. According to this picture, cascades will not develop inside the nucleus as long as the excited hadron has a lifetime greater than the time required to cross the nucleus. The projectile excitation thus decays outside the nucleus, giving rise to an inclusive spectrum characteristic of the beam particle, while the target excitations decay to produce a single-particle inclusive spectrum in the target rapidity region ($x < 0$) equal to $\bar{\nu}$ times the single-target-nucleon distribution, where $\bar{\nu}$ stands for the number of mean free paths in the nucleus for inelastic collisions. Since $\bar{\nu} \approx cA^{1/3}$, the height of the target distribution (and hence the multiplicity) grows as $A^{1/3}$. The available nuclear data are also quite suggestive of an $A^{1/3}$ dependence and according to the result found in Ref. 19, assigning $c \approx 0.66$ yields a very reasonable fit for the range of nuclei covered.

To modify our distributions to include nuclear effects in the CPM picture, we first note that the forward fragment distribution ($x > 0$, small y^{proj}) in nuclei will be identical to the distribution from a single-nucleon target. Nuclear effects are prominent only in the target-fragmentation region. In reality, however, one must interpolate the curve smoothly between the two extremes (cf., the two-phase modification of CPM),[22] thus extending nuclear effects into the central region (see Fig. 3). We may note immediately that since only fast forward-moving fragments contribute appreciably to the integrals for $Z_{p\pi^\pm}$ (the central region as well as the target-fragmentation region being strongly suppressed), our calculations should not be affected seriously by nuclear corrections. If we assume that both fragmentation regions extend over approximately 2.5–2.8 rapidity units, then the length of the central plateau, which corresponds to $\sqrt{s} \approx 30$ GeV, is about 3 units (the lowest reasonable energy was chosen so as to yield an upper limit on the corrected quantities). Using nitrogen as the predominant atmospheric component, we estimate an increase of merely 2% for $Z_{p\pi^+}$ and 1% for $Z_{p\pi^-}$. Table III presents the results of our calculations along with cascade model results (computed following the method outlined in Ref. 2) for comparison purposes. The two models apparently lead to rather different conclusions. Where the cascade model predicts a rather large decrease in the ratio $Z_{p\pi^+}/Z_{p\pi^-}$ (the decrease in the values of the individual Z's is a feature of the particular model used here; different IPM treatments could yield higher Z's), the CPM leads to a slight increase in the values of $Z_{p\pi^\pm}$, while leaving the ratio practically unchanged. Indeed, it is expected that multiple interactions taking place in the nucleus will tend to produce particles with less charge asymmetry. The effect of coherent production is to decrease the Z ratio in the central region and increase it in the target-fragmentation region, the over-all effect then depending on whether scaling has been reached or not. Thus, an eventual reduction in $Z_{p\pi^+}/Z_{p\pi^-}$ can be expected, the magnitude of which will nevertheless stay rather small.

The effect of nuclear corrections on the muon ratio (in the framework of CPM) is thus expected to be rather small (of the order of 2–3%), especially when compared to measurement errors in the distributions themselves as well as the other sources mentioned before. Since this situation would not have changed by much even if a more sophisticated calculation had been employed, we may then conclude that if CPM is a valid interpretation of high-energy intranuclear processes, the error introduced by ignoring the presence of nuclear effects in the air is not serious.

VII. DISCUSSION AND CONCLUSIONS

Subject to a number of uncertainties, we find that the calculated muon ratio consistently exceeds the measured values over the whole energy range considered. The magnitude of the deviation from

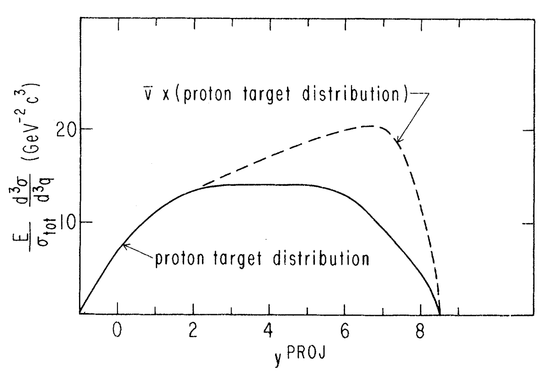

FIG. 3. Nuclear single-particle rapidity distribution for a CPM superimposed on the single-nucleon distribution.

experiment is large enough to render it significant despite the large error put on the theoretical ratio (note that this error is not to be regarded as a standard deviation but rather as a somewhat rough estimate of the major uncertainties involved in the calculations). Furthermore, nuclear corrections have little effect if we are to believe that indeed a coherent-production model rather than an incoherent-production model best describes high-energy interactions in the nucleus—a view that seems to be supported by the experimental data so far. A similar situation exists for the neutron-to-proton ratio, but in this case it is not clear whether this reflects on a real disagreement between theory and experiment or whether it is merely a consequence of the marked sensitivity of the n/p ratio to uncertainties in the input data.

At this point it might be instructive to critically compare our prediction with previously calculated values for the muon ratio at sea level. Table IV summarizes most of the results reported over the last few years. Since these results seem rather widely divergent, we also list the major approximations and assumptions that went into each derivation. Thus in Refs. 1, 2, and 3, kaon contributions were neglected, and so the muon ratio obtained there is lower by about 9%. Even so, the value quoted in Ref. 2 is too low, which we attribute to an invalid approximation in the calculation of the fractional moments (see Ref. 14). In Ref. 3, nuclear effects were incorporated but only through a cascade model, which will generally lower the μ^+/μ^- ratio more drastically relative to a coherent model. It is harder to explain the source of disagreement with the results found in Ref. 6 unless it is a direct consequence of the low-energy nuclear-target data which were used there. While we admit the existence of a discrepancy here, we would like to have higher-energy nuclear data so that a more significant comparison can be made. Our results do agree, however, with the ones reported recently by Adair[5] which were based on a Monte Carlo calculation and took into account most of the major effects listed. We also

TABLE III. Proton-pion fractional moments when nuclear corrections are incorporated via a cascade model and a CPM.

	Proton target	Including nuclear corrections with a cascade model	Including nuclear corrections with a CPM
$Z_{p\pi^+}$	0.0392	0.02604	0.0400
$Z_{p\pi^-}$	0.0239	0.01725	0.0243
$Z_{p\pi^+}/Z_{p\pi^-}$	1.64	1.51	1.646

TABLE IV. Comparison of theoretical predictions for the muon ratio at sea level and for $\theta = 0°$. Also listed are some of the major approximations that went into the various calculations.

	This calculation	Frazer *et al.*, Ref. 1	Garraffo *et al.*, Ref. 2	Yekutieli and Rotter, Ref. 3	Liland and Pilkuhn, Ref. 4	Adair, Ref. 5	Ashley *et al.*, Ref. 6
μ^+/μ^- at $E_\mu \approx 10$ TeV	1.54	1.56	1.38	1.255	1.56	1.53	1.26
Error	0.17	None given	None given	0.03	None given	0.11	0.07–0.11
Nuclear corrections included	Yes	No	Yes	No	Yes	Yes	Yes
Kaon contribution included	Yes	No	No	No	Yes	Yes	Yes
High-energy ISR data used	Yes	No	Yes	Yes	No	Yes	No
Energy dependence included	Yes	No	No	No	Yes	No	Yes

find a reasonable agreement with the calculations of Ref. 4.

As far as we can see, minor improvements on some of the approximations made along the way (such as neglecting pion-kaon interchange, nucleon production by mesons, etc.) will not be sufficient to reconcile the measured and predicted charge ratios. We may, however, notice, if we confine our attention to primaries of energy less than 2 TeV/nucleon ($E_\mu \leq 200$ GeV) that even in this energy range, where both ISR and primary-spectrum data are available, the theoretical predictions are still higher by a factor of 10–12% than the measured values. If this is to be taken as a measure of the uncertainty in present data, the extrapolated values at higher energies would still be too large by about 6–7% which cannot be accounted for. Nevertheless, it would be nice to have high-energy data on both protons and nuclear targets which extend over the whole x and y^{proj} regions so that the errors inherent in the Z's can be reduced and the magnitude of nuclear corrections experimentally estimated. As matters now stand, the discrepancy found reflects on the two quantities to which the charge ratio is most sensitive and which are the ones mainly responsible for the magnitude and direction of the discrepancy, namely, the pion-moment ratio $Z_{p\pi^+}/Z_{p\pi^-}$ ($=\delta_\pi$) and the primary-spectrum n/p ratio. This leads us to propose three major possibilities which may lead to an improvement of the agreement with experiment.

(i) If the cosmic-ray n/p ratio at high energies (above 1 TeV/nucleon) were near 1:4 rather than 1:9, this would reduce both n/p and μ^+/μ^- ratios at sea level by about the right amount (note that the experimental errors on the n/p ratio, as measured below 1 TeV/nucleon, are only about 20%—far below the amount needed to render them consistent with the radical change proposed here). A higher number of neutrons can be carried, for example, in heavy primaries (which should then constitute a much larger fraction of the primary-ray composition), or as Adair has suggested, in deuterons replacing about 20% of the protons.

(ii) Any mechanism that reduces the pion excess. A value of $\delta_\pi \sim 1.45$ is needed to reduce the charge ratio by about 10% relative to its value when the calculated $\delta_\pi \sim 1.67$ is used. Such a mechanism may involve, for example, a breakdown of scaling at very high energies in the beam-fragmentation region—a phenomenon not included in any of the present models. A breakdown in scaling behavior may also affect the n/p ratio through a reduction in the value of Z_{pp}.

(iii) Any source of direct leptons, e.g., a copious production of heavy leptons, or other types of particles which may decay into leptons, at high energies.

All this is particularly interesting in view of the fact that a similar discrepancy was obtained for the cosmic-ray μ/e ratio by Fishbane *et al.*,[23] which could also be accounted for by some or all of the major possibilities listed above.

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[10]Both λ_N and λ_π decrease in fact with the energy as the p-air total cross section increases. However, this yields only a mild effect on the muon ratio, which depends really only on the ratio λ_N/λ_π, so that a similar energy-dependent factor in both λ_N and λ_π tends to cancel out.

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[14]Our results still differ considerably from those quoted in Ref. 3 above, even though the same ISR data were used in both calculations (with just minor differences in the treatment of the high-x regions). Thus we get a pion excess of $(Z_{p\pi^+} - Z_{p\pi^-})/(Z_{p\pi^+} + Z_{p\pi^-}) = 0.2566$ (when Z's computed in terms of x variable are used), as compared to an excess of 0.183 only calculated by Yekutieli and Rotter. This difference (which is crucial to the μ^+/μ^- calculations) can be attributed to different methods of estimation. We believe that it is not a good approximation to use s (the c.m. energy squared) as a constant of integration since it depends in fact on the incident energy which should be integrated over [it is the secondary particle's energy that is being held fixed—see Eq. (3)]. Setting s = constant increases the effect of contributions from the pionization region ($x \approx 0$), which we find are very small. We take our assertion to be further supported by the fact that a similar pion excess is obtained when y^{proj} is integrated over (0.242 vs. 0.2566) and that the contribution of the central region is a relatively minor one.

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