Some strong and electromagnetic meson decays*

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Some strong and electromagnetic decay processes of mesons are studied in the context of Mitra's version of the relativistic quark model involving broken SU(6) × O(3) group structure in conjunction with Schwinger's technique of partial symmetry and the hypothesis of vector-meson dominance. In particular, we calculate the decay rates and the corresponding branching ratios for the processes $(A_1, A_2) \to \gamma\pi$, $\omega\pi\pi$; $B \to \gamma\pi$, $\rho\pi\pi$; $\omega \to 3\pi$; $\eta \to \pi^0\gamma\gamma$, $\pi\pi\gamma$; and $\pi^0 \to 4\gamma$. Comparison of the predicted results with experiment is discussed.

I. INTRODUCTION

In an earlier paper[1] [hereafter referred to as (I)], we studied some two-body electromagnetic decay processes of pseudoscalar and vector mesons, e.g., $P(\pi^0, \eta, X^0) \to 2\gamma$ and $V(\rho, \omega, \phi) \to P(\pi^0, \eta)\gamma$, in addition to some important strong three-body decay modes of X^0, E, and A_1 mesons in the context of Mitra's[2] version of the relativistic quark model with SU(6)× O(3) broken group structure, Schwinger's technique of partial symmetry,[3] and the hypothesis of vector-meson dominance[4] of the electromagnetic interactions of hadrons. In the present investigation we continue the program by examining some additional decay processes in order to assess the true progress of Mitra's formulation of the model in a more comprehensive manner. For this purpose, we calculate the decay rates and the corresponding branching ratios (from the viewpoint of comparison with experiment) of some strong and electromagnetic decay processes involving multibody final states. The strong decay modes studied are the ones corresponding to the processes $(A_1, A_2) \to \omega\pi\pi$, $B \to \rho\pi\pi$, and $\omega \to 3\pi$ while the electromagnetic processes considered are $(A_1, A_2, B) \to \gamma\pi, \eta \to (\pi^0\gamma\gamma, \pi\pi\gamma)$, and $\pi^0 \to 4\gamma$.

The relativistic coupling structures,[2] constructed essentially in a phenomenological way, are based on the idea of single quark transition with the emission of an elementary meson (regarded as a quantum of radiation) corresponding to the basic $\bar{q}qP$ vertex. The form factor multiplying the couplings has the structure[2,5]

$$f_L(\vec{k}^2) = g_L \mu^{-L-1} (\mu/\omega_k)^{L\pm1} (\mu/m_\pi)^{1/2} (2M) , \tag{1}$$

which not only introduces the desired symmetry breaking through physical masses of the hadrons but also gives a better form of parametrization, in that the entire supermultiplet transition $(L \to L\pm 1)$ is governed by a single dimensionless coupling constant g_L.

In Sec. II we discuss the three-body strong decay modes, $(A_1, A_2) \to \omega\pi\pi$, $B \to \rho\pi\pi$, and $\omega \to 3\pi$, assuming the decay amplitude to be dominated by the vector mesons, ρ and ω, in the intermediate channels. Section III is concerned with the evaluation of the very rare decay $\pi^0 \to 4\gamma$ using partial symmetry and vector-meson dominance. Other electromagnetic decay processes considered are $\eta \to \pi^0\gamma\gamma$, $\pi\pi\gamma$, and $(A_1, A_2, B) \to \gamma\pi$. Conclusions of the results are discussed in Sec. IV.

II. STRONG DECAYS

First, we apply the model to the study of the three-body strong decay modes of mesons. The couplings necessary for the evaluation of these decay processes have been worked out by Mitra,[2] and we shall essentially follow his formalism to calculate the decay rates and the branching ratios. Recent measurements[6,7] on the $\omega\pi\pi$ decay mode of A_2 have generated considerable interest in this process. The $A_2(1320)$ meson has been well established as a constituent member of the tensor multiplet ($J^{PC} = 2^{++}$) with $I = 1$ and belongs to the $L = 1$ in the supermultiplet classification scheme. For our purpose, we shall assume that the properties of the $A_2(1320)$ will be those derived from a single resonance, the mass spectra being well described by a Breit-Wigner shape. In the framework of the model, the decay process $A_2 \to \omega\pi\pi$ is assumed to proceed via a $\rho\pi$ mechanism such that

$$A_2 \to \rho\pi \to \omega\pi\pi .$$

This involves the $A_2\rho\pi$ and $\omega\rho\pi$ vertices given respectively by[2]

$$A_2\rho\pi: \quad \frac{1}{m_{A_2}} \partial_\sigma A_{\mu\mu'} k_{\mu'} \epsilon_{\mu\nu\lambda\sigma} k_\nu \rho_\lambda \pi , \tag{2}$$

$$\omega\rho\pi: \quad \frac{1}{m_\omega} \partial_\mu \epsilon_{\mu\nu\lambda\sigma} \omega_\nu \pi_\lambda \partial_\sigma \pi . \tag{3}$$

It is now a matter of straightforward application of (2) and (3) with appropriate SU(3) coefficients and form factors to calculate the decay width for the process $A_2 \to \omega\pi\pi$. We make use of the covariant phase-space formalism suggested by Kumar[8] for three-particle final states and operative on the invariant T-matrix element squared dominated by the ρ-meson pole. The resulting expression for the decay width can be cast in the form

$$\Gamma(A_2 \to \omega\pi\pi) = \left(\frac{C^2}{4\pi}\right)\left(\frac{g_0^2}{4\pi}\right)\left(\frac{g_1^2}{4\pi}\right) m_\pi^{-4} m_{A_2}^{-3} I_p \ , \quad (4)$$

where C is the over-all SU(3) coefficient and the coupling constants g_0 and g_1 are given by[2]

$$\frac{g_0^2}{4\pi} = 0.03, \ \frac{g_1^2}{4\pi} = 0.08 \ . \quad (5)$$

The quantity I_p is a phase-space integral given explicitly by

$$I_p = \int_{(m_\pi + m_\omega)^2}^{(m_{A_2} - m_\pi)^2} \frac{ds}{s} \frac{[-4m_\pi^2 s + (m_{A_2}^2 - s - m_\pi^2)^2][-4m_\pi^2 m_\omega^2 + (s - m_\omega^2 - m_\pi^2)^2]}{(s - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2}$$
$$\times \lambda^{1/2}(s, m_\pi^2, m_\omega^2)\lambda^{1/2}(m_{A_2}^2, s, m_\pi^2) \ , \quad (6)$$

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$.

Since the range of integration in (6) lies well above $s = m_\rho^2$, the result is quite insensitive to Γ_ρ (or any reasonable energy-dependent parametrization of Γ_ρ). We find

$$\Gamma(A_2 \to \omega\pi\pi) = 35.2 \text{ MeV} \ , \quad (7)$$

and with $\Gamma(A_2 \to \rho\pi) = 82$ MeV,[9] a branching ratio

$$R = \Gamma(A_2 \to \omega\pi\pi)/\Gamma(A_2 \to \rho\pi) = 0.43 \ . \quad (8)$$

This is in fair agreement (to within a factor of about 1.5) with the experimental value

$$R = \begin{cases} 0.28 \pm 0.09: & \text{Diaz } et\ al.,^6 \\ 0.29 \pm 0.08: & \text{Karshon } et\ al.^7 \end{cases}$$

We also briefly report on the $A_1 \to \omega\pi\pi$ and $B \to \rho\pi\pi$ decay processes. The $A_1(1100)$ meson is supposed to belong to the $J^{PC} = 1^{++}$ nonet and is taken to be an $L = 1$ excitation in the supermultiplet scheme. However, there are doubts about its interpretation as a resonance, and the enhancement appears to occur mainly in the $\rho\pi$ channel. Assuming this to be a resonant state, the $\omega\pi\pi$ final state is achieved via a $\rho\pi$ intermediate state, $A_1 \to \rho\pi \to \omega\pi\pi$, involving $A_1\rho\pi$ and $\omega\rho\pi$ vertices. The $A_1\rho\pi$ vertex[2]

$$A_1\rho\pi: \ \frac{1}{\sqrt{2}} (k_\mu \rho^\mu k_\nu + \mu^2 \rho_\nu) A^\nu \pi \ , \quad (9)$$

in conjunction with the $\omega\rho\pi$ vertex given by Eq. (3), is used to determine the decay rate $A_1 \to \omega\pi\pi$. The resulting decay width turns out to be

$$\Gamma(A_1 \to \omega\pi\pi) = 21.6 \text{ MeV} \ , \quad (10)$$

so that[9]

$$\frac{\Gamma(A_1 \to \omega\pi\pi)}{\Gamma(A_1 \to \rho\pi)} = 0.33 \ . \quad (11)$$

The decay of the $B(1220)$ meson into the $\rho\pi\pi$ system can be computed in a similar way. The state $B(1220)$ with $J^{PC} = 1^{+-}$ seems to be a well-established resonance with $L = 1$ excitation. The $\rho\pi\pi$ final state is obtained via an $\omega\pi$ channel such that $B \to \omega\pi \to \rho\pi\pi$. We need the vertices $\omega\rho\pi$ [given by Eq. (3)] and $B\omega\pi$ given by[2]

$$B\omega\pi: \ B_\nu k_\mu \omega^\mu k^\nu \pi \quad (12)$$

to evaluate this decay rate. We find

$$\Gamma(B \to \rho\pi\pi) = 18.3 \text{ MeV} \quad (13)$$

and the branching ratio[9]

$$\frac{\Gamma(B \to \rho\pi\pi)}{\Gamma(B \to \omega\pi)} = 0.14 \ . \quad (14)$$

The predictions (11) and (14) await experimental confirmation.

Yet another process of interest is the $\omega \to 3\pi$ decay which is supposed to proceed via a $\rho\pi$ state: $\omega \to \rho\pi \to \pi\pi\pi$. The vertices needed are $\omega\rho\pi$ [given by Eq. (3)] and $\rho\pi\pi$ given by[2]

$$\rho\pi\pi: \ i\epsilon_{abc}\rho_\mu^a \pi^b \partial^\mu \pi^c \ . \quad (15)$$

The decay width for $\omega \to 3\pi$ is then found to be

$$\Gamma(\omega \to 3\pi) = 12.7 \text{ MeV} \ , \quad (16)$$

in good agreement with the experimental value[10] and indicates an improvement over the corresponding value of about 15 MeV obtained by Von Royen and Weisskopf.[11]

III. ELECTROMAGNETIC DECAYS

Now we apply the model prescriptions to the study of some rare electromagnetic decay processes of mesons. As is well known, the decay of the neutral pion into photons is an outstanding process from the theoretical point of view. While the electromagnetic decay of π^0 into a pair of photons, e.g., $\pi^0 \to 2\gamma$, has been studied in sufficient detail in (I), we shall now be concerned with the four-body decay process $\pi^0 \to 4\gamma$. Even though this extremely rare radiative decay is not forbidden by any known laws of interactions, no experimental determination for it seems to have been reported earlier in the literature. Recently, Abrams and collaborators[12] have searched for this decay using the process $K^\pm \to \pi^\pm \pi^0$ as a source of kinematically defined neutral pions. The result of this investigation is to place an upper limit on the branching ratio:

$$R = \frac{\Gamma(\pi^0 \to 4\gamma)}{\Gamma(\pi^0 \to 2\gamma)} < 6.1 \times 10^{-5} \text{ (90\% C. L.)}.$$

Recently Schult and Young[13] have attempted to calculate this decay rate and the corresponding branching ratio by using several theoretical models. These authors find typical values of R in the range from 4×10^{-14} to 10^{-16}, which is obviously many orders of magnitude below the experimental value and is also in sharp contrast even to the crudest estimate for $R \sim \alpha^2 \sim 10^{-5}$.

In this investigation, we attempt to calculate the decay rate for the process $\pi^0 \to 4\gamma$ and estimate the corresponding branching ratio $R = \Gamma(\pi^0 \to 4\gamma)/\Gamma(\pi^0 \to 2\gamma)$ within the general prescriptions of the relativistic quark model by making extensive use of the techniques of partial symmetry[3] and the vector-dominance hypothesis.[4] In accordance with this formulation, the four-body electromagnetic decay process $\pi^0 \to 4\gamma$ is regarded as proceeding via vector mesons ρ, ω, and ϕ in the intermediate state. A schematic diagram indicating the various stages of the process is depicted in Fig. 1. Here V and V' represent the ρ and ω mesons, respectively, while P stands for either π^0, η, or X^0 mesons. It should, however, be noted that we include only the π^0 meson (and neglect η or X^0 mesons), which is expected to yield maximum contribution due to its smaller mass.

In order to construct the model amplitude for this process, the radiative coupling structures

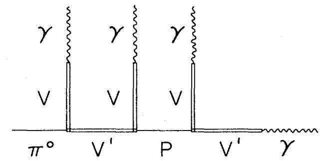

FIG. 1. A typical diagram for the $\pi^0 \to 4\gamma$ decay process. Here V and V' represent the vector mesons (ρ or ω), and P stands for a pseudoscalar meson (π^0, η, or X^0).

needed are those governing vertices of the type $V\gamma P$ and $\gamma\gamma P$, which can be easily obtained from the VVP coupling by successively replacing the vector-meson fields (V_μ) by the corresponding photon field (A_μ) in accordance with vector dominance. The details for the evaluation of these effective electromagnetic couplings have already been worked out in (I).

Application of these coupling structures enables us to calculate the decay width for the process $\pi^0 \to 4\gamma$. For this purpose, we make use of the covariant phase-space formalism suggested by Kumar[8] for four-body final states and operative on the invariant T-matrix element squared. The relevant T-matrix element squared, dominated by ω and π poles, can be written in the form

$$|T|^2 = \beta \frac{[s_2(s_1 - s_2)(m_\pi{}^2 - s_1)]^2}{[(m_\omega{}^2 - s_1)^2 + m_\omega{}^2\Gamma_\omega{}^2][(m_\pi{}^2 - s_2)^2 + m_\pi{}^2\Gamma_\pi{}^2]}, \tag{17}$$

where β is a kinematical factor given explicitly by

$$\beta = \frac{2(32\pi)^3(g_0{}^2/4\pi)^3(e^2/4\pi)^4}{(g_{\rho\pi\pi}{}^2/4\pi)^4 m_\pi{}^6}, \tag{18}$$

with

$$e^2/4\pi = \alpha \simeq 1/137 \text{ and } g_{\rho\pi\pi}{}^2/4\pi = 3.6. \tag{19}$$

The quantities s_1 and s_2 are two Mandelstam-type variables explained in Ref. 8. The resulting expression for the decay width $\Gamma(\pi^0 \to 4\gamma)$ can be written in the form

$$\Gamma(\pi^0 \to 4\gamma) = A \int_0^{m_\pi{}^2} \frac{ds_1}{s_1} \frac{(m_\pi{}^2 - s_1)^3}{[(m_\omega{}^2 - s_1)^2 + m_\omega{}^2\Gamma_\omega{}^2]} \int_0^{s_1} ds_2 \frac{s_2{}^2(s_1 - s_2)^3}{[(m_\pi{}^2 - s_2)^2 + m_\pi{}^2\Gamma_\pi{}^2]}, \tag{20}$$

where A is another factor having the value

$$A = (8\times10^{-10}) m_\pi{}^{-9}. \tag{21}$$

The phase-space integration over the variables s_1 and s_2 is computed numerically. As a result of our calculation, the decay width for $\pi^0 \to 4\gamma$ turns out to be

$$\Gamma(\pi^0 \to 4\gamma) = 6.6\times10^{-15} \text{ MeV}. \tag{22}$$

In order to make comparison of this result with experiment, we need information on the two-photon decay of the neutral pion. As reported in (I), the decay width for the process $\pi^0 \to 2\gamma$ is

$$\Gamma(\pi^0 \to 2\gamma) = 7.4\times10^{-6} \text{ MeV}, \tag{23}$$

in close agreement with the experimental value[10] of $(7.8 \pm 0.9)\times 10^{-6}$ MeV. This immediately leads to the branching ratio

$$R = \frac{\Gamma(\pi^0 \to 4\gamma)}{\Gamma(\pi^0 \to 2\gamma)} = 0.9\times 10^{-9}, \qquad (24)$$

which is roughly five orders of magnitude higher than the corresponding value (in the range 4×10^{-14} to 10^{-16}) obtained by Schult and Young. It is, however, still four orders of magnitude below the experimental upper limit and the naive α^2 estimate. Such a rate suppression is accounted for by the low-energy theorem and the presence of angular momentum barriers. The difference with the results of Ref. 13 may be understood in terms of the difference in assumed coupling structures. Schult and Young impose the low-energy theorem by explicitly including factors of the final photon four-momenta, thus guaranteeing that the amplitude vanishes when any one of these four-momenta is zero. The amplitude of Eq. (17) also has this property, but without the need for explicit factors of photon four-momenta, for, when $k_1 = 0$, s_1 becomes just $m_\pi{}^2$; when $k_2 = 0$, s_1 is equal to s_2; and when either k_3 or $k_4 = 0$, s_2 vanishes. Equation (17) is also consistent with partially conserved axial-vector current (PCAC), if $m_\pi{}^2$ is recognized as the square of the pion four-momentum. When the latter vanishes, $m_\pi{}^2$ is replaced by zero, and s_1 also vanishes.

Encouraged by the performance of the model, we take a brief look at the $\pi^0\gamma\gamma$ and $\pi\pi\gamma$ decay modes of the η meson. Using the coupling structures outlined in (I), we find

$$\Gamma(\eta \to \pi^0\gamma\gamma) = 0.31 \text{ eV}. \qquad (25)$$

If we compare this with $\Gamma(\eta \to \gamma\gamma)$ calculated in (I), we get

$$R = \frac{\Gamma(\eta \to \pi^0\gamma\gamma)}{\Gamma(\eta \to \gamma\gamma)} = 1.1\times 10^{-3}, \qquad (26)$$

in comparison with the experimental value[10] of 0.05 to 0.10. This is in disagreement with our calculation, though the experimental situation is not very clear. We note that our result for this ratio is about two times higher than that obtained by Von Royen and Weisskopf.[11]

The process $\eta \to \pi\pi\gamma$ is analogous to the $\eta \to \pi\gamma\gamma$ process. The decay width for this mode turns out to be

$$\Gamma(\eta \to \pi\pi\gamma) = 0.09 \text{ keV}, \qquad (27)$$

so that the branching ratio

$$\frac{\Gamma(\eta \to \pi\pi\gamma)}{\Gamma(\eta \to \gamma\gamma)} = 0.32, \qquad (28)$$

in comparison with the experimental value of about 0.13. Since the relative strength of the $\eta \to \pi\pi\gamma$ process is still not definitely established by experiments, our result does not seem to be unreasonable.

Finally, for completeness, we consider the $\gamma\pi$ modes of the A_1, A_2, and B mesons. The technique of computing these processes is essentially the same as for the other radiative decays, in that the ρ and ω fields in the coupling structures (2), (9), and (12) are replaced by the corresponding photon field in the spirit of vector-meson dominance. The resulting widths are predicted to be

$$\Gamma((A_1, A_2, B) \to \gamma\pi) = (0.76,\ 0.81,\ 0.61) \text{ MeV}, \qquad (29)$$

respectively, in good agreement with the calculations of Berger and Feld.[14] While the predictions for $(A_1, B) \to \gamma\pi$ await experimental confirmation, the decay width for $A_2 \to \gamma\pi$ compares reasonably well with the experimental value of about 0.5 MeV measured by Eisenberg *et al.*[15]

IV. CONCLUSIONS

In this paper we have attempted to investigate some rare strong and electromagnetic decay processes of mesons using Mitra's formulation of the relativistic quark model. We find that our calculations of the decay rates for these processes are in reasonably good agreement with the experimental data. It may be stressed at this point that this simple quark model of mesons ($q\bar{q}$) and baryons (qqq) structure provides a single consistent way of calculating a large number of decay processes.[1,2,16] The elegance of the present approach is reflected in the model's simplicity and compactness which, unlike the SU(6), serves as a unifying basis for treating the meson and baryon couplings on the same footing (based on the idea of single-quark transition).

The formulation of Mitra gives us a consistent and coherent description of meson decays in terms of a composite particle model in which the decays proceed by single-quark transitions accompanied by radiation of single-meson quanta. Whether or not one believes in this picture, the model provides a phenomenological prescription that incorporates the essential features of SU(6)×O(3) symmetry and gives symmetry breaking through unambiguously defined form factors. The result is a scheme in which a large number of decay processes can be computed in terms of a few phenomenological coupling constants. The present paper, along with Ref. 1, extends the application of the model to multiparticle final states by considering these as due to a succession of single quantum emissions. Electromagnetic decays are easily incorporated by using the ideas of vector dominance. The results are very encouraging; considering the small number of free parameters, the general patterns of the whole range of meson decays treated by this scheme to date are strikingly similar to the experimentally observed patterns.

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