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Proton-antiproton annihilations into four kaons just above threshold*

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Cross-section limits are presented for proton-antiproton annihilation into all possible four-kaon final states in the region from threshold to 1.1 GeV/c lab momentum. Comparisons are made with cross sections for other four-body final states. It is shown that, based on phase-space considerations, the four-neutral-kaon final state manifests an anomalously high relative rate, suggesting the possible influence of a threshold enhancement.

I. INTRODUCTION

No other low-energy elementary-particle collision process is as rich in allowed final states and as varied in possible observable phenomena as proton-antiproton annihilation. Although the present ability of particle theory to explain these phenomena leaves much to be desired, it is nevertheless of considerable potential theoretical interest and a substantial experimental challenge to explore the full range of allowed reactions. This paper reports results of a search for some very rare processes, the annihilation into four kaons, in the energy region just above threshold for these final states. In particular the reactions sought are

$$\bar{p}p \to K^+K^-K^+K^- , \quad (1)$$

$$\bar{p}p \to K^+K^-K^0\bar{K}^0 , \quad (2)$$

$$\bar{p}p \to K^0\bar{K}^0K^0\bar{K}^0 . \quad (3)$$

We know of only one other experimental observation of the four-kaon final states.[1] Although our search yielded only four events [all in final states corresponding to reaction (3)] some interesting cross-section limits can be assigned to these processes, as discussed below. In fact, despite the limited statistics, it appears possible to draw the conclusion that a threshold enhancement may be contributing significantly to the rate for the all-neutral final state. It is conceivable that this phenomenon is related to the S^* enhancement in the K_SK_S system, which has been reported several times.[2]

II. DATA ANALYSIS

The results reported here are based on a study of 220 000 pictures of antiprotons incident on the BNL 30-in. hydrogen bubble chamber at six distinct beam momenta; interactions span the laboratory momentum range from approximately 0.67 to 1.12 GeV/c, corresponding to center-of-mass energy from 1.98 to 2.12 GeV. [The thresholds for reactions (1)–(3) range from $E_{\text{c.m.}} = 1.975$ to 1.991 GeV.] Events were analyzed with the programs TVGP and SQUAW. A detailed study of scanning-processing efficiencies for various topologies was made, and a beam track count was used to normalize cross sections. The approximate sensitivity for the entire exposure was ~0.81 μb/event for the 4-prong events, and ~0.60 μb/event for events with V's.

The reactions (1)–(3) may be manifested in a wide variety of topologies, including 4-prong, 2-prong plus 1 or 2 V's, and 0-prong plus 1, 2, 3, or 4 V's. All these topologies were examined in accumulation of data for this report.

A. $K^+K^-K^+K^-$

In approximately 100 000 4-prong events analyzed, no events were found to fit the above charged kaon hypothesis with confidence level $> 10^{-4}$. This permits assignment of an upper limit on the average cross section over the region from threshold to ~1.1 GeV/c, $\sigma(K^+K^-K^+K^-) < 1.9\ \mu$b (90% confidence level). This result is listed in Table I, together with results from other channels discussed below.

B. $K^+K^-K^0\bar{K}^0$

In 2958 and 380 events containing 2 prongs plus 1 and 2 V's, respectively, we found no events fitting this hypothesis. This yields individual limits on the cross sections for $K^+K^-K_SK_S$ and $K^+K^-K_SK_L$, as well as on the total $K^+K^-K^0\bar{K}^0$ cross section, as given in Table I. (These results contain corrections for missing neutral decays, as well as the assumption of invariance of strong processes under interchange of K_S and K_L.)

C. $K^0\bar{K}^0K^0\bar{K}^0$

The exposure yielded 449 and 142 events containing 0 prongs plus 1 and 2 V's, respectively. A previous analysis of these events, resulting in data on the $K^0\bar{K}^0$ final state, has already been published.[3] An examination of the missing mass in the 1V events reveals several possible candidates for the four-neutral-kaon hypothesis [in particular, the events with missing mass $\geq 3m_K$ at the

TABLE I. Cross-section limits for various final states. All limits are 90% confidence level.

Final state	Average cross section (in μb) from threshold to $E_{\text{c.m.}} = 2.12$ GeV
$K^+K^-K^+K^-$	<1.9
K^+K^- 1V	<1.4
K^+K^- 2V	<1.4
$K^+K^-K_SK_S$	<1.5
$K^+K^-K_SK_L$	<2.0
$K^+K^-K^0\bar{K}^0$ (all modes)	<4.3
0 prongs + 2V	>1.0, <4.0
0 prongs + 3V	>0.3, <2.3
0 prongs + 4V	<1.4
$K_SK_SK_SK_S$	>0.1, <2.8
$K_SK_SK_SK_L$	>0.3, <4.1
$K_SK_SK_LK_L$	>0.5, <6.5
$4K^0$ (all modes)	>5.4, <14.7
$4K^0$ (all modes, if uncorrelated)	>3.4, <11.2
$4K^0$ (all modes, if pure $C = +1$)	>4.1, <11.9
$4K^0$ (all modes, if pure $C = -1$)	>3.8, <12.5
$4K$ (all charges)	>5.4, <18.3

upper end of the histogram in Fig. 1(a) of Ref. 3]. However, since it is not possible to resolve the identification of the final state with any degree of confidence, we exclude the 1V events from further consideration.

In the 2V events, an examination of the missing mass and of the mass recoiling against each V yields three possible candidates for a four-kaon final state. [These can be seen as the three clustered events in the upper right-hand corner of the scatter plot in Fig. 1(b) of Ref. 3.] Again an unambiguous identification of the final state cannot be made, but the following analysis provides virtually conclusive evidence that these events indeed contain four kaons.

After elimination of events fitting $K^0\bar{K}^0$ and $K^0\bar{K}^0\pi^0$ we are left with 50 unidentified 2V events. In Fig. 1 we plot for these events the sum of the squares of the missing mass and the masses recoiling against each V. For a four-kaon event this quantity, M^2, must be $> 22 m_K^2$. Clearly, events involving a pair of kaons and several neutral pions may also satisfy this condition. Hence we fit this distribution to that expected, according to phase space, from the channels: $4K^0$, and $2K^0$ plus 2, 3, 4, 5, 6, and 7 π^0. (Except for our two highest momenta, we are below threshold for $8\pi^0$. Furthermore, the minimum allowed value of M^2 for this configuration exceeds the observed values by several times the error in the quantity.)

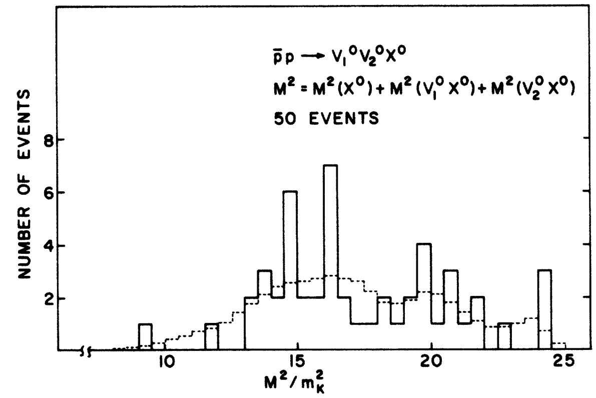

FIG. 1. Histogram of the sum of the squares of the missing mass and the mass recoiling against each V, in 0-prong plus 2V events. Events fitting $2K^0$ and $2K^0\pi^0$ have been eliminated. The dashed histogram shows result of a 6-parameter fit, based on phase space, to the channels $4K^0$ and $2K^0$ plus 2, 3, 4, 5, 6, and 7 π^0, as described in the text.

The number of events in each channel is left free (subject to the constraint of the total number of events) to be determined by the fit. Table II gives the fitted number of events for a 6-parameter fit. We find that the resultant number of events in the $2K^0 7\pi^0$ channel is effectively zero. Whether or not this channel is included (i.e., whether we fit to 6 or 5 free parameters) the fitted result for the $4K^0$ channel is 3.0 ± 1.9 events. These three events are clearly seen at the very end of the spectrum. The obvious advantage of plotting this unusual combination of masses is to achieve maximum separation of these events from the background continuum. We note, for example, that there is a fourth event in this plot with $M^2 > 22 m_K^2$ which is not in the end cluster. The fit does not clearly recognize this as a four-kaon event and it is not accepted as such.

The dashed histogram in Fig. 1 shows the result of the fit and exhibits a small high-mass bump of area ~3 events, corresponding to the $4K^0$ contribution, which, however, does not coincide closely with the actual observed events. This is because the phase-space distribution used in the calculation was weighted according to flux at each of the beam momenta of the exposure, whereas all the observed $4K^0$ events occurred only at the higher momenta ($E_{\text{c.m.}} \geq 2.06$ GeV) with correspondingly higher upper limit for the quantity M^2.

One 3V event was found (at $E_{\text{c.m.}} = 2.12$ GeV) which fit the four-kaon hypothesis with a confidence level >10%. (It should be noted that this identification is unambiguous in any case, since there is insufficient energy to produce an additional pion.) No 4V events were detected. Consideration of the contributions of different possible final states to the various topologies results in the values given in Table I. The statistical method used involved construction of a multidimensional likelihood function of the parameters, i.e., cross sections, to be estimated, and then integration of this function to yield marginal distributions in each of the relevant cross sections. The confidence limits were obtained from the latter distributions. Although this technique is not without ambiguity (particularly with regard to the *a priori* probabilities, or "noninformative priors", chosen), it was felt to be sufficient and adequate for the present case. Results presented here, based on a uniform prior distribution, are essentially unaffected by use of other rules, as discussed, for example, in Ref. 4.

TABLE II. Number of events obtained in various channels from fitting the effective mass histogram of Fig. 1 with a phase-space model.

Channel	Fitted number of events
$4K^0$	3.0 ± 1.9
$2K^0$ plus $2\pi^0$	8.5 ± 8.1
$2K^0$ plus $3\pi^0$	14.9 ± 16.1
$2K^0$ plus $4\pi^0$	14.8 ± 13.4
$2K^0$ plus $5\pi^0$	0.0 ± 3.9
$2K^0$ plus $6\pi^0$	8.9 ± 4.3
$2K^0$ plus $7\pi^0$	0.0 ± 2.1

Owing to the variety of possible neutral final states, the limits obtained are relatively loose. It might be expected that more stringent limits can be obtained if special hypotheses are made concerning these final states. Three such examples are given in Table I. The first assumes all neutral kaons are uncorrelated, i.e., each has a 50% probability of being K_S or K_L. The second and third cases assume annihilation proceeds through pure $C = +1$ and -1 channels, respectively. It is observed that the resulting limits are relatively unaffected by these hypotheses.

III. DISCUSSION OF RESULTS

It may be expected that the 4K cross sections rise rapidly in the energy region just above threshold; the phase space increases $\sim (E_{\text{c.m.}} - E_{\text{c.m.}}^{\text{threshold}})^{3.5}$ (as illustrated by the curve labeled "phase space" in Fig. 2). Hence it is of interest to specify cross-section values at the upper end of the region. Approximately one-half of the beam flux above threshold used for this study was taken in the center-of-mass energy interval 2.06 to 2.12 GeV; consequently cross sections for this region may be obtained by multiplying those in the table by a factor ~2. Our result for the $4K^0$ final state averaged over the entire energy interval is $\approx 8^{+4}_{-3}$ μb, whereas averaged over the upper half of this region (where in fact all observed events occurred) it is $\approx 16^{+8}_{-6}$ μb.

A. The relative rate for 4*K*

The $\bar{p}p$ annihilation cross section in the region studied is ~70 mb. Hence the total 4K cross section given in Table I implies a relative rate for these final states of approximately 10^{-4}. This may be compared with the relative rate ~0.1 for annihilation to all strange-particle final states.[5]

B. Comparison with a previous experiment

Chapman *et al.*[1] report cross sections of 4 μb for $K^+K^-K_SK_S$ at 2.2 GeV/c ($E_{\text{c.m.}} = 2.50$ GeV) based on one 2V event, and 6.5 ± 2.5 μb for $K^+K^-K_SK_L$, based on a sample of 17 events fitting this hypothesis in the region 1.6 to 2.2 GeV/c ($E_{\text{c.m.}} = 2.29$ to 2.50 GeV). Both of these values are somewhat larger than our upper limits for these channels near threshold. They also report a single 3V event, but no indication is given whether this is consistent with a four-neutral-kaon hypothesis. Based on their reported sensitivity of ~3.8 events/μb for the entire exposure, absence of any 3V events of this type would imply an upper limit of ~0.6 μb for $\sigma(3V)$ compared with our lower limit ~0.3 μb near threshold.

This group also reports an average cross section of 3.8 μb for $K^+K^-K^+K^-$, based on six events in the region 1.6 to 2.2 GeV/c.

C. Comparison with other final states

Cross-section measurements of other specific four-body final states are available in our energy region from this exposure. In particular, the average value of $\sigma(\pi^+\pi^-\pi^+\pi^-)$ in this region is ≈3.6 mb,[6] and the average value of $\sigma(K^+K^-\pi^+\pi^-)$ is ≈0.5 mb.[7] There is very copious resonance production observed in both of these final states, amounting to approximately 75% in each case. A striking fact is that the observed relative rate for the two is very nearly equal to the ratio of the corresponding total Lorentz-invariant phase-space volumes. The values of total phase space for various final states, relative to the four-pion phase space taken as unity, are listed in Table III. (The phase-space entries in Table III are the total Lorentz-invariant phase-space integrals for the relevant mass combinations, averaged over the indicated energy range and normalized to the four-pion state. No corrections have been made for Bose-Einstein symmetry effects, and no isotopic spin weight factors are included. One method of dealing with the presence of identical bosons is to include a factor of $\frac{1}{2}$ for each pair of identical particles. This would have the effect of decreasing the values for $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-K^+K^-$, and $K^0\overline{K}{}^0K^0\overline{K}{}^0$ by a factor of 4 relative to the other two final states. As a consequence the phase-space ratio of $K^+K^-\pi^+\pi^-$ relative to $\pi^+\pi^-\pi^+\pi^-$ would be approximately four times larger than the experimentally

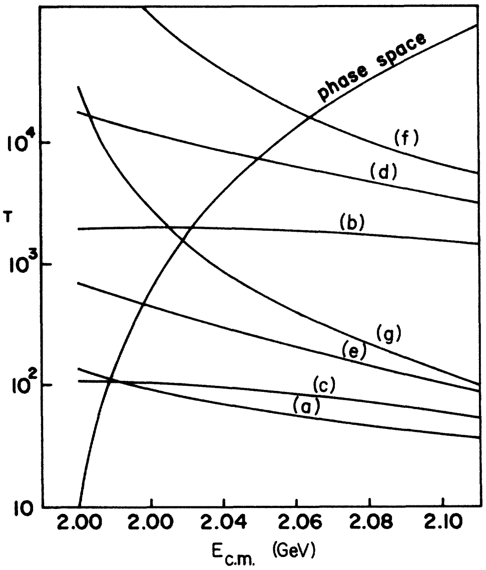

FIG. 2. The total enhancement ϵ_T as a function of $E_{\text{c.m.}}$. Curve (a) gives the result for effective range formula with $a = 1.3$ F, $r_e = 1.4$ F. Curves (b) through (g) show results for Breit-Wigner resonance with parameters as follows (E_R, Γ in GeV, R in F). Curves (b) and (c): $E_R = 1.08$, $\Gamma = 0.2$, $R = 0.2$, 0.5 respectively; curves (d) and (e): $E_R = 1.05$, $\Gamma = 0.2$, $R = 0.2$, 0.5 respectively; curves (f) and (g): $E_R = 1.00$, $\Gamma = 0.05$, $R = 0.2$, 0.5, respectively. Also shown, for comparison, is the dependence of total phase space on $E_{\text{c.m.}}$ (normalization arbitrary).

TABLE III. Phase space relative to 4π state and "predicted" cross sections.

Final state	Average total phase space[a] relative to 4π state $E_{\text{c.m.}}$ (GeV) 2.00–2.12	2.06–2.12	"Predicted" cross section[b] (μb) scaled according to phase space $E_{\text{c.m.}}$ (GeV) 2.00–2.12	2.06–2.12
$\pi^+\pi^-\pi^+\pi^-$	1	1		
$K^+K^-\pi^+\pi^-$	0.14	0.15	130	140
$K^+K^-K^+K^-$	1.6×10^{-4}	2.7×10^{-4}	0.14	0.24
$K^+K^-K^0\overline{K}{}^0$	1.2×10^{-4}	2.1×10^{-4}	0.11	0.19
$K^0\overline{K}{}^0K^0\overline{K}{}^0$	9.3×10^{-5}	1.6×10^{-4}	0.08	0.14

[a] See text for a description of the phase space as used here.

[b] Based on nonresonant 4π cross section. Typical uncertainties ≲20%.

observed ratio. None of the other conclusions drawn in this paper would be affected, however, by such Bose-Einstein considerations. The ignored isotopic spin weight factors could imply modifications, but only by factors of order unity. Another method of treating the Bose-Einstein effects, in the context of the statistical model, is discussed in Ref. 8.) Values are given for the average over the region $2.00 < E_{\text{c.m.}} < 2.12$ GeV, as well as for the upper half of this region. Also given in Table III are predicted cross-section values obtained by scaling the average nonresonant $\pi^+\pi^-\pi^+\pi^-$ cross sections[6] according to phase space. Comparing Table I and Table III we note that, with the exception of the $4K^0$ state, the four-kaon cross-section limits obtained are consistent with expectations from phase space alone.

For the $4K^0$ state the observed rate is approximately 100 times larger than predicted. Although this factor is quite rough (i.e., the statistical error is large and there are perhaps ambiguities of factors of order unity associated with particulars such as isotopic spin weight factors), nevertheless the conclusion is strongly suggested that some type of resonantlike enhancement is manifested in the four-neutral-kaon rate. This conclusion is particularly compelling if one notes that states with a high multiplicity of strange particles are usually significantly suppressed with respect to nonstrange particle states.

IV. THE ANOMALOUS $4K^0$ RATE

One possible explanation of the seemingly high rate for the four-neutral-kaon final state may be the well-known threshold enhancement observed in the K_SK_S system, the S^*. Although the properties of this phenomenon are still not very well defined and its origin is poorly understood, it appears well established by several observations.[2] In particular, present evidence is divided between a threshold effect caused by a large scattering length, or a resonant enhancement somewhat above threshold ($1.04 \leq M_{K_SK_S} \leq 1.10$ GeV, $0.15 \leq \Gamma \leq 0.30$ GeV). [The parameters quoted above from Ref. 2 are based on fits to the K_SK_S mass distribution in $\pi^-p \to K_SK_Sn$. Another analysis,[9] using a coupled-channel formalism to study $\pi\pi$ and $K\bar{K}$ production in π^+p experiments, yields substantially different results, i.e., a pole almost precisely at $K\bar{K}$ threshold (~0.996 GeV) with a relatively small imaginary part corresponding to $\Gamma \sim 0.055$ GeV. Evidence for a narrow K^+K^- enhancement shows up in the latter experiment. The discrepancy between fitted parameters, however, would appear to leave unresolved the question of whether the two experiments, both nominally dealing with the S^*, are in fact looking at the same phenomenon.] Either of these interpretations is qualitatively consistent with the phenomenon observed here. If this enhancement were predominantly in K_SK_S (and K_LK_L), but not in the K^+K^- system, it would explain the absence of any anomalous rate in $K^+K^-K^+K^-$. The appearance of a high rate in $K^0\bar{K}^0K^0\bar{K}^0$ but not in $K^+K^-K^0\bar{K}^0$ could then be attributed to the occurrence of the enhancement factor squared in the former process, and this may be further multiplied due to the several permutations of K_SK_S pairs in this state.

A quantitative treatment of the enhancement is less obvious, but in principle the theory of final-state interactions provides a basis.

A. Implications of final-state interactions

Most treatments in the literature of enhancements due to final-state interactions[10,11] have dwelt principally on applications to energy dependences of processes, or resultant deviations in the shapes of distributions, rather than enhancements in absolute rate. Nevertheless, theory provides an expression for an absolute enhancement due to an interaction between two particles in the final state, which is determined by the value of the scattering wave function at zero separation.[12]

In particular, the enhancement factor is given by

$$\epsilon(k) = \left| \frac{\psi_k^{(-)}(r)}{\psi_k^{\text{free}}(r)} \right|^2_{r=0}, \tag{4}$$

where $\psi_k^{(-)}$ is the solution of the two-body scattering problem with wave-vector k, and ψ_k^{free} is the corresponding wave function in the absence of interaction.

This result presupposes, among other things, a division of the dynamics into two separate interactions, one responsible for the primary production of the particles and the other for the final-state interaction. Although this separation is somewhat arbitrary in the case where strong interactions account for both parts of the process, it is at least conceptually realizable. In the present case we can visualize an annihilation mechanism leading to a multiparticle final state, followed by rescattering of pairs of final particles. Another implicit assumption is that production of the pair of final particles occurs over a relatively small range, which is presumably satisfied for the case at hand.

Since the features of kaon-kaon scattering are still quite obscure, as indicated above, we satisfy ourselves with a relatively crude treatment to investigate whether an enhancement of the magnitude suggested above can be realized with scattering parameters of reasonable dimensions. Toward this end we investigate two different possible situations: (i) a strong attractive force giving rise to a large scattering length in the KK system, and (ii) a KK resonance somewhere above threshold. For simplicity in (ii) we assume an elastic resonance and explore several different sets of resonance parameters. Furthermore, to be specific, we assume the attractive force is in the K_SK_S (and K_LK_L) S-wave system, and calculate over-all enhancements for the $K_SK_SK_LK_L$ final state as a function of $E_{\text{c.m.}}$. Hence the final-state two-body interaction enhancement appears essentially squared.

B. Effective-range hypothesis

It can be shown that the enhancement, Eq. (4), is given by the inverse of the absolute square of the Jost function. As discussed by Gillespie,[11] if the phase shift is given by the effective-range formula

$$k\cot\delta = \frac{1}{a} + \frac{1}{2}r_e k^2 , \tag{5}$$

where a and r_e are the scattering length and effective range, respectively, and if α, β are defined by the equations

$$\frac{r_e}{2} = \frac{1}{\alpha-\beta} , \qquad \frac{1}{a} = \frac{\alpha\beta}{\alpha-\beta} , \tag{6}$$

then for $\alpha, \beta > 0$ the Jost function is given by

$$f(k) = \frac{k-i\beta}{k-i\alpha} \tag{7}$$

and the enhancement by

$$\epsilon(k) = \frac{k^2+\alpha^2}{k^2+\beta^2} . \tag{8}$$

As a special case we take $a = 1.3$ F and $r_e = 1.4$ F. (These are, respectively, the magnitudes of the real part of the scattering length and the effective range, as obtained from fits to K_SK_S data[2,13]; these fits yield a very small value for Ima, which we are then justified in ignoring for our purposes.) Inserting such an enhancement factor for each affected KK pair and integrating over phase space, we obtain a total enhancement ϵ_T over pure phase space as given in curve (a) of Fig. 2.

C. Breit-Wigner resonance hypothesis

In the case of a resonance, enhancement is not necessarily determined by the wave function at zero separation. However, as pointed out by Watson,[10] if the range of the final-state interaction is shorter than that of the primary interaction, then one may conveniently use the asymptotic form of the scattering solution

$$\psi_k^{(-)}(r) \sim \frac{e^{-i\delta}\sin(kr+\delta)}{kr} . \tag{9}$$

A measure of the enhancement can be obtained by evaluating the ratio (4) at an appropriate distance R characteristic of the primary interaction. For the phase shift δ we take a simple Breit-Wigner form,

$$\delta = \tan^{-1}\left[\frac{\Gamma}{2(E_R-E)}\left(\frac{k}{k_R}\right)\right] , \tag{10}$$

where Γ, E_R, and k_R are full-width, energy, and wave-vector at resonance, and the factor k/k_R is included to provide appropriate S-wave threshold behavior. Curves (b) through (g) in Fig. 2 give resulting enhancements as functions of $E_{\text{c.m.}}$ for various selections of the parameters E_R, Γ, and R.

We note that the enhancement is quite sensitive to the characteristic length R, behaving essentially like R^{-4}. Enhancements of the magnitude suggested by the data are readily obtainable for $R \sim 0.5$ F. Regardless of whether the particular selected model and parameters are realistic, it seems clear that enhancement effects of this kind could overcome suppression by phase space to give rise to an essentially constant $4K$ cross section near threshold. The effect of a narrow P-wave ϕ meson ($E_R = 1.020$ GeV, $\Gamma = 0.004$ GeV) was also calculated, and was found to yield enhancements one to two orders of magnitude smaller than the S-wave resonances considered.

V. CONCLUSIONS

Experimental cross-section limits have been obtained for $\bar{p}p$ annihilation to four kaons from threshold to $E_{\text{c.m.}} = 2.1$ GeV; only the neutral-kaon channels show an observable cross section in this region. The limits have been shown to be consistent with expectations based on phase-space considerations and the measured cross sections for other four-body final states, with the exception of the all neutral $4K$ channels which exhibit a rate of approximately ~100 times larger than predicted. An analysis using the theory of final-state interactions shows that such an enhancement may arise from a large KK scattering length or resonance near threshold.

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