$\bar{K}K$ system in $\pi^-p \to K^-K^+n$ at 6 GeV/c*

A. J. Pawlicki, D. S. Ayres, R. Diebold, A. F. Greene,† S. L. Kramer, and A. B. Wicklund
Argonne National Laboratory, Argonne, Illinois 60439
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The reaction $\pi^-p \to K^-K^+n$ has been studied at 6 GeV/c with a 16000 event data sample obtained with the Argonne National Laboratory Effective Mass Spectrometer. Moments of the K^-K^+ decay angular distributions are shown as functions of the K^-K^+ effective mass from threshold to 1500 MeV, and as functions of momentum transfer from t_{min} to -0.4 GeV^2. The t-channel moments $\langle Y_l^m \rangle$ with $m=0$ dominate over the $m=1$ moments, while moments with $m \geq 2$ (and those with $l \geq 6$) are negligible; this indicates that the dominant t-channel exchange production amplitudes have helicity zero at the meson vertex. The slope of $d\sigma/dt$ decreases as the K^-K^+ mass increases. There is a broad D-wave enhancement near 1350 MeV. Interfering f and A_2^0 D-wave Breit-Wigner resonances are not sufficient to explain all of the Y_4^0 distribution above 1350 MeV and an additional D-wave contribution may be present. Ignoring this complication, the best f-A_2^0 fit to the Y_4^0 spectrum yields a branching ratio $(f \to \bar{K}K)/(f \to \text{all}) = (3.1 \pm 1.0)\%$.

I. INTRODUCTION

We report a high statistics measurement of the K^-K^+ decay angular distribution in

$$\pi^-p \to K^-K^+n \quad (1)$$

at 6 GeV/c, for K^-K^+ masses from threshold through the f-A_2^0 region. Previous measurements of the moments for this reaction[1-4] have been limited by low statistics or restricted to low K^-K^+ masses (<1300 MeV). There is considerable interest in precise data for several reasons. In the $\bar{K}K$ threshold region, $\pi\pi$ scattering studies need data on the S wave in $\pi\pi \to \bar{K}K$ for coupled channel analyses.[4] At higher masses, interference effects between f and A_2^0 production amplitudes have been predicted.[5] Consequently, the $f \to \bar{K}K$ branching ratio cannot be reliably determined without separating the f and A_2^0 contributions to the $\bar{K}K$ mass spectrum.[6] Data[7-10] also exist for the related reaction

$$\pi^-p \to K_S^0K_S^0n\,; \quad (2)$$

the combined sample from all previous experiments on reaction (2) is about as large as our 16 000 event sample for reaction (1).

Lipkin[11] has pointed out that if A_0 and A_1 are the amplitudes for pions to produce isospin 0 and isospin 1 $\bar{K}K$ systems, then the amplitude for reaction (1) is given by A_0+A_1, while that for

$$\pi^-p \to \bar{K}^0K^0n \quad (3)$$

is A_0-A_1. Thus, these reactions differ only in the sign of the isospin 0-isospin 1 interference term. Denoting the various t-channel exchange production amplitudes by the symbols for mesons with quantum numbers of the $\bar{K}K$ system, the cross sections for reactions (1) and (3) are given by

$$\frac{d^2\sigma}{dt\,dM}(\bar{K}K)^0 = |S^* \pm \delta^0|^2 + |\phi \pm \rho^0|^2 + |f \pm A_2^0|^2 + \cdots\,, \quad (4)$$

where the isospin 1 amplitudes enter with the plus sign in reaction (1) and with the minus sign in reaction (3). The even G-parity amplitudes, S^*, ρ^0, and f, can be produced by one-pion exchange and are expected to dominate at small t. The $\bar{K}^0K^0$ experiments[7-10] have looked only at the $K_S^0K_S^0$ final state; only even angular momenta contribute to this final state.[12]

II. EXPERIMENTAL METHOD

The sample of 16 000 $\pi^-p \to K^-K^+n$ events was obtained using the Argonne National Laboratory Effective Mass Spectrometer at the Zero Gradient Synchrotron. The spectrometer configuration[13-16] and trigger logic[13,14] have been described previously. The K^- and K^+ traversed the spectrometer, which measured their momenta and angles; the recoil neutron was not observed. Veto counters around the 20-in. liquid hydrogen target suppressed triggers from unwanted final states such as $K^-K^+\Delta^0$, and a large Čerenkov counter C_π vetoed those containing fast charged pions.

For K^-K^+ masses M_{KK} above 1500 MeV, some of the K's are fast enough to be vetoed by C_π, and backgrounds from other reactions are relatively worse than at lower masses; geometrical acceptance also decreases with increasing K^-K^+ mass. For these reasons, we have confined our study to those events with $M_{KK} < 1500$ MeV.

Each event was analyzed assuming that a particle-antiparticle pair was produced together with a recoil neutron, i.e., that the reaction was of the type $\pi^- p \to A^- A^+ n$. The mass squared of A, M_A^2, could then be calculated. The resulting distribution is shown in Fig. 1; a large K^-K^+n signal is observed. The major background, due to $\pi^-\pi^+X$ events for which C_π failed to count, is well understood because a companion experiment[17,18] studied $\pi^-p \to \pi^-\pi^+n$. A least-squares fit to the M_A^2 spectrum, allowing $\pi^-\pi^+X$, K^-K^+n, and a linear background, gives the curves shown in Fig. 1. The decay angular distribution of the $\pi^-\pi^+$ background was taken from the companion $\pi^-\pi^+n$ experiment, with the same C_π acceptance cuts as for K^-K^+n. The linear background was assumed to have the same decay angular distribution under the K^-K^+n peak as in the control region $0.15 < M_A^2 - M_K^2 < 0.25$ GeV^2; see Fig. 1. The corrections to the Y_4^0 moment, for example, are <15% for $M_{KK} < 1450$ MeV, rising to ~30% for $1450 < M_{KK} < 1500$ MeV.

The normalized moments of the K^-K^+ decay angular distributions, $\langle Y_l^m \rangle$, are defined by the relation

$$\frac{d^4\sigma}{dt\,dM\,d\cos\theta\,d\phi} = \sum_{l=0}^{l_{\max}} \sum_{m=0}^{l} \langle Y_l^m \rangle \operatorname{Re} Y_l^m(\cos\theta, \phi)\, \frac{d^2\sigma}{dt\,dM}, \quad (5)$$

where t is the momentum transfer to the K^-K^+ system, M is M_{KK}, the K^-K^+ mass, and θ, ϕ are the angles defining the K^- direction in the t-channel coordinate frame. To determine the angular distribution coefficients in Eq. (5), a maximum-likelihood fit was made to the events in each mass and t bin, as described in section 4.4 of Ref. 19. The spectrometer efficiency used for the fit was calculated analytically as a function of the kinematic invariants, using the same fiducial cuts imposed on the events. The log of the likelihood function was expanded quadratically in the angular distribution coefficients about a reasonable set of initial values. Solutions were obtained by setting the first derivatives equal to zero. This procedure was iterated and was found to converge in one or two steps. Monte Carlo experiments were performed under a variety of conditions (different angular distributions, numbers of events, reactions, fiducial volumes, etc.) to test both the efficiency calculations and the maximum-likelihood procedure.

The efficiency calculation included corrections for geometric acceptance (10% to 60%, depending on decay angles, at $M_{KK} = 1325$ MeV, for example), decays in flight (~35%), attenuation in spectrometer material and liquid hydrogen (~9%), and kaon interactions in C_π vetoing good events (~4%). Other corrections applied include those for accidental vetoing (~3%), spark chamber and counter

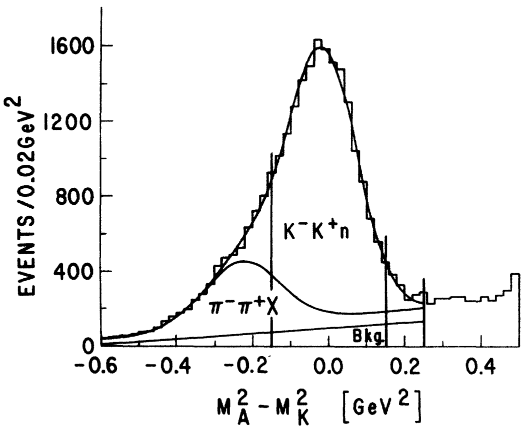

FIG. 1. $M_A^2 - M_K^2$ spectrum for events with $M_{KK} < 1.55$ GeV and $-t < 0.3$ GeV^2. Vertical lines indicate K^-K^+ peak cut and control region cut. Curves shown are the result of the fit described in the text.

inefficiencies (~8%), μ^- and e^- contamination in the π^- beam (~4%), and events lost by the cut on the K^-K^+n peak (~11%). The over-all normalization uncertainty is ±10%.

A test of the experimental method was made by comparing data from the companion $\pi^-p \to \pi^-\pi^+n$ experiment at 6 GeV/c to a 5.1 GeV/c bubble-chamber experiment[20] on $\pi^+n \to \pi^+\pi^-p$. The comparison was made for $1040 < M_{\pi\pi} < 1400$ MeV, $-t < 0.4$ GeV^2, and the two sets of moments were found to be in good agreement.

III. RESULTS

The final moments[21] for $\pi^-p \to K^-K^+n$, corrected for backgrounds, are shown in Figs. 2–5 as functions of M_{KK} and t in the t-channel frame. The moments exhibit a number of notable features.

Low-mass S wave. There is a low-mass $\langle Y_0^0 \rangle$ enhancement which rises very sharply from threshold; this has been seen previously[1,4,7,8] and shown to be spin zero.

P wave. The Y_1^0 moment indicates the presence of P wave throughout our mass range. An analysis similar to that presented for the D wave in the next section suggests that for $M_{KK} < 1200$ MeV the P wave contributes ~15% of the cross section. An analysis of the 9.8 GeV/c data of Ref. 4 by Morgan[22] shows the low-mass P wave to be consistent with the tail of the ρ^0 meson decaying into K^-K^+, with a ρKK coupling that agrees with the SU(3) prediction. The $\phi \to K^-K^+$ contribution is quite small.[14]

D-wave peak. The cross section and the $l = 4$ moments show broad maxima at $M_{KK} \approx 1350$ MeV.

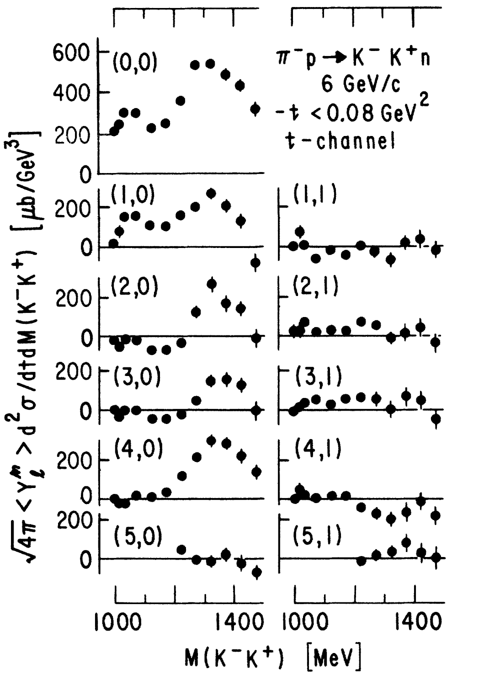

FIG. 2. K^-K^+ decay angular distribution moments (l, m in parentheses) calculated in the t-channel frame as functions of M_{KK} for $-t < 0.08$ GeV^2. Errors shown are statistical only, and do not include the over-all ± 10% normalization uncertainty or the systematic uncertainty (typically much smaller than the statistical uncertainty shown) due to the linear background subtraction.

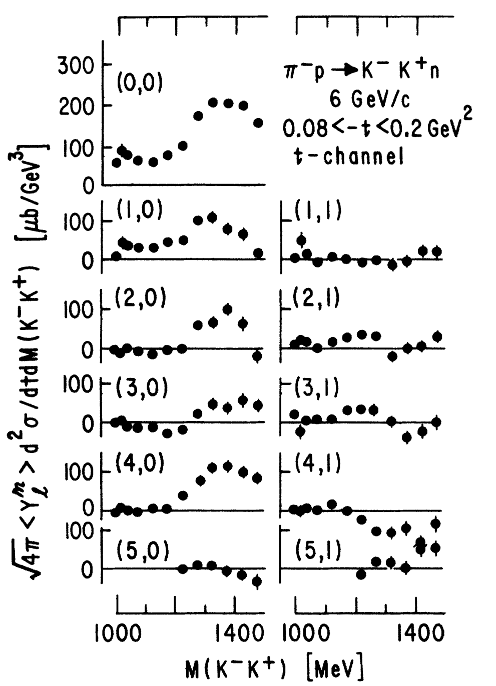

FIG. 3. K^-K^+ t-channel moments for $0.08 < -t < 0.20$ GeV^2.

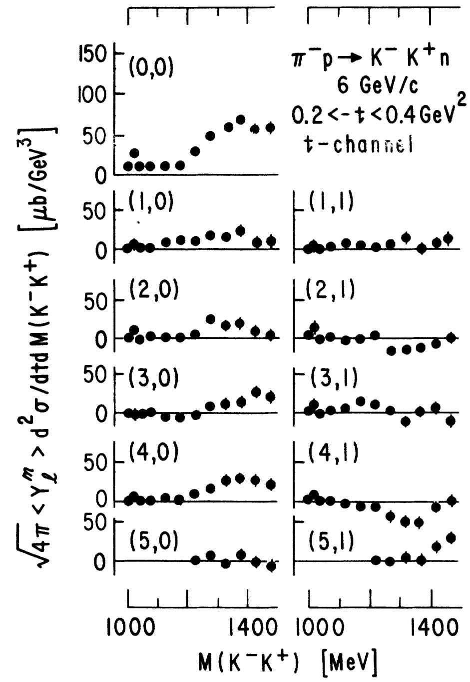

FIG. 4. K^-K^+ t-channel moments for $0.2 < -t < 0.4$ GeV^2.

Spin ⩾ 3 effects. The $l \geq 6$ moments (not shown) are everywhere negligible and the $l = 5$ moments are small, indicating spin ⩾ 3 production is not important below 1500 MeV.

Dominance of $m = 0$ amplitudes. The $m \geq 2$ moments (not shown) are everywhere consistent with zero and the $m = 1$ moments are much smaller than the $m = 0$ moments. The simplest explanation for these effects is that t-channel exchange production amplitudes with $m = 0$ dominate over those with $m \geq 1$.

Mass-dependent slope. Table I and Fig. 5 show the slope of $d\sigma/dt$ decreasing as M_{KK} increases; using the parameterization Ae^{Bt}, B falls from 16 GeV^{-2} near threshold to 9 GeV^{-2} at $M_{KK} = 1450$ MeV.

Slopes in the f-A_2^0 region. The slope in the f-A_2^0 region (~12 GeV^{-2}) agrees with that for $\pi^+ n \to f p$ at 5.1 GeV/c (see Ref. 23) and is much steeper than that of $\pi^+ n \to A_2^0 p$ (~2.0 GeV^{-2}).[23] Measured A_2^0 production[23] (corrected for the $A_2^0 \to K^-K^+$ branching ratio[24]) is too small, by a factor of at least three for $-t < 0.08$ GeV^2, to account for the Y_4^0 enhancement in the f-A_2^0 region. Hence, for small t, f production dominates over A_2^0 production in our reaction, despite the fact that the Y_0^0 and Y_4^0 moments peak at $M_{KK} \approx 1350$ MeV, rather than at the f mass of ~1280 MeV.[25]

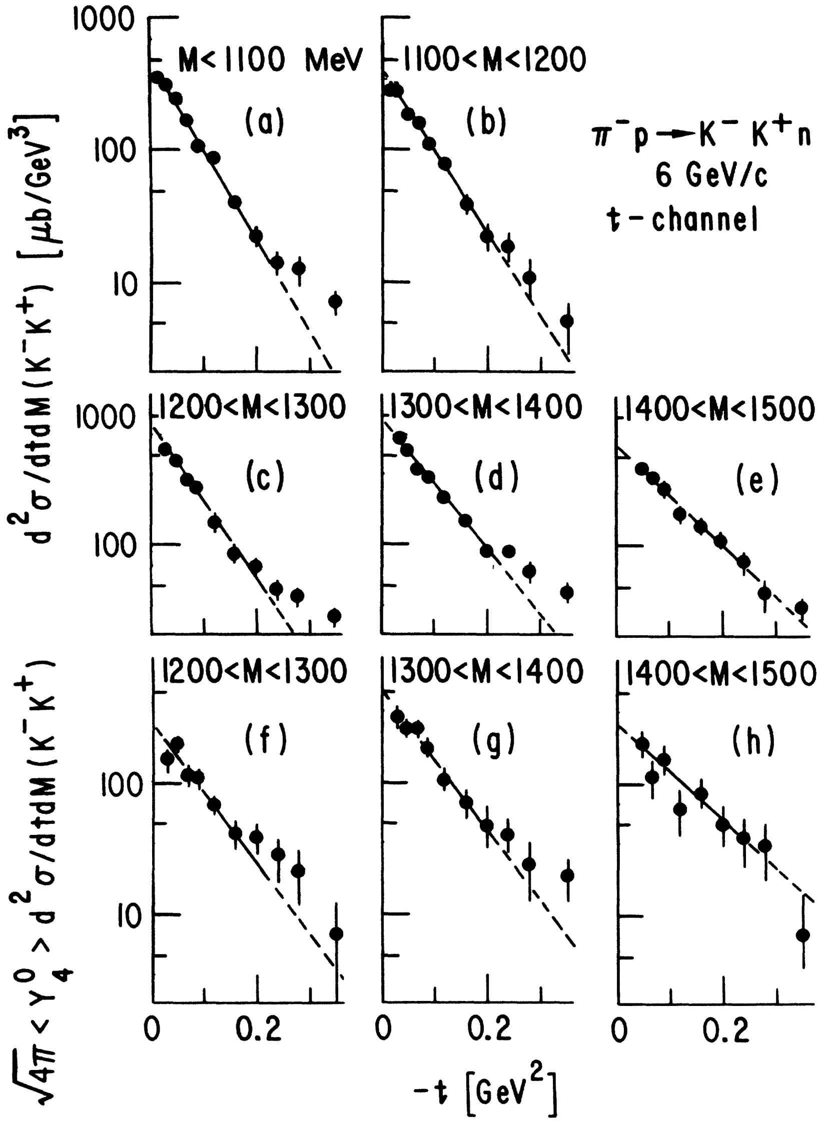

FIG. 5. (a)–(e) K^-K^+ differential cross sections as functions of t for the indicated M_{KK} ranges. (f)–(h) K^-K^+ t-channel Y_4^0 moments. The straight lines are the results of the exponential fits given in Table I.

TABLE I. Slopes and intercepts derived from exponential fits to the differential cross sections and Y_4^0 moment for $0.02 < -t < 0.22$ GeV^2: $d^2\sigma/dtdM = Ae^{Bt}$; $(4\pi)^{1/2}\langle Y_4^0\rangle d^2\sigma/dtdM = Ce^{Dt}$. The errors shown do not include the ±10% over-all normalization uncertainty.

M_{KK} (MeV)	Intercept A (μb/GeV^3)	Slope B (GeV^{-2})	Intercept C (μb/GeV^3)	Slope D (GeV^{-2})
<1100	533 ± 24	16.0 ± 0.5	...	...
1100–1200	444 ± 23	14.6 ± 0.7	...	...
1200–1300	817 ± 50	13.4 ± 0.7	287 ± 44	12.2 ± 1.6
1300–1400	932 ± 63	11.5 ± 0.7	523 ± 83	12.3 ± 1.5
1400–1500	607 ± 64	8.8 ± 0.9	291 ± 64	8.3 ± 1.9

IV. DISCUSSION

A. D-wave production amplitudes

One can unambiguously determine the magnitudes of the D-wave production amplitudes near 1300 MeV, given certain plausible assumptions.[26,27] For each of the five spin-two-meson helicity states there are two (incoherent) nucleon helicity amplitudes, a total of ten amplitudes. Using only meson helicity indices, we write the amplitudes[27] as D_0, D_-, D_{2-}, D_+, and D_{2+} (where, e.g., $|D_0|^2 \equiv |D_0^{\text{nonflip}}|^2 + |D_0^{\text{flip}}|^2$). Asymptotically, at high energy, D_0, D_-, and D_{2-} project out the $m=0$, 1, and 2 unnatural-parity exchange contributions; D_+ and D_{2+} asymptotically project out the $m=1$ and 2 natural-parity exchange contributions. If we assume that (a) $m=2$ amplitudes are negligible; (b) F-wave and higher amplitudes are negligible, and (c) the unnatural-parity amplitudes D_0 and D_- are coherent in phase and have the same ratio of flip to nonflip at the nucleon vertex, the amplitudes can be obtained from the relations[27]

$$\sigma\langle Y_4^0\rangle = \tfrac{6}{7}|D_0|^2 - \tfrac{4}{7}(|D_-|^2 + |D_+|^2)\ , \qquad (6)$$

$$\sigma\langle Y_4^1\rangle = \tfrac{4}{7}(15)^{1/2}\,\text{Re}(D_-^* D_0)\ , \qquad (7)$$

$$\sigma\langle Y_4^2\rangle = \tfrac{2}{7}(10)^{1/2}(|D_-|^2 - |D_+|^2)\ , \qquad (8)$$

where $\sigma \equiv (4\pi)^{1/2}\, d^2\sigma/dt\,dM$.

The fractions of D-wave cross section due to the $m=1$ amplitudes are shown in Table II, averaged over the peak region $1250 < M_{KK} < 1400$ MeV. The D_0 amplitude dominates in the Gottfried-Jackson (t-channel) frame, as expected for f production by one-pion exchange. The natural-parity exchange $m=1$ fraction is zero to within the errors of 10%. The unnatural-parity $m=1$ fraction ranges from (1 or 2)% at small t to 7% at $-t = 0.3$ GeV^2.

TABLE II. The natural- and unnatural-parity exchange $m=1$ fractions of the D-wave cross section for $1250 < M_{KK} < 1400$ MeV, $f_\pm = |D_\pm|^2/(|D_0|^2 + |D_-|^2 + |D_+|^2)$.

$-t$ (GeV^2)	f_+	f_-
<0.08	0.03 ± 0.07	0.015 ± 0.007
0.08–0.20	−0.12 ± 0.09	0.047 ± 0.019
0.20–0.40	0.10 ± 0.10	0.073 ± 0.027

B. f-A_2^0 interference analysis

Irving and Michael[5] have discussed f-A_2^0 interference effects and have made predictions on the basis of model amplitudes derived from duality and vector-meson production studies. We have attempted to fit the Y_4^0 mass spectrum assuming that only f and A_2^0 production contribute to $\langle Y_4^0\rangle$. We write

$$(4\pi)^{1/2}\langle Y_4^0\rangle \frac{d^2\sigma}{dt\,dM} = 2M[C_f^{\,2} D_f^{\,2} e^{b_f t} + C_A^{\,2} D_A^{\,2} e^{b_A t} + 2\xi\, C_f D_f C_A D_A e^{(b_f + b_A)t/2} \times \cos(\phi + \delta_f - \delta_A)]\ , \qquad (9)$$

where $C_f^{\,2}$, $C_A^{\,2}$ are incoherent sums of squares of f and A_2^0 production amplitudes extrapolated to $t=0$, multiplied by the K^-K^+ decay widths; b_f, b_A are slopes taken from data for $\pi^- p \to f n$ ($f \to \pi^-\pi^+$) (see Ref. 18) and $\pi^+ n \to A_2^0 p (A_2^0 \to \pi^+\pi^-\pi^0)$,[23] respectively; D_f, D_A are D-wave Breit-Wigner shape functions;[28] ξ is a coherence factor ($0 \le \xi \le 1$), allowing for the presence of different helicity amplitudes at both the meson and nucleon vertices; ϕ is the production amplitude phase difference; δ_f, δ_A are the Breit-Wigner decay amplitude phases.[28] The masses and total widths of the f and A_2^0 were fixed[25] and fits made from threshold to 1450 MeV using four free parameters: $C_f^{\,2}$, $C_A^{\,2}$, ξ, and ϕ.

The results are shown in Fig. 6 and Table III. We note the following: (a) Although ϕ depends somewhat on ξ, it is consistent with the Irving and Michael[5] prediction of ~70°. (b) The Y_4^0 moment for $A_2^0 \to K^-K^+$ is maximal when compared to the Y_0^0 data of Armenise *et al.*,[23] i.e., consistent with the upper limit $\langle Y_4^0\rangle \le \frac{6}{7}\langle Y_0^0\rangle$ for a pure D-wave cross section; see Eq. (6). This implies that the D_0 amplitude dominates A_2^0 production, consistent with the dominance of unnatural-parity exchange in $\pi^+ n \to A_2^0 p$ found by Gordon *et al.*[29] (c) An (unpublished) analysis of our 6-GeV/c $\pi^-\pi^+ n$ data in the f region[18] shows that f production for $-t \lesssim 0.2$ GeV^2 is also dominated by a D_0 amplitude, as expected for one-pion exchange. Since both f and A_2^0 are produced mainly with $m=0$, ξ is approximately the coherence at the nucleon vertex. The 90% confidence level lower limit for the coherence is $\xi \ge 0.4$. Incoherent fits ($\xi = 0$) were unacceptable, as shown in Table III.

C. Comparison to $K_S^0 K_S^0$ results

The apparent success of the fits to the Y_4^0 moment may be misleading. Despite the reasonable χ^2's (Table III), there is a systematic excess of Y_4^0 moment over the best fit values for $M_{KK} > 1350$ MeV (Fig. 6). A structure in $K_S^0 K_S^0$ at $M_{KK} \approx 1440$ MeV has been previously suggested by Beusch *et al.*,[7,8] whose preliminary data at 9 GeV/c for the Y_4^0 moment in reaction (2) have much the same shape as our Y_4^0 spectrum in reaction (1). This result disagrees[30] with our prediction for the

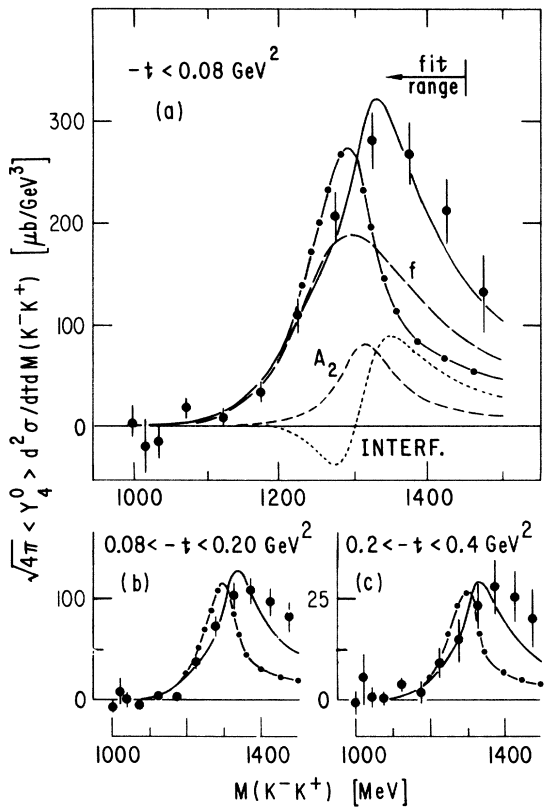

FIG. 6. Fits to the $\langle Y_4^0 \rangle$ mass spectra in the three t ranges: (a) $-t < 0.08$ GeV^2; (b) $0.08 < -t < 0.20$ GeV^2; (c) $0.2 < -t < 0.4$ GeV^2. In each case only the data for $M_{KK} < 1450$ MeV were fitted. The solid line is the fit; the dot-dashed line is the prediction for $\pi^- p \to \bar{K}^0 K^0 n$. (a) also shows the three terms of Eq. (9) separately for $-t < 0.08$ GeV^2.

$\bar{K}^0 K^0$ spectrum shape, also shown in Fig. 6. The prediction is simply obtained by changing the sign of the interference term in Eq. (9).[31]

If the K^-K^+ and $\bar{K}^0K^0$ spectra are indeed similar in the region below 1500 MeV, then the f-A_2^0 interference term must be small. Without the interference term one is left with a large excess $\langle Y_4^0 \rangle$ at higher masses beyond the contribution expected from the f and A_2^0 tails. The effect does not appear to be due to spins higher than 2 since the $l \geq 6$ moments are negligible and the $l = 5$ moments are small for $M_{KK} < 1500$ MeV. Assuming that D-wave Breit-Wigners correctly describe the f and A_2^0 resonant amplitudes, the preliminary $K_S^0 K_S^0$ data together with our results imply that there must be an additional contribution to the D-wave amplitude. Such a contribution would alter the fit results given in Table III.

TABLE III. f-A_2^0 interference parameters from fits to the Y_4^0 moments vs M_{KK} for $M_{KK} < 1450$ MeV. For $-t > 0.20$ GeV^2, C_f^2 is poorly determined and has been fixed at the result for $-t < 0.08$ GeV^2. The errors on ξ are 90% confidence level limits.

$-t$ (GeV^2)	<0.08	0.08–0.20	0.2–0.4
C_f^2 (μb/GeV)	19.4 ±5.6	17.5 ±7.4	19.4 (fixed)
C_A^2 (μb/GeV)	1.58 ± 0.67	1.02 ±0.33	0.41 ±0.39
ξ	$1.0^{+0.0}_{-0.6}$	$1.0^{+0.0}_{-0.6}$	$1.0^{+0.0}_{-0.6}$
φ ($\xi = 1.0$) (deg)	52 ± 21	58 ± 22	57 ± 31
χ^2 ($\xi = 1.0$)/D.F.	8.5/7	17.3/7	6.1/8
φ ($\xi = 0.75$) (deg)	69 ± 38	82 ± 36	63 ± 31
χ^2 ($\xi = 0.75$)	8.5	17.3	6.8
χ^2 ($\xi = 0$)	18.8	29.4	11.3

D. $f \to \bar{K}K$ branching ratio

We have calculated the $f \to \bar{K}K$ branching ratio implied by the best f-A_2^0 fit. Since at low t the f dominates the $\langle Y_4^0 \rangle$ moment below ~1350 MeV, the complication at higher $\bar{K}K$ masses should not significantly affect our result. The $\pi^-\pi^+n$ data[18] for $-t < 0.08$ GeV^2 were fitted in a similar way.[32] Comparing the $\pi^-\pi^+$ result to the K^-K^+ result for $-t < 0.08$ GeV^2 (Table III), we obtain the branching ratio $(f \to K^-K^+/f \to \pi^-\pi^+) = (2.8 \pm 0.9)\%$, leading to[33] $(f \to \bar{K}K/f \to \text{all}) = (3.1 \pm 1.0)\%$. Note that the K^-K^+ and $\pi^-\pi^+$ data were taken with the same apparatus and the analyses had many elements in common; many potential systematic errors (such as overall normalization) are therefore minimized in this measurement of the branching ratio. The previous results[2,3,5,24,34] have a large spread, with an average[24] of $(4 \pm 3)\%$, consistent with our result. The SU(3) prediction[35] of 3.3% is also in good agreement with the present result.

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†Present address: Fermi National Accelerator Laboratory, Batavia, Ill. 60510.

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[25]The absolute mass scale is known to ± 2 MeV, so the 1350 MeV peak is not a misplaced f peak. The effective mass resolution is ± 3 MeV in this mass range. The f mass and width in the physical region ($M=1282\pm6$ MeV, $\Gamma=199\pm16$ MeV) were empirically found by fitting a D-wave Breit-Wigner curve to unpublished data on the Y_4^0 moment vs $M_{\pi\pi}$ from the companion $\pi^-\pi^+n$ experiment (Ref. 18) at 6 GeV/c for $-t<0.08$ GeV^2, $1040<M_{\pi\pi}<1400$ MeV. The A_2^0 mass and width ($M=1324$ MeV, $\Gamma=104$ MeV) were taken from G. Conforto *et al.*, Phys. Lett. 45B, 154 (1973).

[26]F. Wagner, in *Proceedings of the Seventeenth International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, England, 1974), p. II-27; A. D. Martin and C. Michael, Nucl. Phys. B84, 83 (1975).

[27]P. Estabrooks *et al.*, in *π-π Scattering—1973*, proceedings of the International Conference on π-π Scattering and Associated Topics, Tallahassee, 1973, edited by P. K. Williams and V. Hagopian (A.I.P., New York, 1973), p. 37.

[28]The D-wave Breit-Wigner shape functions used are based on the formalism given in Appendixes A and B of Ref. 5. For the $A_2^0\to K^-K^+$ decay, we have

$$D_A\equiv D_A(M)=\frac{[(M_A/\pi)\Gamma_{KK}^A(M)/\Gamma_{KK}^A(M_A)]^{1/2}}{|M_A{}^2-M^2-iM_A\Gamma_{\text{total}}^A(M)|}\,,$$

where $M=M_{KK}$, M_A=the A_2^0 central mass. The mass-dependent width functions $\Gamma(M)$ are given by

$$\Gamma^A(M)=(q/q_A)^5\Gamma^A(M_A)D_2(q_AR)/D_2(qR),$$

where q is the decay momentum in the relevant channel (K^-K^+ channel for Γ_{KK}^A, $\rho\pi$ channel for Γ_{total}^A), with $q_A=q$ evaluated at $M=M_A$. The function $D_2(x)=9+3x^2+x^4$. The Breit-Wigner phase

$$\delta_A=\arg[M_A{}^2-M^2+iM_A\Gamma_{\text{total}}^A(M)].$$

The $D(M)$ shape function is thus seen to carry all of the mass dependence but no information about the branching ratio; the branching ratio information is carried by the $C_f{}^2$ and $C_A{}^2$ factors of Eq. (9).

[29]H. A. Gordon *et al.*, Phys. Rev. Lett. 33, 603 (1974).

[30]Empirically, f and A_2^0 production have about the same $p_{\text{lab}}{}^{-2}$ energy dependence [see Ref. 5 and J. T. Carroll *et al.*, Phys. Rev. Lett. 25, 1393 (1970)], so this shape comparison is reasonable.

[31]This prediction also applies to $\pi^+n\to K^-K^+p$ (see Ref. 11). A better way to study the isospin 0-isospin 1 interference terms and the nature of the 1440 MeV structure would be to measure both reaction (1) and $\pi^+n\to K^-K^+p$ in the same apparatus, thereby achieving good relative normalization while minimizing systematic differences.

[32]The data were fitted to the form given by Eq. (9) with the A_2^0 terms set to zero and using the D_f shape function appropriate to the $\pi^-\pi^+$ decay of the f. A χ^2 of 16.8 for 13 degrees of freedom was obtained.

[33]We used the ratio $(f\to\pi\pi)/(f\to\text{all})=83\%$ from Ref. 24.

[34]M. Aderholz *et al.*, Nucl. Phys. B11, 259 (1969).

[35]N. P. Samios *et al.*, Rev. Mod. Phys. 46, 49 (1974).