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Renormalizable massive Yang-Mills theory with intrinsic symmetry breakdown*

J. P. Hsu and J. A. Underwood
Center for Particle Theory, The University of Texas, Austin, Texas 78712
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We use the method of the Lagrange multiplier field to construct, within the framework of indefinite-metric field theory, a model of a massive Yang-Mills field, the mass of which derives from an *intrinsic* symmetry breakdown, rather than a spontaneous symmetry breakdown. The rules for the Feynman diagrams are given. The dynamical structure of the interacting fields is intrinsically related to a local non-Abelian gauge symmetry which renders the theory renormalizable by standard power counting. The part of the amplitude due to the interaction of unphysical particles in the intermediate states, which violates unitarity and gauge symmetry, is isolated and removed, leaving the unitary physical S matrix. The unitarity of the resulting theory, together with its independence with respect to the parameter ξ, is demonstrated in calculations at the one-loop and the two-loop levels.

I. INTRODUCTION

It is widely believed that a requirement for the renormalizability of a theory of the massive Yang-Mills field is that the Yang-Mills quanta should acquire their mass through a spontaneous breakdown of local isospin gauge symmetry. The principal reasoning behind any conjecture in this vein lies with 't Hooft's establishment[1] of the renormalizability of such gauge theories. The generality of such an assertion, however, is limited with the observation of certain renormalizable theories, discussed in this paper and in Ref. 2, characterized by an indefinite metric, not employing the spontaneous-symmetry-breakdown mechanism. Among the significant features of the theory in Ref. 2, not generally shared by those gauge theories employing multiplets of Higgs mesons, is that of asymptotic freedom. Now, it is worth noting that in elaborating this theory the exercise of some care is required in the assigning of statistical weights to higher-order Feynman diagrams. This is due, in fact, to a lack of independence among the unphysical modes participating dynamically in the various processes.[3] It is possible to circumvent this circumstance by including a pair of auxiliary fields in the structure of the Lagrangian: one an unphysical isovector field $\vec{\phi}$ of spin zero, the mass of which coincides with that of the spin-zero part $\vec{f}_s$ of the Yang-Mills field $\vec{f}_\mu$; and the other a physical isoscalar field $\mathfrak{U}$ of zero spin and vanishing rest mass. The more usual gauge formulation for the massive Yang-Mills field[1,4] centers on a gauge-invariant Lagrangian incorporating a quartic polynomial of scalar fields. The parameters appearing as coefficients in this polynomial are of such a character that one of the scalar fields (let us say $\mathfrak{U}$) spontaneously develops a vacuum expectation value. The Lagrangian that displays the particle content of the theory is obtained by making a shift of the form $\mathfrak{U}\to\mathfrak{U}+\text{const.}$ in the structure of the original Lagrangian. This mechanism, in breaking down the gauge symmetry, generates a mass for the vector field. By contrast, the Lagrangian to be exhibited shortly does not contain such a quartic polynomial of scalar fields. The mass of the vector field is, indeed, introduced in the Lagrangian at the beginning. The Lagrangian derives from the original gauge-invariant Lagrangian through the shifting of $\mathfrak{U}$ by the value $2M/g$, twice the ratio of the mass of the Yang-Mills field and the attendant coupling constant. The mass of the vector boson originates, then, not through a spontaneous breakdown in the gauge symmetrv. but rather through what may be regarded as an *intrinsic* symmetry breakdown.

Within the framework of indefinite-metric field theories,[5] the propagator of the massive vector meson is *quadratic convergent* and the resultant Lagrangian [cf. Eq. (15) below] does not contain terms of dimension higher than four. Thus, the renormalizability of the theory is *manifest.* The Green's functions, on the other hand, are not unitary in general on account of the unphysical poles appearing in the propagators. This is precisely what happens in the *R-gauge* formulation of the more usual gauge theories.[1,6] The divergences in this formulation are no stronger than those cropping up in the well-known renormalizable theories.[1] The significant questions appear in connection with handling the unitarity of the theory.

A Lagrange multiplier field $\vec{\chi}$ is introduced in the Lagrangian in a manner which eventually does not affect any physical properties of the system, and such that all unitarity excess derives from the source terms in the $\vec{\chi}$ equation of motion. A fictitious Lagrangian $\mathfrak{L}_{ff}$ is constructed by using the $\vec{\chi}$ equation directly. The unitarization of the

S matrix is realized by incorporating Feynman rules, derived from $\mathcal{L}_{ff}$, involving a pair of fictitious scalar fermions D and D', which cancel the effects of the unphysical particles order by order in the perturbation series.[7,1] The presence of the fields $\vec{\phi}$ and $\mathcal{U}$ allows deriving the $\vec{\chi}$ equation of motion without invoking the constraint equation; hence, the fields participating in the dynamics are treated as statistically independent.[5] The gauge freedom of the theory is embodied in the arbitrariness of the ratio ξ of the squared masses of the physical and unphysical parts of the Yang-Mills field, yet the physical S matrix is ξ independent.

II. FORMALISM

Let us consider a physical system consisting of a massive Yang-Mills field $\vec{f}_\mu$, an unphysical spin-0 isovector field $\vec{\phi}$, and a spin-0 isoscalar field $\mathcal{U}$, all obeying Bose statistics, with a Lagrangian density of the form

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_1 + \mathcal{L}_\xi,\\ \mathcal{L}_1 &= -\tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu} + \tfrac{1}{2}M^2\vec{f}_\mu{}^2 + \tfrac{1}{2}\partial_\mu\mathcal{U}\partial^\mu\mathcal{U}\\ &\quad + \tfrac{1}{2}\partial_\mu\vec{\phi}\cdot(\partial^\mu + g\vec{f}^\mu\times)\vec{\phi} - \tfrac{1}{2}g\vec{f}_\mu\cdot(\mathcal{U}\partial^\mu\vec{\phi} - \vec{\phi}\partial^\mu\mathcal{U})\\ &\quad - \tfrac{1}{8}g^2\vec{f}_\mu{}^2(\vec{\phi}^2 + \mathcal{U}^2) + \tfrac{1}{2}gM\vec{f}_\mu{}^2\mathcal{U} + M\vec{\phi}\,\partial_\mu\vec{f}^\mu, \qquad (1)\\ \mathcal{L}_\xi &= -\tfrac{1}{2}\xi(\partial_\mu\vec{f}^\mu + M\vec{\phi}/\xi)^2,\end{aligned}$$

where $\vec{f}_{\mu\nu} \equiv \partial_\nu\vec{f}_\mu - \partial_\mu\vec{f}_\nu - g\vec{f}_\mu\times\vec{f}_\nu$ is the Yang-Mills field strength tensor. The term $\mathcal{L}_\xi$ gives the field $\vec{f}_s \propto \partial_\mu\vec{f}^\mu$ a well-defined equation of motion, and hence is a legitimate way of introducing the negative-metric spin-0 part $\vec{f}_s$ of the 4-vector field $\vec{f}_\mu$ in the theory.[8] We introduce a Lagrange multiplier field $\vec{\chi}$ by replacing the last term in $\mathcal{L}$ according to

$$\mathcal{L}_\xi \to \mathcal{L}_L = M\vec{\chi}\cdot(\partial_\mu\vec{f}^\mu + M\vec{\phi}/\xi) + (\tfrac{1}{2}M^2/\xi)\vec{\chi}^2. \qquad (2)$$

This replacement has no physical content [cf. Eq. (8) below]. The Euler-Lagrange equations deriving from $\mathcal{L}' = \mathcal{L}_1 + \mathcal{L}_L$ are

$$\partial_\mu\vec{f}^{\mu\nu} + M\partial^\nu\vec{\chi} + M\partial^\nu\vec{\phi} - g\vec{f}^{\mu\nu}\times\vec{f}_\mu - M^2\vec{f}^\nu + \tfrac{1}{2}g\partial^\nu\vec{\phi}\times\vec{\phi} + \tfrac{1}{2}g(\mathcal{U}\partial^\nu\vec{\phi} - \vec{\phi}\partial^\nu\mathcal{U}) - gM\vec{f}^\nu\mathcal{U} + \tfrac{1}{4}g^2\vec{f}^\nu(\vec{\phi}^2 + \mathcal{U}^2) = 0, \qquad (3)$$

$$\partial_\mu(\partial^\mu\vec{\phi} + \tfrac{1}{2}g\vec{f}^\mu\times\vec{\phi} - \tfrac{1}{2}g\mathcal{U}\vec{f}^\mu) - \tfrac{1}{2}g\partial_\mu\vec{\phi}\times\vec{f}^\mu - \tfrac{1}{2}g\vec{f}_\mu\partial^\mu\mathcal{U} - \tfrac{1}{4}g^2\vec{f}_\mu{}^2\vec{\phi} - M\partial_\mu\vec{f}^\mu - M^2\vec{\chi}/\xi = 0, \qquad (4)$$

$$\partial_\mu(\partial^\mu\mathcal{U} + \tfrac{1}{2}g\vec{f}^\mu\cdot\vec{\phi}) + \tfrac{1}{2}g\vec{f}_\mu\cdot\partial^\mu\vec{\phi} - \tfrac{1}{4}g^2\vec{f}_\mu{}^2\mathcal{U} - \tfrac{1}{2}gM\vec{f}_\mu{}^2 = 0, \qquad (5)$$

and the constraint equation, by variation of $\mathcal{L}' = \mathcal{L}_1 + \mathcal{L}_L$ with respect to $\vec{\chi}$, is

$$\partial_\mu\vec{f}^\mu + M(\vec{\chi} + \vec{\phi})/\xi = 0. \qquad (6)$$

Taking the divergence of (3) and using (4) and (5) one derives, after some manipulation, the $\vec{\chi}$ equation of motion

$$(\Box + M^2/\xi)\vec{\chi} + g\vec{f}_\mu\times\partial^\mu\vec{\chi} + \tfrac{1}{2}(M/\xi)g\mathcal{U}\vec{\chi} + \tfrac{1}{2}(M/\xi)g\vec{\chi}\times\vec{\phi} = 0, \qquad (7)$$

which shows the $\vec{\chi}$ field also to have a mass $M/\sqrt{\xi}$. One sees that the form of the interaction terms conducts the derivation of the $\vec{\chi}$ equation of motion around the necessity of employing the constraint equation, while leaving the $\vec{\chi}$ equation with only renormalizable source terms.

The equation (7) for the Lagrangian multiplier field $\vec{\chi}$ shows that $\vec{\chi}$ couples to other fields. Thus, the physical states cannot be defined consistently for all times and the physical amplitude due to the Lagrangian (1) is not unitary because it contains an extra amplitude contributed by the interactions of $\vec{\chi}$ in the intermediate states. This has been discussed by, for example, Rudolph and Dürr,[5] and Fradkin and Tyutin.[5] (See also Ref. 2.) In order to have a unitary theory, we must isolate and remove this extra amplitude completely.

The amplitude due to the Lagrangian (1), which is not unitary, may be expressed as the functional integral

$$\begin{aligned}\mathfrak{A} &= \int \exp\left[i\int d^4x(\mathcal{L}' + \mathcal{L}_s)\right] d[\vec{f}_\mu, \vec{\phi}, \vec{\chi}, \mathcal{U}] \quad (\mathcal{L}' = \mathcal{L}_1 + \mathcal{L}_L)\\ &= \int \exp\left[i\int d^4x(\mathcal{L} + \mathcal{L}_s)\right] d[\vec{f}_\mu, \vec{\phi}, \mathcal{U}], \qquad (8)\end{aligned}$$

where we have integrated over $\vec{\chi}$ and omitted the constant factor, with $\mathcal{L}_s$ the external source for the physical fields, i.e., the spin-0 field $\mathcal{U}$ and the spin-1 part of the field $\vec{f}_\mu$. As with the massless Yang-Mills field,[2] the Lagrangian $\mathcal{L}$ in (8) involves two unphysical degrees of freedom—here $\vec{\phi}$ and $\vec{f}_s$, with the same mass $M/\sqrt{\xi}$. Furthermore, as in Ref. 2, the equation (7) for the field $\vec{\chi}$ indicates that the amplitude X due to the interactions of $\vec{f}_s$ and $\vec{\phi}$ in the intermediate states can be effectively expressed by

$$X = \int \exp\left[i\int d^4x\,\mathcal{L}(\vec{\chi}, \vec{\chi}^*)\right] d[\vec{\chi}, \vec{\chi}^*], \qquad (9)$$

$$\mathcal{L}(\vec{\chi},\vec{\chi}^*) = -\vec{\chi}^*\cdot[(\Box+M^2/\xi)\vec{\chi}+g\vec{f}_\mu\times\partial^\mu\vec{\chi} + (\tfrac{1}{2}M/\xi)g\mathfrak{u}\vec{\chi}+(\tfrac{1}{2}M/\xi)g\vec{\chi}\times\vec{\phi}], \tag{10}$$

where we have constructed the Lagrangian $\mathcal{L}(\vec{\chi},\vec{\chi}^*)$ with complex fields to imitate the interactions of the two real fields $\vec{\phi}$ and $\vec{f}_s$. Since $\mathcal{L}(\vec{\chi},\vec{\chi}^*)$ is a quadratic form we can easily perform the integration to obtain

$$X = \frac{\text{const}}{\det(\delta^{ab}+(\Box+M^2/\xi)^{-1}\{(\tfrac{1}{2}M/\xi)g\mathfrak{u}\delta^{ab}-\epsilon^{abc}[gf^c_\mu\partial^\mu-(\tfrac{1}{2}M/\xi)g\phi^c]\})}, \tag{11}$$

where the constant factor is irrelevant to physics. Thus, the extra amplitude is completely isolated formally in a functional determinant factor. In this case one may express the unitarized amplitude formally as

$$\mathfrak{A}_u = \text{const}\times\int X^{-1}\exp\left[i\int d^4x(\mathcal{L}+\mathcal{L}_s)\right]d[\vec{f}_\mu,\vec{\phi},\mathfrak{u}]. \tag{12}$$

The factor X^{-1} can be expressed by introducing fictitious scalar fermions D and D',

$$X^{-1} = \int\exp\left(i\int d^4x\,\mathcal{L}_{ff}\right)d[\vec{D},\vec{D}'], \tag{13}$$

$$\mathcal{L}_{ff} = -\vec{D}'\cdot[(\Box+M^2/\xi)\vec{D}+g\vec{f}_\mu\times\partial^\mu\vec{D} + (\tfrac{1}{2}M/\xi)g\mathfrak{u}\vec{D}+(\tfrac{1}{2}M/\xi)g\vec{D}\times\vec{\phi}]. \tag{14}$$

In the unitarized amplitude (12), with X^{-1} expressed by (13), the extra amplitude due to the two unphysical fields $\vec{f}_s$ and $\vec{\phi}$ in the Lagrangian $\mathcal{L}$ is canceled by the two fictitious fields $\vec{D}$ and $\vec{D}'$ in the computation of the physical S matrix. The unitarized theory follows completely from the effective Lagrangian $\mathcal{L}_{\text{eff}}$,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}+\mathcal{L}_{ff}, \tag{15}$$

with the particles f_s, ϕ, D, and D' unphysical and hence not appearing in the external states of physical processes.

III. FEYNMAN RULES

The Lagrangian $\mathcal{L}_{\text{eff}}$ in (15) can be quantized according to the usual canonical quantization procedure within the framework of indefinite-metric field theories.[2,5] Using $\mathcal{L}_{\text{eff}}$ one is able to derive the following Feynman rules:

$$f_\mu:\ \frac{-i\delta_{ab}[g_{\mu\nu}-k_\mu k_\nu(1-\xi^{-1})/(k^2-M^2/\xi)]}{k^2-M^2}, \tag{16}$$

$$D,D':\ \frac{i\delta_{ab}}{k^2-M^2/\xi}, \tag{17}$$

$$\phi:\ \frac{i\delta_{ab}}{k^2-M^2/\xi}, \tag{18}$$

$$\mathfrak{u}:\ \frac{i}{k^2}, \tag{19}$$

$$f^a_\mu(p)f^b_\nu(q)f^c_\lambda(k):\quad -g\epsilon_{abc}[g_{\mu\nu}(p-q)_\lambda+g_{\nu\lambda}(q-k)_\mu+g_{\lambda\mu}(k-p)_\nu], \tag{20}$$

$$f^\alpha_\mu(p)f^\beta_\nu(q)f^\gamma_\lambda(k)f^\delta_\sigma(l):\quad -ig^2[\epsilon_{a\alpha\beta}\epsilon_{a\gamma\sigma}(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda}) + \epsilon_{a\alpha\gamma}\epsilon_{a\delta\beta}(g_{\mu\sigma}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\sigma}) + \epsilon_{a\alpha\delta}\epsilon_{a\beta\gamma}(g_{\mu\nu}g_{\lambda\sigma}-g_{\mu\lambda}g_{\nu\sigma})], \tag{21}$$

$$f^\alpha_\mu(p)\phi^\beta(q)\phi^\gamma(k):\ \tfrac{1}{2}g\epsilon_{\alpha\beta\gamma}(q-k)_\mu, \tag{22}$$

$$f^\alpha_\mu(p)\phi^\beta(q)\mathfrak{u}(k):\ \tfrac{1}{2}g\delta_{\alpha\beta}(q-k)_\mu, \tag{23}$$

$$f^\alpha_\mu(p)f^\beta_\nu(q)\mathfrak{u}(k):\ igM\delta_{\alpha\beta}g_{\mu\nu}, \tag{24}$$

$$f^\alpha_\mu(p)f^\beta_\nu(q)\phi^\gamma(k)\phi^\delta(l):\ \tfrac{1}{2}ig^2\delta_{\alpha\beta}\delta_{\gamma\sigma}g_{\mu\nu}, \tag{25}$$

$$f^\alpha_\mu(p)f^\beta_\nu(q)\mathfrak{u}(k)\mathfrak{u}(l):\ \tfrac{1}{2}ig^2\delta_{\alpha\beta}g_{\mu\nu}, \tag{26}$$

$$f^\alpha_\mu(p)D^\beta(q)D'^\gamma(k):\ -g\epsilon_{\alpha\beta\gamma}k_\mu, \tag{27}$$

$$\mathfrak{u}(p)D^\alpha(q)D'^\beta(k):\ -\tfrac{1}{2}iMg\delta_{\alpha\beta}/\xi, \tag{28}$$

$$\phi^\alpha(p)D^\beta(q)D'^\gamma(k):\ -\tfrac{1}{2}iMg\epsilon_{\alpha\beta\gamma}/\xi \tag{29}$$

(α, β, γ, δ, a, b, c are isospin indices), where we have used the convention that all 4-momenta are outgoing from the vertices. All fermion vertices are bilinear in the fermions, and therefore the scalar fermions appear only in closed loops. We note that there is a factor of -1 for each scalar-fermion loop. These Feynman rules indicate that the theory is renormalizable by standard power counting.

IV. UNITARITY

At this stage one is able to check if the theory is unitary. One-loop unitarity is tested by calculating the imaginary part of those diagrams contributing to second order in the $\vec{f}$ self-energy which possess a 2-particle cut. One must show that the contributions to the imaginary part in the unitarity relation

$$\text{Im}T_{fi} = \sum_n T^*_{fn}T_{ni} \tag{30}$$

arising from those diagrams with unphysical particles in the intermediate states sum to zero, and, thus, that in the unitarity sum only the physical degrees of freedom contribute. We decompose the $\vec{f}_\mu$ propagator into the physical spin-1 part $\vec{f}$ and the unphysical spin-0 part $\vec{f}_s$. Denoting by A_i the amplitude for the process $f^a(p_1) \to I_i \to f^a(p_1)$, and by $Z[I_i]$ the amplitude for the decay process $f^a(p_1) \to I_i$, with I_i one of six distinct intermediate states having at least one unphysical particle, we have

$$\operatorname{Im} A_i = \int a_i \delta(p_2{}^2 - M_2{}^2)\delta(p_3{}^2 - M_3{}^2)\delta^4(p_1 - p_2 - p_3) \times \theta(p_{20})\theta(p_{30}) d^4p_2 d^4p_3,$$

where

$$a_1 \equiv -|Z[f^b(p_2)f_s^c(p_3)]|^2 = 0, \tag{31}$$

$$a_2 \equiv +\tfrac{1}{2}|Z[f_s^b(p_2)f_s^c(p_3)]|^2 = \tfrac{1}{2}\sum_{b,c} |-g\epsilon_{acb} e\cdot p_2|^2, \tag{32}$$

$$a_3 \equiv +\tfrac{1}{2}|Z[\phi^b(p_2)\varphi^c(p_3)]|^2 = \tfrac{1}{2}\sum_{b,c} |-g\epsilon_{acb} e\cdot p_2|^2, \tag{33}$$

$$a_4 \equiv -|Z[D^b(p_2)D_c'(p_3)]|^2 = -\sum_{b,c} |-g\epsilon_{acb} e\cdot p_2|^2, \tag{34}$$

$$a_5 \equiv +|Z[\mathfrak{U}(p_2)\varphi^b(p_3)]|^2 = \sum_b |g\delta_{ab} e\cdot p_3|^2, \tag{35}$$

$$a_6 \equiv -|Z[f_s^b(p_2)\mathfrak{U}(p_3)]|^2 = -\sum_b |ig\delta_{ab} e\cdot p_3|^2, \tag{36}$$

and therefore $\operatorname{Im} A = \sum_i \operatorname{Im} A_i = 0$, where e_μ is the polarization vector of $f^a(p_1)$, $e\cdot p_1 = 0$, and we have used the property of symmetric phase-space integration when $M_2 = M_3$. The over-all minus sign in a_1, a_6, and a_4 is due to the negative metric of f_s or the scalar-fermion loop.

In the case of two-loop unitarity the phase space can be partitioned into six distinct sectors depending on the masses of the particles in the intermediate states. The contributions to $\operatorname{Im} A$ from the various sectors must vanish independently for any ξ. We display here the calculation for the sector $\sum p_j{}^2 = 3M^2/\xi$. We denote the amplitude for the direct transition $f^a(p_1) \to I_i$ by $Z[I_i]$ and the amplitude for the two-step transition $f^a(p_1) \to w(p_2)v(k) \to I_i$ by $Y_i[w(p_2)v(k)]$, where w and v are some fields and $i = 1, 2, \ldots$. For the contribution of three unphysical particles in the intermediate step of the two-loop self-energy diagrams of $f^a(p_1)$ we have for $I_1 \equiv f_s^b(p_2)f_s^c(p_3)f_s^d(p_4)$

$$\begin{aligned} a_1 \equiv -\frac{1}{3!}\sum_{b,c,d} | &Y_1[f_s^b(p_2)f_\mu^e(k)] + Y_1[f_s^c(p_3)f_\mu^e(k)] + Y_1[f_s^d(p_4)f_\mu^e(k)] \\ &+ Y_1[f_s^b(p_2)\mathfrak{U}(k)] + Y_1[f_s^c(p_3)\mathfrak{U}(k)] + Y_1[f_s^d(p_4)\mathfrak{U}(k)] + Z[I_1]|^2 \\ = -\frac{1}{3!}\sum_{b,c,d} &\left| \frac{-ig^2}{2M}\{[3\epsilon_{\beta ca}\epsilon_{\beta db} + 2K(\delta_{ad}\delta_{bc}T_{14} - \delta_{ab}\delta_{cd}T_{12})]E_2 + [3\epsilon_{\beta ba}\epsilon_{\beta dc} + 2K(\delta_{ad}\delta_{bc}T_{14} - \delta_{ac}\delta_{bd}T_{13})]E_3\} \right|^2, \end{aligned} \tag{37}$$

for $I_2 \equiv f_s^b(p_2)\phi^c(p_3)\varphi^d(p_4)$

$$\begin{aligned} a_2 \equiv -\frac{1}{2!}\sum_{b,c,d} | &Y_2[f_s^b(p_2)f_\mu^e(k)] + Y_2[\phi^c(p_3)\mathfrak{U}(k)] + Y_2[\phi^d(p_4)\mathfrak{U}(k)] \\ &+ Y_2[f_s^b(p_2)\mathfrak{U}(k)] + Y_2[\varphi^c(p_3)\phi^e(k)] + Y_2[\varphi^d(p_4)\phi^e(k)] + Z[I_2]|^2 \\ = -\frac{1}{2!}\sum_{b,c,d} &\left| \frac{ig^2M}{2\xi}[\delta_{ad}\delta_{bc}T_{14}(E_2 + E_3) - \delta_{ac}\delta_{bd}T_{13}E_3] \right|^2, \end{aligned} \tag{38}$$

for $I_3 \equiv f_s^b(p_2)\varphi^c(p_3)f_s^d(p_4)$

$$\begin{aligned} a_3 \equiv \frac{1}{2!}\sum_{b,c,d} | &Y_3[f_s^b(p_2)\mathfrak{U}(k)] + Y_3[f_s^d(p_4)\mathfrak{U}(k)] + Y_3[\phi^c(p_3)\mathfrak{U}(k)]|^2 \\ = \frac{1}{2!}\sum_{b,c,d} &\left| -\frac{g^2}{2M}\{[\epsilon_{\beta ac}\epsilon_{\beta bd} + K(\delta_{ad}\delta_{cb}T_{14} - \delta_{ab}\delta_{cd}T_{12})]E_2 + [\epsilon_{\beta ab}\epsilon_{\beta cd} + K(\delta_{ad}\delta_{cb}T_{14} - 2\delta_{ac}\delta_{bd}T_{13})]E_3\} \right|^2, \end{aligned} \tag{39}$$

for $I_4 \equiv f_s^b(p_2)D'^c(p_3)D^d(p_4)$

$$a_4 \equiv \sum_{b,c,d} | Y_4[f_s^b(p_2)f_\mu^e(k)] + Y_4[D'^c(p_3)D^e(k)] + Y_4[D^d(p_4)D'^e(k)] + Y_4[f_s^b(p_2)\mathfrak{U}(k)]|^2$$

$$= \sum_{b,c,d} \left| \frac{ig^2}{2M}\{\epsilon_{\beta ca}\epsilon_{\beta db}(1+KM_{13})E_3 - \epsilon_{\beta da}\epsilon_{\beta cb}(1-KM_{14})(E_2+E_3) - \epsilon_{\beta ba}\epsilon_{\beta dc}[(1+KM_{12})E_2+2E_3] + \delta_{ab}\delta_{cd}KT_{12}E_2\} \right|^2, \quad (40)$$

and for $I_5 \equiv \phi^b(p_2)D'^c(p_3)D^d(p_4)$

$$a_5 \equiv -\sum_{b,c,d} | Y_5[D'^c(p_3)D^e(k)] + Y_5[D^d(p_4)D'^e(k)] + Y_5[\phi^b(p_2)\phi^e(k)] + Y_5[\phi^b(p_2)\mathfrak{U}(k)]|^2$$

$$= -\sum_{b,c,d} \left| \frac{g^2M}{2\xi}[\delta_{ab}\delta_{cd}T_{12}E_2 + \epsilon_{\beta ca}\epsilon_{\beta db}M_{13}E_3 + \epsilon_{\beta da}\epsilon_{\beta cb}M_{14}(E_2+E_3) - \epsilon_{\beta ba}\epsilon_{\beta dc}M_{12}E_2] \right|^2, \quad (41)$$

where $K \equiv M^2/\xi$, $T_{ij} = 1/(p_i - p_j)^2$, $M_{ij} = 1/[(p_i - p_j)^2 - M^2]$, and $E_i \equiv e_\mu \cdot p_i^\mu$ with $i, j = 1, 2, 3, 4$. As before we have

$$\mathrm{Im}A_i = \int a_i \delta(p_2^2 - M^2/\xi)\delta(p_3^2 - M^2/\xi)\delta(p_4^2 - M^2/\xi)\, \delta^4(p_1 - p_2 - p_3 - p_4)\theta(p_{20})\theta(p_{30})\theta(p_{40})d^4p_2d^4p_3d^4p_4,$$

from which it follows that $\mathrm{Im}A = \sum_i \mathrm{Im}A_i = 0$ because p_2, p_3, and p_4 are symmetric in the phase-space integration. We have also carried out the calculation for the sector $\sum p_j^2 = M^2/\xi$ with the same result. Furthermore, using the path integral method, one can give a general and formal proof of the unitarity and gauge independence of the theory. (See Sec. V.)

We note that the amplitudes a_1–a_5 in Eqs. (37)–(41) for the imaginary part of the two-loop self-energy of the vector boson f^a are different from those obtained by using the Feynman rules in Ref. 2 or 9. To be specific, if one uses the Feynman rules in Ref. 2 or 9 there will be no amplitudes a_2 and a_3, and the amplitudes a_1, a_4, and a_5 will be different. This is due to the fact that the Lagrangian (1) involves scalar fields and, therefore, is different from the massive Yang-Mills Lagrangian in Ref. 2. An important difference between this paper and Ref. 2 is that the Feynman rules related to fictitious scalar fermions in this paper are derived without using any constraint, while those in Ref. 2 are derived by using a constraint. The use of a constraint in Ref. 2 leads to complication of unitarity beyond one-loop diagrams as discussed in the "note added" in that paper and also discussed in Ref. 9 by Mohapatra *et al*. In the present work, the Lagrangian (1) includes scalar fields in such a way that it has a symmetry structure (cf. Ref. 13) and, furthermore, the use of a constraint is avoided and hence the complication of unitarity beyond one-loop diagrams in Ref. 2 is resolved at the same time.

V. REMARKS AND CONCLUSION

The mass term $M^2\vec{f}_\mu{}^2/2$ destroys the usual gauge invariance of the Lagrangian $\mathcal{L}$ in (1), yet the resultant theory is independent of the parameter ξ. This is a reflection of the fact that the unphysical particles with mass $M/\sqrt{\xi}$ in the Lagrangian are not observable, just as the gauge independence of a non-Hermitian field derives from the nonobservability of its phase. Although the Lagrangian $\mathcal{L}_1$ in (1) is not invariant under the *usual* gauge transformation, the structure of couplings in (1) is still highly symmetric and this theory can be described as possessing a new type of "gauge invariance." We emphasize that in deriving the field equation (7) for $\vec{\chi}$, all the complicated nonrenormalizable source terms (which cannot be tolerated in a renormalizable theory) cancel completely, just as in the gauge-invariant case (i.e., $\mathcal{L}$ with $M=0$). In this sense, the intimate relation between dynamics and symmetry in the gauge-invariant Lagrangian $\mathcal{L}$ with $M=0$ is not disturbed at all by the presence of the terms with the factor M in $\mathcal{L}_1$ because of the *intrinsic* symmetry breakdown discussed in Sec. I. This is why the present theory for the massive Yang-Mills field is renormalizable.[3,10] We may remark that there is no genuine infrared divergence in the theory even though the $\mathfrak{U}$ field is massless. This has been checked at the one-loop level.

If one applies the Lagrange multiplier formalism of Sec. II to the Georgi-Glashow theory and the Weinberg unified theory[11] and repeats the steps

(1)–(15),[12] one obtains the same effective Lagrangians as those in the usual R-gauge formalism.[6] This indicates the general validity of our formalism. On the basis of this formulation, we have verified the unitarity and gauge independence of several nontrivial S-matrix elements. This indicates that the fictitious Lagrangian (14) is indeed working. [In fact, one can give a general and formal proof of the unitarity and gauge independence of the renormalized physical amplitudes to all orders by using the "distorted" local gauge symmetry[4] of the Lagrangian $\mathcal{L}_1$ in (1) and the path integral.[13]]

To conclude, we have shown, to the two-loop level, that one can construct a unitary and renormalizable theory of the massive Yang-Mills field with an *intrinsic* symmetry breakdown and hence without resorting to the device of spontaneous symmetry breakdown.

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