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String approaches to hadron structure*

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The two principal string approaches to a more realistic dual resonance model are discussed. Firstly, Nambu's proposal of 1974, identifying the Dirac magnetic-monopole string with the dual string, after spontaneous breakdown and the Higgs mechanism in a strong-coupling limit, is investigated. The mathematics underlying Nambu's rather intuitive derivation has been further investigated by Balachandran, Rupertsberger, and Schechter (who put in the vector mass by hand) and independently by Jevicki and Senjanović (who fully exploit spontaneous breaking and the Higgs mechanism). Here we show that in a leading approximation to the Nambu monopole action the phenomenologically desirable linearity of the leading Regge trajectory seems to be badly violated. Secondly, alternative quantization procedures for the original 1970 Nambu relativistic string action (area of the world sheet) are treated; in particular, a timelike identification of the string time τ which has recently been advocated by Patrascioiu, by Rohrlich, and by Goddard, Hanson, and Ponzano. The most novel of these discussions seems to be that of Rohrlich, who uses a quite different representation for the canonical algebra and the Poincaré group. Here we demonstrate, however, that the usual unphysical level spectrum with a massless first excited state emerges as a fully consistent solution even in this approach, and that probably no other solution exists. Finally, the various Nambu string approaches are compared to other attempts to discover the "right" model of strong interactions.

I. INTRODUCTION

The reinterpretation of dual resonance models as a theory of interacting strings reveals an appealing physical and intuitive picture of the hadron internal structure. From the string viewpoint it has been possible to reproduce faithfully all results, at least at the tree-diagram level, previously known in the conventional dual theory.

The usefulness of the string approach would, however, be even further enhanced if it could lead us in some new direction towards more realistic models. It is natural, therefore, that extensive efforts have been made by theorists towards this goal; the two principal approaches are (i) the magnetic monopole string and (ii) the use of alternative quantization procedures. In the present paper we shall treat both of these possibilities.

In the first method, the basic idea due to Nambu[1] is to identify the Dirac monopole string[2] with the dual string. For massless photons the Dirac string is unphysical but if one arranges that the gauge vector field acquire mass through spontaneous breakdown and the Higgs mechanism then the string becomes a physical entity and in the strong-coupling limit of very large vector masses (m_V) and Higgs scalar masses (m_S) becomes essentially identical to the dual string. The strong-coupling limit involved is of the type first suggested by Nielsen and Olesen;[3] the mathematics underlying Nambu's result has been studied by Balachandran, Rupertsberger, and Schechter[4] (who put in the vector mass by hand) and independently by Jevicki and Senjanović[5] (who use the Higgs mechanism). In Sec. II we study the classical leading Regge trajectory of the magnetic monopole string. As already mentioned, in the "sharp" string limit of infinite m_V and m_S the trajectory is linear but it is shown that keeping the next-order terms in m_S^{-1} and m_V^{-1} where the string is not purely one-dimensional leads to nonlinearity. It is found that for reasonable values of the relevant parameters, the degree of nonlinearity is phenomenologically unacceptable when compared, for example, with the degree of linearity implied by the observed masses of the ρ, f, and g mesons.

The second method reverts to the original Nambu string action[6] proportional to the area of the world sheet, and emphasizes identification of the string-time variable τ with the coordinate component x_0 rather than with the lightlike x_+. This has been studied by Patrascioiu,[7] by Rohrlich[8], and by Goddard, Hanson, and Ponzano.[9] The most novel of these is the center-of-mass approach advocated by Rohrlich;[8] in Sec. III we analyze the structure of the Hilbert space spanned by the center-of-mass physical states. It is shown that for general ground state mass this space comprises a purely transverse space together with just *one* extra physical state at the first excited level. If the first excited state is massless, however, the space is purely transverse and consistent with Poincaré invariance for space-time dimension $d=26$ as in the conventional case. For more general ground-state mass, it appears that the single extra state is inadequate to allow a realization of the Poincaré generators.

Finally, in Sec. IV we compare the string approach to alternative methods of finding the "right" model for strong interactions.

II. MAGNETIC MONOPOLE STRING AND NONLINEAR REGGE TRAJECTORIES

For a string between two massive magnetic monopoles of mass M and magnetic charges $g, -g$ in a Higgs-type theory with vector mass m_V Higgs scalar mass m_S, Nambu[1] has arrived at the following expression for the classical action which will be the starting point of our present discussion.

$$S=-\frac{g^2 m_V^2}{4}\int_0^\pi d\tilde{\sigma}\int_{-\infty}^{\infty} d\tilde{\tau}\int_0^\pi d\sigma\int_{-\infty}^{\infty} d\tau\, J\begin{pmatrix}\tilde{x}_\mu & \tilde{x}_\nu\\ \tilde{\sigma} & \tilde{\tau}\end{pmatrix}\Delta_{m_V}(\tilde{x}-x)\, J\begin{pmatrix}x_\mu & x_\nu\\ \sigma & \tau\end{pmatrix}$$
$$-g^2\int_{-\infty}^{\infty} d\tau_0\int_{-\infty}^{\infty} d\tau_\pi\, \dot{x}_\mu(0,\tau_0)\,\Delta_{m_V}(x(0,\tau_0)-x(\pi,\tau_\pi))\,\dot{x}_\mu(\pi,\tau_\pi)$$
$$+M\left(\int_{-\infty}^{\infty} d\tau_0[\dot{x}^2(0,\tau_0)]^{1/2}+\int_{-\infty}^{\infty} d\tau_\pi[\dot{x}^2(\pi,\tau_\pi)]^{1/2}\right), \tag{1}$$

where J is a Jacobian and

$$\Delta_{m_V}(x)=\frac{1}{(2\pi)^4}\int\frac{d^4k\, e^{ik\cdot x}}{k^2-m_V^2} \tag{2}$$

is the Green's function for the vector field. The scalar mass m_S does not occur explicitly but is related to the transverse cutoff in k_μ as explained below.

The physical meaning of the three terms in Eq. (1) is as follows: the first is the Yukawa interaction between surface elements, the second is the Yukawa interaction between the magnetic currents of the monopoles, and the third is the mechanical mass term for the monopoles.

The expression is formal to the extent that the first two terms diverge but as explained by Nambu[1] the divergence is controlled by a cutoff in the transverse momentum $\vec{k}$ at $k^2=m_S^2$. Then, by choosing at a point (σ,τ) on the world sheet a coordinate system where

$$x'_\mu=\delta_{\mu 3}, \tag{3}$$

$$\dot{x}_\mu=\delta_{\mu 0}, \tag{4}$$

one can evaluate

$$\int d\sigma\, d\tau\,\Delta(\tilde{x}-x)J\begin{pmatrix}x_3 & x_0\\ \sigma & \tau\end{pmatrix}\doteq-\frac{1}{4\pi}\ln\left(1+\frac{m_S^2}{m_V^2}\right) \tag{5}$$

so that the first term in Eq. (1) becomes precisely

$$\frac{1}{2\pi\alpha'}\int d\sigma\, d\tau[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}, \tag{6}$$

where

$$\alpha'=\frac{8}{g^2m_V^2}\left[\ln\left(1+\frac{m_S^2}{m_V^2}\right)\right]^{-1}. \tag{7}$$

The second term in Eq. (1), describing the Yukawa interaction between the monopole currents, can be treated similarly. Then we find that the coefficient multiplying this term becomes

$$\frac{1}{(2\pi)^4}\int\frac{d^4k\, dx_0 e^{ik\cdot x}}{k^2-m_V^2}\doteq-\frac{1}{(2\pi)^3}\int\frac{d^3k}{\vec{k}^2+m_V^2} \tag{8}$$

$$=-\frac{1}{(2\pi)^3}4\pi\int_0^{m_S}\frac{k^2dk}{\vec{k}^2+m_V^2} \tag{9}$$

$$=-\frac{1}{(2\pi)^2}\left[m_S-m_V\tan^{-1}\left(\frac{m_S}{m_V}\right)\right], \tag{10}$$

where we chose the frame with $\dot{x}_\mu(0,\tau_0)=\delta_{\mu 0}$.

Now, from the Nielsen-Olesen approach,[3] we expect that $m_S\simeq m_V$, and both are large; therefore, $m_V^2\gg m_S$, and the second term becomes negligible compared to the first. Thus, in this lowest approximation the action becomes

$$S=\int_{\tau_i}^{\tau_f} d\tau\left[\int_0^\pi d\sigma\,\mathcal{L}+L_1+L_2\right], \tag{11}$$

with

$$\mathcal{L}=\frac{1}{2\pi\alpha'}[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}, \tag{12}$$

$$L_1=M[\dot{x}^2(0,\tau)]^{1/2}, \tag{13}$$

$$L_2=M[\dot{x}^2(\pi,\tau)]^{1/2}. \tag{14}$$

Although the nonlinearity of this system disallows superposition of special solutions and hence precludes finding the exact solution, one can find the classical leading Regge trajectory.[10] From experience with the $M=0$ case we know that this corresponds to the rigid-rotator mode which may be solved exactly as follows.

The equation of motion follows from Hamilton's principle of stationary action

$$0=\delta S=\int d\tau\left[\int d\sigma\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\delta\dot{x}+\frac{\partial\mathcal{L}}{\partial x'}\delta x'\right)+\frac{\partial L_1}{\partial\dot{x}(0,\tau)}\delta\dot{x}(0,\tau)+\frac{\partial L_2}{\partial\dot{x}(\pi,\nu)}\delta\dot{x}(\pi,\tau)\right], \tag{15}$$

giving

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right)+\frac{\partial}{\partial\sigma}\left(\frac{\partial\mathcal{L}}{\partial x'}\right)=0 \quad (0<\sigma<\pi), \tag{16}$$

$$\frac{\partial\mathcal{L}}{\partial x'}=-\frac{\partial}{\partial\tau}\left(\frac{\partial L_1}{\partial\dot{x}}\right) \quad (\sigma=0), \tag{17}$$

$$\frac{\partial\mathcal{L}}{\partial x'}=\frac{\partial}{\partial\tau}\left(\frac{\partial L_2}{\partial\dot{x}}\right) \quad (\sigma=\pi). \tag{18}$$

Now we adopt the gauge where $\tau=x_0$ to eliminate the time component. Then we have

$$L_1=M[1-\dot{\vec{x}}^2(0,\tau)]^{1/2}, \tag{19}$$

$$L_2=M[1-\dot{\vec{x}}^2(\pi,\tau)]^{1/2}, \tag{20}$$

$$\mathcal{L}=[(\dot{\vec{x}}\cdot\vec{x}')^2+\vec{x}'^2(1-\dot{\vec{x}}^2)]^{1/2}. \tag{21}$$

Separation of the σ,ν variables is according to

$$\vec{x}=\rho(\sigma)(\cos\omega\tau,\sin\omega\tau,0), \tag{22}$$

corresponding to uniform rotation about the 3 axis. Because of reparametrization invariance of Eq. (1) the results will not depend on the form of $\rho(\sigma)$ provided it is analytic.

One then finds

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{L}}{\partial\dot{\vec{x}}}\right)=-\frac{\partial}{\partial\sigma}\left(\frac{\partial\mathcal{L}}{\partial\vec{x}'}\right) \tag{23}$$

$$=\frac{\omega^2\rho\rho'}{2\pi\alpha'(1-\omega^2\rho^2)^{1/2}}(\cos\omega\tau,\sin\omega\tau,0) \tag{24}$$

so that Eq. (16) is satisfied identically. At $\sigma=0$ one finds

$$\frac{\partial}{\partial\tau}\left(\frac{\partial L_1}{\partial\dot{\vec{x}}}\right)=\frac{M\omega^2\rho}{(1-\rho^2\omega^2)^{1/2}}(\cos\omega\tau,\sin\omega\tau,0), \tag{25}$$

$$\frac{\partial\mathcal{L}}{\partial\vec{x}'}=\frac{(1-\omega^2\rho^2)^{1/2}}{2\pi\alpha'}(\cos\omega\tau,\sin\omega\tau,0), \tag{26}$$

and then Eqs. (17), (18) imply that

$$\omega\rho(0)=-\omega\rho(\pi)=\omega R \quad \text{(say)}, \tag{27}$$

where

$$\omega R=[1+(\pi\alpha' M\omega)^2]^{1/2}-\pi\alpha' M\omega. \tag{28}$$

The root of the quadratic is chosen such that $\omega R<1$, corresponding to velocity of the end points smaller than that of light.

The four-momentum density is

$$\mathcal{P}_\mu=\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}=\frac{x'_\mu(\dot{x}\cdot x')-\dot{x}_\mu x'^2}{2\pi\alpha'[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}}+\frac{Mx_\mu}{\sqrt{\dot{x}^2}}[\delta(\sigma)+\delta(\pi-\sigma)]. \tag{29}$$

In particular, the energy is

$$E=\int_0^\pi \mathcal{P}_0 d\sigma \tag{30}$$

$$=\int_0^\pi d\sigma\left(\frac{\rho'}{2\pi\alpha'(1-\rho^2\omega^2)^{1/2}}+\frac{M}{(1-\rho^2\omega^2)^{1/2}}[\delta(\sigma)+\delta(\pi-\sigma)]\right) \tag{31}$$

$$=\frac{1}{\pi\alpha'\omega}\left[\sin^{-1}(\omega R)+\frac{2\pi\alpha' M\omega}{(1-R^2\omega^2)^{1/2}}\right]. \tag{32}$$

The angular momentum is

$$J=\int_0^\pi d\sigma(x_1\mathcal{P}_2-x_2\mathcal{P}_1) \tag{33}$$

$$=\int_0^\pi d\sigma\frac{\omega^2\rho^2}{(1-\rho^2\omega^2)^{1/2}}\left(\frac{\rho'}{2\pi\alpha'}+M[\delta(\sigma)+\delta(\pi-\sigma)]\right) \tag{34}$$

$$=\frac{1}{2\pi\alpha'\omega^2}\left[\sin^{-1}(\omega R)+\omega R(1-\omega^2R^2)^{1/2}\right]+\frac{2M\omega R^2}{(1-R^2\omega^2)^{1/2}}. \tag{35}$$

A convenient parametrization is

$$\xi=2\pi\alpha' M/R, \tag{36}$$

$$Y(\xi)=\frac{2}{\pi}\sin^{-1}(1+\xi)^{-1/2}, \tag{37}$$

whereupon

$$J=\frac{R^2}{2\pi\alpha'}\left[(1+\xi)\frac{\pi}{2}Y+\sqrt{\xi}\right], \tag{38}$$

$$\alpha' E^2=\frac{R}{4\alpha'}(1+\xi)Y^2\left(1+\frac{2\sqrt{\xi}}{\pi Y}\right). \tag{39}$$

At low $\alpha' E^2$ the leading trajectory is curved downwards, and the intercept is lowered. Let us agree to adopt the viewpoint (for the lack of something better) that quantum fluctuations will *raise* the classical intercept by one unit in J, as in the $M=0$ case. Then we may choose a value[11,12] of M (~550 MeV), such that $J(E=0)=-\frac{1}{2}$ classically, as a reasonable value.

With such a choice the Regge trajectory appears on a Chew-Frautschi plot as indicated in Figs. 1(a) and 1(b). The two principal branches are indicated and we see that the degree of nolinearity is phenomenologically unacceptable compared to the timelike region of, say, the observed ρ-f-g trajectory where a linear formula $J=\frac{1}{2}+0.88M^2$ gives the correct masses of the three mesons ρ, f, g all to within an accuracy $\pm 3\%$ in mass.

Of course, this is only a leading approximation to the action, Eq. (1), and we may put the magnetic monopole mass $M=0$, for example. But then to the next order in $m_S{}^{-1}, m_V{}^{-1}$ the Yukawa term of Eq. (1) has an exactly similar effect. The nonlinear trajectories imply that any S-matrix element is likely to be, at best, much more complicated than the familiar dual resonance models.

III. ALTERNATIVE QUANTIZATION PROCEDURES

Let us return to the original Nambu action[6]

$$S=\frac{1}{2\pi\alpha'}\int d\sigma d\tau[(\dot{x}\cdot x')^2-\dot{x}^2x'^2]^{1/2}. \quad (40)$$

It is generally convenient (and presumably nonrestrictive) to exploit the (σ,τ) reparametrization invariance such that

$$\dot{x}^2+x'^2=0, \quad (41)$$

$$\dot{x}\cdot x'=0. \quad (42)$$

But there still remains a freedom in the identification of τ. Several authors[7,8,9] have suggested the identification of τ with x_0 (time) since one suspects that the reason one finds only the *standard solution* (i.e., corresponding to the Veneziano model spectrum of massless first excited state and space-time dimension $d=26$) after the identification $\tau=(1/\sqrt{2})(x_0+x_L)$, despite the fact that solutions with intercept $\alpha(0)<1$ and $d<26$ are consistent with the gauge conditions and positivity, is because the lightlike identification is itself restrictive. Thus, one would like to obtain the $\alpha(0)<1$, $d<26$ solutions by avoiding the lightlike identification of the string time τ.

Along this line of thought, there have been contributions by Patrascioiu,[7] Rohrlich,[8] and Goddard, Hanson, and Ponzano.[9]

Patrascioiu[7] has pointed out the singular nature of the lightlike gauge but did not explicitly construct a Poincaré-invariant physical Hilbert space.

Rohrlich[8] has suggested enmeshing the canonical algebra and the Poincaré algebra as follows. We take $\overline{X}_\mu, \overline{P}_\mu$ as "center-of-mass" or global coordinates of the system satisfying the noncovariant commutator

$$[\overline{X}_\mu,\overline{P}_\nu]=ig_{\mu\nu}-i\frac{\overline{P}_\mu}{\overline{P}_0}g_{\nu 0} \quad (43)$$

and where $(p^\mu=\dot{x}^\mu)$

$$\overline{X}^\mu=\frac{1}{\pi}\int_0^\pi d\sigma\, x^\mu(\sigma,\tau), \quad (44)$$

$$\overline{P}^\mu=\frac{1}{\pi}\int_0^\pi d\sigma\, p^\mu(\sigma,\tau), \quad (45)$$

$$x^\mu(\sigma,\tau)=\overline{X}^\mu+\overline{P}^\mu\tau+\frac{i}{\sqrt{\pi}}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{\alpha_n^\mu}{n}\cos n\sigma\, e^{-in\tau}. \quad (46)$$

Here the space components x^k, p^k, α_n^k satisfy the usual commutation relations (with α_n^k commuting

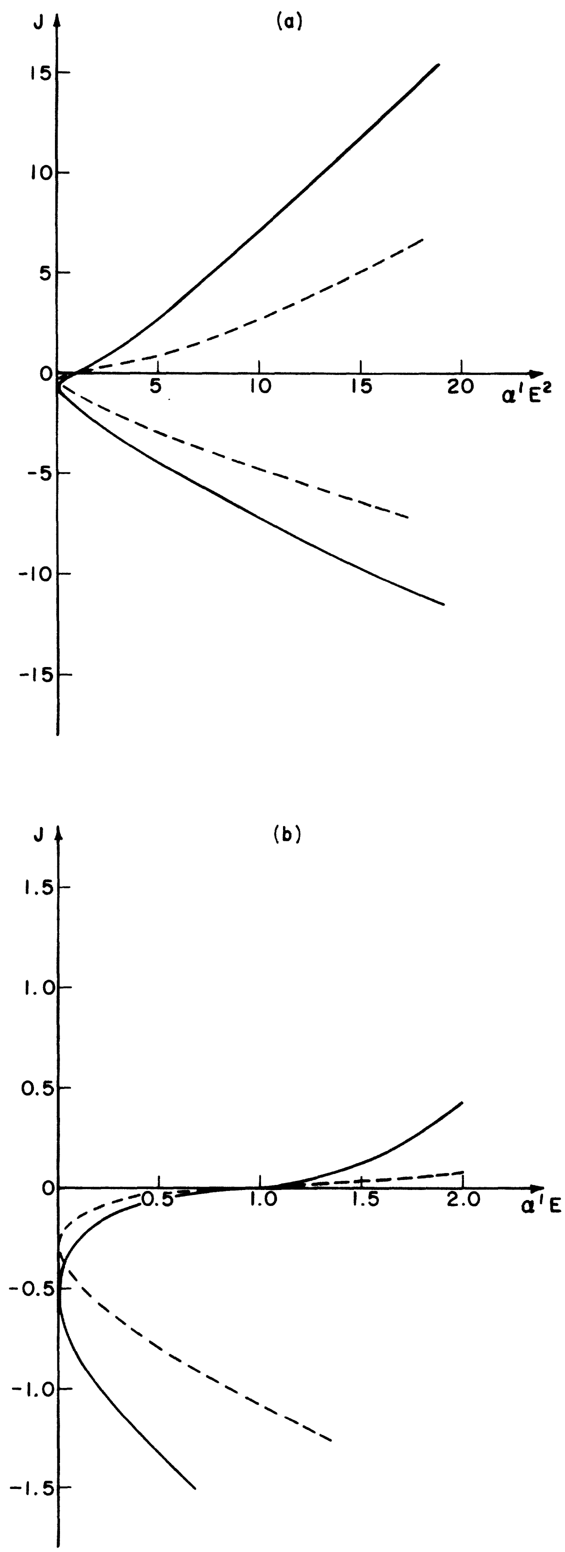

FIG. 1. (a) Chew-Frautschi plot corresponding to Eqs. (38) and (39) of the text; (b) same as (a) with enlarged scale.

with $\overline{X}^\mu, \overline{P}^\mu$). The Poincaré algebra is guaranteed by asserting that the Lorentz generators are

$$M^{\mu\nu}=\frac{1}{\pi}\int_0^\pi d\sigma(\tfrac{1}{2}\{x^\mu,p^\nu\}_+-\tfrac{1}{2}\{x^\nu,p^\mu\}_+). \tag{47}$$

The coordinate conditions

$$\dot{x}\cdot x'=p\cdot x'+x'\cdot p=0, \tag{48}$$

$$\dot{x}^2+x'^2=p^2+x'^2=0 \tag{49}$$

become then

$$L_n+\frac{1}{2\sqrt{\pi}}(\alpha_n\cdot\overline{P}+\overline{P}\cdot\alpha_n)=0, \tag{50}$$

$$M^2=-\overline{P}^2=L_0, \tag{51}$$

$$L_n=:\tfrac{1}{2}\sum_{m\neq 0}\alpha_m\cdot\alpha_{n-m}:. \tag{52}$$

In the case where we identify $\tau\sim x^0$, then $\alpha_n^0=0$ and one arrives eventually at (for $\overline{P}^k=0$, center-of-mass system);

$$\mathcal{L}_n|\phi\rangle=0,\quad n\geqslant 1, \tag{53}$$

$$\mathcal{L}_n=:\tfrac{1}{2}\sum_{m\neq 0}\underline{\alpha}_m\cdot\underline{\alpha}_{n-m}: \tag{54}$$

as the gauge conditions on the physical states. Normal ordering of $\mathcal{L}_0$ gives an arbitrary c number m_0^2

$$M^2=m_0^2+\mathcal{L}_0. \tag{55}$$

Thus far, therefore, Rohrlich's approach suffers no restriction on m_0^2 or on the space-time dimension d. We return to this later.

Goddard, Hanson, and Ponzano[9] have also studied the identification $\tau\sim x^0$. By leaning heavily on the dual model physical state construction they show that the standard solution can be obtained. In their work, however, a simplicity assumption is made concerning the Poisson brackets of the physical-state creation and annihilation operators; a certain auxiliary vector is chosen to be lightlike, but from experience with the dual model[13] we know that this auxiliary vector is associated with the mass of the first excited state. Hence one suspects that this assumption has prejudiced the solution and that by relaxing it the $\alpha(0)<1, d<26$ solutions might be found in a more thorough treatment.

The most novel of these three papers is that of Rohrlich,[8] since a different representation of the Poincaré group is used. Hence we will here complete the analysis of this case.

It is useful to decompose the Fock space $\mathfrak{F}$ spanned by the operators

$$[a^{i(m)},a^{j(n)\dagger}]=\delta_{mn}\delta_{ij}, \tag{56}$$

$$\left.\begin{aligned} a^{i(n)}&=\sqrt{n}\,\alpha_n^i \quad (57)\\ a^{i(n)\dagger}&=\sqrt{n}\,\alpha_{-n}^i \quad (58)\end{aligned}\right\} n\geqslant 1$$

into irreducible subspaces[14] with respect to the generalized projective algebra. Here $i,j=1,2,\ldots,s$ with $s=(d-1)$, the number of spacelike dimensions.

Let us write a general Fock-space state as

$$|f,N\rangle=\prod_{r,i}\frac{(a^{i(r)\dagger})^{\lambda_r^i}}{(\lambda_r^i!)^{1/2}}|0\rangle \tag{59}$$

so that

$$\mathcal{L}_0|f,N\rangle=N|f,N\rangle, \tag{60}$$

$$N=\sum_{r,i}r\lambda_r^i; \tag{61}$$

then the number of states at level N is $d^{(s)}(N)$, where

$$[p(x)]^s=\prod_{r=1}^\infty(1-x^r)^{-s}=\sum_{N=0}^\infty d^{(s)}(N)x^N. \tag{62}$$

We now define

$$\mathcal{L}_1'=\mathcal{L}_1+\hat{n}\cdot\vec{a}^{(1)}, \tag{63}$$

$$\mathcal{L}_1'^\dagger=\mathcal{L}_1^\dagger+\hat{n}\cdot\vec{a}^{(1)\dagger}, \tag{64}$$

where $\hat{n}$ is an arbitrary unit vector. Then, since

$$[\mathcal{L}_m,a^{(1)i}]=-(m+1)^{1/2}a^{(m+1)i}, \tag{65}$$

$$[\mathcal{L}_m^\dagger,a^{(1)i}]=(m-1)^{1/2}a^{(m-1)i\dagger}, \tag{66}$$

we have

$$[\mathcal{L}_m,\mathcal{L}_1']=(m-1)\mathcal{L}_{m+1}-(m+1)^{1/2}\hat{n}\cdot\vec{a}^{(m+1)}, \tag{67}$$

$$[\mathcal{L}_m^\dagger,\mathcal{L}_1']=-(m+1)\mathcal{L}_{m-1}^\dagger+(m-1)^{1/2}\hat{n}\cdot\vec{a}^{(m-1)\dagger}. \tag{68}$$

Now consider the subspace built on the vacuum state as follows (the notation is clumsy but accurate):

$$|\sigma,N,0,\boldsymbol{\alpha}\rangle=\prod_{i=2}^\infty(\mathcal{L}_i^\dagger)^{\mu_i}[(\mathcal{L}_1^\dagger)^{(\mu_1-1)\theta(\mu_1-3/2)}\times(\mathcal{L}_1'^\dagger)^{\theta(\mu_1-1/2)}]|0\rangle, \tag{69}$$

$$N=\sum_{i=1}^\infty i\mu_i. \tag{70}$$

The notation in Eq. (69) means that if $\mu_1=0$, the expression in square brackets is unity, if $\mu_1=1$ it is $\mathcal{L}_1'^\dagger$, if $\mu_1\geqslant 2$ it is $(\mathcal{L}_1^\dagger)^{\mu_1-1}\mathcal{L}_1'^\dagger$. Now the number of states $|\sigma,N,0,\boldsymbol{\alpha}\rangle$ with given N is simply $d^{(1)}(N)$. The first state ($N=1$, $\mu_1=1$) is physical; all others are spurious.

At $N=1$, we now construct the remaining physical states orthogonal to states built on the vacuum state. On these we then build irreducible subspaces of the type

$$|\sigma,N+1,1,\boldsymbol{\alpha}\rangle=\prod_{i=1}^\infty(\mathcal{L}_i^\dagger)^{\mu_i}|\phi,1,\boldsymbol{\alpha}\rangle. \tag{71}$$

Proceeding iteratively we set up a general irreducible subspace on a physical state $|\phi,N',\boldsymbol{\alpha}\rangle$ at level $\mathfrak{L}_0 = N$ by

$$|\sigma, N+N', N, \alpha\rangle = \prod_{i=1}^{\infty} (\mathfrak{L}_i^{\dagger})^{\mu_i} |\phi, N', \alpha\rangle. \qquad (72)$$

The subspaces are irreducible in the following sense: By operation with the generators of the generalized projective algebra we can get from any state of the subspace to any other state but we cannot move outside of the subspace nor can we move from a state in $\mathfrak{F}$, but outside of the subspace, back into the subspace.

The number of $|\phi, N, \alpha\rangle$ states at the level N is ρ_N, where

$$d^{(s)}(N) = \sum_{m=0}^{N} \rho_m d^{(1)}(N-m), \qquad (73)$$

which has the solution

$$\rho_N = d^{(s-1)}(N) \qquad (74)$$

since, in general, it is easy to show that

$$d^{(s)}(N) = \sum_{n=0}^{N} d^{(s-r)}(n) d^{(r)}(N-n). \qquad (75)$$

All of the $|\sigma, N+N', N', \alpha\rangle$ states are spurious except one,

$$\mathfrak{L}_1'^{\dagger}|0\rangle, \qquad (76)$$

so that the number of physical states at level $L_0 = N$ satisfying Eq. (53) is given by

$$d^{(s-1)}(N) + \delta_{N,1}. \qquad (77)$$

Thus, we have the curious situation that the solutions of the center-of-mass gauge conditions are isomorphic to purely transverse states, except that there is just *one* additional physical state, at the first excited level. We know physically that this extra state must be present for a massive vector first excited state; *a priori* one might have expected more additional states, but the above analysis is rigorous and accurate.

So far we have taken general m_0^2 in Eq. (55). For the special case $m_0^2 = -1$, however, an anomalous situation exists. It is simplest to discuss the anomalous case first. Here the first excited state is massless and we *cannot* transform to $\overline{P}^k = 0$. Hence one must use Eq. (50) rather than Eq. (53) at this level whereupon the additional physical state goes away, leaving a purely transverse physical Hilbert space $\mathcal{P}$ spanned by $(s-1)$-dimensional operators

$$[A^{(r)i}, A^{(s)j\dagger}] = \delta_{rs}\delta_{ij} \qquad (78)$$

as usual. It is well known how to realize the Poincaré generators on these operators: For the subgroup $O(S-1)$, one writes

$$M^{ij} = \sum_{n=1}^{\infty} (A^{(n)i\dagger}A^{(n)j} - A^{(n)j\dagger}A^{(n)i}) \qquad (79)$$

and for the remaining generators of $O(S)$ (involving a longitudinal direction L, arbitrarily chosen)

$$M^{Li} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} (\tilde{\mathfrak{L}}_n^{\dagger} A^{(n)i} - A^{(n)i\dagger}\tilde{\mathfrak{L}}_n), \qquad (80)$$

where the $\mathfrak{L}_n$ are constructed as in Eq. (52) but replacing the S-dimensional α_m by the $(S-1)$-dimensional $\vec{A}^{(m)}$, that is,

$$\tilde{\mathfrak{L}}_n = \sum_{r=1}^{\infty} [r(r+n)]^{1/2} \vec{A}^{(r)\dagger}\cdot\vec{A}^{(r+n)} - \tfrac{1}{2}\sum_{r=1}^{n-1} [r(n-r)]^{1/2}\vec{A}^{(r)}\cdot\vec{A}^{(n-r)}. \qquad (81)$$

One then finds that (cf. Rebbi, Ref. 6)

$$[M^{Li}, M^{Lj}] = 2[\tilde{\mathfrak{L}}_0 - \tfrac{1}{24}(s-1)]M^{ij} - [\tfrac{1}{24}(s-1) - 1]\sum_{n=1}^{\infty} n^2 (A^{(n)i\dagger}A^{(n)j} - A^{(n)j\dagger}A^{(n)i}) \qquad (82)$$

so that the algebra closes only if $S = 25$ ($d = 26$).

Therefore, the purely transverse physical space is consistent with Poincaré invariance only if $S = 25$. This standard solution is hence one consistent solution of the Rohrlich approach. The interesting question then is the following: Is there any solution when $m_0^2 \neq -1$? Although we cannot prove the impossibility of such a solution rigorously, let us now show that it is unlikely.

We have the problem of modifying the generators, Eq. (81), such that the final term in Eq. (82) is removed. The only extra freedom is given by the occurrence of *one* extra physical state, which we write

$$A^{(1)L\dagger}|0\rangle. \qquad (83)$$

We may try to extend the $O(S-1)$ algebra implied by Eq. (80) to an enlarged $O(S)$ algebra in the weak sense that

$$\langle\phi'|[\hat{M}^{Li}, \hat{M}^{Lj}]|\phi\rangle = \sum_{\lambda} [\langle\phi'|\hat{M}^{Li}|\phi_\lambda\rangle\langle\phi_\lambda|\hat{M}^{Lj}|\phi\rangle - \langle\phi'|\hat{M}^{Lj}|\phi_\lambda\rangle\langle\phi_\lambda|\hat{M}^{Li}|\phi\rangle] = -\langle\phi'|\hat{M}^{ij}|\phi\rangle. \qquad (84)$$

One may therefore attempt to write

$$\hat{M}^{Li} = M^{Li} + (A^{(1)L\dagger}A^{(1)i} - A^{(1)i\dagger}A^{(1)L}). \qquad (85)$$

But detailed study shows that one cannot modify $\hat{M}^{ij}$ in any way that Eq. (84) is satisfied. This leads to the conclusion that the single additional state, designated in (83), is inadequate to allow the $O(S-1) \to O(S)$ extension.

We cannot rule out the possibility of a solution for the following reason. We have taken, as the $\mathcal{L}_n$ in Eq. (81), a *bilinear* representation of the generalized projective algebra identical to that occurring in the gauge conditions Eq. (53); there seems to be no *a priori* reason why these representations should be identical and hence the representation in Eq. (81) could be quadrilinear (for example). It is not impossible, although we believe it unlikely, that consideration of such representations could lead to a $\alpha(0)<1$, $d<26$ solution.

IV. SUMMARY AND DISCUSSION

The two principal methods, currently under study, of obtaining a more physical dual resonance theory from the string approach are the magnetic monopole string and the alternative quantization procedures.

The magnetic monopole string is based on a strong-coupling limit of a spontaneously broken local field theory. One possibility here is that by stopping short of the strong-coupling limit—that is, finite m_V and m_S—one will maintain consistency in the original space-time dimension $d=4$. The string will then be "fuzzy" rather than a "sharp" one-dimensional object. But the price paid is that the action and equations of motion become nonlinear and this precludes a general solution. More serious, as we have seen, the original linear Regge trajectories which are attractive phenomenologically become badly distorted.

The alternative quantization procedures suppose that the restriction to a massless first excited state in the relativistic quantum string is imposed by the lightlike identification of the string time τ. The simplest solution to a timelike identification of τ is, however, always of the same character, although in the different formulations of this there is always a technical algebraic assumption that might allow $\alpha(0)<1$, $d<26$ solutions to have escaped. But are these more complicated solutions worth finding? The answer is probably not, since it seems very likely that they correspond to the unphysical types of solution hinted at already by studies[15] of the operator formalism in the original Fock space. For a more realistic model, one almost certainly needs to add extra degrees of freedom which are *absent* in the elegant Nambu action, Eq. (40).

To put the string approach in context we should recall that the four methods currently under study for obtaining an even better dual resonance model are (i) writing S-matrix elements with physical intercepts directly,[16] (ii) spontaneously breaking existing dual models by exploiting the tachyon as a virtue,[17] (iii) writing new realizations of the generalized projective algebra in an operator basis, and (iv) investigating strings. It is natural that the last is the most attractive since it is most closely allied to field theory; from the S-matrix viewpoint, however, the first two have certain advantages such as ensuring good analytic and asymptotic properties.

What is needed in the string approach appears to be a "sharp" one-dimensional string with some unknown additional degrees of freedom going beyond anything described in the present paper or any of its references. It is already established beyond question that the Nambu action provides a profound understanding of the known dual model; its general features such as the one-dimensionality and the topological properties of the string interactions will probably persist. The "right" model of strong interactions requires an essential modification to the string (extra degrees of freedom) that no author at present seems able to provide.

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