Point transformation and renormalization in the unitary gauge for non-Abelian fields

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It is shown by means of a model that the renormalization and unitary gauges for non-Abelian fields can be connected by a point transformation, and this fact is used to construct a formal proof of renormalization in the unitary gauge. The formal proof is then verified by demonstrating that for a fourth-order on-shell scattering process the S matrix calculated directly in the unitary gauge is exactly equal to that calculated in the renormalization gauge. The calculation is refined to the point where it becomes purely graphical and this allows one to see by inspection how the cancellation of divergences occurs in the unitary gauge, and to trace this cancellation to the spontaneous-symmetry-breaking mechanism in the Lagrangian.

I. INTRODUCTION

In a previous paper,[1] which we shall refer to as paper I, it was shown for an Abelian model that the renormalization and unitary gauges[2] could be connected by a point transformation, and this fact was used to construct a formal proof to all orders of renormalizability in the unitary gauge, and to verify the proof in fourth order by means of an explicit calculation. In the present paper we show that the same procedure can be applied to a non-Abelian model.

In the Abelian model the mechanism by which the cancellation of divergences in the unitary gauge (and ghosts in the renormalization gauge) takes place was made transparent by reducing the calculation to a purely graphical one. In the present paper we repeat the reduction of the proof to a purely graphical one, but we also go a step further. That is to say, we group the graphs so that they exhibit in a very obvious way the crucial role that is played by the scalar-vector interaction which is induced by the spontaneous-symmetry-breaking mechanism, and so would not be present in a "naive," gauge-noninvariant Lagrangian.

For the explicit calculation, by which we verify the equality of the S matrix in the renormalization and unitary gauge in fourth order, we shall use the canonical interaction Hamiltonian formalism. The reason that we prefer to use the canonical formalism, rather than, say, a dispersion calculation,[3] is that it allows us to insert renormalization counterterms in the Lagrangian in a simple gauge-invariant way, and to calculate from first principles not only the Feynman graphs with structure, but also to calculate all the renormalization constants in each gauge. The calculation of the renormalization constants actually requires also that we take into account the fact that the gauge transformation is for second-quantized fields, and it is here that the point transformation plays a role, because, as we shall see, the point transformation allows us to calculate the correction due to second quantization in a simple way.

In practice, what we shall show is that the *difference* between the S matrices calculated in the unitary and renormalization gauges is zero. It turns out that to evaluate the contribution to this difference from various graphs, we do not have to actually calculate any Feynman integrals, but only to use their invariance under translations in momentum space. Thus, any form of regularization which preserves this translational invariance of the Feynman graphs will suffice. We rely on the fact that at least one such regularization scheme (i.e., dimensional regularization) has been shown to exist.[2]

The non-Abelian model which we shall consider is a very simple one in which we have an SU(2) gauge group carried by a fermion doublet (e, ν), a vector-meson triplet $\vec{A}$, and a scalar triplet $\vec{\phi}$. The fermion–vector-meson interaction is vectorial, and the nonconservation of the fermion current $\vec{J}_\mu$ results not from axial-vector coupling, but rather from a mass difference $m = m_e - m_\nu$ which we assume for the fermions. The fourth-order process which we shall use to verify the formal equivalence of the S matrices in the unitary and renormalization gauges will be e-ν scattering.

II. BASIC LAGRANGIAN AND COUNTERTERMS

The unified gauge theory which we shall consider is given by the broken SU(2)-gauge Lagrangian

$$\mathcal{L} = -\tfrac{1}{8}\,\mathrm{tr}F_{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{D} - m_0)\psi + \frac{g}{2}\bar{\psi}\pi\psi + \tfrac{1}{4}\,\mathrm{tr}[D_\mu,\pi]^2 + \frac{\kappa^2}{8}\left(\mathrm{tr}\pi^2 - \frac{g^2}{2m^2}\mathrm{tr}\pi^4\right),$$

$$\psi = \begin{pmatrix} e \\ \nu \end{pmatrix}, \quad D_\mu = \partial_\mu - \frac{ie}{2}\vec{A}_\mu \cdot \vec{\tau}, \tag{2.1}$$

$$F_{\mu\nu} = \frac{2i}{e}[D_\mu, D_\nu], \quad \pi = \vec{\tau}\cdot\vec{\phi} - \frac{m}{g}\tau_3.$$

Under a gauge transformation the fields transform as follows:

$$\psi \to S\psi, \quad D_\mu \to S D_\mu S^{-1}, \quad \pi \to S\pi S^{-1},$$

where (2.2)

$$S = e^{i\vec{\lambda}(x)\cdot\vec{\tau}}.$$

Here $\vec{\tau}$ are the Pauli matrices, $\langle\vec{\phi}\rangle = 0$, m_0, e, g, m, and κ are constant parameters, and the last term is arranged so that it has no linear terms in the fields, but has a mass term $-\frac{1}{2}\kappa^2\phi_3{}^2$ for ϕ_3. The field $A_\mu^{(3)}$ is massless and is coupled to a conserved current, and hence may be regarded as a photon field. The fields ϕ_1, ϕ_2 are the massless Goldstone bosons, and the unitary and renormalization gauges will be taken to be the gauges $\phi_1 = \phi_2 = \partial_\mu A_\mu^{(3)} = 0$ and $\partial_\mu \vec{A}_\mu = 0$, respectively.

The mass-renormalization counterterms which we choose for the Lagrangian (2.1) are

$$\delta\mathcal{L} = \delta m_0 \bar{\psi}\psi - \delta m \frac{g}{2m}\bar{\psi}\pi\psi + \frac{\delta a}{4}\mathrm{tr}[D_\mu, \pi]^2 + \delta b\,\mathrm{tr}\pi^2 + \delta c\,\mathrm{tr}\pi^4 - \frac{\delta\mu}{2}(A_\mu^{(3)})^2. \tag{2.3}$$

These counterterms are manifestly invariant under the broken gauge transformation which takes us from the renormalization to the unitary gauge, since the first five terms are manifestly invariant under any broken gauge transformation, and $A_\mu^{(3)}$ is the same in both gauges. [We could actually take the self mass of the photon field $A_\mu^{(3)}$ to be zero by using the usual gauge-invariance arguments of QED, but we find it more convenient to insert the counterterm $\delta\mu$ in (2.3).] The first two terms in (2.3) provide mass-renormalization terms for e and ν, and thereby also induce the charge renormalization $-\delta m\,(g/2m)\bar{\psi}\vec{\tau}\cdot\vec{\phi}\psi$. The δa and δb, and δc terms provide mass counterterms for the $A_\mu^{(1,2)}$ and $\vec{\phi}$ fields, respectively, inducing in the same way some meson charge renormalizations and interactions (we shall see later that the linear terms in $\delta b\,\mathrm{tr}\pi^2$ cancel against normal ordering and self-mass terms). We omit ϕ^4 renormalization terms in (2.3) (other than those induced in δc) as these do not contribute to the process which we shall consider. Finally, having written the term $\frac{1}{4}\delta a\,\mathrm{tr}[D_\mu, \pi]^2$ in this form in (2.3) to display its manifest invariance, we note that it contains the kinetic term $\frac{1}{2}\delta a(\partial_\mu\vec{\phi})^2$. Hence, it will be more convenient in practice to make a wave-function renormalization $(1+\delta a)^{1/2}\vec{\phi} \to \vec{\phi}$ to obtain a conventional kinetic term for ϕ when $\delta\mathcal{L}$ is added to $\mathcal{L}$. This renormalization is accomplished in the Lagrangian by making the substitution

$$\pi = \vec{\tau}\cdot\vec{\phi} - \frac{m}{g}\tau_3 \to (1+\delta a)^{-1/2}\left[\vec{\tau}\cdot\vec{\phi} - \frac{m}{g}(1+\delta a)^{1/2}\tau_3\right] \equiv (1+\delta a)^{-1/2}\tilde{\pi}. \tag{2.4}$$

III. ANOMALOUS RENORMALIZATION, POINT TRANSFORMATION, AND RENORMALIZABILITY IN THE UNITARY GAUGE

There is one further renormalization that we must perform before using the Lagrangian $\mathcal{L} + \delta\mathcal{L}$ in an explicit calculation. This stems from the fact that the broken-gauge transformation which takes us from the renormalization to the unitary gauge is not really a classical transformation. A convenient way to take this into account is to regard the transformation as a point, rather than a gauge, transformation, as follows: Let η, $W_\mu^{(1,2)}$, B_μ, and σ be the values of ψ, $A_\mu^{(1,2)}$, $A_\mu^{(3)}$, and $\vec{\phi}$ in the unitary gauge (where $\phi_1 = \phi_2 = \partial_\mu B_\mu = 0$) and ψ, $\vec{U}_\mu$, $\vec{\phi}$ be their values in the renormalization gauge ($\partial_\mu \vec{U}_\mu = 0$). Then it is easy to verify that at least classically these two sets of variables are connected by a point transformation, namely,

$$\psi = e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}\eta, \tag{3.1a}$$

$$\vec{\tau}\cdot\vec{\phi} - \frac{m}{g}\tau_3 = e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}\left(\sigma - \frac{m}{g}\right)\tau_3 e^{-(ig/2m)\vec{\tau}\cdot\vec{\rho}}, \tag{3.1b}$$

$$\vec{\tau}\cdot\vec{U}_\mu = e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}(\tau^{(1)}W_\mu^{(1)} + \tau^{(2)}W_\mu^{(2)} + \tau^{(3)}B_\mu)e^{-(ig/2m)\vec{\tau}\cdot\vec{\rho}} + i e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}\partial_\mu e^{-(ig/2m)\vec{\tau}\cdot\vec{\rho}}, \tag{3.1c}$$

where the $\vec{\rho}$ are three fields which are determined in terms of the given variables in the two gauges by Eqs. (3.1b) and (3.1c). In second quantizations, however, these equations do not quite suffice, because Eq. (3.1b) does not guarantee that the fields $\vec{\phi}$ and σ can have zero vacuum expectation value simultaneously (and in general they will not). Accordingly, we must modify Eq. (3.1b) to

$$\vec{\tau}\cdot\vec{\phi} - \frac{m}{g}\tau_3 = e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}\left(\sigma - \frac{\hat{m}}{g}\right)\tau_3 e^{-(ig/2m)\vec{\tau}\cdot\vec{\rho}}, \tag{3.2}$$

where $\hat{m}$ is a constant determined by the condition $\langle\vec{\phi}\rangle = \langle\sigma\rangle = 0$ or

$$m\tau_3 = \langle e^{(ig/2m)\vec{\tau}\cdot\vec{\rho}}\,\hat{m}\tau_3\, e^{-(ig/2m)\vec{\tau}\cdot\vec{\rho}}\rangle . \tag{3.3}$$

We then have four equations for the four unknowns $(\vec{\rho},\hat{m})$, instead of three equations for the three unknowns $\vec{\rho}$ which we had in the classical case. For the process which we shall consider, it suffices to determine $\hat{m}$ up to second order in g. To this order we obtain from (3.3)

$$\hat{m} = m\left(1 + \frac{g^2}{m^2}\langle\rho^+\rho^-\rangle\right). \tag{3.4}$$

Solving (3.1b) to lowest order we find

$$\rho_1 = -\phi_2 , \quad \rho_2 = \phi_1 , \tag{3.5}$$

which with (3.4) determines $\hat{m}$ to the required order.

Thus, the correction due to second quantization is to make the replacement $m \to \hat{m}$ in $\bar{\pi}$ for the unitary gauge. Since this replacement is to be made in the unitary gauge only, we call it an *anomalous* mass renormalization. If we include the anomalous mass renormalization, we see that the renormalization and unitary gauge Lagrangians are connected by a point transformation. Hence, by the formal equivalence theorem for point transformations,[4] the S matrices constructed with the two Lagrangians will be exactly the same to all orders. It follows that the S matrix will be renormalizable in the unitary gauge since it is renormalizable, by definition, in the renormalization gauge. *Thus, we obtain a formal proof of renormalizability in the unitary gauge (and unitarity in the renormalization gauge) to all orders by reducing the proof to the formal equivalence theorem for point transformations.*

In our later calculation we shall verify that this formal equality of the S matrices does indeed hold for fourth-order Feynman graphs.

IV. FULL LAGRANGIAN

Collecting the results of the last two sections, we see that we can write a total Lagrangian

$$L = -\tfrac{1}{8}\,\mathrm{tr}F_{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{D} - M_0)\psi + G\bar{\psi}\bar{\pi}\psi + \tfrac{1}{4}\,\mathrm{tr}[D_\mu, \bar{\pi}]^2 + B\,\mathrm{tr}\bar{\pi}^2 + C\,\mathrm{tr}\bar{\pi}^4 - \tfrac{1}{2}\delta\mu(A_\mu^{(3)})^2 , \tag{4.1}$$

where

$$\begin{aligned} M_0 &= m_0 - \delta m_0 , \\ B &= (1+\delta a)^{-1}(\tfrac{1}{8}\kappa^2 + \delta b) , \\ C &= (1+\delta a)^{-2}\left(\frac{-g^2}{16m^2}\kappa^2 + \delta c\right) , \\ \tilde{G} &= \frac{g}{2}\left(1 - \frac{\delta m}{m}\right)(1+\delta a)^{-1/2} \end{aligned} \tag{4.2}$$

and where

$$\begin{aligned} \bar{\pi} &= \vec{\tau}\cdot\vec{\phi} - mg^{-1}(1+\delta a)^{1/2}\tau_3 , \quad \partial_\mu \vec{A}_\mu = 0 \\ \bar{\pi} &= \tau_3\phi_3 - \hat{m}g^{-1}(1+\delta a)^{1/2}\tau_3 , \quad \partial_\mu A_\mu^{(3)} = 0 \end{aligned} \tag{4.3}$$

in the renormalization and unitary gauges, respectively.

Let us consider first the kinetic part of this Lagrangian

$$L_0 = -\tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}_{\mu\nu} + \bar{\psi}(i\not{\partial} - m_0 - \tfrac{1}{2}m\tau_3)\psi + \tfrac{1}{2}\mu^2[(A_\nu^{(1)})^2 + (A_\nu^{(2)})^2] + \tfrac{1}{2}\,|\partial_\mu\vec{\phi}|^2 - \tfrac{1}{2}\kappa^2(\phi^{(3)})^2 , \tag{4.4}$$

where $f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the vector-meson mass μ is given by $\mu = (e/g)m$.

In the renormalization gauge we have $A_\mu^{(1,2)} = U_\mu^{(1,2)}$, where $\partial_\mu U_\mu^{(1,2)} = 0$, whereas in the unitary gauge we have $\phi_1 = \phi_2 = 0$ but $\partial_\mu A_\mu^{(1,2)} \neq 0$. If, in the unitary gauge, we adopt the Stueckelberg formalism and write

$$A_\nu^{(1,2)} = U_\nu^{(1,2)} \pm \frac{1}{\mu}\,\theta^{(2,1)}_{,\nu} , \tag{4.5}$$

L_0 becomes

$$L_0^U = -\tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}_{\mu\nu} + \bar{\psi}(i\not{\partial} - m_0 - \tfrac{1}{2}m\tau_3)\psi + \tfrac{1}{2}\mu^2[(U_\nu^{(1)})^2 + (U_\nu^{(2)})^2] + \tfrac{1}{2}[(\partial_\mu\phi^{(3)})^2 + (\partial_\mu\theta^{(1)})^2 + (\partial_\mu\theta^{(2)})^2] - \tfrac{1}{2}\kappa^2(\phi^{(3)}) , \tag{4.6}$$

which is the same as in the renormalization gauge, except that $\varphi^{(1,2)}$ have been replaced by $\theta^{(1,2)}$. Thus, the kinetic part of the Lagrangian is formally the same in both gauges.

In the sequel it will be convenient to consider e rather than μ as the dependent variable, i.e.,

$$e = (\mu/m)g , \tag{4.7}$$

as we can then make an expansion in powers of μ^2 and obtain results for each order of μ^2 separately.

V. INTERACTION HAMILTONIANS

We now come to the main part of the paper which is the comparison of e-ν scattering to fourth order in the two gauges. For this purpose, we need to write down the interaction Hamiltonian

$$\mathcal{H}_{\rm int} = -L_{\rm int} + h_{\rm int} \tag{5.1}$$

in each gauge, where $h_{\rm int}$ is a term which comes from the presence of derivative couplings, the so-called Faddeev-Popov term.[5] This term can be calculated using the path-integral method of Faddeev and Popov, or by deriving $\mathcal{H}_{\rm int}$ directly from the Lagrangian, using the canonical formalism and cancelling the "surface terms" against the non-covariant parts of the propagators, so as to obtain covariant Feynman rules.[6] Let us first consider the renormalization gauge, for which we let $A_\mu^{(1,2)}=U_\mu^{(1,2)}$, $A_\mu^{(3)}=B_\mu$, $\phi^\pm=(1/\sqrt{2})(\phi^{(1)}\pm i\phi^{(2)})$, $\phi^{(3)}=\sigma$ where $\partial_\mu U_\mu^\pm=\partial_\mu B_\mu=0$. Then from the Lagrangian L in (4.1) we easily obtain

$$L_{\rm int}^R=\bar\psi\left[\frac{e}{2}\begin{pmatrix}\not B & \sqrt{2}\,\not U^- \\ \sqrt{2}\,\not U^+ & -\not B\end{pmatrix}+\tilde G\begin{pmatrix}\sigma & \sqrt{2}\,\phi^- \\ \sqrt{2}\,\phi^+ & -\sigma\end{pmatrix}\right]\psi+ie\left[f_{\mu\nu}(B)U_\mu^-U_\nu^++f_{\mu\nu}(U^+)B_\mu U_\nu^-+f_{\mu\nu}(U^-)B_\nu U_\mu^+\right]$$
$$-\frac{e^2}{2}\left[(U^+\cdot U^-)^2+2U^+\cdot U^-B^2-(U^+)^2(U^-)^2-2B\cdot U^-B\cdot U^+\right]-2ie\left[(U_\mu^-\phi^+-U_\mu^+\phi^-)\partial_\mu\sigma+B_\mu\phi^-\partial_\mu\phi^+\right]$$
$$+\frac{e^2}{2}\left[\left(\sigma^2-\frac{2m}{g}\sigma+\frac{m^2\delta a}{g^2}\right)(|U^+|^2+|U^-|^2)+B^2(|\phi^+|^2+|\phi^-|^2)\right.$$
$$\left.+2U^+\cdot U^-\phi^+\phi^--2\left(\sigma-\frac{m}{g}-\frac{m\delta a}{2g}\right)B_\mu(U_\mu^-\phi^++U_\mu^+\phi^-)\right]$$
$$+\bar\psi(\delta m_0+\tfrac{1}{2}\delta m\tau_3)\psi+B\,{\rm tr}\vec\pi^2+C\,{\rm tr}\vec\pi^4+\tfrac{1}{2}\kappa^2\sigma^2-\tfrac{1}{2}\delta\mu B^2\,, \tag{5.2}$$

where we have used $\partial_\mu U_\mu^\pm=0$. Then expanding this expression up to the order required for fourth-order e-ν scattering, and normal ordering the result, we obtain

$$-L_{\rm int}^R\simeq -:\bar\psi\left[\frac{e}{2}\begin{pmatrix}\not B & \sqrt{2}\,\not U \\ \sqrt{2}\,\not U^+ & -\not B\end{pmatrix}+\frac{g}{2}\left(1-\frac{\delta m}{m}-\frac{\delta a}{2}\right)\begin{pmatrix}\sigma & \sqrt{2}\,\phi^- \\ \sqrt{2}\,\phi^+ & -\sigma\end{pmatrix}\right]\psi$$
$$-ie\left[f_{\mu\nu}(B)U_\mu^-U_\nu^++f_{\mu\nu}(U^+)B_\mu U_\nu^-+f_{\mu\nu}(U^-)B_\nu U_\mu^+\right]-\frac{e\kappa^2}{2\mu}:\sigma^3:-\frac{e\kappa^2}{\mu}\sigma:\phi^+\phi^-:$$
$$+2ie(U_\mu^-\phi^+-U_\mu^+\phi^-)\partial_\mu\sigma+2ie\,B_\mu\phi^-\partial_\mu\phi^++2e\mu\sigma:U^+\cdot U^-:-e\mu B_\mu(U_\mu^-\phi^++U_\mu^+\phi^-)$$
$$-\lambda\sigma-\bar\psi\begin{pmatrix}\delta_R m_e & 0 \\ 0 & \delta_R m_\nu\end{pmatrix}\psi-\tfrac{1}{2}\delta_R k_1\sigma^2-\tfrac{1}{2}\delta_R k_2(|\phi^+|^2+|\phi^-|^2)+\tfrac{1}{2}\delta_R\mu_1B^2+\tfrac{1}{2}\delta_R\mu_2(|U^+|^2+|U^-|^2)\,. \tag{5.3}$$

Finally in this gauge we have the standard result[6,7]

$$h_{\rm int}=ie\left[\Omega^{(3)}(U_\mu^+\partial_\mu C^--U_\mu^-\partial_\mu C^+)+B_\mu(\Omega^-\partial_\mu C^+-\Omega^+\partial_\mu C^-)+(U_\mu^-\Omega^+-U_\mu^+\Omega^-)\partial_\mu C^{(3)}\right]\,,$$
$$\langle T(\Omega^i,\Omega^j)\rangle=\langle T(C^i,C^j)\rangle=0\,, \tag{5.4}$$
$$\langle T(\Omega^i(x),C^j(y))\rangle=i\delta^{i,j}\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2+i\epsilon}\,,$$

where Ω^i, C^i are ghost fields [and, as a consequence, any Ω-C loop acquires an extra factor of (-1) in a Feynman calculation]. The renormalization constants appearing in (5.3) are given by

$$\delta_R m_e=\delta m_0+\tfrac{1}{2}\delta m\,,$$
$$\delta_R m_\nu=\delta m_0-\tfrac{1}{2}\delta m\,,$$
$$\delta_R k_1=4\delta b+\frac{24m^2}{g^2}\delta c+\kappa^2\delta a+2e^2\langle U^+\cdot U^-\rangle-\frac{3g^2\kappa^2}{2m^2}\langle\sigma^2\rangle-\frac{g^2\kappa^2}{m^2}\langle\phi^+\phi^-\rangle\,,$$
$$\delta_R k_2=4\delta b+\frac{8m^2}{g^2}\delta c+e^2\langle B^2\rangle+e^2\langle U^+\cdot U^-\rangle-\frac{g^2\kappa^2}{2m^2}\langle\sigma^2\rangle-\frac{2g^2\kappa^2}{m^2}\langle\phi^+\phi^-\rangle\,, \tag{5.5}$$
$$\delta_R\mu_1=\delta\mu+\frac{3e^2}{2}\langle U^+\cdot U^-\rangle-2e^2\langle\phi^+\phi^-\rangle\,,$$
$$\delta_R\mu_2=-\mu^2\delta a+\frac{3e^2}{4}\langle U^+\cdot U^-\rangle+\frac{3e^2}{4}\langle B\cdot B\rangle-e^2\langle\sigma^2\rangle-e^2\langle\phi^+\phi^-\rangle\,,$$

where δm_0, δm, δa, δb, and δc are the original constants inserted in (2.3). If we demand that $\mathcal{H}_{\rm int}$ have no term linear in the fields, i.e., that $\lambda\sigma=0$, then we have the condition $\lambda=0$ where

$$0=\lambda=-\frac{m}{g}\left(4\delta b+\frac{8m^2}{g^2}\delta c\right)-2e\mu\langle U^+\cdot U^-\rangle+\frac{g}{2}(\langle\bar{e}e\rangle-\langle\bar{\nu}\nu\rangle)+\frac{g\kappa^2}{2m}(3\langle\sigma^2\rangle+2\langle\phi^+\phi^-\rangle), \tag{5.6}$$

which fixes the value of the renormalization parameter $4\delta b+(8m^2/g^2)\delta c$, which is the free parameter appearing in the $\phi^{\pm}$ mass counterterm.

Turning now to the unitary gauge, and using the Stueckelberg[8] formalism (4.5), and also $A^{(3)}_\mu=B_\mu$ and $\phi^{(3)}=\sigma$ as before, we see from (4.1) that

$$\begin{aligned}L^U_{\text{int}}=\bar{\psi}&\left[\frac{e}{2}\begin{pmatrix}\not{B} & \sqrt{2}\,\not{U}^-+\frac{i}{\mu}\sqrt{2}\,\theta^-_{,\nu}\gamma_\nu\\ \sqrt{2}\,\not{U}^+-\frac{i}{\mu}\sqrt{2}\,\theta^+_{,\nu}\gamma_\nu & -\not{B}\end{pmatrix}+\tilde{G}\begin{pmatrix}\sigma & 0\\ 0 & -\sigma\end{pmatrix}\right]\psi\\
&+ie\left[f_{\mu\nu}(B)\left(U^-_\mu+\frac{i}{\mu}\theta^-_{,\mu}\right)\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)+f_{\mu\nu}(U^+)B_\mu\left(U^-_\nu+\frac{i}{\mu}\theta^-_{,\nu}\right)+f_{\mu\nu}(U^-)B_\nu\left(U^+_\mu-\frac{i}{\mu}\theta^+_{,\mu}\right)\right]\\
&-\frac{e^2}{2}\left\{\left[\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)\left(U^-_\nu+\frac{i}{\mu}\theta^-_{,\nu}\right)\right]^2+2\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)\left(U^-_\nu+\frac{i}{\mu}\theta^-_{,\nu}\right)B^2\right.\\
&\qquad\left.-\left(U^+_{,\nu}-\frac{i}{\mu}\theta^+_{,\nu}\right)^2\left(U^-_\alpha+\frac{i}{\mu}\theta^-_{,\alpha}\right)^2-2B_\alpha\left(U^-_\alpha+\frac{i}{\mu}\theta_{,\alpha}\right)B_\nu\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)\right\}\\
&+e^2\left[\left(\sigma^2-\frac{2m}{g}\sigma+\frac{m^2\delta a}{g^2}+2\langle\theta^+\theta^-\rangle\right)\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)\left(U^-_\nu+\frac{i}{\mu}\theta^-_{,\nu}\right)\right]\\
&+\bar{\psi}\left\{\delta m_0+\frac{1}{2}\left[-\left(1-\frac{\delta m}{m}\right)\hat{m}+m\right]\tau_3\right\}\psi+B\,\text{tr}\,\bar{\pi}^2+C\,\text{tr}\,\bar{\pi}^4+\frac{\kappa^2}{2}\sigma^2-\frac{\delta\mu}{2}B^2.\end{aligned} \tag{5.7}$$

Expanding (5.7) to the required order and normal ordering,

$$\begin{aligned}-L^U_{\text{int}}\simeq-:\bar{\psi}&\left[\frac{e}{2}\begin{pmatrix}\not{B} & \sqrt{2}\,U^-+\frac{i}{\mu}\sqrt{2}\,\theta^-_{,\nu}\gamma_\nu\\ \sqrt{2}\,\not{U}^+-\frac{i}{\mu}\sqrt{2}\,\theta^+_{,\nu}\gamma_\nu & -\not{B}\end{pmatrix}+\frac{g}{2}\left(1-\frac{\delta m}{m}-\frac{\delta a}{2}\right)\begin{pmatrix}\sigma & 0\\ 0 & -\sigma\end{pmatrix}\right]\psi:\\
&-ie\left[f_{\mu\nu}(B)\left(U^-_\mu+\frac{i}{\mu}\theta^-_{,\mu}\right)\left(U^+_\nu-\frac{i}{\mu}\theta^+_{,\nu}\right)+f_{\mu\nu}(U^+)B_\mu\left(U^-_\nu+\frac{i}{\mu}\theta^-_{,\nu}\right)+f_{\mu\nu}(U^-)B_\nu\left(U^+_\mu-\frac{i}{\mu}\theta^+_{,\mu}\right)\right]\\
&+2e\mu\sigma\left[:U^+_\nu U^-_\nu:-\frac{i}{\mu}(U^-_\nu\theta^+_{,\nu}-U^+_\nu\theta^-_{,\nu})+\frac{1}{\mu^2}:\theta^+_{,\nu}\theta^-_{,\nu}:\right]-\frac{e\kappa^2}{2\mu}:\sigma^3:\\
&-\frac{g^2}{m^2}:\theta^+_{,\nu}\theta^-_{,\nu}:\left(-\frac{3}{4}\langle U^+\cdot U^-\rangle-\frac{3}{4\mu^2}\langle\theta^+_{,\nu}\theta^-_{,\nu}\rangle-\frac{3}{4}\langle B^2\rangle+\frac{m^2}{g^2}\delta a+2\langle\theta^+\theta^-\rangle+\langle\sigma^2\rangle\right)\\
&-\tilde{\lambda}\sigma-\bar{\psi}\begin{pmatrix}\delta_U m_e & 0\\ 0 & \delta_U m_\nu\end{pmatrix}\psi-\tfrac{1}{2}\delta'_U k_1\sigma^2+\tfrac{1}{2}\delta_U\mu_1 B^2+\tfrac{1}{2}\delta_U\mu_2(|U^+|^2+|U^-|^2).\end{aligned} \tag{5.8}$$

Using standard results we see that in this gauge are no Faddeev-Popov ghosts, but that there is a Faddeev-Popov or Lee-Yang term of the form

$$\begin{aligned}h_{\text{int}}&=2i\delta^4(0)\ln\left(1-\frac{g\sigma}{\hat{m}}\right)\\
&\simeq-\frac{2ig}{m}\delta^4(0)\sigma-\frac{ig^2}{m^2}\delta^4(0)\sigma^2+\cdots.\end{aligned} \tag{5.9}$$

The term proportional to σ in the unitary-gauge interaction Hamiltonian is given by $[\tilde{\lambda}+(2ig/m)\times\delta^4(0)]\sigma=\lambda\sigma$, where λ is the coefficient of the linear term in the renormalization gauge. By choosing the renormalization constant to make the linear term vanish in the renormalization gauge, it *automatically* vanishes in the unitary gauge.

The counterterms for the unitary-gauge interaction Hamiltonian are given by

$$\delta_U m_e = \delta m_0 + \tfrac{1}{2}\delta m - \frac{g^2}{2m}\langle \theta^+\theta^-\rangle\,,$$

$$\delta_U m_\nu = \delta m_0 - \tfrac{1}{2}\delta m + \frac{g^2}{2m}\langle \theta^+\theta^-\rangle\,,$$

$$\delta_U k_1 = \delta'_U k_1 + \frac{2ig^2}{m^2}\delta^4(0)$$

$$= 4\delta b + \frac{24m^2}{g^2}\delta c + \delta a\kappa^2 + 2e^2\langle U^+\cdot U^-\rangle - \frac{3e^2\kappa^2}{2\mu^2}\langle\sigma^2\rangle - \frac{3e^2\kappa^2}{\mu^2}\langle \theta^+\theta^-\rangle + \frac{2e^2}{\mu^2}\langle \theta^+_{,\nu}\theta^-_{,\nu}\rangle + \frac{2ig^2}{m^2}\delta^4(0)\,,$$

$$\delta_U \mu_1 = \delta\mu + \frac{3e^2}{2}\langle U^+\cdot U^-\rangle + \frac{3e^2}{2\mu^2}\langle \theta^-_{,\nu}\theta^+_{,\nu}\rangle\,, \qquad (5.10)$$

$$\delta_U \mu_2 = -\mu^2\delta a + \frac{3e^2}{4}\langle U^+\cdot U^-\rangle + \frac{3e^2}{4}\langle B\cdot B\rangle - 2e^2\langle \theta^+\theta^-\rangle + \frac{3e^2}{4\mu^2}\langle \theta^+_{,\nu}\theta^-_{,\nu}\rangle - e^2\langle\sigma^2\rangle\,,$$

where we have absorbed the term proportional to σ^2 in (5.9) into the σ mass counterterm.

From (5.5) and (5.10) it is clear that the input parameter can be chosen so that the second-order self-masses of σ, ψ, U, and B vanish in either gauge. However, if we require that they should be chosen so that the self-masses of these particles vanish in both gauges simultaneously, we see that we have the consistency conditions

$$\delta_U m_e = \delta_R m_e - \frac{g^2}{2m}\langle \theta^+\theta^-\rangle\,, \qquad (5.11a)$$

$$\delta_U m_\nu = \delta_R m_\nu + \frac{g^2}{2m}\langle \theta^+\theta^-\rangle\,, \qquad (5.11b)$$

$$\delta_U k_1 = \delta_R k_1 + \frac{g^2\kappa^2}{m^2}\left(-3\langle \theta^+\theta^-\rangle + \langle \phi^+\phi^-\rangle\right) + \frac{2e^2}{\mu^2}\langle \theta^+_{,\nu}\theta^-_{,\nu}\rangle + 2i\frac{g^2}{m^2}\delta^4(0)\,, \qquad (5.11c)$$

$$\delta_U \mu_1 = \delta_R\mu_1 + 2e^2\langle \phi^+\phi^-\rangle + \frac{3e^2}{2\mu^2}\langle \theta^-_{,\nu}\theta^+_{,\nu}\rangle\,, \qquad (5.11d)$$

$$\delta_U \mu_2 = \delta_R\mu_2 - 2e^2\langle \theta^+\theta^-\rangle + e^2\langle \phi^+\phi^-\rangle + \frac{3e^2}{4\mu^2}\langle \theta^+_{,\nu}\theta^-_{,\nu}\rangle\,, \qquad (5.11e)$$

where δ_U and δ_R denote the self-mass calculated from the loops, i.e., without the use of counterterms. Furthermore, we have the condition that in the unitary gauge the loop contribution to the self-mass of the θ field must vanish, since there are no counterterms for this field, and in the renormalization gauge we have the condition

$$\delta_R k_2 = 4\delta b + \frac{8m^2}{g^2}\delta c + e^2\langle B\cdot B\rangle + e^2\langle U^+\cdot U^-\rangle - \frac{2g^2\kappa^2}{m^2}\langle \phi^+\phi^-\rangle - \frac{g^2\kappa^2}{2m^2}\langle\sigma^2\rangle\,, \qquad (5.12)$$

since the parameter $4\delta b + (8m^2/g^2)\delta c$ for the ϕ field in the renormalization gauge has already been fixed by (5.6). We have verified by direct calculation that all these consistency conditions are satisfied, and as an example we derive (5.11e) in Appendix A. We should like to emphasize that all the terms proportional to $\langle \theta^+\theta^-\rangle$ in (5.11) originate from the anomalous mass renormalization exhibited by the point transformation in Sec. III.

VI. FEYNMAN VERTICES

The Feynman vertices for the Hamiltonians $\mathcal{H}^R_{\rm int}$ and $\mathcal{H}^U_{\rm int}$ are shown in Fig. 1. We wish to compare the Feynman graphs for fourth-order e-ν scattering calculated with these two sets of vertices. For this purpose, we shall consider only the difference between the graphs in the two gauges. The advantage of considering only the difference is that the differences between the couplings in the two gauges are such that they just cancel propagators, and so the difference between two graphs with a given number of propagators is a graph with a lower number of propagators (and a larger number of lines entering at each vertex), i.e., what we call a "short-circuited" graph. For example, from the algebraic identity

$$\sqrt{2}\,\not k + \sqrt{2}\,m = \sqrt{2}\left[-(\not q - \not k - m_e) + (\not q - m_\nu)\right]$$

we obtain the graphical identity shown in Fig. 2(a) and hence the "short circuit" shown in Fig. 2(b). If one defines suitable auxiliary vertices for all such short-circuited graphs then, as we shall see in the next section, the "short-circuited" graphs all cancel by inspection, and so the S matrices are the same in the two gauges. The required auxiliary vertices are as shown in Fig. 3. Where any ambiguity might arise, we have always chosen the momentum to be in the direction that the negatively charged particle would travel.

VII. EQUALITY OF S MATRICES IN THE TWO GAUGES

In this section we give a graphical proof of the equality of the S matrices for e-ν scattering in the two gauges by expanding the differences of all the contributing graphs in terms of short-circuited graphs, which can then be seen to vanish by inspection.

There are three typical kinds of graphs, namely, two-particle exchange graphs, graphs with loops,

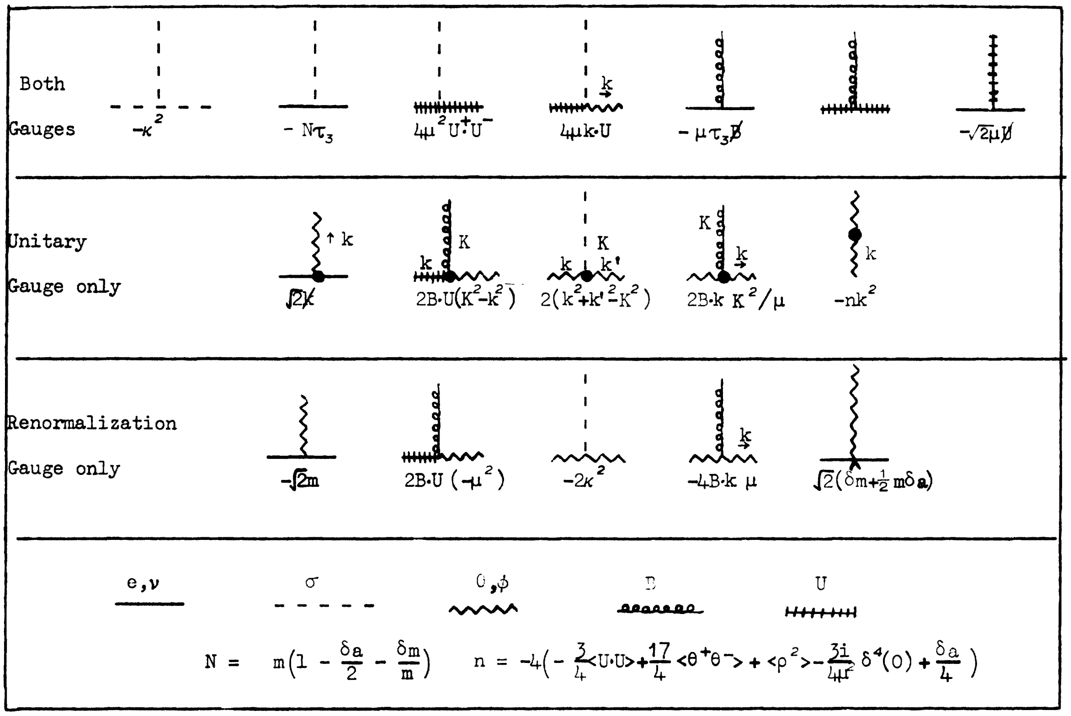

FIG. 1. Feynman vertices for the Hamiltonians $\mathcal{H}^{R}_{\text{int}}$ and $\mathcal{H}^{U}_{\text{int}}$.

and graphs with triangles (third-order vertices). We have already indicated in the last section how the two-particle graphs can be expanded. To show how the other two types of graph are expanded, we take a typical example of each kind and carry out the expansion in detail in Appendixes B and C, respectively. All other expansions are obtained using the same general procedure, except for the fermion-wave-function renormalization graphs, which are obtained by the same procedure as in paper I for the Abelian model. Hence the details of the expansions are omitted.

The differences between the graphs contributing to e-ν scattering in fourth order in the two gauges actually fall into five groups, for each of which separately we have equality of the S matrices. The groups are the following:

(1) neutral scalar exchange, which occurs only in order zero in μ^2,

(2) neutral vector exchange in order zero in μ^2,

(3) neutral vector exchange in order μ^2 and μ^4,

(4) charge scalar exchange in order zero in μ^2,

(5) charge exchange (both scalar and vector) in order μ^2 and μ^4.

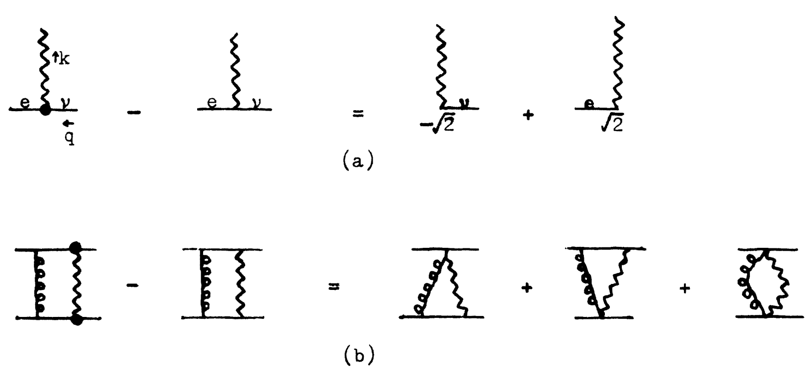

FIG. 2. Graphical identity and "short-circuit" diagrams.

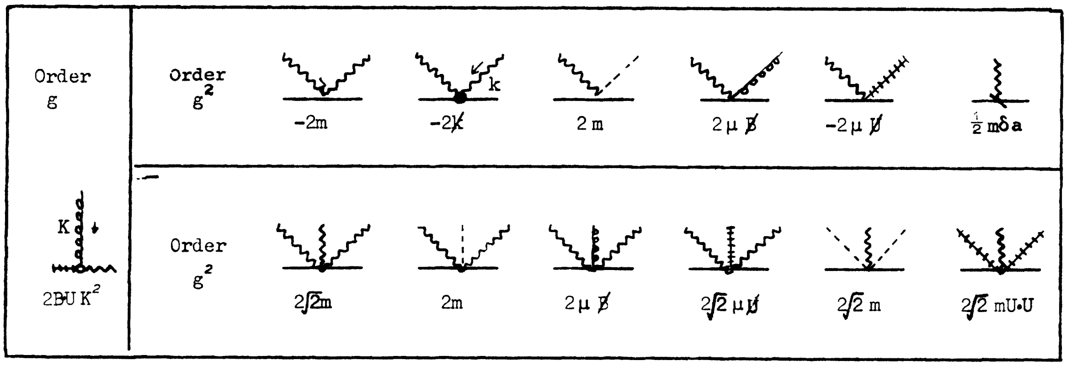

FIG. 3. Auxiliary vertices.

The graphs corresponding to these five groups are displayed in Figs. 4–6. In these figures Δ denotes the difference between a graph in the unitary gauge and the same graph in the renormalization gauge. Figure 7 illustrates graphically the meaning of Δ for a typical Feynman graph. In Figs. 4 and 6 the loops with the letter g inside are the Faddeev-Popov ghost contributions. It is interesting to note that the Faddeev-Popov contributions are among the few contributions of order μ^4. There is a factor $\frac{1}{2}$ attached to all meson loops because half their contribution belongs to the corresponding upper vertices, but in some cases this factor $\frac{1}{2}$ does not appear explicitly because it is canceled by a Wick combinatorial factor 2. In all the figures the inverted graphs (i.e., the graphs with third-order upper and first-order lower vertices and denoted by I for the two-particle exchanges) are omitted on the understanding that similar results hold for them also, and in the case of Fig. 6 the left to right mirror image graphs are omitted on the same understanding. In all loop graphs, it is understood that the on-shell self-mass has been subtracted in both gauges, as justified by the results stated in (5.11) and (5.12).

One can see by inspection that the auxiliary graphs in Figs. 4–6 exactly cancel, thus establishing the equality of the S matrices to fourth order in the two gauges. What is more interesting than the simple fact that the graphs cancel, however, is that we can see explicitly the mechanism by which the cancellation comes about and relate it to the spontaneous-symmetry-breaking mechanism for the Lagrangian. That is to say, we can see by inspection that what happens in each table is that the difference between the most structured contributions to the scattering, namely, the two-particle contributions, is completely canceled by the contribution from the meson-meson interactions, which, if we recall, were induced without any free parameters by the spontaneous-symmetry-breaking mechanism. The remainder of the meson-meson interaction contains only vertex renormalizations, which are then canceled by other nonstructured contributions such as the fermion wave-function renormalization and the anomalous charge renormalization induced by the counterterm $-\delta m(g/2m)\bar{\psi}\pi\psi$ of Sec. II. In this way we see very clearly how the meson-meson interaction which is induced in a unique way by spontaneous symmetry breaking, and would not be present in a "naive" gauge-noninvariant Lagrangian, contributes in such a way as to make the unitary gauge renormalizable and the renormalization gauge unitary.

APPENDIX A

The graphs contributing to the U-particle self-mass which are different in the unitary and renormalization gauges are shown respectively in Figs. 8(a) and 8(b). Evaluating these graphs we obtain the following contributions:

Unitary gauge [Fig. 8(a)]:

$$-4\int \frac{P_{\mu\nu}(k)(k^2-\mu^2)^2}{k^2k'^2}\,. \tag{A1}$$

Renormalization gauge [Fig. 8(b)]:

$$-4\int \frac{P_{\mu\nu}(k)\mu^4}{k^2k'^2} - 8\mu^2\int\frac{k_\mu k_\nu}{k^2k'^2}\,, \tag{A2}$$

where the second graph in Fig. 8(b) is the Faddeev-Popov contribution and $\int \equiv \int d^4k/(2\pi)^4$, $P_{\mu\nu}(k) = g_{\mu\nu} - (k_\mu k_\nu)/k^2$, $k'=k-q$, $q^2=\mu^2$. Subtracting (A2) from (A1) we obtain

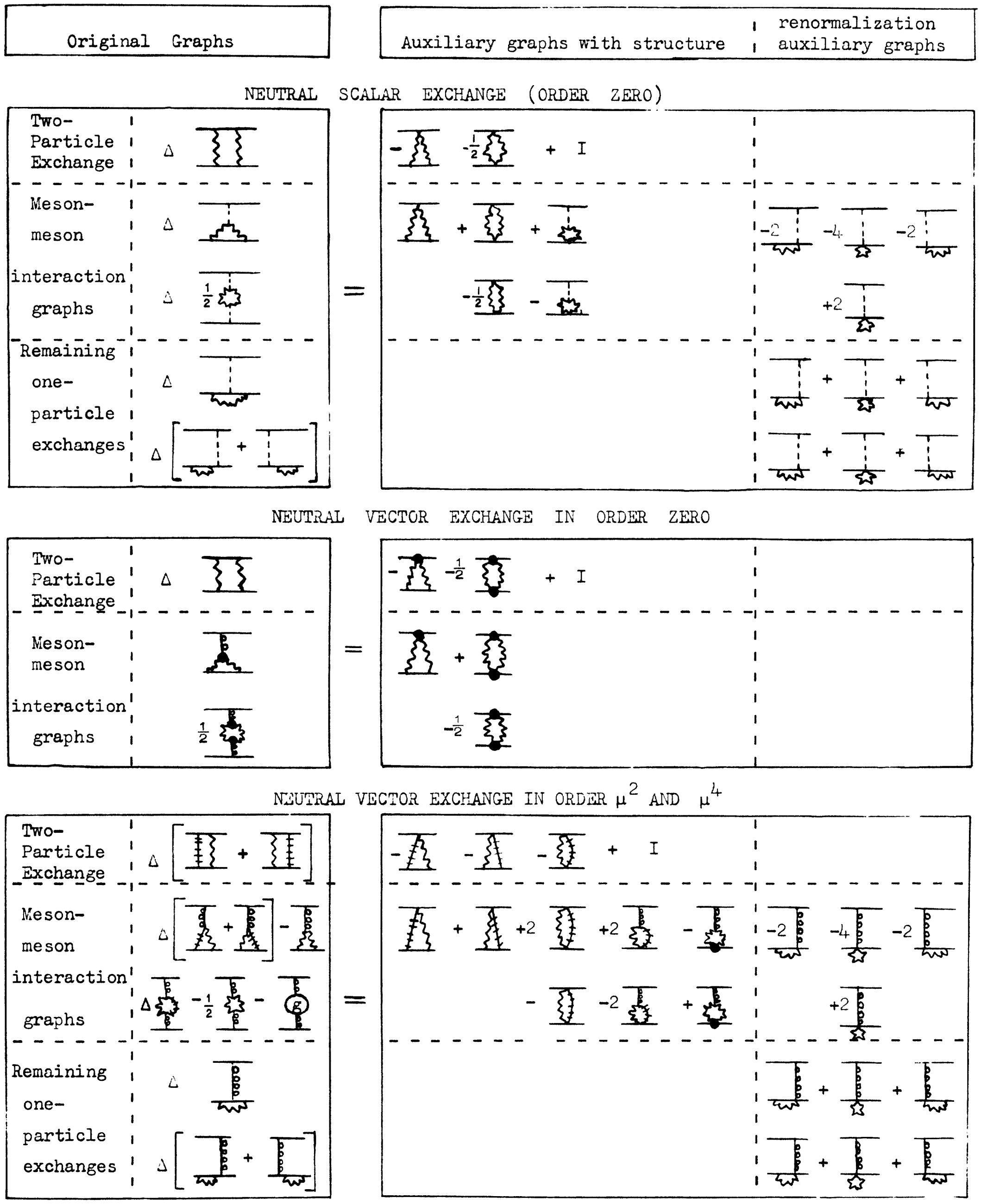

FIG. 4. Neutral exchanges.

$$i(\delta_U \mu_2 - \delta_R \mu_2) g_{\mu\nu}$$

$$= -4 \int \frac{P_{\mu\nu}(k)[k^4 - 2k^2\mu^2]}{k^2 k'^2} + 8\mu^2 \int \frac{k_\mu k_\nu}{k^2 k'^2}$$

$$= -4 \int \frac{g_{\mu\nu}(k^4 - 2k^2\mu^2)}{k^2 k'^2} + 4 \int \frac{k_\mu k_\nu}{k^2} \frac{k^4}{k^2 k'^2}$$

$$= -4 g_{\mu\nu} \int \left(\frac{k^2}{k'^2} - \frac{2\mu^2}{k'^2} \right) + 4 \int \frac{k_\mu k_\nu}{k'^2} . \qquad \text{(A3)}$$

Using the symmetry of the regularization procedure to shift the variable of integration from k to $k' = k - q$ and then dropping the primes we obtain

$$i(\delta_U \mu_2 - \delta_R \mu_2) g_{\mu\nu} = -4 g_{\mu\nu} \int \left(\frac{(k+q)^2}{k^2} - \frac{2\mu^2}{k^2} \right) + 4 \int \frac{(k_\mu + q_\mu)(k_\nu + q_\nu)}{k^2} . \qquad \text{(A4)}$$

Finally, dropping the terms proportional to q_μ and

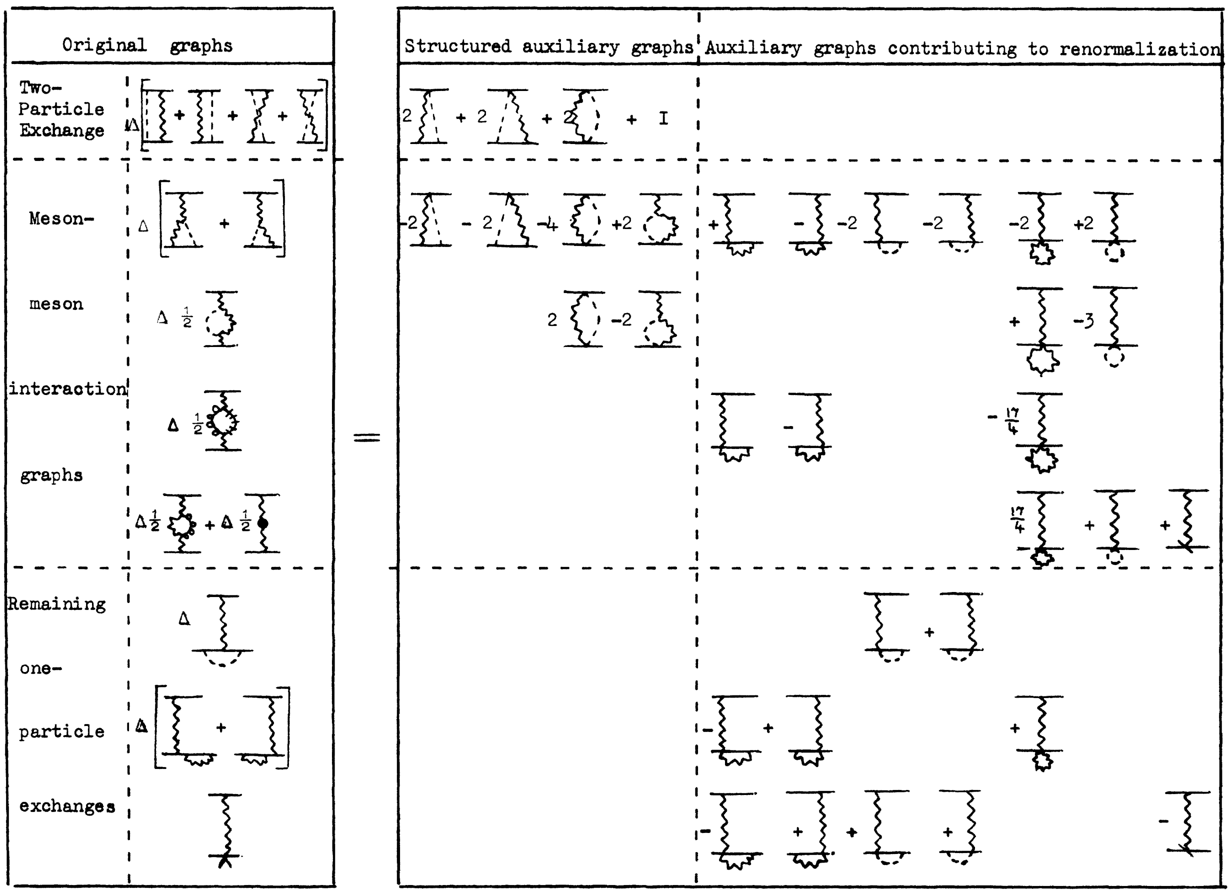

FIG. 5. Charged scalar exchange in order zero.

q_ν because $q\cdot U=0$, and using symmetric integration we obtain

$$i(\delta_U\mu_2-\delta_R\mu_2)g_{\mu\nu}=g_{\mu\nu}\int\left(-3+\frac{4\mu^2}{k^2}\right)$$
$$=g_{\mu\nu}[-3\delta^4(0)-4i\mu^2\langle\theta^+\theta^-\rangle] \tag{A5}$$

as required.

APPENDIX B

As an example of a meson-loop calculation we consider the difference shown in Fig. 9 which yields the contribution

$$\frac{4\mu^2}{\delta^4}J_\alpha P_{\alpha\mu}(\delta)J_\beta P_{\beta\nu}(\delta)\int\frac{P_{\mu\nu}(k')[(\delta^2-k'^2)^2-\mu^4]}{k^2(k'^2-\mu^2)},$$

where J_α, J_β are neutral fermion currents and $k'=k+\delta$. This can be written

$$\frac{4\mu^2}{\delta^4}J_\mu(\delta)J_\nu(\delta)\int\frac{P_{\mu\nu}(k')}{k^2(k'^2-\mu^2)}[(k'^2-\mu^2)(k'^2+\mu^2-2\delta^2)+\delta^4-2\delta^2\mu^2]$$
$$=\frac{4\mu^2}{\delta^4}J_\mu(\delta)J_\nu(\delta)\int\frac{P_{\mu\nu}(k')}{k^2}(k'^2+\mu^2-2\delta^2)+4\mu^2J_\mu(\delta)J_\nu(\delta)\left(1-\frac{2\mu^2}{\delta^2}\right)\int\frac{P_{\mu\nu}(k')}{k^2(k'^2-\mu^2)}. \tag{B1}$$

The first term can be further simplified by writing

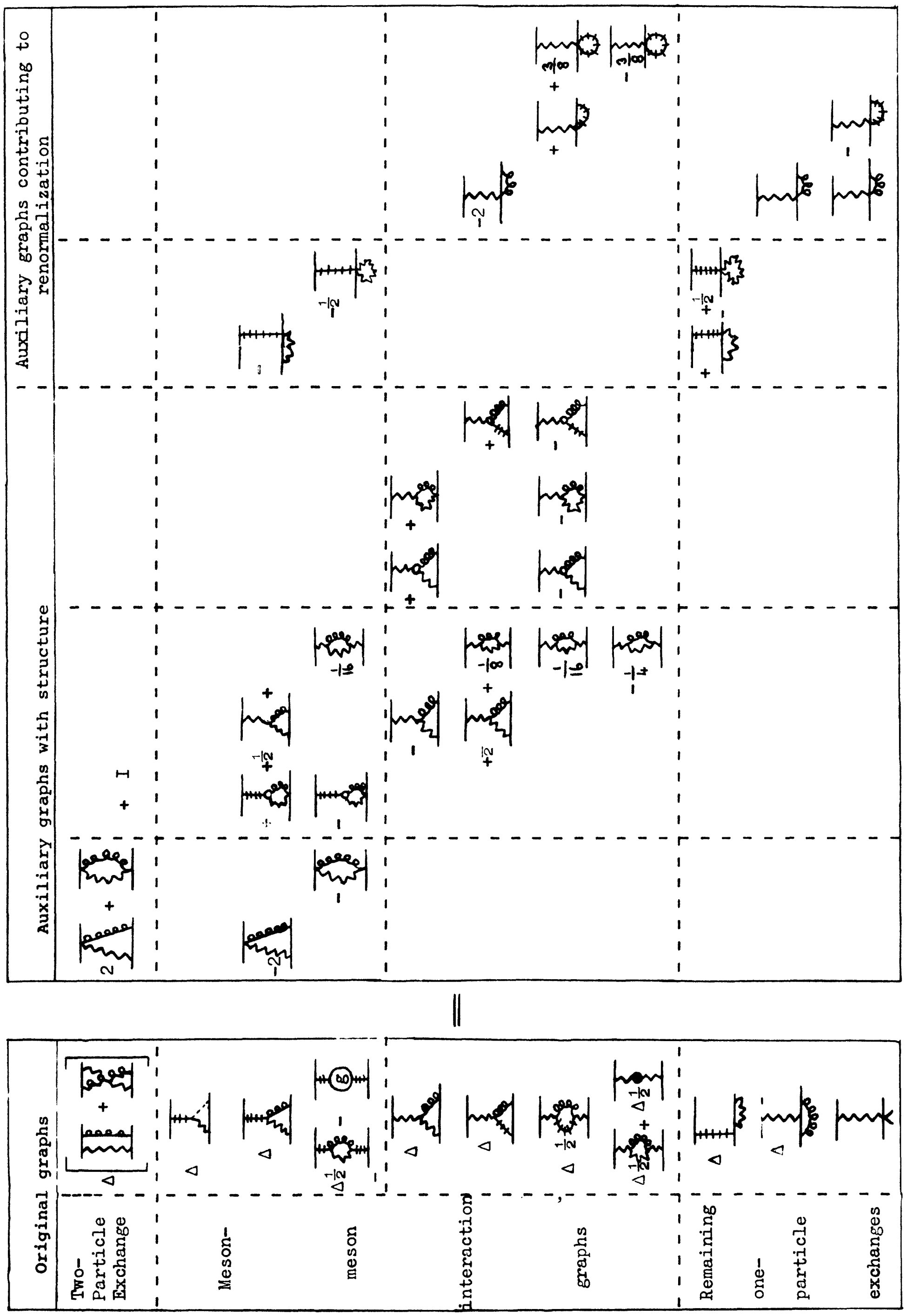

FIG. 6. Charged exchange in order μ^2 and μ^4.

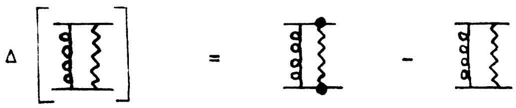

FIG. 7. Δ of a typical Feynman graph.

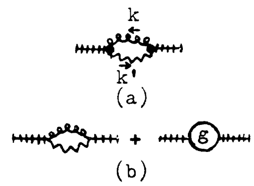

FIG. 8. Three Feynman diagrams contributing to the U-particle self-mass.

$$\int \frac{P_{\mu\nu}(k')}{k^2}(k'^2+\mu^2-2\delta^2) = \int \left(g_{\mu\nu} - \frac{k'_\mu k'_\nu}{k'^2}\right)\frac{1}{k^2}(k'^2+\mu^2-2\delta^2)$$

$$= \int \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k'^2}\right)\frac{1}{k^2}(k'^2+\mu^2-2\delta^2)\,, \tag{B2}$$

where in the last equation we have simply dropped the primes on k'_μ, k'_ν because $(k'_\mu - k_\mu)J^\mu = \delta_\mu J^\mu = 0$. Therefore (B2) becomes

$$\int \frac{P_{\mu\nu}(k')}{k^2}(k'^2+\mu^2-2\delta^2) = \int g_{\mu\nu}\left(\frac{k'^2}{k^2} + \frac{\mu^2-2\delta^2}{k^2}\right) - \int k_\mu k_\nu\left(\frac{1}{k^2} + \frac{\mu^2-2\delta^2}{k^2k'^2}\right), \tag{B3}$$

and using symmetrical integration this expression reduces to

$$\int \frac{P_{\mu\nu}(k')}{k^2}(k'^2+\mu^2-2\delta^2) = g_{\mu\nu}\int\left(\frac{3}{4} + \frac{\mu^2-\delta^2}{k^2}\right) - (\mu^2-2\delta^2)\int\frac{k_\mu k_\nu}{k^2k'^2}\,. \tag{B4}$$

Thus, the final result is

$$\Delta = \frac{4\mu^2}{\delta^4}\left[g_{\mu\nu}\int\left(\frac{3}{4} - \frac{\mu^2}{k^2}\right) - \delta^2 g_{\mu\nu}\int\frac{1}{k^2} - \mu^2\int\frac{k_\mu k_\nu}{k^2k'^2} + 2\delta^2\int\frac{k_\mu k_\nu}{k^2k'^2} - 2\mu^2\delta^2\int\frac{P_{\mu\nu}(k')}{k^2(k'^2-\mu^2)} + \delta^4\int\frac{P_{\mu\nu}(k')}{k^2(k'^2-\mu^2)}\right]. \tag{B5}$$

Equation (B5) is expressed in terms of graphs in Fig. 10.

APPENDIX C

As an example of a triangle graph, we consider the graph shown in Fig. 11(a). Evaluating this graph, we get

$$\frac{4m\mu^2}{\eta_2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)}{k^2}\,\frac{P_{\mu\alpha}(k')}{k'^2-\mu^2}\left[(k'^2-k^2)-\mu^2\right], \tag{C1}$$

where

$$J_{\alpha\beta} = \overline{U}(P)\gamma^\alpha\frac{1}{\not P + \not k' - m_e}\gamma^\beta U(q)\,.$$

Cancelling the denominator $k'^2-\mu^2$ with the corresponding term in the numerator, we obtain

$$-\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)}{k^2}\,\frac{P_{\mu\alpha}(k')}{k'^2-\mu^2}k^2 + \frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)}{k^2}P_{\mu\alpha}(k')\,. \tag{C2}$$

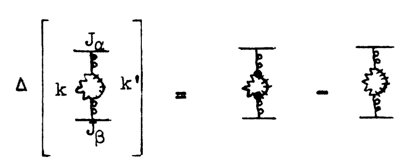

FIG. 9. Δ of a meson loop.

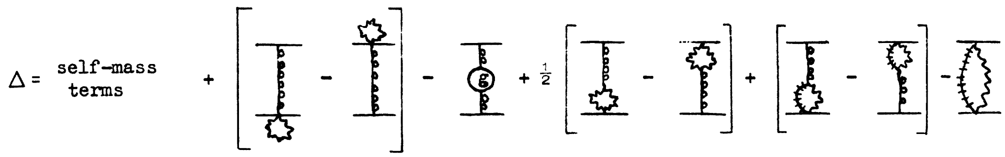

FIG. 10. Equation (B5) expressed graphically.

Expanding $P_{\mu\alpha}(k')$, we get

$$-\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)}{k^2}\frac{P_{\mu\alpha}(k')}{k'^2-\mu^2}k^2+\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\alpha\beta}(k)}{k^2}-\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)k'_\mu k'_\alpha}{k^2k'^2}\,. \tag{C3}$$

But now from the definitions of $J_{\alpha\beta}$ and $P_{\beta\mu}$ we have the identities

$$k'_\alpha J_{\alpha\beta}=mJ_\beta+\overline{U}(P)\gamma_\beta U(q)\,,$$

where

$$J_\beta=\overline{U}(P)\frac{1}{\not P+\not k-m_e}\gamma_\beta U(q) \tag{C4}$$

and

$$P_{\beta\mu}(k)k'_\mu=P_{\beta\mu}(k)\eta_\mu\,. \tag{C5}$$

Hence, for the last term in (C3) we can write

$$-\frac{4m\mu^2}{\eta_2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)k'_\mu k'_\alpha}{k^2k'^2}=-\frac{4m^2\mu^2}{\eta^2}\int J_\beta\frac{P_{\beta\mu}(k)k'_\mu}{k^2k'^2}-\frac{4m\mu^2}{\eta_2}\overline{U}(\rho)\gamma^\beta U(q)\int\frac{P_{\beta\mu}(k)\eta_\mu}{k^2k'^2}\,. \tag{C6}$$

Furthermore, since the last integral in (C6) can be a function of η_β only, we have

$$\int\frac{P_{\beta\mu}(k)\eta_\mu}{k^2k'^2}=\frac{\eta_\beta}{\eta^2}\int\frac{\eta_\nu P_{\nu\mu}(k)\eta_\mu}{k^2k'^2}\,. \tag{C7}$$

Combining all of these results and using $\overline{U}(P)\gamma_\beta U(q)\eta_\beta=m\overline{U}(P)U(q)$, we obtain the following contribution corresponding to Fig. 11(a),

$$-\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\beta\mu}(k)}{k^2}\frac{P_{\mu\alpha}(k')}{k'^2-\mu^2}k^2+\frac{4m\mu^2}{\eta^2}\int J_{\alpha\beta}\frac{P_{\alpha\beta}(k)}{k^2}$$
$$-\frac{4m^2\mu^2}{\eta^2}\int J_\beta\frac{P_{\beta\mu}(k)k'_\mu}{k^2k'^2}-\frac{4m^2\mu^2}{\eta^4}\overline{U}(P)U(q)\int\frac{\eta_\nu P_{\nu\mu}(k)\eta_\mu}{k^2k'^2}\,,$$

which is equal to the contribution from Fig. 11(b). These results are used in Fig. 6.

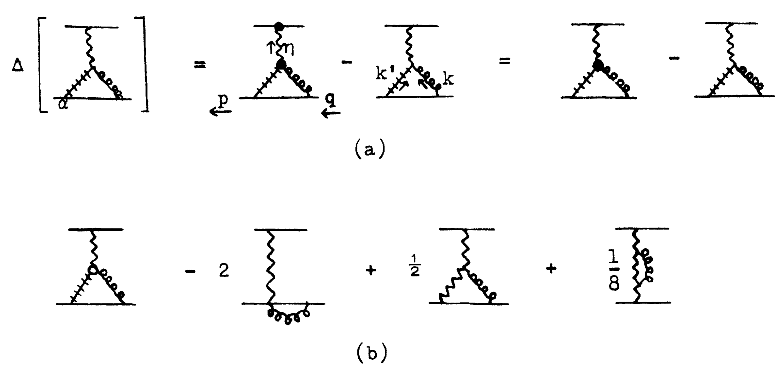

FIG. 11. Δ of a triangle graph.

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