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Photon emission by an electron in a bichromatic field

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A general expression for the probability of photon emission by an electron in a unidirectional bichromatic classical field is calculated making use of the exact solution to the Dirac equation for the electron in the field. In the limit that one component of the field is weak, a cross section for scattering a photon out of this component is derived. This has a complicated form but numerical values have been obtained in the particular case of an electron of energy 5 GeV travelling through radiation from a ruby laser (considered weak) and an intense CO_2 laser. It is found that, as a function of the intensity of the CO_2 laser, the cross section first increases, but eventually decreases to values below the Compton cross section. The spectrum of the scattered radiation is also calculated; is shows intensity-dependent shifts attributable to a field-induced drift velocity of the electron.

I. INTRODUCTION

In this paper we are concerned with photon emission by an electron in an intense bichromatic electromagnetic field. Our original interest in this problem stemmed from the demonstration by Bemporad *et al.*[1] that γ rays could be produced through backward scattering of ruby laser photons by energetic electrons. The question arose as to whether the yield of γ rays could be improved by adding a supplementary low-frequency field which would stimulate the double Compton scattering process. It turns out that this question is too narrow, because although stimulated double Compton scattering occurs, increasing the production of γ rays, many other processes also occur which affect the over-all γ-ray yield. Obviously, it is this over-all yield which is important experimentally, and one is hence obliged to study the more general problem enunciated at the beginning of this paragraph.

We choose to represent the electron by the exact solution to the Dirac equation for a charged particle in a unidirectional classical field.[2] This electron is then coupled to a quantized field and the probability of single-photon emission is calculated. The scattering cross section σ, correct to all orders in the classical field strength, follows. Plane waves are used throughout the calculation in spite of the fact that in so doing one cannot in an unequivocal way decouple the electron from the classical field at the beginning and end of the scattering process as would be the case in a real experiment. We feel justified in this procedure, however, on the basis of the work by Neville and Rohrlich.[3] They treated the problem of Compton scattering by an electron encountering an intense pulse of monochromatic radiation using rigorously separable wave packets, and obtained the same result as Brown and Kibble[4] who used infinitely long wave trains.

In the general expression for the cross section one can, on the basis of the frequency of the emitted photon, identify different contributions to σ arising from the Compton process, the double Compton process, etc. Nevertheless, the formulas for these individual contributions are too complex to permit general statements concerning their behavior with respect to the field parameters. Some simplification results in the case that one field component is weak, but, even then, understanding of the formulas only comes through numerical analysis of specific examples. Because of our interest in γ-ray production the case of an electron of energy 5 GeV traveling through radiation from a ruby laser and an intense CO_2 laser has been studied. It is found that, as a function of the intensity of the CO_2 laser, the cross section for scattering a photon out of the ruby laser beam first increases, but eventually decreases to values below the Compton cross section. The maximum increase is about 10% of the initial value, and, as a consequence, the γ-ray yield can be improved at most by this amount.

The problem of photon emission by an electron in a bichromatic field has already received some consideration in the literature. Kronig,[5] and Prakash and Vachaspati,[6] investigated the possibility of enhancing the cross section for the scattering of x rays by irradiating the electron with light from a laser, and predicted a substantial effect. Later, however, this conclusion was retracted by Kronig and Höfelt[7] who, in a new calculation, found no effect at all. Lebedev[8] also studied the general problem, but for a particle satisfying the Klein-Gordon equation. His results are unfortunately incomplete, a cross term between two equally important parts of the transition amplitude having been omitted from the square of the latter. A formula for the intensity of radiation by a Fermi particle interacting with two electromagnetic waves has been given by Klimenko and Khudomyasov,[9] but the implications of the formula were not extensively discussed. Oleinik,[10] using the exact Green's function for an electron in a monochromatic classical field, studied the scattering of photons by the electron out of a quantized field. He found resonances in the scattering cross section when the frequency of the incident photons was integrally related to the classical field frequency. There is no evidence for such resonances in our calculation.

Of course, if in the general expression for σ the amplitude of one field component is set equal to zero, ond finds the cross section for scattering a photon out of a monochromatic field, a problem already dealt with by several authors.[4,11,12]

II. THEORETICAL DEVELOPMENT

A. Electron wave function

The Dirac equation for an electron in a classical electromagnetic field A has the well-known form

$$[\gamma\cdot(i\partial-eA)-m]\psi=0\,, \tag{2.1}$$

where e and m are the charge and the rest mass of the electron, respectively. In Eq. (2.1) the γ matrices satisfy the anticommutation relations

$$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}\,, \tag{2.2}$$

with the metric tensor $g^{\mu\nu}=0$ for $\mu\neq\nu$ and $g^{00}=-g^{11}=-g^{22}=-g^{33}=1$.[13] Units have been chosen so that $\hbar=c=1$.

For a field propagating in an arbitrary direction characterized by the null four-vector n, the solution to (2.1) has been shown to be[2]

$$\psi_p(x)=C\left(1+\frac{1}{2}\,\frac{e}{p\cdot n}\,\gamma\cdot n\,\gamma\cdot A\right)\times\exp\left[-ip\cdot x+i\,\frac{1}{2}\,\frac{1}{p\cdot n}\int d\phi(e^2A^2-2eA\cdot p)\right]\times w(\vec{p})\,. \tag{2.3}$$

Here p is a constant four-vector, which would represent the momentum of the electron were the external field switched off, and ϕ is the scalar product $n\cdot x$. The quantity $w(\vec{p})$ is a spinor which satisfies the equation $(\gamma\cdot p-m)w(\vec{p})=0$, and the normalization condition $\bar{w}(\vec{p})w(\vec{p})=m/E$, with $E=p^0$. In deriving (2.3) it has been assumed that A obeys the Lorentz condition $\partial_\mu A^\mu=0$, and further that it is a function of ϕ only. Later, for simplicity it will be assumed that A propagates in the z direction; that is, $n=(1;0,0,1)$. The normalization constant C is to be determined shortly.

The wave function (2.3) was obtained by Volkov imposing that a function $\psi(x)=\exp(-ip\cdot x+F)w(\vec{p})$, with $F=F(\phi)$, be a solution to Eq. (2.1). However, in the case that A is monochromatic it can also be derived in principle by summing a perturbation series. Such a derivation throws some light on the interpretation of the various factors in the Volkov solution and will be discussed in Appendix A.

From the wave function, Eq. (2.3), one can calculate the probability current for an electron in the field A,

$$j^\mu=\bar{\psi}_p\gamma^\mu\psi_p=CC^*\left[\frac{p^\mu}{E}-\frac{1}{2}\,\frac{e^2A^2}{Ep\cdot n}\,n^\mu+\frac{e}{p\cdot n}(p\cdot An^\mu-p\cdot nA^\mu)\right]. \tag{2.4}$$

Averaging this time-dependent quantity over a few cycles of the applied field one obtains the constant probability current

$$\langle j^\mu\rangle=CC^*\left(\frac{p^\mu}{E}+\chi n^\mu\right), \tag{2.5}$$

where χ stands for the quantity $-e^2\langle A^2\rangle/2Ep\cdot n$. From the zeroth component of $\langle j^\mu\rangle$, that is, the time-average probability density

$$\langle j^0\rangle=\langle\psi_p^\dagger\psi_p\rangle=CC^*(1+\chi)\,, \tag{2.6}$$

one deduces the normalization constant

$$C=[V(1+\chi)]^{-1/2}\,, \tag{2.7}$$

where V is the quantization volume. The spatial components of $\langle j^\mu\rangle$, on the other hand, yield the average velocity components of the electron as modified by the presence of the applied field. In particular,

$$v_z^{(A)}=\langle j^3\rangle V=\frac{v_z+\chi}{1+\chi}\,, \tag{2.8}$$

where $v_z=p_z/E$. That is, the field has caused the particle to drift in the z direction. It will be shown later that this modification to the particle's velocity has important consequences insofar as the spectrum of scattered radiation is concerned when the field is intense.

The orthogonality condition satisfied by the wave functions (2.3) is found by evaluating the integral

$$I = \int d^3x\, \psi_{p'}^{\dagger}(x)\, \psi_p(x) \tag{2.9}$$

and taking a time average of the result. One finds

$$\langle I \rangle = \frac{(2\pi)^3}{V}\, \delta(\vec{q}\,' - \vec{q}), \tag{2.10}$$

where

$$q = p - \frac{1}{2}\, \frac{e^2\langle A^2 \rangle}{p \cdot n}\, n. \tag{2.11}$$

This quantity, called the quasimomentum,[14] equals the time average of the expectation value of the kinetic momentum operator $p - eA$. Its square defines an effective mass for the electron in the field:

$$m^{*2} = q^2 = m^2 - e^2\langle A^2 \rangle. \tag{2.12}$$

This effective mass has been invoked to explain frequency shifts in high-intensity Compton scattering.[4]

B. Scattering amplitude and cross section

We would now like to calculate the probability that the electron emits a photon. To this end the electron is coupled to a quantized field $\mathfrak{A}$ through the interaction Hamiltonian

$$H_I = -e\gamma \cdot \mathfrak{A}, \tag{2.13}$$

where

$$\mathfrak{A} = \frac{\epsilon}{(2\omega' V)^{1/2}} (a^{\dagger} e^{ik'\cdot x} + a e^{-ik'\cdot x}). \tag{2.14}$$

In (2.14) ϵ is the polarization vector, $a^{\dagger}$ and a are the creation and annihilation operators, and ω' is the frequency of the field. The probability amplitude equals

$$S_{fi} = -i\langle \Psi_f | H_I | \Psi_i \rangle, \tag{2.15}$$

where the symbol $|\Psi_i\rangle$ stands for the initial state of the electron and the vacuum state of the field, while $|\Psi_f\rangle$ stands for the final electron state and the one-photon state of the quantized field.

The introduction into Eq. (2.15) of the wave function ψ_p, together with the expression (2.13) for H_I, yields

$$\begin{aligned} S_{fi} &= \frac{ie}{(2\omega' V)^{1/2}} \int d^4x\, \bar{\psi}_{p'}(x)\, \gamma \cdot \epsilon\, \psi_p(x) e^{ik'\cdot x} \\ &= \frac{ieC'^*C}{(2\omega' V)^{1/2}} \int d^4x\, \bar{w}(\vec{p}\,')\left(1 + \frac{1}{2}\, \frac{e}{p'\cdot n}\, \gamma \cdot A\, \gamma \cdot n\right) \gamma \cdot \epsilon \left(1 + \frac{1}{2}\, \frac{e}{p \cdot n}\, \gamma \cdot n\, \gamma \cdot A\right) w(\vec{p}) e^{iS}, \end{aligned} \tag{2.16}$$

where the exponent S stands for the following sum of terms:

$$\begin{aligned} S = p'\cdot x &- \frac{1}{2}\, \frac{1}{p'\cdot n} \int d\phi (e^2A^2 - 2eA\cdot p') \\ &- p\cdot x + \frac{1}{2}\, \frac{1}{p\cdot n} \int d\phi (e^2A^2 - 2eA\cdot p) + k'\cdot x. \end{aligned} \tag{2.17}$$

To proceed further, a specific choice of the potential A has to be made. It is assumed that the scalar potential A^0 equals zero and that the vector potential $\vec{A}$ is the sum of two circularly polarized components propagating in the z direction with frequency ω_a and ω_b, respectively. That is,

$$\begin{aligned} \vec{A} = \frac{1}{\sqrt{2}} [& A_a(\vec{\xi} e^{-ik_a\cdot x} + \vec{\xi}^* e^{ik_a\cdot x}) \\ &+ A_b(\vec{\xi} e^{-ik_b\cdot x} + \vec{\xi}^* e^{ik_b\cdot x})], \end{aligned} \tag{2.18}$$

where the vector $\vec{\xi}$ equals $(\hat{x} - i\hat{y})/\sqrt{2}$, $\hat{x}$ and $\hat{y}$ being unit vectors in the x and y directions. From this choice of A it follows that

$$\begin{aligned} A^2 &= -\vec{A}^2 \\ &= -\{A_a^{\,2} + A_b^{\,2} + 2A_aA_b \cos[(\omega_a - \omega_b)\phi]\}. \end{aligned} \tag{2.19}$$

with $\phi = t - z$, and that the quasimomentum q, Eq. (2.11), equals

$$q = p + \frac{1}{2}\, \frac{e^2}{p\cdot n} (A_a^{\,2} + A_b^{\,2}) n. \tag{2.20}$$

Introducing (2.18) into (2.17) and making use of the definition of the quasimomentum, one finds

$$\begin{aligned} S &= (q' - q + k')\cdot x \\ &\quad + \alpha \sin(k_a - k_b)\cdot x - \beta_a \sin k_a\cdot x - \beta_b \sin k_b \cdot x, \end{aligned} \tag{2.21}$$

where

$$\alpha = \frac{A_aA_b}{\omega_a - \omega_b} \left(\frac{1}{p'\cdot n} - \frac{1}{p\cdot n}\right), \tag{2.22a}$$

$$\beta_a = \frac{eA_a}{\omega_a}\left(\frac{p_x'}{p'\cdot n} - \frac{p_x}{p\cdot n}\right), \quad (2.22b)$$

$$\beta_b = \frac{eA_b}{\omega_b}\left(\frac{p_x'}{p'\cdot n} - \frac{p_x}{p\cdot n}\right). \quad (2.22c)$$

For simplicity it has been assumed that the scattering takes place in the (x, z) plane.

Using the well-known expansion of an exponential in terms of Bessel functions,

$$e^{iz\sin\theta} = \sum_{m=-\infty}^{\infty} e^{im\theta} J_m(z), \quad (2.23)$$

one can rewrite the factor e^{iS} appearing in (2.16) as follows:

$$e^{iS} = \sum_{l,m,n} J_l(\alpha) e^{il(k_a-k_b)\cdot x} J_m(\beta_a) e^{-imk_a\cdot x} \times J_n(\beta_b) e^{-ink_b\cdot x} e^{i(q'-q+k')\cdot x}$$
$$= \sum_{r,s} \mathcal{J}_{r,s}\, e^{i(q'-q-rk_a-sk_b+k')\cdot x}. \quad (2.24)$$

The second line of (2.24) is obtained from the first line by introducing the new indices $r=-l+m$, and $s=l+n$, and the function

$$\mathcal{J}_{r,s} = \sum_l J_l(\alpha) J_{r+l}(\beta_a) J_{s-l}(\beta_b). \quad (2.25)$$

In the special case that $\omega_a = j\omega_b$, with $j = 2, 3, 4, \ldots,$ one can instead introduce the function

$$\mathcal{K}_t = \sum_l \sum_r J_l(\alpha) J_{r+l}(\beta_a) J_{t-jr-l}(\beta_b), \quad (2.26)$$

and the exponential e^{iS} may be expressed as follows:

$$e^{iS} = \sum_t \mathcal{K}_t\, e^{i(q'-q-tk_b+k')\cdot x}, \quad (2.27)$$

where t is an integer. The subsequent analysis would be the same, *mutatis mutandis*, as that for the case in which the frequencies are not integrally related.

Introducing (2.18) and (2.24) into (2.16) it will be seen that the matrix element S_{fi} is the sum of a number of terms, each one of which contains a δ function of the form $\delta(q'-q-jk_a-lk_b+k')$, with j and l integers. After some simplification one finds

$$S_{fi} = \frac{ieC'^*C}{(2\omega'V)^{1/2}} \sum_{r,s} \overline{w}(\vec{p}')[B_0\gamma\cdot\epsilon + B_1(\zeta_i\gamma\cdot\epsilon\,\gamma\cdot n\,\gamma\cdot\xi + \zeta_f\gamma\cdot\xi\,\gamma\cdot n\,\gamma\cdot\epsilon) + B_2(\zeta_i\gamma\cdot\epsilon\,\gamma\cdot n\,\gamma\cdot\xi^* + \zeta_f\gamma\cdot\xi^*\gamma\cdot n\,\gamma\cdot\epsilon)$$
$$+ B_3(\gamma\cdot\xi\,\gamma\cdot n\,\gamma\cdot\epsilon\,\gamma\cdot n\,\gamma\cdot\xi^* + \gamma\cdot\xi^*\gamma\cdot n\,\gamma\cdot\epsilon\,\gamma\cdot n\,\gamma\cdot\xi)]w(\vec{p})\,\delta(q'-q-rk_a-sk_b+k'). \quad (2.28)$$

In (2.28) the following definitions have been introduced:

$$B_0 = \mathcal{J}_{r,s}, \quad (2.29a)$$

$$B_1 = \frac{1}{\sqrt{2}}(A_a\mathcal{J}_{r-1,s} + A_b\mathcal{J}_{r,s-1}), \quad (2.29b)$$

$$B_2 = \frac{1}{\sqrt{2}}(A_a\mathcal{J}_{r+1,s} + A_b\mathcal{J}_{r,s+1}), \quad (2.29c)$$

$$B_3 = \tfrac{1}{2}\zeta_f\zeta_i[(A_a^2 + A_b^2)\mathcal{J}_{r,s} + A_aA_b(\mathcal{J}_{r-1,s+1} + \mathcal{J}_{r+1,s-1})], \quad (2.29d)$$

with ζ_i and ζ_f equal to $e/2p\cdot n$ and $e/2p'\cdot n$, respectively. The probability of emission is now calculated by taking the square of S_{fi}, averaging over the initial electron spin states, summing over the final spin states, and summing over the polarizations of the emitted photon. One obtains the following result:

$$P = \left[\frac{(2\pi)^4 eC'C}{4m}\right]^2 \frac{1}{V} \times \sum_{r,s} (\omega')^{-1}\delta(q'-q-rk_a-sk_b+k')^2 Z_{r,s}, \quad (2.30)$$

where

$$Z_{r,s} = C_1\mathcal{J}_{r,s}^2 + C_2\mathcal{J}_{r,s}\mathcal{L}_{r,s} + C_3[(A_a\mathcal{J}_{r-1,s} + A_b\mathcal{J}_{r,s-1})^2 + (A_a\mathcal{J}_{r+1,s} + A_b\mathcal{J}_{r,s+1})^2], \quad (2.31)$$

with

$$C_1 = -8m^2 - 4e^2(A_a^2 + A_b^2)\Gamma, \quad (2.32a)$$

$$\Gamma = \frac{p\cdot n}{p'\cdot n} + \frac{p'\cdot n}{p\cdot n}, \quad (2.32b)$$

$$C_2 = -8\left[\frac{2e^2A_aA_b}{\alpha} + (\omega_a - \omega_b)(p\cdot n - p'\cdot n)\right], \quad (2.32c)$$

$$C_3 = 2e^2\Gamma\,, \tag{2.32d}$$

$$\mathcal{L}_{r,s} = \sum_l l J_l(\alpha) J_{r+l}(\beta_a) J_{s-l}(\beta_b)\,. \tag{2.32e}$$

The transition rate, dP/dt, is then integrated over the final quasimomentum of the electron and the final momentum of the emitted photon, yielding the total transition rate

$$R = \left(\frac{eC'C}{8\pi m}\right)^2 V \times \sum_{r,s} \int \cdots \int d^3q'\, d\omega'\, d\Omega'\, \omega' \times \delta(q' - q - rk_a - sk_b + k') Z_{r,s}\,, \tag{2.33}$$

where the symbol $d\Omega'$ stands for the element of solid angle into which the photon is emitted. The integration over ω' requires some care, because in the argument of the energy part of the δ function,

$$\Xi = E' + \frac{e^2(A_a^2 + A_b^2)}{2p'\cdot n} - E - \frac{e^2(A_a^2 + A_b^2)}{2p\cdot n} - r\omega_a - s\omega_b + \omega'\,, \tag{2.34}$$

both E' and $p'\cdot n$ are a function of ω'. Such an implicit dependence on ω' is taken into account introducing into the integrand the factor

$$\left(\frac{d\Xi}{d\omega'}\right)^{-1} = \frac{E'\omega'}{(r\omega_a + s\omega_b)p\cdot n}\left(1 + \frac{\rho}{E'p'\cdot n}\right), \tag{2.35}$$

with

$$\omega' = \frac{(r\omega_a + s\omega_b)p\cdot n}{E - |\vec{p}|\cos\theta_{\vec{p}\vec{k}'} + (1-\cos\theta)(r\omega_a + s\omega_b + \rho/p\cdot n)} \tag{2.36}$$

and

$$\rho = \tfrac{1}{2}e^2(A_a^2 + A_b^2)\,. \tag{2.37}$$

In (2.36) the symbol θ stands for the angle between $\vec{k}'$, the emitted photon momentum, and the z axis.

The cross section for scattering is now defined as the ratio of the total transition rate to the relative flux of electrons and a-type photons,

$$N = (1 - v_z^{(A)})\,\omega_a A_a^2/V\,,$$

that is,

$$\sigma = R/N = \frac{2\pi r_0^2}{1 - v_z}\frac{m^2}{Ep\cdot n} \times \sum_{r,s}\int d\theta \sin\theta\, \frac{\omega'^2}{\omega_a(r\omega_a + s\omega_b)} Z'_{r,s}\,, \tag{2.38}$$

where $Z'_{r,s} = Z_{r,s}/(2eA_a)^2$ and r_0 is the classical electron radius. This definition of cross section has been chosen because of our eventual interest in the scattering of photons out of the a component of the field.

Owing to the complexity of $Z_{r,s}$ [Eq. (2.31) with Eqs. (2.32a)–(2.32e)] it is not at all transparent how the cross section σ depends on various parameters such as A_a and A_b. However, in two cases more tractable forms of σ emerge.

1. Monochromatic case

In the case that A_b equals zero the parameters α and β_b and the function $\mathcal{L}_{r,s}$ all equal zero; further, the function $\mathcal{J}_{r,s}$ simplifies to $J_r(\beta_a)\delta_{s0}$. It follows from (2.31) that

$$Z'_{r,s} = \left(\Gamma\left\{J_r'^2(\beta_a) + \left[\left(\frac{r}{\beta_a}\right)^2 - 1\right] J_r^2(\beta_a)\right\} - 2\left(\frac{m}{eA_a}\right)^2 J_r^2(\beta_a)\right)\delta_{s0}\,, \tag{2.39}$$

which, introduced into (2.38) gives the cross section for scattering out of an intense monochromatic field A_a. This is the same expression as that obtained by other workers.[4,11,12] The behavior of the cross section as a function of the field intensity will be illustrated in Sec. III. It may be readily shown that one obtains from (2.39) the usual expression for the low-intensity Compton cross section[15] in the limit that A_a vanishes.

2. Case of weak a component

By allowing A_a to tend to zero in (2.38) one obtains the dependence of the cross section for scattering a photon out of a weak field on the intensity of another field. The result is applicable to the problem of enhancement of γ-ray production mentioned in the Introduction. In this limit an examination of Eqs. (2.25), (2.31), and (2.32) shows that the only nonvanishing terms are those for which $r=1$ and $l=-1$ or 0. One obtains for $Z'_{r,s}$

$$Z'_{1,s} = \mathcal{Y} + \mathfrak{z}\,, \tag{2.40}$$

with

$$\mathcal{Y} = D_1 J_s^2 + D_2 J_s'^2 + D_3 J_s J_s' \tag{2.41}$$

and

$$\mathfrak{z} = \left(\frac{\alpha A_b}{2A_a}\right)^2\left[\Gamma\left(J_{s+1}'^2 + \frac{2\beta_b}{\alpha} J_{s+1}' J_{s+1}\right) + D_4 J_{s+1}^2 + D_5 J_s J_{s+1}\right], \tag{2.42}$$

where the argument of the Bessel functions is β_b and where

$$D_1=\Gamma\left[\frac{1}{2}+\frac{s}{2}\frac{\omega_b}{\omega_a}+\left(\frac{s}{2}\frac{\omega_b}{\omega_a}\right)^2-\left(\frac{\beta_b\omega_b}{2\omega_a}\right)^2-\frac{\alpha}{2}\frac{A_b}{A_a}\right]-\left[\frac{1}{2}\left(\frac{\beta_b\omega_b}{\omega_a}\right)^2\left(\frac{m}{eA_b}\right)^2+\frac{s\omega_b}{\omega_a}\right], \tag{2.43a}$$

$$D_2=\left(\frac{\beta_b\omega_b}{2\omega_a}\right)^2\Gamma, \tag{2.43b}$$

$$D_3=\Gamma\left[\frac{\beta_b\omega_b}{2\omega_a}-\frac{\alpha\beta_a}{2}\left(\frac{A_b}{A_a}\right)^2\right]+\frac{\beta_b\omega_b}{\omega_a}, \tag{2.43c}$$

$$D_4=\Gamma\left[\left(\frac{s+1}{\beta_b}\right)^2-1\right]-2\left(\frac{m}{eA_b}\right)^2+\frac{2(\omega_a-\omega_b)[(p\cdot n)^2+(p'\cdot n)^2]}{(eA_b)^2(p\cdot n-p'\cdot n)}, \tag{2.43d}$$

$$D_5=\frac{-2\beta_a}{(eA_b)^2\alpha}[(p\cdot n-p'\cdot n)(\omega_a-\omega_b)-2m^2]. \tag{2.43e}$$

The cross section obtained by introducing (2.40) into Eq. (2.38) still has a very complicated form, and to understand the effect of the second field on σ one is obliged to resort to numerical analysis on a digital computer. This has been done and our results will be presented in Sec. III. It is clear by inspection, however, that no resonance behavior is indicated in any of the terms (2.43); Oleinik's[8] predictions are consequently not confirmed.[16]

In the case that $\tilde{A}=eA_b/m$ is small compared to unity an expansion of σ in powers of $\tilde{A}$ is useful. This will be written down, taking $\vec{p}$ equal to zero and assuming that the frequencies ω_a and ω_b are very small compared to m. We are considering scattering by an electron initially at rest out of, for example, visible and infrared laser beams. The ratio $R=\omega_b/\omega_a$ is taken to be less than unity. With these assumptions, the frequency of the emitted photon equals

$$\omega'=\frac{\omega_a+s\omega_b}{1+\frac{1}{2}\tilde{A}^2(1-\cos\theta)}, \tag{2.44}$$

and the argument of the Bessel functions in Eqs. (2.41) and (2.42) equals

$$\beta_b=-\frac{\tilde{A}\sin\theta}{1+\frac{1}{2}\tilde{A}^2(1-\cos\theta)}\left(s+\frac{\omega_a}{\omega_b}\right). \tag{2.45}$$

It will be convenient to denote the contribution to the cross section for a given value of s by $2\pi r_0^2\sigma^{(s)}$. Then, replacing the Bessel functions by their well-known power series, and performing the integration in Eq. (2.38),[17] one finds for the reduced cross section $\sigma^{(s)}$, with $s=0,\pm1,\pm2,$

$$\sigma^{(0)}=\frac{4}{3}-\frac{2}{5}\left(\frac{\tilde{A}}{R}\right)^2\left(1+\frac{5}{3}R+\frac{10}{3}R^2+\frac{10}{3}R^3+\cdots\right)+\frac{1}{105}\left(\frac{\tilde{A}}{R}\right)^4(6+21R+119R^2+161R^3+217R^4+259R^5+\cdots)+\cdots, \tag{2.46a}$$

$$\sigma^{(1)}=\frac{1}{5}\left(\frac{\tilde{A}}{R}\right)^2(1+4R+6R^2+4R^3+\cdots)-\frac{1}{105}\left(\frac{\tilde{A}}{R}\right)^4(4+31R+137R^2+339R^3+487R^4+458R^5+\cdots)+\cdots, \tag{2.46b}$$

$$\sigma^{(-1)}=\frac{1}{5}\left(\frac{\tilde{A}}{R}\right)^2\left(1-\frac{2}{3}R+\frac{8}{3}R^2-\frac{2}{3}R^3+\cdots\right)-\frac{1}{105}\left(\frac{\tilde{A}}{R}\right)^4(4-3R+67R^2-73R^3+39R^4+4R^5+\cdots)+\cdots, \tag{2.46c}$$

$$\sigma^{(2)}=\frac{1}{105}\left(\frac{\tilde{A}}{R}\right)^4(1+12R+60R^2+160R^3+240R^4+192R^5+\cdots)+\cdots, \tag{2.46d}$$

$$\sigma^{(-2)}=\frac{1}{105}\left(\frac{\tilde{A}}{R}\right)^4(1-5R+25R^2-55R^3+44R^4-31R^5+\cdots)+\cdots. \tag{2.46e}$$

The quantity $2\pi r_0^2\sigma^{(0)}$ is the cross section for emitting a photon of frequency equal to approximately ω_a, that is, the cross section for Thomson scattering. It will be noticed that it equals the sum of the usual Thomson cross section, $\frac{8}{3}\pi r_0^2$, and terms depending on the intensity of the b component of the field. The dominant correction, $-2\pi r_0^2\times\frac{2}{5}(\tilde{A}/R)^2$, indicates that the application of the second field reduces the intensity of Thomson scattering. This reduction arises from consecutive absorption and emission of b-type photons, as is shown in Appendix B.

The quantity $2\pi r_0^2\sigma^{(-1)}$ may be interpreted as the cross section for stimulated double Compton scattering. In this process a photon of type a is scattered into two photons, one of frequency ω_b, the other of frequency $\omega'\approx\omega_a-\omega_b$. The latter photon is accessible to an observer, whereas the former is emitted into the b component of the field. One may obtain the leading term of (2.46c) using the theory of stimulated processes and the known double Compton cross section, as is shown in Appendix C. The quantity $\sigma^{(+1)}$, on the other hand, corresponds to a process in which two photons are absorbed by the electron, one of type a, the other of type b, and only one photon is emitted.

The total cross section, to the fourth power in $\tilde{A}$, is obtained by summing the reduced cross sections listed in (2.46):

$$\sigma = 2\pi r_0^2[\tfrac{4}{3} + \tilde{A}^2(\tfrac{2}{5} - \tfrac{2}{3}R + \cdots) - \tilde{A}^4(\tfrac{5}{21} + \tfrac{2}{5}R + \cdots) + \cdots]. \qquad (2.47)$$

For both R and $\tilde{A}$ small compared to unity it will be seen that the effect of the b field is to increase the cross section. This increase stems from the fact that the sum, $2\pi r_0^2(\sigma^{(+1)} + \sigma^{(-1)})$, exceeds the first correction to Thomson scattering; that is, the increase in photon emission at the sum and difference frequencies, $\omega' = \omega_a \pm \omega_b$, is not exactly canceled by the decrease in emission at $\omega' = \omega_a$. Kronig and Höfelt[7] found instead that σ was independent of $\tilde{A}$; the discrepancy between their result and (2.47) arises presumably from their numerous approximations.

III. NUMERICAL RESULTS

A. Bichromatic case

The range of validity of Eq. (2.47) is limited to values of $\tilde{A}$ small compared to unity. To evaluate the cross section for larger values of $\tilde{A}$, numerical evaluation of Eq. (2.38) is required. As a consequence one is obliged to choose values for ω_a, ω_b, and the initial velocity of the electron v_z. Because of the already-mentioned interest in the production of γ rays by backward scattering of ruby laser photons from an energetic electron beam, the free electron energy was taken to be equal to 5.11 GeV. Radiation from a CO_2 laser was selected as the supplementary b component of the field.

The variation of the cross section with $\tilde{A}$ is shown in Fig. 1. It will be seen that σ first increases with $\tilde{A}$, as would be expected from (2.47), but that it reaches a maximum and then decreases. The maximum enhancement of the cross section equals about 10%. In Fig. 2 is shown the contribution to the total cross section for different values of the index s in the case $\tilde{A} = 1$. It will be noticed that processes in which the electron absorbs a large number of b-type photons (i.e., high s values) are important. In Fig. 3 are shown the reduced cross sections $\sigma^{(s)}$ for $s = 0, \pm 1, \pm 2$, as functions of $\tilde{A}$. For values of $\tilde{A}$ much less than unity the shape of these graphs is the same as one would expect on the basis of the formulas for $\sigma^{(s)}$, Eqs. (2.46).

In addition, the differential cross section for emitting a photon of frequency ω' into an interval $d\omega'$, that is, the spectrum, $P(\omega')d\omega'$, was calculated. The latter is obtained from (2.38) chang-

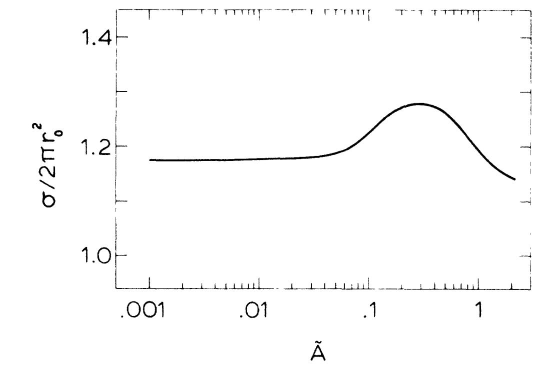

FIG. 1. Cross section for scattering a photon out of a weak field as a function of the amplitude $\tilde{A} = eA_b/m$ of a supplementary field. The energies of the ingoing electron and the weak (ruby laser) and supplementary (CO_2 laser) field photons equal respectively 5.11 GeV, 1.79 eV, and 0.118 eV. The electron and the two fields are initially propagating in opposite directions.

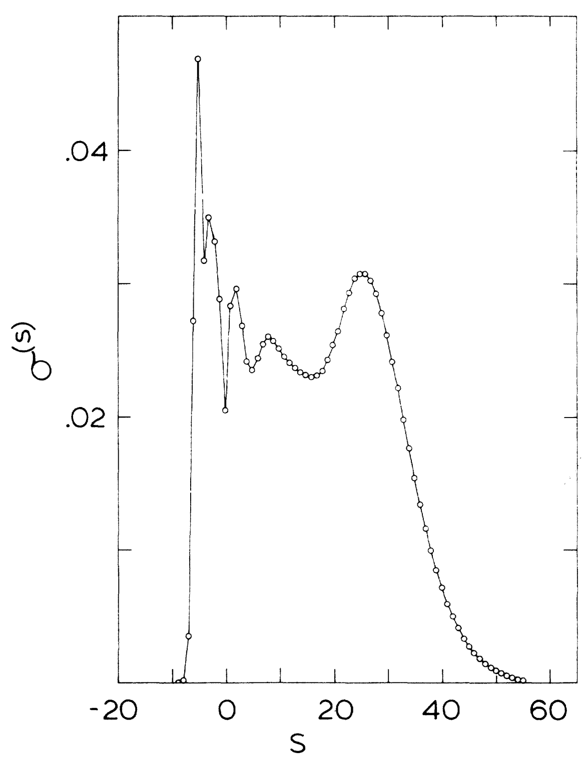

FIG. 2. The reduced cross section as a function of the index s for $\tilde{A} = 1$.

ing the variable of integration from θ to ω' through Eq. (2.36):

$$P(\omega') = 2\pi r_0^2 \sum_s \{[\omega_a(\omega_a + s\omega_b)/\gamma + \rho(1-\beta) - \beta]Ep\cdot n\}^{-1} \times Z'_{1,s}\,. \qquad (3.1)$$

In (3.1) β stands for the speed of the electron v_z/c and $\gamma = (1-\beta^2)^{-1/2}$. The spectrum of the radiation is illustrated in Fig. 4 for several values of $\tilde{A}$. The shape changes significantly as $\tilde{A}$ increases; however, the energy at which $P(\omega')$ falls to negligible values does not increase with $\tilde{A}$. This is unexpected in view of the fact that, as mentioned above, for large $\tilde{A}$, processes in which the electron absorbs a large number of b-type photons become important. The explanation resides in the fact that when the electron enters the field it is slowed down by the intense b component, as may be deduced from (2.8), and the frequency of the emitted photon is decreased.

B. Monochromatic case

Because of the development of powerful CO_2 lasers it was deemed interesting to consider replacing the ruby laser normally used in the γ-ray production set-up with a CO_2 laser, and rely on high harmonic generation to produce energetic γ rays. The results of this investigation are illustrated in Figs. 5 and 6, showing the cross section and spectrum, respectively, calculated from Eqs. (2.39) and (2.38). The initial electron energy

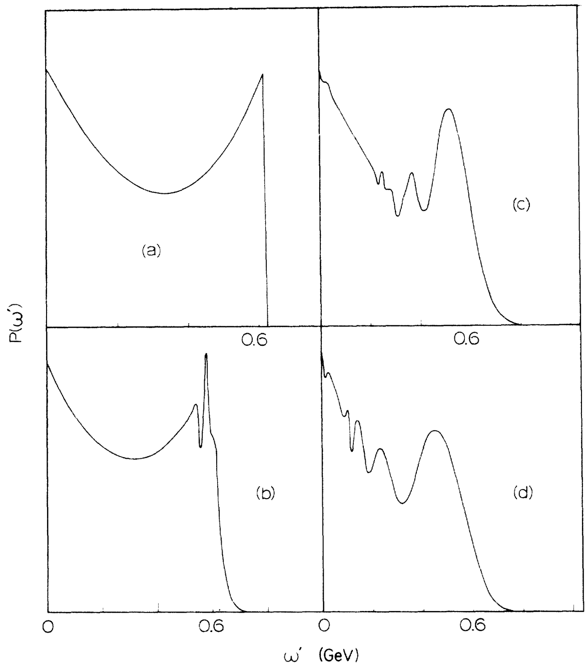

FIG. 4. The spectrum of the scattered radiation for $\tilde{A}=0$, 0.25, 1.0, and 2.0 in (a), (b), (c), and (d), respectively. The energies of the ingoing electron and the weak and supplementary field photons equal respectively 5.11 GeV, 1.79 eV, and 0.118 eV.

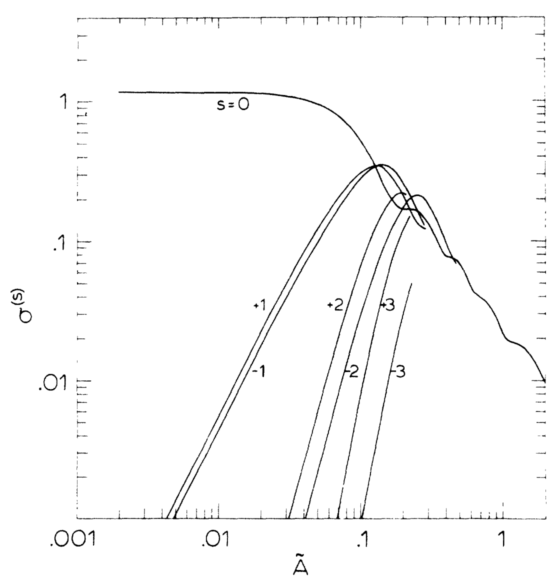

FIG. 3. The reduced cross section $\sigma^{(s)}$, for $s=0$, ± 1, ± 2, and ± 3 as a function of $\tilde{A}$. Because the different curves interlace for $\tilde{A} \gtrsim 0.1$, only $\sigma^{(0)}$ has been plotted in this region.

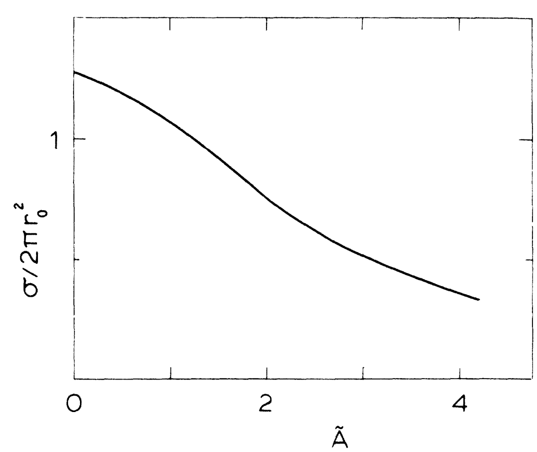

FIG. 5. The total cross section for scattering a photon out of radiation from a CO_2 laser as a function of the amplitude $\tilde{A} = eA/m$. The initial electron energy equals 25.6 GeV. The electron and the field propagate initially in opposite directions.

was taken as equal to 25.6 GeV. It was found that the total cross section decreased with the intensity of the field and that the spectrum displayed a similar characteristic to that of the bichromatic field case; that is, although production of high harmonics becomes important, the corresponding γ-ray energies are much lower than expected because of the already-mentioned electron drift. It appears, consequently, that a powerful but low-frequency laser is not useful in this application.

APPENDIX A

Following the notation of Bjorken and Drell[13] one can write the solution to Eq. (2.1) as the perturbation expansion

$$\psi(x)=\Psi(x)+e\int d^4y\, S_F(x-y)\gamma\cdot A(y)\Psi(y)+e^2\int\int d^4y\, d^4y'\, S_F(x-y)\gamma\cdot A(y)S_F(y-y')\gamma\cdot A(y')\Psi(y')+\cdots . \quad \text{(A1)}$$

In (A1)

$$\Psi(x)=(2\pi)^{-3/2}(m/E)^{1/2}e^{-ip\cdot x}w(\vec{p}) \quad \text{(A2)}$$

is the Dirac wave function for a free electron and

$$S_F(x-y)=\int\frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{p^2-m^2+i\epsilon}(\gamma\cdot p+m) \quad \text{(A3)}$$

is the free electron propagator. Assume a monochromatic field A propagating in the z direction and given by

$$A(x)=\frac{A_b}{\sqrt{2}}(\xi e^{-ik_b\cdot x}+\xi^* e^{ik_b\cdot x}), \quad \text{(A4)}$$

with $\xi=(0;1/\sqrt{2},-i/\sqrt{2},0)$. Then the lowest-order correction to the free wave function is

$$C^{(1)}=(2\pi)^{-3/2}\left(\frac{m}{2E}\right)^{1/2}eA_b\int d^4p'\, e^{-ip'\cdot x}\frac{\gamma\cdot p'+m}{p'^2-m^2+i\epsilon}[\gamma\cdot\xi\delta(p'-p-k_b)+\gamma\cdot\xi^*\delta(p'-p+k_b)]w(\vec{p})$$

$$=\frac{e}{2p\cdot k_b}[i2A_bp_x\sin k_b\cdot x+\gamma\cdot k_b\gamma\cdot A]\Psi(x), \quad \text{(A5)}$$

corresponding to processes in which a single photon is either absorbed or emitted from the field. The second-order correction to the wave function corresponds to the two-photon processes illustrated in Fig. 7. The correction corresponding to the first two diagrams is easily calculated and found to be

$$C_1^{(2)}=\left(\frac{eA_b}{2p\cdot k_b}\right)^2[e^{-i2k_b\cdot x}p\cdot\xi(p\cdot\xi+\gamma\cdot k_b\gamma\cdot\xi)+e^{i2k_b\cdot x}p\cdot\xi^*(p\cdot\xi^*-\gamma\cdot k_b\gamma\cdot\xi^*)]\Psi(x) . \quad \text{(A6)}$$

However, the correction corresponding to the last two diagrams is not trivial to calculate because of a propagator which becomes infinite. To avoid such a difficulty, one assumes that the amplitude of the field A is a function $A_b(\omega)$ very sharply peaked around a value ω_b with $\int d\omega A_b(\omega)=A_b$. This artifice allows one to perform all necessary integrations and leads to the following contribution to the second-order correction[18]:

$$C_2^{(2)}=-\frac{1}{2}(eA_b)^2\left[\frac{|p\cdot\xi|^2}{(p\cdot k_b)^2}+\frac{1}{2(p\cdot k_b)^2}(p\cdot\xi^*\gamma\cdot k_b\gamma\cdot\xi-p\cdot\xi\gamma\cdot k_b\gamma\cdot\xi^*)+i\frac{k_b\cdot x}{p\cdot k_b}\right]\Psi(x). \quad \text{(A7)}$$

The sum of Ψ, (A5), (A6), and (A7) gives the wave function of the electron in the field correct to the second power in A,

$$\psi=\left[\left(1+\frac{e}{2p\cdot k_b}\gamma\cdot k_b\gamma\cdot A\right)\left(1+i\frac{eA_bp_x}{p\cdot k_b}\sin k_p\cdot x\right)-i\frac{(eA_b)^2k_b\cdot x}{2p\cdot k_b}-\frac{1}{2}\left(\frac{eA_bp_x}{p\cdot k_b}\sin k_b\cdot x\right)^2\right]\Psi(x). \quad \text{(A8)}$$

This solution is identical with the Volkov wave function (2.3), up to powers linear and quadratic in A, as may be seen by expanding the exponential factor

$$\exp\left[i\frac{1}{2}\frac{1}{p\cdot n}\int d\phi(e^2A^2-2eA\cdot p)\right], \quad \text{(A9)}$$

occurring in (2.3).

APPENDIX B

In this appendix the lowest-order correction to Thomson scattering due to the presence of a classical field is discussed. The model taken is that of an electron in a classical circularly polarized electromagnetic field of frequency ω_b absorbing a photon of frequency ω_a from a quantized field and emitting a photon of frequency ω' with $\omega' \approx \omega_a$. It is assumed that ω_b is much less than ω_a. The calculation is carried out in the standard way using second-order perturbation theory in the quantized field, choosing, however, expression (A8) to represent the initial and final electron wave function.

We are interested in corrections to the lowest order in $\tilde{A}/R$. With this in mind, and since R is assumed much less than unity, one is justified in taking the free electron propagator as intermediate propagator. The transition amplitude then equals

$$S_{fi} = -ie^2 \left\langle 1, 0 \left| \int\int d^4y_2 d^4y_1 \bar{\psi}_f(y_2)\gamma\cdot\mathfrak{A}(y_2) S_F(y_2 - y_1)\gamma\cdot\mathfrak{A}(y_1)\psi_i(y_1) \right| 0, 1 \right\rangle, \tag{B1}$$

where the symbol $|n, m\rangle$ represents the state of the quantized field with n photons of frequency ω' and m photons of frequency ω_a. The symbol $\mathfrak{A}$ stands for the quantized field operator. It is assumed that the scattering takes place in the x, z plane, and that the classical field propagates in the $+z$ direction. It is further assumed that the initial electron momentum equals zero. The only part of ψ_f which turns out to be relevant in this problem equals

$$\psi_f' = \left[1 - \left(\frac{eA_b p_x}{2p\cdot k_b}\right)^2\right]\Psi, \tag{B2}$$

the correction being the first term of (A7). This correction is due to consecutive absorption and emission of b-type photons from the classical field. The other terms in (A8) either result in the incorrect frequencyfor ω' or else they give rise to terms in $|S_{fi}|^2$ which either have zero trace or the wrong dependence on $\tilde{A}/R$. From the form of (B2) it is clear that the corrected Thomson differential cross section equals simply

$$\frac{d\sigma}{d\Omega'} = \left[1 - \frac{1}{2}\left(\frac{eA_b p_x}{p\cdot k_b}\right)^2\right]\frac{d\sigma_T}{d\Omega'}$$
$$= \left[1 - \frac{1}{2}\left(\frac{\tilde{A}}{R}\right)^2 \sin^2\theta\right]\frac{d\sigma_T}{d\Omega'}, \tag{B3}$$

where

$$\frac{d\sigma_T}{d\Omega'} = \frac{r_0^2}{2}(1 + \cos^2\theta) \tag{B4}$$

is the usual differential Thomson cross section. Integrating over the solid angle one obtains for the complete cross section

$$\sigma = 2\pi r_0^2\left[\frac{4}{3} - \frac{2}{5}\left(\frac{\tilde{A}}{R}\right)^2\right], \tag{B5}$$

in agreement with the first two terms of (2.46a).

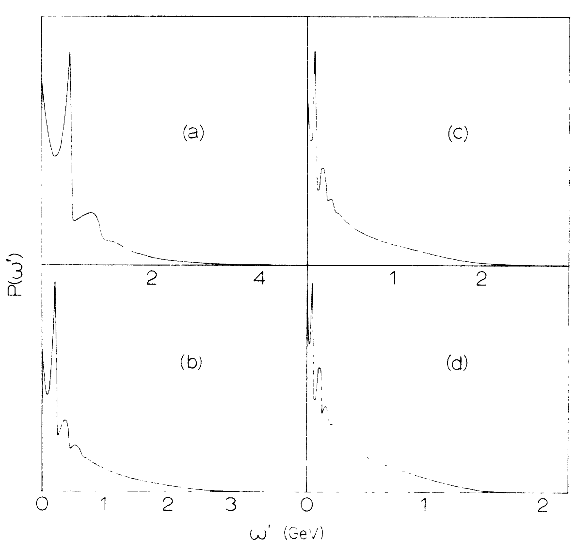

FIG. 6. The spectrum of the scattered radiation from a CO_2 laser for $\tilde{A} = 1, 2, 3$, and 4 in (a), (b), (c), and (d), respectively. The incident electron energy equals 25.6 GeV.

APPENDIX C

The cross section for stimulated double Compton scattering may be calculated by multiplying the cross section for spontaneous double Compton scattering by the number of photons in the stimulating field. Assume a beam of photons of frequency ω_a propagating in the z direction incident on an electron at rest, with $\omega_a \ll m$. Consider the

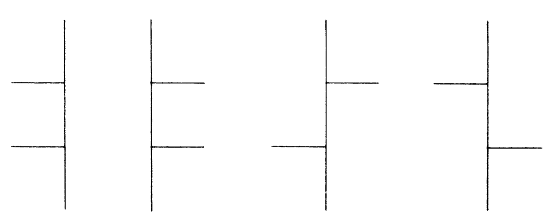

FIG. 7. Two-photon processes corresponding to second-order corrections to the electron wave function.

process in which the electron emits a photon of frequency ω', $\omega' \approx \omega_a$, at an angle θ with respect to the z axis, and at the same time emits a photon of frequency ω, $\omega \ll \omega_a$, along the z axis. The differential cross section for this process,

$$d\sigma_D = \frac{\alpha}{\pi}\, d\sigma_T(\omega_a, \omega')\, \frac{d\omega}{\omega}\, \frac{d\Omega}{4\pi} \left(\frac{\omega' \sin\theta}{m}\right)^2, \quad \text{(C1)}$$

may be obtained from formula (11.41) of Ref. 15. In (C1) $d\sigma_T$ is the differential Compton cross section (B4), $d\Omega$ is the element of solid angle into which the photon of frequency ω is emitted, and α is the fine-structure constant. Assume that a classical field of amplitude $A_b(\omega)$, very sharply peaked around $\omega = \omega_b$, $\omega_b \ll \omega_a$, is propagating along the z axis. The number of photons associated with this wave per unit frequency interval and per unit solid angle may be expressed as

$$n(\omega) = (2\pi)^3 A_b{}^2(\omega)/(2\omega\Delta\Omega). \quad \text{(C2)}$$

The differential cross section for stimulated double Compton scattering hence equals

$$d\sigma = \int d\omega\, n(\omega)\, d\sigma_D$$
$$= \frac{\tilde{A}^2}{2}\left(\frac{\omega'}{\omega_b}\right)^2 \sin^2\theta\, d\sigma_T, \quad \text{(C3)}$$

where $\tilde{A}^2 = (e/m)^2 \int A_b{}^2(\omega)\, d\omega$. Performing the integration over θ one obtains for the total stimulated cross section

$$\sigma = \frac{2\pi r_0{}^2}{5}\left(\frac{\tilde{A}}{R}\right)^2, \quad \text{(C4)}$$

with $R = \omega_b/\omega_a$. The same expression has been obtained as the first term in (2.46c).

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