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Domain structure of a Reggeon field theory with three couplings

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The high-energy behavior of a Reggeon field theory with a three-Reggeon coupling and two four-Reggeon couplings is considered. Using the renormalization-group equations which are calculated in lowest order by Bardeen *et al.* for this model, we study the way the couplings evolve to the fixed points in the high-energy limit. We find that there are three fixed points (plus a fourth at infinity) to which the couplings can evolve. The space of couplings divides into four domains, two of which are two-dimensional and two of which are three-dimensional. The couplings in one of the three-dimensional domains are found to evolve to infinity and the couplings in the other three domains are found to evolve to the fixed-point couplings. We show the boundaries of these domains and some characteristic evolution curves. The implications of the domain structure of the model are discussed.

INTRODUCTION

The renormalization group has proved to be a very intriguing tool for the study of Reggeon field theory.[1] Using the renormalization group, one is in principle able to extract the infrared behavior of the theory, a limit in which, in general, an infinite number of bare diagrams would contribute. One is able to accomplish this feat by showing that in this limit the effective renormalized coupling becomes small; therefore, all one need do is calculate this small effective coupling and use perturbation theory.

Reggeon field theory has the possibility of giving a direct physical interpretation to both the bare theory and the renormalized theory.[2] At low energies (~100 GeV) one can argue that only a few Reggeon diagrams will contribute to a given process, and that at higher energies more diagrams will contribute. While a naive attempt to sum this series leads to trouble,[3] the use of the renormalization group and Reggeon field theory in some sense allows us to sum them and obtain a sensible result. The bare couplings in a Reggeon field theory are those that one would obtain from fitting present energy data, and the couplings calculated from the renormalization group are those that one would obtain by fitting ultra-high-energy data. If this use of the renormalization group is correct and one were presented with data over this tremendous energy range and proceeded to fit it with Reggeon diagrams one would find the following: At low energies one would be able to find fits with relatively few diagrams, going to higher energies would require more diagrams to get a good fit, and the couplings would be energy-dependent. Going finally to the highest energies the fits would become simpler, only a few diagrams would be necessary and the coupling would be quite small.

This of course is the best of all possible worlds and there are a number of things that could go wrong. First the use of perturbation theory to find the effective infrared coupling may not be justified. Secondly, the effective infrared coupling may not be small, implying that the use of perturbation theory to calculate the ultra-high-energy behavior is not justified, and furthermore that the initial use of perturbation theory may not have been valid. Finally, there is the possibility that starting with the experimental bare coupling one is not carried by the renormalization group to the stable infrared coupling.

In this paper we will consider a model with three couplings, one three-Reggeon coupling and two four-Reggeon couplings, and study the detailed structure of the evolution of these couplings from their bare values to their effective infrared values. We find one new infrared fixed point, and discuss in some detail the relation between the pure φ^3 theory and the pure φ^4 theory. Finally, we attempt to analyze the implication of bare couplings at present energies.

REGGEON FIELD THEORY

The Reggeon field theory that we are considering here has been calculated in detail in Ref. 4. This theory has (a) a linear Pomeron trajectory, (b) a triple-Pomeron coupling g, and (c) two types of four-Pomeron coupling h_1 and h_2. The reasons for studying this model in detail are several: (i) phenomenology indicates that linear trajectories are preferred, (ii) there is strong experimental evidence for the existence of a bare triple-Reggeon coupling, (iii) the four-Reggeon coupling is the only other coupling that might be relevant to the ultra-high-energy behavior, and there is experimental evidence that the four-Reggeon coupling is sizable at present energies,[10] and (iv) the model has sufficient complexity to see new phenomena; in particular it has a nontrivial domain structure.[5]

The bare perturbation graphs for this theory (with a cutoff to remove infinites) will essentially reproduce the basic Gribov calculus.[2] Each of these graphs is a representation that is valid in a particular kinematic region, for example, the graph in Fig. 1(a) is valid in a region where the energy across the one Reggeon is large. The graph in Fig. 1(b) will require a higher-minimum energy, and a measure of its relative importance or size is $g^2(0)\ln s$.[6] This idea of a perturbation series of diagrams with successive diagrams becoming more important with increasing energy has been discussed in detail by Frazer, Snider, and Tan[6] in the context of the multiperipheral model with a triple-Reggeon coupling. The basic point, which has been elaborated on by White[2] in the context of the Reggeon field theory, is that the relative size of the successive graphs is measured by $g^2\ln s$ so that as s (the center-of-mass energy squared) increases, the effective coupling increases and one needs more diagrams to maintain the same accuracy.

The Reggeon field theory not only allows you to sum this series in the ultra-high-energy limit, but using the renormalization group shows you that the ultra-high-energy behavior can be expressed in terms of a few perturbation graphs, which have a new effective coupling that has evolved from the bare coupling, in a manner that can be investigated in detail.

The essence of the renormalization group is the following relation between Reggeon Green's functions

$$\Gamma(\xi(1-J), g(0), \ldots) = f\,\Gamma(1-J, g(-\ln\xi), \ldots)$$

which says that the behavior near $J=1$ ($\xi\to 0$) is given by the behavior for J away from one but with the bare coupling $g(0)$ replaced by $g(\infty)$ (multiplied by some calculable function f). The evolution of g from $g(0)$ to $g(\infty)$ is given by the differential equation

$$\frac{dg(t)}{dt} = -\beta_g(g(t)),$$

where $t=-\ln\xi$. In practice, β_g is obtained from renormalized perturbation theory in some approximation. The solution to the equation, $g(t)$, gives the evolution of the coupling constant g from the bare coupling $g(0)$ to the effective coupling for J closer and closer to one, i.e., $-\ln\xi\to\infty$.

In the particular model that we are considering there are three couplings: the triple-Reggeon coupling g, a four-Reggeon coupling h_1 involving one incoming and three outgoing Reggeons or vice versa, and a four-Reggeon coupling h_2 for two incoming Reggeons and two outgoing Reggeons. Thus we have three coupled differential equations that describe the evolution of the coupling:

$$\frac{dg^2(t)}{dt} = -\beta_{g^2}(g^2(t), h_1(t), h_2(t)),$$

$$\frac{dh_1(t)}{dt} = -\beta_{h_1}(g^2(t), h_1(t), h_2(t)),$$

$$\frac{dh_2(t)}{dt} = -\beta_{h_2}(g^2(t), h_1(t), h_2(t)).$$

For the model we are considering, β_{g^2}, β_{h_1}, and β_{h_2} have been calculated in perturbation theory in the one-loop approximation[4] and are given by

$$\beta_{g^2} = -g^2 + 4.886g^4 + 2g^2h_1 + g^2h_2,$$

$$\beta_{h_1} = 12.527g^2h_1 + 4.54g^2h_2 + 1.5h_1h_2,$$

$$\beta_{h_2} = 13.772g^2h_1 + 10.372g^2h_2 + 0.5h_2{}^2 + 2h_1{}^2.$$

DOMAIN STRUCTURE

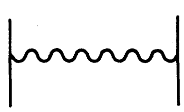
We consider here the detailed evolution of the bare couplings $g^2(0)$, $h_1(0)$, and $h_2(0)$ to the so-called fixed-point couplings $g^2(\infty)=\bar{g}^2$, $h_1(\infty)=\bar{h}_1$, and $h_2(\infty)=\bar{h}_2$. A fixed-point coupling is, as the name suggests, a coupling which does not change as t changes, i.e.,

$$\frac{d\bar{g}^2}{dt}=0, \quad \frac{d\bar{h}_1}{dt}=0, \quad \frac{d\bar{h}_2}{dt}=0$$

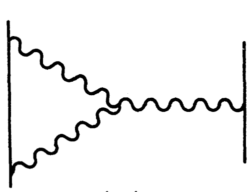
or equivalently

$$\beta_{g^2}(\bar{g}^2, \bar{h}_1, \bar{h}_2)=0, \quad \beta_{h_1}(\bar{g}^2, \bar{h}_1, \bar{h}_2)=0, \quad \beta_{h_2}(\bar{g}^2, \bar{h}_1, \bar{h}_2)=0.$$

These fixed-point couplings are candidates for the



(a)



(b)

FIG. 1. Reggeon perturbation graphs.

effective ultra-high-energy couplings. It is a relatively straightforward process to solve these three sets of equations for the fixed points; there are three fixed points:

Case (a)

$$\bar{g}^2=0, \quad \bar{h}_1=0, \quad \bar{h}_2=0;$$

Case (b)

$$\bar{g}^2=0.2047, \quad \bar{h}_1=0, \quad \bar{h}_2=0;$$

Case (c)

$$\bar{g}^2=0.196\,18, \quad \bar{h}_1=0.288\,87, \quad \bar{h}_2=-0.536\,21.$$

As a result of these three fixed points our space of couplings (g^2, h_1, h_2 space) is divided up into four domains: A, B, C, and D. If our bare coupling is in domain A the coupling will evolve to fixed point (a), and similarly for domains B and C. If the bare coupling is in domain D then the coupling will evolve to infinity.

We can study the nature of these domains by considering the linearized form of our differential equation about each point, or by performing numerical computer studies; we have done both. The linearized equations have the form

$$\frac{d}{dt}\begin{pmatrix} g_2-\bar{g}^2 \\ h_1-\bar{h}_1 \\ h_2-\bar{h}_2 \end{pmatrix} = -\begin{pmatrix} & & \\ & \beta_L & \\ & & \end{pmatrix}\begin{pmatrix} g^2-\bar{g}^2 \\ h_1-\bar{h}_1 \\ h_2-\bar{h}_2 \end{pmatrix},$$

where β_L is a matrix obtained by linearizing β_{g^2}, β_{h_1}, and β_{h_2} about the fixed point ($\bar{g}^2, \bar{h}_1, \bar{h}_2$). This is an eigenvalue problem, and the dimensionality of the domain is related to the number of positive eigenvalues.[5] If we have one positive eigenvalue there is a one-dimensional domain or line in the coupling space on which we can start and be led to the associated fixed point. This line represents a very special relation between bare couplings. While we know of no reason why these couplings should be related in this way, such relations between Reggeon couplings do occur in nature.[7] Furthermore, such relations are known to occur in the context of other field theories; for example, couplings in models with supersymmetry have the correct relationship between couplings to put them on the type of line described above.[8]

If two of the eigenvalues of the linearized equation are positive there is a two-dimensional domain or surface which contains the bare couplings that lead to the associated fixed point. This would again require a special relation between couplings. Finally, if all the eigenvalues are positive there is a three-dimensional region of the coupling space that has bare couplings that lead to the associated fixed points.

Of the three sets of fixed points in this problem, (a) and (c) have two-dimensional domains and (b) has a three-dimensional domain. For case (a) the problem cannot be linearized in h_1 and h_2; however, the problem had been studied analytically[9] in the h_1h_2 plane and found to be stable in a portion of that plane (same as two positive eigenvalues). We have studied that plane numerically and, of course, find the same result as shown in Fig. 2. All points in the triangular region above the origin make up domain A. The equation can be linearized in g^2 and the eigenvalue is -1, indicating that the solution is unstable in the g^2 direction, and that domain A lies entirely in the h_1h_2 plane. The fixed point (c) can be linearized and has two positive eigenvalues indicating that domain C is two-dimensional. Locally about the point (c) the domain C is the plane constructed from the two eigenvalues associated with the two positive eigenvalues. The unstable direction is given by the eigenvector associated with the negative eigenvalue, i.e., (0.001 725 5, −0.501 222 7, 1.000). Finally the fixed point (b) has all positive eigenvalues, and the domain B is a region of three-space. We show a contour plot of the boundaries of the domain in Fig. 3. In Fig. 4 we show the projection of various evolution curves as projected on the h_1h_2 plane. The curves start at $g^2=0.1$ and various values of h_1 and h_2 on the boundary of the graph. Near the domain boundary we see that the evolution curves are quite erratic as might be expected. This is the result of the fact that the eigenvector of the smallest eigenvalue for domain B which controls the approach to the fixed point is nearly colinear with the negative eigenvalue for domain C which controls the motion out to infinity.

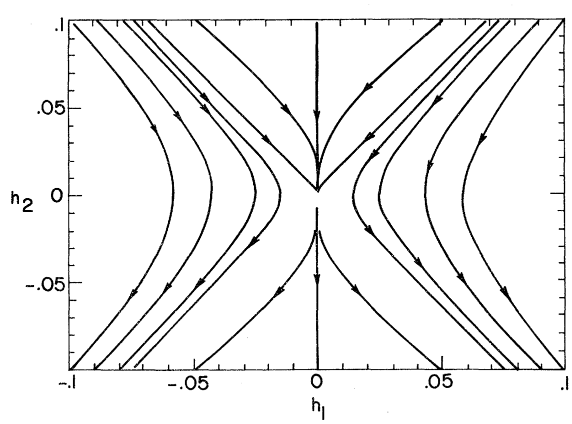


FIG. 2. Coupling-constant evolution curves in the $g^2=0$ plane.

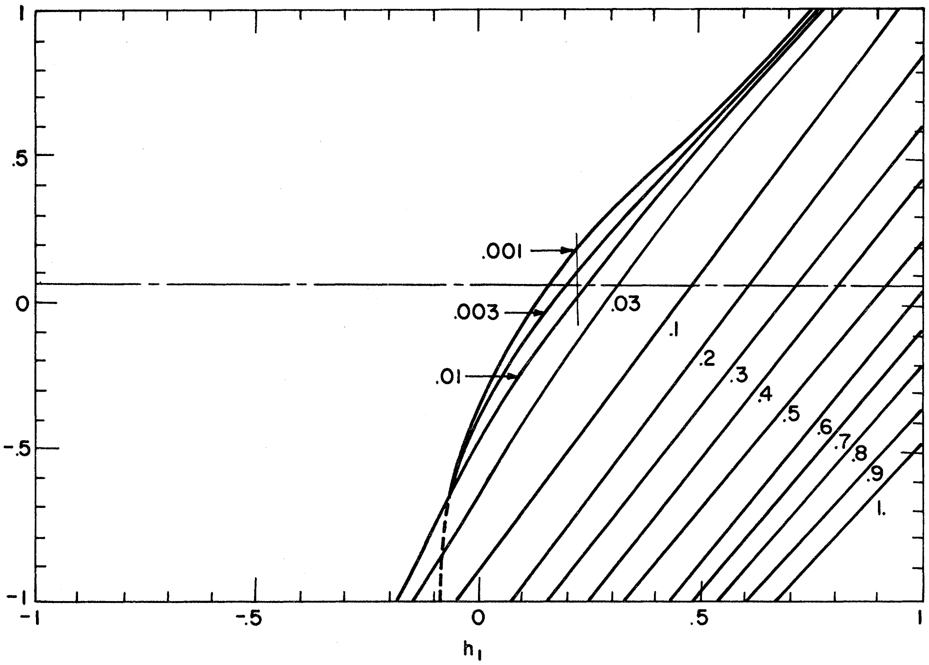


FIG. 3. g^2 contours of domain B boundary.

Finally, let us attempt to estimate the location of the bare coupling as dictated by the data. In Ref. 2, White has estimated the bare triple-Reggeon coupling. With our slightly different normalization this gives

$$g_{\exp}^2 \cong 0.006.$$

From double-diffraction data one can estimate the size of the four-Reggeon coupling[10] h_2 (actually, the thing estimated is the discontinuity of the off-shell coupling). It is found that

$$(h_2/g^2)_{\exp} \cong 10.$$

Therefore

$$h_2 \cong 0.064.$$

In Fig. 3 we have the contour of the boundary of domain B. The line $h_2 = 0.064$ is drawn along with the place it crosses the domain boundary. We see that for $h_1 \leq 0.2$ we are in domain B and the coupling will evolve to fixed point (b). Furthermore, one might expect some generalized crossing symmetry to imply $h_1 \approx h_2$, which would clearly put it in domain B.

The interesting point here in this model is that the domain boundary is quite close to the fixed point (b), and that even though the bare couplings are small [in the sense that fixed-point couplings (b) are small], the bare coupling could be in domain D and evolve to ∞, implying the failure of the renormalization-group procedure.

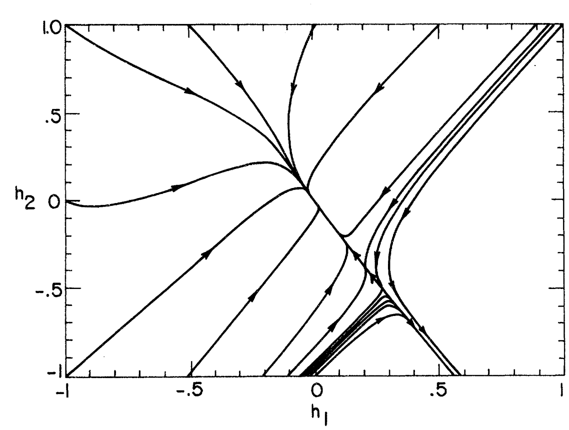


FIG. 4. Coupling-constant evolution curves projected onto the $g^2 = 0$ plane. Curves start at $g^2 = 0.1$ and various values of h_1 and h_2 on the edge of the graph.

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