Dynamical breaking of general covariance and massive spin-2 mesons

M. J. Duff*
Department of Mathematics, King's College, London WC2R 2LS, England
(Received 22 May 1975)

In a system of massive fermions in interaction with massless tensor and scalar mesons, the existence of a massless vector bound state in the fermion-antifermion channel gives a mass to the spin-2 particle and leads to a gauge-invariant and unitary S matrix which is completely free of massless poles.

Attempts[1] have recently been made to understand the spontaneous breakdown of gauge symmetries in purely dynamical terms, without recourse to the Higgs-Kibble phenomenon. In these schemes the Goldstone boson is not represented by a field in the Lagrangian but is a true bound state. The vector gauge particle then acquires a mass through a pole at $q^2=0$ in its proper self-energy[2] coming from this massless bound-state exchange.

It is of interest to ask whether a massless spin-2 particle might also acquire a mass through such a dynamical process, especially since the Higgs-Kibble mechanism works only for spin-1.[3] The spin-2 situation is rather more subtle than the spin-1, however, as may be seen by a naive counting of the number of polarization states available. Here an extra three degrees of freedom are necessary to turn a massless tensor meson (two degrees) into a massive one (five degrees), and a scalar Goldstone boson can provide only one. It is essential therefore that the massless bound state be a vector (two degrees). Yet two plus two is still only four, and there will be ghost problems unless we begin with a tensor-scalar theory. In this way we can give a mass to the spin-2 particle and show that not only does the massless vector excitation decouple from the physical S matrix but so does the scalar. Moreover, this model then shares with those of Ref. 1 the empirical advantage that no scalar mesons, unobserved in experiments, are left over.

Consider a system of massive fermions in interaction with massless tensor mesons. This masslessness and Lorentz invariance together imply the existence of a gauge symmetry,[4] by virtue of which the proper tensor-fermion-antifermion vertex function, $\Gamma_{\mu\nu}(p,p')$, obeys the Ward identity

$$q_\mu \Gamma_{\mu\nu}(p,p+q)=0 \tag{1}$$

when the fermions are on the mass shell. Furthermore, this identity can only be satisfied by taking into account the self-interaction of the spin-2 particles, and the gauge symmetry in question must contain general coordinate invariance.[5]

The proper tensor self-energy part, denoted $\Pi_{\mu\nu\rho\sigma}(q)$, also obeys the Ward identity[6]

$$q_\mu \Pi_{\mu\nu\rho\sigma}(q)=0 \tag{2}$$

and hence may be written in the general form

$$\Pi_{\mu\nu\rho\sigma}(q)=(d_{\mu(\rho}d_{\nu\sigma)}-\tfrac{1}{3}d_{\mu\nu}d_{\rho\sigma})q^2\Pi_1(q^2)+\tfrac{2}{3}d_{\mu\nu}d_{\rho\sigma}q^2\Pi_2(q^2), \tag{3}$$

where

$$d_{\mu\nu}=\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}. \tag{4}$$

Next we consider massless scalar mesons whose coupling to the fermions is given by $\Gamma_{\mu\mu}/\sqrt{6}$. The factor $1/\sqrt{6}$ will prove crucial[7] in the ghost elimination. Closed-loop effects will provide a tensor-scalar mixing and yield the combined tensor-scalar self-energy

$$\frac{1}{6}\begin{pmatrix} 6\Pi_{\mu\nu\rho\sigma} & \sqrt{6}\,\Pi_{\mu\nu\beta\beta} \\ \sqrt{6}\,\Pi_{\alpha\alpha\rho\sigma} & \Pi_{\alpha\alpha\beta\beta}\end{pmatrix}. \tag{5}$$

The complete tensor-scalar propagator then becomes

$$\begin{pmatrix} G_{\mu\nu\rho\sigma} & G_{\mu\nu} \\ G_{\rho\sigma} & G\end{pmatrix}, \tag{6}$$

where

$$G_{\mu\nu\rho\sigma}(q)=-\frac{1}{q^2(1+\Pi_1)}(\delta_{\mu(\rho}\delta_{\nu\sigma)}-\tfrac{1}{3}\delta_{\mu\nu}\delta_{\rho\sigma})+\frac{(1+\Pi_2)}{6q^2}\delta_{\mu\nu}\delta_{\rho\sigma}+\cdots, \tag{7}$$

$$G_{\mu\nu}(q)=-\frac{\Pi_2}{q^2\sqrt{6}}\delta_{\mu\nu}+\cdots, \tag{8}$$

and

$$G(q)=-\frac{1-\Pi_2}{q^2}. \tag{9}$$

Here the dots denote gauge terms like $q_\mu q_\nu q_\rho q_\sigma q^{-6}$ and $q_\mu q_\nu q^{-4}$, etc., which vanish, by Eq. (1), when the propagator couples to a conserved current.

Now our principal assumption is that the fermion-antifermion scattering amplitude contains a massless vector-boson bound state. Its propagator is given by

$$\Delta_{\mu\nu}(q)=\frac{\delta_{\mu\nu}}{q^2}-(1-\xi)\frac{q_\mu q_\nu}{q^4} \tag{10}$$

with ξ an arbitrary gauge parameter. We identify this massless particle with the Goldstone boson that accompanies the dynamical breaking of the global gauge symmetry. As in Ref. 1, this massless exchange imparts a pole at $q^2=0$ to the self-energy and we find that as $q^2\to 0$

$$\Pi_1(q^2)\sim -\frac{m^2}{q^2}\,,\quad \Pi_2(q^2)\sim\frac{m^2}{q^2}(3\xi-2)\,, \tag{11}$$

where $\sqrt{2}m$ is the coefficient describing the coupling of the tensor particle to the Goldstone boson state. Note that Π_2 is gauge dependent. From Eq. (7), we see that the spin-2 propagator has developed a pole at $\Pi_1(q^2)=-1$, and the tensor meson has acquired a mass (equal to m in the pole approximation).

However, we still have to worry about the apparent massless ghost contribution in Eq. (7) (especially since Π_2 depends on ξ), and also the effects of the massless Goldstone pole. Let us consider the fermion-antifermion scattering process $i\to f$ and denote the on-shell scattering amplitude by T^{fi}. It can be decomposed into three terms[8]

$$T^{fi}=T^{fi}_{(1)}+T^{fi}_{(2)}+T^{fi}_{(3)}\,. \tag{12}$$

$T^{fi}_{(1)}$ has neither tensor-scalar nor Goldstone poles in the s channel. $T^{fi}_{(2)}$ has an s-channel Goldstone pole but no tensor-scalar pole; it is given by

$$T^{fi}_{(2)}=P^f_\mu\,\Delta_{\mu\nu}P^i_\nu\,, \tag{13}$$

where P^i_μ and P^f_μ are the vertices connecting the initial and final states to the Goldstone boson, and are defined to be regular at $q^2=0$. Spin-1 gauge invariance demands

$$q_\mu P_\mu(p,p+q)=0 \tag{14}$$

for on-shell fermions and therefore

$$T^{fi}_{(2)}=\frac{P^f_\mu P^i_\mu}{q^2}\quad. \tag{15}$$

Lastly, $T^{fi}_{(3)}$ contains the tensor-scalar pole in the s channel and is given by

$$\begin{aligned}T^{fi}_{(3)}&=(\Gamma^f_{\mu\nu},\Gamma^f_{\alpha\alpha}/\sqrt{6})\begin{pmatrix}G_{\mu\nu\rho\sigma} & G_{\mu\nu}\\ G_{\rho\sigma} & G\end{pmatrix}\begin{pmatrix}\Gamma^i_{\rho\sigma}\\ \Gamma^i_{\beta\beta}/\sqrt{6}\end{pmatrix}\\ &=-\frac{1}{q^2(1+\Pi_1)}(\Gamma^f_{\mu\nu}\Gamma^i_{\mu\nu}-\tfrac{1}{3}\Gamma^f_{\mu\mu}\Gamma^i_{\nu\nu})\,.\end{aligned} \tag{16}$$

Note that $T^{fi}_{(3)}$ is completely free of the scalar ghost poles and all dependence on Π_2 has disappeared. Thus unitarity and gauge invariance are simultaneously secured. We still have the Goldstone pole in $T^{fi}_{(2)}$, but the crucial point is that $\Gamma^f_{\mu\nu}$ and $\Gamma^i_{\mu\nu}$ are not regular at $q^2=0$ but themselves have Goldstone poles. We find, from the Dyson-Schwinger equations, that on the mass shell,

$$\Gamma^f_{\mu\nu}=\tilde{\Gamma}^f_{\mu\nu}+i\sqrt{2}\,\frac{m}{q^2}q_{(\mu}P^f_{\nu)}\,, \tag{17}$$

where $\tilde{\Gamma}^f_{\mu\nu}$ is regular at $q^2=0$. On using Eqs. (1) and (14), $T^{fi}_{(3)}$ now becomes

$$T^{fi}_{(3)}=-\frac{1}{q^2(1+\Pi_1)}\times\left(\tilde{\Gamma}^f_{\mu\nu}\tilde{\Gamma}^i_{\mu\nu}-\tfrac{1}{3}\tilde{\Gamma}^f_{\mu\mu}\tilde{\Gamma}^i_{\nu\nu}-\frac{m^2}{q^2}P^f_\mu P^i_\mu\right)\,. \tag{18}$$

Equation (11) then shows that the singularity at $q^2=0$ in $T^{fi}_{(2)}$ of Eq. (15) is exactly canceled by singularity in $T^{fi}_{(3)}$ of Eq. (18).

In summary, the complete on-shell amplitude T^{fi} is both spin-2 and spin-1 gauge independent, is free of ghost poles, is free of Goldstone poles, but does have a pole at $\Pi_1(q^2)=-1$ (i.e., $q^2=m^2$ in the pole approximation) with residue appropriate to that of a pure spin-2 massive particle.[9]

We do not identify this massive tensor particle with gravitation. Rather, in analogy with the unified spin-1 gauge theories, we envisage a multiplet of spin-2 gauge particles, some of which become massive by the process described above, while one (the graviton) remains massless.[10] This is similar in spirit to f-g theory,[11] though the mass generation is, of course, quite different.

Finally, the S-matrix arguments we have outlined here may be summarized by a gauge-invariant phenomenological Lagrangian[12] which exhibits the excitation spectrum and the various spin 2, 1, $\frac{1}{2}$, and 0 couplings. Moreover, there exists a choice of gauge for the tensor field in which the vectors and scalars decouple from the Lagrangian.[13]

This, and other details, will be published elsewhere.

My interest in this problem was first aroused by Professor Abdus Salam and Dr. C. J. Isham. I am particularly grateful to Dr. J. Strathdee for useful discussions.

*Work supported by the Science Research Council.

[1]R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. M. Cornwall and R. E. Norton, *ibid.* 8, 3338 (1973); see also Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).

[2]J. Schwinger, Phys. Rev. 128, 2425 (1962).

[3]D. Boulware and S. Deser, Phys. Rev. D 6, 3368 (1972).

[4]S. Weinberg, Phys. Rev. 135, B1049 (1964).

[5]D. Boulware and S. Deser, Ann. Phys. (N. Y.) 89, 193 (1975).

[6]Provided that, as consistency demands, the renormalized cosmological constant is set equal to zero. M. J. Duff, in *Quantum Gravity: An Oxford Symposium*, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Oxford Univ. Press, Oxford, 1975).

[7]This has been noted in not dissimilar circumstances by Boulware and Deser (see Ref. 3), who point out the connection with the $\omega=0$ limit of Brans-Dicke theory.

[8]We follow the notation of S. Weinberg, Rev. Mod. Phys. 46, 255 (1974).

[9]We have nothing to say about the problem of renormalizability, except that the high-energy behavior of the propagator in Eq. (7) indicates that the massive theory will be no more divergent than the massless theory.

[10]E.g., an SO(2)-type model like that of Cornwall and Norton (see Ref. 1). Another possibility is to take zero bare-mass fermions with axial tensor mesons in analogy with the model of Jackiw and Johnson (see Ref. 1).

[11]C. J. Isham, Abdus Salam, and J. Strathdee, Phys. Rev. D 3, 687 (1971); B. Zumino, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970).

[12]We hope by this Lagrangian approach to investigate whether our dynamically broken tensor-scalar theory avoids the well-known "sixth degree of freedom" problem of Boulware and Deser (see Ref. 3).

[13]As an illustration, consider part of the linearized Lagrangian:

$$-\tfrac{1}{2}m^2({\phi_{\mu\nu}}^2-{\phi_{\mu\mu}}^2)-\tfrac{1}{4}{F_{\mu\nu}}^2+\sqrt{2}m(\phi_{\mu\nu}\partial_\mu A_\nu-\phi_{\mu\mu}\partial_\nu A_\nu).$$

Under the gauge transformation $\phi_{\mu\nu}\to\phi_{\mu\nu}+\sqrt{2}m^{-1}\partial_{(\mu}A_{\nu)}$, the vector decouples.