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Electromagnetic mass differences from an effective Lagrangian*

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An effective-Lagrangian model that gave finite lowest-order dynamical electromagnetic corrections to the masses of vector mesons is generalized to a chiral O(4) model, which includes A_1 mesons and pions. Aside from a PCAC (partial-conservation-of-axial-vector-current) "tadpole" term the Lagrangian is chiral-O(4)-symmetric, but a term responsible for supplying a part of the A_1 mass is not gauge-symmetric. Both pion and A_1 mass differences contain logarithmically divergent terms, as in chiral current-algebra calculations. The soft-pion part of the pion mass difference is, however, 3.2 MeV, instead of 5.0 MeV as given by current-algebra and other effective-Lagrangian calculations.

I. INTRODUCTION

In a recent article[1] the vector meson was described by a Yang-Mills field which acquired mass by the Higgs-Kibble mechanism.[2] When the electromagnetic (em) interaction was introduced by the method of Kroll, Lee, and Zumino[3] the em mass difference of the charged and neutral ρ meson was found to be finite and consistent with experiment. It is interesting to ask if this procedure can be extended by embedding the vector-meson Lagrangian in a chiral Lagrangian which includes axial-vector mesons and pions. Such a model, presented here, contains two chiral four-vector fields of opposite parity. One of these acts as a Higgs field to generate the mass of the vector meson; the other also resembles a Higgs field in supplying a part of the axial meson mass but, because of the presence of a chiral-symmetric (but not gauge-symmetric) axial-vector mass term, it survives as the physical pion. Our model thus provides a link between the chiral σ model and a pure chiral gauge model.

Unlike the gauge model,[4] where extra scalar masses provide a natural cutoff, the present model does not cure the divergence difficulty of the chiral σ model, and both pion and axial meson have logarithmically divergent em mass differences. In the case of the pion, this divergence is a "hard-pion" correction, proportional to the square of the ratio of pion to vector meson mass. For the vector meson, as in I, the mass difference remains finite, owing to its "soft" propagator.

The effective Lagrangian is written in a chiral O(4) formalism, making use of the isomorphism of the groups O(4) and SU(2)×SU(2), as discussed elsewhere.[5] Indices a, b, c, ... will be used as superscripts to denote the internal symmetry, with $a=1\ldots4$, etc., while Greek subscripts are used as Lorentz indices. The internal index 4 will always be understood to carry with it the opposite parity, as compared with indices 1, 2, and 3. This characterizes the group as *chiral* O(4). In particular, we introduce the scalar O(4) vector ψ^a and the pseudoscalar O(4) vector π^a, which transform under parity as

$$\begin{aligned}\psi^a(\vec{x},t)&\to(-1)^{\delta_{a4}}\psi^a(-\vec{x},t),\\ \pi^a(\vec{x},t)&\to-(-1)^{\delta_{a4}}\pi^a(-\vec{x},t).\end{aligned}\tag{1}$$

Analogously, the antisymmetric spin-one tensor V_μ^{ab} has components which are vector and axial-vector fields (i,j,k are restricted to the values 1, 2, 3):

$$V_\mu^{ij}\equiv\epsilon^{ijk}V_\mu^k,\quad V_\mu^{k4}\equiv A_\mu^k,\tag{2}$$

V_μ^k being a polar vector and A_μ^k its axial-vector chiral partner. We shall also use the dual tensor

$$\tilde{V}_\mu^{ab}\equiv\tfrac{1}{2}\epsilon^{abcd}V_\mu^{cd},\tag{3}$$

whose components are

$$\tilde{V}_\mu^{ij}=\epsilon^{ijk}A_\mu^k,\quad \tilde{V}_\mu^{k4}=V_\mu^k.\tag{4}$$

Consider now the chiral O(4) gauge-symmetric Lagrangian

$$\mathcal{L}_S=-\tfrac{1}{8}V_{\mu\nu}^{ab}V_{\mu\nu}^{ab}+\tfrac{1}{2}\mathfrak{D}_\mu\psi^a\mathfrak{D}_\mu\psi^a+\tfrac{1}{2}\mathfrak{D}_\mu\pi^a\mathfrak{D}_\mu\pi^a+U(\psi,\pi),\tag{5}$$

with

$$V_{\mu\nu}^{ab}=\partial_\mu V_\nu^{ab}-\partial_\nu V_\mu^{ab}-g[V_\mu,V_\nu]^{ab},\tag{6a}$$

$$\mathfrak{D}_\mu\psi^a=\partial_\mu\psi^a-\tfrac{1}{2}gV_\mu^{ab}\psi^b-\tfrac{1}{2}g\tilde{V}_\mu^{ab}\pi^b,\tag{6b}$$

$$\mathfrak{D}_\mu\pi^a=\partial_\mu\pi^a-\tfrac{1}{2}gV_\mu^{ab}\pi^b-\tfrac{1}{2}g\tilde{V}_\mu^{ab}\psi^b.\tag{6c}$$

Here, $-U(\psi,\pi)$ is a chiral O(4) gauge-symmetric quartic "potential," depending only on ψ^a and π^a. We note that if $\vec{A}_\mu$, $\vec{\pi}$, and ψ^4 are set equal to zero in Eq. (5), then we recover the symmetric Lagrangian [Eq. (1)] of I. Under infinitesimal local chiral O(4) transformations, with rotation parameters $\theta^{ab}=-\theta^{ba}$, we have

$$g\delta V^{ab}_{\mu}=g[\theta, V_{\mu}]^{ab}+\partial_{\mu}\theta^{ab} \tag{7a}$$

and

$$g\delta\tilde{V}^{ab}_{\mu}=g[\tilde{\theta}, V_{\mu}]^{ab}+\partial_{\mu}\tilde{\theta}^{ab}, \tag{7b}$$

where

$$\tilde{\theta}^{ab}=\tfrac{1}{2}\epsilon^{abcd}\theta^{ab}.$$

From Eq. (7a) it follows that

$$\delta V^{ab}_{\mu\nu}=[\theta, V_{\mu\nu}]^{ab}. \tag{8}$$

$\mathcal{L}_s$ is invariant under the local transformations (7) and

$$\delta\psi^a=\tfrac{1}{2}(\theta^{ab}\psi^b+\tilde{\theta}^{ab}\pi^b), \tag{9a}$$

$$\delta\pi^a=\tfrac{1}{2}(\theta^{ab}\pi^b+\tilde{\theta}^{ab}\psi^b), \tag{9b}$$

since these lead to analogous transformations for $\mathfrak{D}_{\mu}\psi^a$ and $\mathfrak{D}_{\mu}\pi^a$ and $\psi^a\psi^a+\pi^a\pi^a$ is an invariant. Indeed, the only other basic invariant under (9) is the pseudoscalar $\psi^a\pi^a$, so that $U(\psi,\pi)$ in $\mathcal{L}_s$ has the form

$$U(\psi,\pi)=a(\psi^a\psi^a+\pi^a\pi^a)+b(\psi^a\psi^a+\pi^a\pi^a)^2+c(\psi^a\pi^a)^2. \tag{10}$$

Besides the local chiral O(4) symmetry defined by Eqs. (7) and (9), $\mathcal{L}_s$ possesses an additional global chiral O(4) symmetry which we will call the *ordinary* chiral group. We designate it as O′(4), with rotation parameters φ^{ab}. The corresponding *local* transformations are

$$g\delta' V^{ab}_{\mu}=g[\varphi, V_{\mu}]^{ab}+\partial_{\mu}\varphi^{ab}, \tag{11a}$$

$$\delta'\psi^a=\varphi^{ab}\psi^b, \tag{11b}$$

$$\delta'\pi^a=\varphi^{ab}\pi^b. \tag{11c}$$

Although $\mathcal{L}_s$ is only globally invariant under O′(4), we will use (11) in order to find the local currents, using the Gell-Mann–Lévy definition

$$J^{ab}_{\mu}=-\partial(\delta'\mathcal{L})/\partial(\partial_{\mu}\varphi^{[ab]}), \tag{12}$$

where $[ab]$ indicates antisymmetrization.

We complete the Lagrangian of strong interactions by adding an O′(4)-symmetric (but not gauge-symmetric) term which will contribute one-half of the chiral vector mass, and a PCAC tadpole term which breaks the chiral symmetry. Thus our effective Lagrangian will be $\mathcal{L}=\mathcal{L}_s+\mathcal{L}'$, with

$$\mathcal{L}'=\tfrac{1}{8}g^2(V^{ab}_{\mu}\pi^b)^2+\gamma m_{\pi}^2\pi^4. \tag{13}$$

II. PHYSICAL FIELDS

In the manner by now familiar, we minimize the "potential energy" terms in $\mathcal{L}$, with the spin-one fields neglected; namely, we minimize $-U(\psi,\pi)-\gamma m_{\pi}^2\pi^4$, assuming the coefficient b in Eq. (10) to be negative, and setting $\langle\psi^4\rangle_0=0$ and $\langle\pi^4\rangle_0=2m/g$, $\pi^4=\sigma+2m/g$. Eliminating "tadpoles" requires that $\gamma=m/g$ and we obtain for the quadratic part of the Lagrangian $\mathcal{L}_0$ ($\mathcal{L}_0\equiv\mathcal{L}-\mathcal{L}_{\rm int}$)

$$\begin{aligned}\mathcal{L}_0=&-\tfrac{1}{4}(\partial_{\mu}\vec{V}_{\nu}-\partial_{\nu}\vec{V}_{\mu})^2+\tfrac{1}{2}(\partial_{\mu}\vec{\psi}-m\vec{V}_{\mu})^2\\&-\tfrac{1}{4}(\partial_{\mu}\vec{A}_{\nu}-\partial_{\nu}\vec{A}_{\mu})^2+\tfrac{1}{2}(\partial_{\mu}\vec{\pi}-m\vec{A}_{\mu})^2\\&+\tfrac{1}{2}m^2\vec{A}_{\mu}^{\,2}+\tfrac{1}{2}(\partial_{\mu}\sigma)^2-\tfrac{1}{2}m_{\sigma}^2\sigma^2\\&+\tfrac{1}{2}(\partial_{\mu}\eta)^2-\tfrac{1}{2}m_{\eta}^2\eta^2-\tfrac{1}{2}\mu^2(\vec{\psi}^2+\vec{\pi}^2).\end{aligned} \tag{14}$$

In this semirealistic model, the σ and η fields will play no significant role, and we can assume that the magnitudes of m_{σ}^2 and m_{η}^2, governed by the parameters b and c, are large enough that σ and η may be neglected in the dynamics of the pion, the ρ, and the A_1 meson.

The mixing of $\vec{A}_{\mu}$ and $\vec{\pi}$ is eliminated by a redefinition of the axial-vector field, accompanied by a renormalization of the pion field:

$$\vec{\pi}=\sqrt{2}\,\vec{\pi}', \tag{15a}$$

$$m\vec{A}_{\mu}=\vec{A}'_{\mu}+\frac{1}{\sqrt{2}}\partial_{\mu}\vec{\pi}'. \tag{15b}$$

We shall regard $\vec{\pi}'$ and $\vec{A}'_{\mu}$ as the physical pion and A_1 meson fields. Dropping σ and η we have, effectively,

$$\begin{aligned}\mathcal{L}_0=&-\tfrac{1}{4}(\partial_{\mu}\vec{V}_{\nu}-\partial_{\nu}\vec{V}_{\mu})^2+\tfrac{1}{2}(\partial_{\mu}\vec{\psi}-m\vec{V}_{\mu})^2\\&-\tfrac{1}{4}(\partial_{\mu}A'_{\nu}-\partial_{\nu}A'_{\mu})^2+m^2\vec{A}'^{\,2}_{\mu}+\tfrac{1}{2}(\partial_{\mu}\vec{\pi}')^2\\&-\tfrac{1}{2}\mu^2\vec{\psi}^2-\mu^2\vec{\pi}'^2.\end{aligned} \tag{16}$$

We identify μ^2 as being $2m_{\pi}^2$ and note that the axial-vector current divergence is $\gamma m_{\pi}^2\vec{\pi}=\sqrt{2}\gamma m_{\pi}^2\vec{\pi}'$. Thus, the pion decay constant f_{π} is given by

$$f_{\pi}=\sqrt{2}m/g. \tag{17}$$

This is actually the well-known KSFR relation, as the pion decay interaction (given below) shows that $g=2g_{\rho\pi\pi}$.

From Eq. (16) it is clear that the physical field A'_{μ} and π' have their usual bare propagators, but that the part of $\mathcal{L}_0$ describing $\vec{V}_{\mu}$ and $\vec{\psi}$ is singular and agrees with the analogous $\mathcal{L}_0$ in I, except for the $\vec{\psi}$ mass term. The same procedure as in I can be followed for fixing the gauge, in spite of the $\vec{\psi}$ mass term, and we shall use the particular gauge in which we have

$$V_{\mu}\ \text{propagator:}\quad -i\,\frac{g_{\mu\nu}-k_{\mu}k_{\nu}/k^2}{k^2-m^2}, \tag{17$'$}$$

$$\psi\ \text{propagator:}\quad i(k^2-\mu^2)^{-1}. \tag{17$''$}$$

At this stage, any other gauge choice is equally possible, but when we introduce the em field it will break down the symmetry to that of an Abelian gauge by mixing with the neutral component of the vector field. We will, therefore, do all our calculations with this gauge.

In our view $\vec{\psi}$ plays the role of a regularizing field for $\vec{V}_\mu$ and has no other dynamical significance. It can be shown (see I) that a redefinition of $\vec{\psi}$ and $\vec{V}_\mu$ is possible such that the redefined scalar field $\vec{\psi}'$ is infinitely massive, and so is not dynamically effective. The redefined vector field is then $\mathcal{V}_\mu = \vec{V}_\mu - m^{-1}\partial_\mu \vec{\psi}$.

III. ELECTROMAGNETIC INTERACTION

In our previous work (I) a point was made of coupling the electromagnetic field $\mathcal{A}_\mu$ to $V_\mu{}^3$. The mass shift of the neutral vector meson was obtained by diagonalization of the Lagrangian, and the dynamical mass shift of the charged meson was calculated. Actually, in a transverse photon gauge, like the Landau gauge used, it does not matter whether the coupling is to $\vec{V}_\mu$ or $\mathcal{V}_\mu$, since $\partial_\mu \vec{\psi}$ does not couple to a transverse photon. In the present model, the result for the mass difference of the vector mesons is identical to that of I, and we will discuss it no further.

Thus, to introduce $\mathcal{A}_\mu$ we add to the Lagrangian of strong interactions the term

$$\mathcal{L}^{\text{em}} = -\tfrac{1}{4}(\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu)^2 + eJ_\mu^{\text{em}}\mathcal{A}_\mu , \tag{18}$$

with

$$J_\mu^{\text{em}} = (m^2/g)\mathcal{V}_\mu{}^3 + (em^2/2g^2)\mathcal{A}_\mu . \tag{19}$$

As often discussed, the apparent photon mass term in (18) actually compensates an imaginary mass pole induced in the photon propagator by the ρ^0-γ mixing, and it is automatically eliminated by diagonalizing away this mixing.

Introducing the physical pion $\vec{\pi}'$ and A_1 meson A_μ' through the substitutions given in Eqs. (15), we obtain the following three-particle vertices (with $\beta = 1/\sqrt{2}m$) from $\mathcal{L}_s + \mathcal{L}'$, Eqs. (5) and (13):

$$V\pi'\pi' \text{ vertex:} \quad -g\vec{V}_\mu \cdot \partial_\mu \vec{\pi}' \times \vec{\pi}' + \beta^2 g \partial_\nu \vec{V}_\mu \cdot \partial_\mu \vec{\pi}' \times \partial_\nu \vec{\pi}' , \tag{20}$$

$$V\pi'A' \text{ vertex:} \quad \beta g \partial_\nu \vec{V}_\mu \cdot (\vec{A}_\mu{}' \times \partial_\nu \vec{\pi}' + \partial_\mu \vec{\pi}' \times A_\nu') - \beta m^2 g \vec{A}_\mu' \cdot \vec{V}_\mu \times \vec{\pi}' + \beta g \partial_\nu \vec{A}_\mu' \cdot (\vec{V}_\mu \times \partial_\nu \vec{\pi}' + \partial_\mu \vec{\pi}' \times \vec{V}_\nu) , \tag{21}$$

$$VA'A' \text{ vertex:} \quad g\partial_\nu \vec{V}_\mu \cdot \vec{A}_\mu' \times \vec{A}_\nu' + g\partial_\nu \vec{A}_\mu' \cdot (\vec{V}_\mu \times \vec{A}_\nu' + \vec{A}_\mu' \times \vec{V}_\nu) . \tag{22}$$

The two relevant four-particle interactions which enter the vector dominance calculation of pion and A_1 meson mass differences are

$$A'A'VV: \quad -\tfrac{1}{2}g^2[2(\vec{V}_\mu \cdot \vec{A}_\mu')^2 - (\vec{V}_\mu \cdot \vec{A}_\nu')(\vec{V}_\nu \cdot \vec{A}_\mu' + \vec{V}_\mu \cdot \vec{A}_\nu') - (\vec{V}_\mu \cdot \vec{V}_\nu)(\vec{A}_\mu' \cdot \vec{A}_\nu') + \vec{V}_\mu{}^2 \vec{A}_\nu'{}^2] , \tag{23}$$

$$\begin{aligned} \pi'\pi'VV: \quad & \tfrac{1}{2}g^2[(\vec{V}_\mu \times \vec{\pi}')^2 + \tfrac{1}{2}(\vec{V}_\mu \cdot \vec{\pi}')^2] \\ & -\tfrac{1}{2}g^2\beta^2[2(\vec{V}_\mu \cdot \partial_\mu \vec{\pi}')^2 - (\vec{V}_\mu \cdot \partial_\nu \vec{\pi}')(\vec{V}_\nu \cdot \partial_\mu \vec{\pi}' + \vec{V}_\mu \cdot \partial_\nu \vec{\pi}') - (\vec{V}_\mu \cdot \vec{V}_\nu)(\partial_\mu \vec{\pi}' \cdot \partial_\nu \vec{\pi}') + \vec{V}_\mu{}^2(\partial_\nu \vec{\pi}' \cdot \partial_\nu \vec{\pi}')] . \end{aligned} \tag{24}$$

The em mass shifts of the pion and the A_1 meson are calculated in lowest order using loop diagrams constructed from the three-particle vertices, Eqs. (20)–(22), the propagators, and the em interaction, Eqs. (18) and (19). There are also loop diagrams arising from the four-particle, or seagull, vertices in Eqs. (23) and (24). As is usual in vector dominance theories, the latter diagrams are convergent in the photon Landau gauge, since they involve integration only over the effective photon propagator[3]

$$G_{\beta\beta'}(p) = im^4(g_{\beta\beta'} - p_\beta p_{\beta'}/p^2)[p^2(p^2 - m^2)^2]^{-1} . \tag{25}$$

In our model soft-pion contributions to the pion em mass difference come only from the term $\frac{1}{2}g^2[(\vec{V}_\mu \times \vec{\pi}')^2 + \frac{1}{2}(\vec{V}_\mu \cdot \vec{\pi}')^2]$ in Eq. (24) and the term $-\beta m^2 g \vec{A}_\mu' \cdot V_\mu \times \vec{\pi}'$ in Eq. (21). The former gives a contribution $3\alpha m^2/4\pi$ to $\delta_\mu{}^2 = m_{\pi+}{}^2 - m_{\pi-}{}^2$, via a seagull loop, and the second gives $(3\alpha m^2/8\pi) \times(-1 + \ln 2)$, via a loop diagram with A_1 intermediate state. The net result is $(3\alpha m^2/8\pi)(1 + \ln 2)$, which gives a mass difference of about 3.2 MeV compared to the usual result[6] (from a gauge-invariant Lagrangian) of about 5.0 MeV.

Hard-pion corrections, apart from the logarithmically divergent term are estimated to be about 1.0 MeV, so our result is within about 10% of the experimental value of 4.6 MeV, if we ignore the logarithmic divergence.

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