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Why the pseudoscalar-meson mixing angle is −10°*

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We propose a simple approach to the problem of mixing in nonets which leads to an understanding of the nature of both the "ideal" mixing angle of 35° found for the vector and tensor mesons and the "nonideal" mixing angle of −10° found for the pseudoscalar mesons. We argue that the pseudoscalar meson mixing angle is "nonideal" because of a near degeneracy in the masses of the strange and nonstrange pseudoscalar mesons. As by-products we predict that more massive nonets will be nearly ideal and that, if charmed quarks "exist," the low-lying charm-anticharm mesons will be very narrow.

I. INTRODUCTION

It has been known from the beginning that many aspects of the violation of SU(3) symmetry can be described in terms of a medium-strong symmetry-breaking interaction which, while breaking SU(3), acknowledges its importance by having well-defined transformation properties under the group. In particular, Gell-Mann[1] and Okubo[2] showed that the hadron spectrum could be understood in terms of an effective[3] interaction $\mathcal{L}_3$ which transforms like the 8-component of an octet. Such an interaction may be represented, most simply, as additional terms in the effective strong-interaction Lagrangian; for example, the mass matrix of the Lagrangian describing an octet of particles might be altered from its SU(3)-invariant form to the forms

$$m_{ij} = m_0\delta_{ij} + af_{8ij} + bd_{8ij} \tag{1}$$

for baryons and

$$\mu_{ij}^2 = \mu^2\delta_{ij} + \beta d_{8ij} \tag{2}$$

for mesons.[4] These results are the Gell-Mann–Okubo (GMO) formulas for octets. Note that because $\mathcal{L}_8$ conserves isospin and hypercharge, the mass matrices are diagonal in the exact SU(3) basis and $\mathcal{L}_8$ simply has the effect of splitting the masses. If, however, there are states in two distinct SU(3) multiplets which have the same strongly conserved quantum numbers, then $\mathcal{L}_8$ can mix the two states and give rise to terms in the mass matrix which mix submatrices belonging to a given representation.[5] For example, two whole octets α and β may mix with a term like

$$\lambda_{\alpha i,\beta j} = r_{\alpha\beta}f_{8ij} + s_{\alpha\beta}d_{8ij}\,. \tag{3}$$

For example, the $\rho(770)$ and $\rho'(1600)$ octets may have such a mixing. Or, more commonly, the eighth component of an octet and a singlet may mix with a term $\lambda_{8,0}$. In a case where mixing occurs, the physically observed masses are those obtained by diagonalizing the relevant sector of the mass matrix.

In those cases where no mixing is possible, the simple GMO formulas are in excellent agreement with the known masses (up to electromagnetic corrections). For the octets this amounts to predicting one mass, while for the decuplet two are predicted. On the other hand, in the case of an octet mixing with a singlet the additional parameter $\lambda_{8,0}$ allows no prediction to be made; the masses of the members of the octet and the mass of the singlet are just sufficient to determine the four unknown parameters.

Our discussion will concentrate on the nonets of pseudoscalar, vector, and tensor mesons. If we denote by M any one of P (the pseudoscalars), V (the vectors), or T (the tensors), and if further we let the subscript 8 denote the 8-component of an octet and the subscript 0 denote a singlet, then the relevant terms in the effective Lagrangian are

$$\mu_8^2M_8^2 + \mu_0^2M_0^2 + \lambda_{8,0}M_0M_8\,. \tag{4}$$

Thus the physical fields are

$$M_{m8} \equiv M_8\cos\theta + M_0\sin\theta\,, \tag{5}$$

$$M_{m0} \equiv M_0\cos\theta - M_8\sin\theta\,, \tag{6}$$

where M_{m8} and M_{m0} are the "mostly octet" and "mostly singlet" physical fields, and θ is the mixing angle between the "exact" SU(3) fields and the physical fields. Since μ_8^2 is given by the GMO relation

$$\mu_8^2 = \frac{4\mu_{Y=1}^2 - \mu_{I=1}^2}{3} \tag{7}$$

(where $\mu_{Y=1}$ is the mass of the $I=\frac{1}{2}$, $Y=\pm1$ members of the octet, and $\mu_{I=1}$ is the mass of the $I=1$, $Y=0$ triplet) one can calculate μ_0^2 from

$$\mu_0^2 = \mu_{m8}^2 + \mu_{m0}^2 - \mu_8^2\,. \tag{8}$$

Knowing μ_0^2 one can then find θ, since

$$\cos 2\theta = \frac{\mu_8^2 - \mu_0^2}{\mu_{m8}^2 - \mu_{m0}^2}, \tag{9}$$

and finally, knowing θ one has

$$\lambda_{8,0} = (\mu_{m8}^2 - \mu_{m0}^2)\sin 2\theta. \tag{10}$$

The values of these parameters for $M = P$, V, and T are displayed in Table I.[6] Even though no mass predictions are made using this program, it is certainly not empty. For one thing, the equations do not necessarily have solutions (for example, if μ_{m0}^2 is less than $\mu_8^2 - \mu_{m8}^2$). In addition, of course, one can now make predictions pertaining to a number of processes since this program has completely specified the SU(3) properties of M_{m8} and M_{m0}; a well-known example is that since

$$j^\mu_{\text{em}} = j^\mu_3 + \frac{1}{\sqrt{3}} j^\mu_8 \tag{11}$$

the electromagnetic current cannot connect V_0 to the vacuum. Thus

$$\frac{\langle 0 | j^\mu_{\text{em}} | \omega \rangle}{\langle 0 | j^\mu_{\text{em}} | \phi \rangle} = \tan\theta^V \tag{12}$$

and one can make the prediction[7]

$$\frac{\Gamma(\omega \to e^+e^-)}{\Gamma(\phi \to e^+e^-)} = \frac{m_\omega}{m_\phi} \tan^2\theta^V, \tag{13}$$

which is well satisfied experimentally.

Nevertheless, this totally empirical approach to the mixing problem is very unsatisfying and it is natural to attempt to understand the situation more deeply. The quark model can offer guidance in this direction, as has long been realized.[8] In the quark model, the states M_8 and M_0 have the representations

$$|M_8\rangle = -\frac{1}{\sqrt{6}} (|\mathcal{P}\mathcal{P}^c\rangle + |\mathfrak{N}\mathfrak{N}^c\rangle - 2|\lambda\lambda^c\rangle), \tag{14}$$

$$|M_0\rangle = \frac{1}{\sqrt{3}} (|\mathcal{P}\mathcal{P}^c\rangle + |\mathfrak{N}\mathfrak{N}^c\rangle + |\lambda\lambda^c\rangle). \tag{15}$$

If we assume that the symmetry breaking occurs at the level of quark mass terms in the form

$$m_0(\bar{\mathcal{P}}\mathcal{P} + \bar{\mathfrak{N}}\mathfrak{N} + \bar{\lambda}\lambda) + \Delta\bar{\lambda}\lambda \tag{16}$$

then to first order the quark model gives

$$\mu_8^2 = 4m_0(m_0 + \tfrac{4}{3}\Delta), \tag{17}$$

$$\mu_0^2 = 4m_0(m_0 + \tfrac{2}{3}\Delta), \tag{18}$$

$$\lambda_{8,0} = 4m_0(-\tfrac{2}{3}\sqrt{2}), \tag{19}$$

so that

$$\tan 2\theta = \frac{\lambda_{8,0}}{\mu_8^2 - \mu_0^2} \tag{20}$$

which implies

$$\tan\theta = \frac{1}{\sqrt{2}} \tag{21}$$

i.e.,

$$\theta = \theta_{\text{ideal}} \equiv 35.3^\circ. \tag{22}$$

θ^V and θ^T are near to the "ideal" mixing angle of 35.3°, and, were it not for the pseudoscalars, one could feel some satisfaction with this result. The fact that θ^P has the value of −10°, however, casts doubts on this explanation since it is certainly not apparent why the pseudoscalar-meson nonet should behave so differently from the vector and tensor nonets.

It is our intention here to make one simple—almost trivial—observation which we believe greatly clarifies the nature of mixing in the nonets. We shall show, among other things, that the seemingly arbitrary pseudoscalar mixing angle of −10° is a very natural mixing angle and is, in fact, the unique angle that results when the conditions for "ideality" are strongly violated.

II. MIXING ANGLES FROM ANOTHER PERSPECTIVE

Our proposal arises from a quark-model picture, which serves to supply a dynamical mechanism for the mixing phenomenon. We note that with mass terms of the type (16), and in the absence of any interactions, the states diagonal in mass are *necessarily* the "strange" and "nonstrange" states

$$|M_s\rangle = |\lambda\lambda^c\rangle, \tag{23}$$

$$|M_{ns}\rangle = \frac{1}{\sqrt{2}} (|\mathcal{P}\mathcal{P}^c\rangle + |\mathfrak{N}\mathfrak{N}^c\rangle). \tag{24}$$

TABLE I. The octet-singlet mixing parameters. The values of μ^2 and λ are given in units of GeV^2.

	μ_{m8}^2	μ_{m0}^2	μ_8^2	μ_0^2	θ	$\lambda_{8,0}$
P	0.301	0.917	0.322 ± 0.003	0.896 ± 0.003	−10.6 ± 0.5°	0.222 ± 0.012
V	1.040	0.613	0.870 ± 0.018	0.783 ± 0.018	+39.2 ± 2.2°	0.418 ± 0.008
T	2.29 ± 0.01	1.61 ± 0.03	2.12 ± 0.01	1.78 ± 0.02	+30.0 ± 2.5°	0.59 ± 0.02

Since

$$|M_s\rangle = (\tfrac{2}{3})^{1/2}|M_8\rangle + (\tfrac{1}{3})^{1/2}|M_0\rangle , \quad (25)$$

$$|M_{ns}\rangle = (\tfrac{2}{3})^{1/2}|M_0\rangle - (\tfrac{1}{3})^{1/2}|M_8\rangle \quad (26)$$

we have immediately the result

$$\tan\theta = \frac{1}{\sqrt{2}} = \tan\theta_{\text{ideal}} . \quad (27)$$

Of course this is simply another, perhaps clearer, method of deriving the result (22). Thus, in the absence of interactions, mass breaking alone forces the result $\theta = \theta_{\text{ideal}}$. If we imagine turning on interactions [for definiteness and ease of exposition we shall imagine henceforth a neutral-SU(3)-singlet—gluon interaction] then two types of interactions can occur in $|qq^c\rangle$ states, as illustrated in Fig. 1. In Fig. 1(a) the interaction takes $|qq^c\rangle$ into itself; such an interaction contributes diagonal terms to the M_s-M_{ns} mass matrix. On the other hand, in Fig. 1(b) the interaction takes $|qq^c\rangle$ into $|q'q'^c\rangle$, and this may contribute to diagonal terms (if $q'=q$) or off-diagonal (mixing) terms if $q' \neq q$. Thus, to the extent that annihilation graphs of the type in Fig. 1(b) are suppressed, the mass matrix will be diagonal in $|M_s\rangle$ and $|M_{ns}\rangle$ and θ will be θ_{ideal}.

In view of this simple interpretation of the ideal mixing angle, we switch at this point to the use of the states $|M_s\rangle$ and $|M_{ns}\rangle$ to describe the mixing problem. Of course this change is of only aesthetic interest unless it leads to some new understanding of the physics involved. We shall immediately find this to be the case.

By making use of the values of μ_8^2, μ_0^2, and $\lambda_{8,0}$ in Table I, one can reexpress the effective Lagrangian in terms of the fields M_s and M_{ns} according to

$$\mu_s^2 M_s^2 + \mu_{ns}^2 M_{ns}^2 + \lambda_{s,ns} M_s M_{ns} . \quad (28)$$

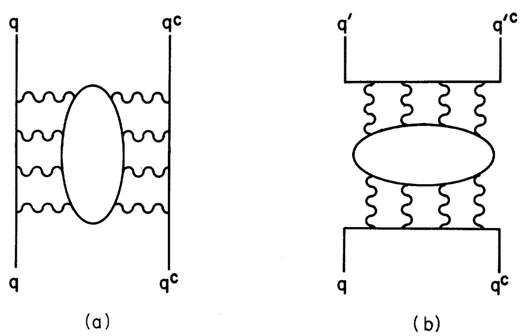

FIG. 1. Contributions to meson masses in a quark-gluon picture.

We display the values of the parameters μ_s^2, μ_{ns}^2, and $\lambda_{s,ns}$ in Table II. Notice particularly the near degeneracy of μ_s^2 and μ_{ns}^2 for the pseudoscalars. If, as before, we write

$$M_{m8} = M_s \cos\phi + M_{ns} \sin\phi , \quad (29)$$

$$M_{m0} = M_{ns} \cos\phi - M_s \sin\phi , \quad (30)$$

where M_{m8} and M_{m0} are the "mostly octet" and "mostly singlet" physical states, then

$$\tan 2\phi = \frac{\lambda_{s,ns}}{\mu_s^2 - \mu_{ns}^2} \quad (31)$$

and

$$\theta = \theta_{\text{ideal}} + \phi . \quad (32)$$

We immediately see that $\theta \approx \theta_{\text{ideal}} = 35°$ results if $\lambda_{s,ns} \ll \mu_s^2 - \mu_{ns}^2$ and that $\theta = \theta_{\text{ideal}} - 45° \approx -10°$ results if $\lambda_{s,ns} \gg \mu_s^2 - \mu_{ns}^2$.[9] Thus, the ideal mixing angle of 35° and the "nonideal" mixing angle of −10° are the two limiting angles which appear in the extreme cases of dominance of mass breaking over mixing and dominance of mixing over mass breaking (between the strange and nonstrange states).

Analyzing the values of the parameters of Table II we find

$$\phi_{P,V,T} = -45.8°, +3.9°, -5.3° . \quad (33)$$

The large value of ϕ_P arises from the fact that for the pseudoscalars mixing dominates mass breaking. Since $\theta = \theta_{\text{ideal}} + \phi$ we of course recover the previously computed values for θ. We have not predicted the θ, of course; we have simply looked at them from another perspective. However, from this perspective we can see that the pseudoscalar mixing angle of −10° is not at all arbitrary but has essentially the same status as the ideal mixing angle of +35°. Actually, the pseudoscalar mixing angle of −10° is much more stable under variations in the values of $\lambda_{s,ns}$ and $\mu_s^2 - \mu_{ns}^2$ than are the vector or tensor mixing angles.

III. DISCUSSION

We feel that the arguments presented above adequately establish that the "nonideal" mixing angle of −10° has the same *a priori* physical relevance

TABLE II. The strange-nonstrange mixing parameters, in units of GeV^2.

	μ_s^2	μ_{ns}^2	$\lambda_{s,ns}$
P	0.619 ± 0.008	0.600 ± 0.008	0.61 ± 0.01
V	1.04 ± 0.02	0.62 ± 0.02	0.06 ± 0.03
T	2.29 ± 0.03	1.61 ± 0.03	-0.12 ± 0.03

as the "ideal" mixing angle of $+35°$. What is still very unclear is why the conditions $\lambda_{s,ns} \gg \mu_s^2 - \mu_{ns}^2$ or $\lambda_{s,ns} \ll \mu_s^2 - \mu_{ns}^2$ should apply in a particular case. One attractive possibility that the data allow is that $\lambda_{s,ns}$ is *generally* quite small, i.e., that annihilation processes are relatively unimportant. (This would correspond to the duality rules for drawing quark scattering diagrams.) After all, $|\lambda^P_{s,ns}| : |\lambda^V_{s,ns}| : |\lambda^T_{s,ns}| = 10:1:2$, and this may be viewed as a normal configuration-dependent variation, coupled perhaps with an energy dependence of the annihilation process.[10] In fact, it is quite clear that $\theta_P \simeq -10°$ primarily because of the near degeneracy in mass of $|M_s\rangle$ and $|M_{ns}\rangle$ for the pseudoscalar mesons. For example, we note that even if $|\lambda^P_{s,ns}|$ had a value of 0.1 GeV^2, which would be more in line with $|\lambda^V_{s,ns}|$ and $|\lambda^T_{s,ns}|$, with $\mu_s^2 - \mu_{ns}^2$ as in Table II, one would still find $\theta_P \simeq -10°$. To the extent that quark models can predict such a degeneracy, we can therefore consider θ_P to be predicted.[11]

With this insight into the "accidental" nature of the pseudoscalar mixing angle, our faith in the ideal mixing angle is restored. Assuming, therefore, that $\lambda_{s,ns}$ remains small, one can venture some guesses as to the behavior of more massive nonets. Since $\mu_s^2 - \mu_{ns}^2 \sim 4\mu\Delta_{eff}$, where Δ_{eff} is the effective mass difference between strange and nonstrange quarks, as one goes to higher masses deviation in Δ_{eff} will be less likely to make $\mu_s^2 - \mu_{ns}^2$ small. Additionally, as one goes to more massive states one can expect smaller deviations in Δ_{eff} to the extent that the mass difference between strange and nonstrange quarks becomes less important in determining their "binding energy." Thus a good guess would be that $\theta \approx \theta_{ideal}$ for higher-mass nonets.

There is another situation, which is of some current interest, where this guesswork may be put on firmer ground. If a charmed quark $\mathcal{P}'$ exists with $m_{\mathcal{P}'} \gg m_\lambda$ then arguments similar to those just advanced indicate that $|M_c\rangle \equiv |\mathcal{P}'\mathcal{P}'^c\rangle$ will have a mass much larger than that of $|M_s\rangle$ or $|M_{ns}\rangle$, so that $\mu_c^2 - \mu_s^2$ and $\mu_c^2 - \mu_{ns}^2$ should be much larger than $\lambda_{c,s}$ and $\lambda_{c,ns}$. Thus, one would expect a physical meson that is very nearly pure $|M_c\rangle$. In particular, if we identify the new ψ particles[12] as nearly pure $|M_c\rangle$ states (and if we assume that $|\lambda_{c,s}|$ and $|\lambda_{c,ns}|$ maintain the pattern established previously by having values of about 0.3 GeV^2), then we would expect the ψ particles to mix with their strange and nonstrange counterparts with angles $\omega \simeq 1°$. Thus the ψ particles should be considerably more "pure" than are the ϕ or f'. If we adopt the rough but convincing view that the ϕ and f' decay rates to final states containing no strange hadrons are accounted for by their small impurities of nonstrangeness, then by a continuation of the same argument we must expect

$$\Gamma(\psi \to \text{hadrons}) \sim 2\left(\frac{\sin\omega}{\sin\phi}\right)^2 \Gamma(\phi \to 3\pi) \sim 90 \text{ keV},$$

as is observed.

IV. CONCLUSIONS

We have advocated the treatment of the mixing problem for nonets in terms of the strange and nonstrange quark configurations. From this perspective we have shown that the "ideal" mixing angle of $+35°$ and the "nonideal" mixing angle of $-10°$ have a common dynamical origin and are of *a priori* similar physical status. We have argued, in particular, that the pseudoscalar-meson mixing angle of $-10°$ arises because of a near degeneracy in the masses of the strange and nonstrange pseudoscalar-meson states. As by-products we have shown that these considerations lead one to expect more massive nonets to be nearly ideal and to predict that if charmed quarks "exist," then the low-lying charm-anticharm mesons will be very narrow.

Note added in proof. We have found a mechanism for the near degeneracy of μ_s^2 and μ_{ns}^2: The existence of an annihilation amplitude due to Fig. 1(b) not only causes mixing but can itself push μ_{ns}^2 away from $\mu_{I=1}^2$ toward μ_s^2. Details of this proposal are forthcoming.[13]

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[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[2]S. Okubo, Prog. Theor. Phys. 27, 949 (1962).

[3]"Effective" because we do not believe that the interactions of the known hadrons are fundamental.

[4]As usual, we adopt the use of the $(\text{mass})^2$ formula for mesons, which is natural in an effective-Lagrangian approach.

[5]This possibility was first mentioned in Ref. 1. See also J. J. Sakurai, Phys. Rev. Lett. 9, 472 (1962); S. Coleman, S. L. Glashow, and D. J. Kleitman, Phys. Rev. 135, 779 (1964).

[6]We identify the $\eta'(958)$ as the ninth member of the pseudoscalar-meson octet and comment later on the possible appropriateness of the $E(1420)$.

[7]R. H. Dalitz, in *Proceedings of the Siena International Conference on Elementary Particle Physics, Siena, Italy, 1963*, edited by G. Bernardini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963), p. 171.

[8]R. H. Dalitz, in *High Energy Physics*, 1965 Les Houches lectures, edited by C. DeWitt and M. Jacob (Gordon and Breach, New York, 1966).

[9]We discard the solution $\theta=\theta_{\text{ideal}}+45°$ as lying too far away from $\theta=0$.

[10]If we had chosen $E(1420)$ as the ninth element of the octet, then we would have found $\lambda^{P}_{s,ns}\sim 1.7$. This value is so different from the values found for the other nonets that we feel this identification is unlikely. Perhaps the $E(1420)$ will prove to be a radially excited 0^- state.

[11]It is probably worth mentioning that this kind of behavior would not be at all bizarre and can easily occur in a variety of models.

[12]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974); J. J. Aubert *et al.*, *ibid.* 33, 1404 (1974).

[13]Nathan Isgur, Phys. Rev. D (to be published).