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Electromagnetic mass splittings of mesons in the charmed-quark model*

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Assuming that the electromagnetic mass splitting of pseudoscalar and vector mesons results from intrinsic quark mass differences plus Coulomb and magnetic interactions, we derive inequalities among the meson masses, including the masses of charmed mesons.

The recent discovery of the J (ψ) particles[1] has been interpreted by many authors[2] as providing indirect evidence for the existence of a charmed quark.[3] Subsequent experiments[4] have provided some evidence, as yet inconclusive, for the existence of charmed particles. In view of these facts, it is useful to consider quark-model predictions for the masses of charmed hadrons. In the first place, such predictions can guide the experimental physicist in his search for charmed particles. Furthermore, if these particles are discovered, a comparison of the predicted mass relations with experiment will provide further tests of the quark model. It is for this latter reason that we here consider the electromagnetic mass splitting of charmed mesons.

There have been numerous recent papers[5] giving mass relations for hadrons based on the groups SU(4) and SU(8). Some of these papers have made explicit use of the quark model, while others have relied principally on certain assumed transformation properties of the hadron mass operator. A common feature of these works is that they are based on perturbations from an SU(4)- or SU(8)-invariant interaction. This is a defect, because the large mass of the ψ (J) meson indicates that SU(4) and SU(8) are badly broken symmetries.

Franklin[6] has considered charmed baryon mass splittings in a model which does not make use of full SU(4) or SU(8) symmetry, and yet gives many of the same mass relations as the latter approaches. Franklin's method is a generalization to charm of previous work by several authors.[7] The main assumption of these papers is that the baryon mass breaking arises from intrinsic mass differences among the quarks plus two-body quark-quark interactions. Unfortunately, when these assumptions are applied to mesons, no useful information is obtained.

In order to obtain relations among meson masses, we assume that the electromagnetic mass splitting arises from the difference in effective masses between a u and d quark plus Coulomb and magnetic moment interactions between the quarks.[8,9] This interaction V_{12} is given by

$$V_{12} = Q_1 Q_2/r_{12} - (2\pi/3) Q_1 Q_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2\, \delta(\vec{r}_{12})/(m_1 m_2), \tag{1}$$

where Q_i is the charge and m_i is the mass of the ith quark. With this interaction, the meson mass differences can be written (we use the symbol for a meson to denote its mass and label the quarks by u, d, s, and c)

$$\pi^+ - \pi^0 = \tfrac{1}{2} C^0_{u\bar{u}} + \tfrac{3}{2} M^0_{u\bar{u}}, \tag{2a}$$

$$K^0 - K^+ = \epsilon - \tfrac{1}{3} C^0_{u\bar{s}} - M^0_{u\bar{s}}, \tag{2b}$$

$$\rho^+ - \rho^0 = \tfrac{1}{2} C^1_{u\bar{u}} - \tfrac{1}{2} M^1_{u\bar{u}}, \tag{2c}$$

$$K^{*0} - K^{*+} = \epsilon - \tfrac{1}{3} C^1_{u\bar{s}} + \tfrac{1}{3} M^1_{u\bar{s}}, \tag{2d}$$

$$D^+ - D^0 = \epsilon + \tfrac{2}{3} C^0_{c\bar{u}} + 2 M^0_{c\bar{u}}, \tag{2e}$$

$$D^{*+} - D^{*0} = \epsilon + \tfrac{2}{3} C^1_{c\bar{u}} - \tfrac{2}{3} M^1_{c\bar{u}}. \tag{2f}$$

Here D and D^* denote the charmed pseudoscalar and vector mesons of isospin $\frac{1}{2}$, ϵ is the difference in effective masses of the d and u quarks, and C^s_{ij} and M^s_{ij} ($S = 1$, 0 being the total spin of the quark-antiquark pair) are the Coulomb and magnetic interaction energies. They are given by

$$\begin{aligned} C^s_{ij} &= \alpha \langle 1/r_{ij} \rangle^s, \\ M^s_{ij} &= (2\pi\alpha/3) |\psi^s_{ij}(0)|^2/(m_i m_j). \end{aligned} \tag{3}$$

We have assumed isospin invariance of the wave functions and neglected ϵ in M^s_{ij}. The relaxation of these assumptions would lead to second-order changes in the mass differences.

Because C^s_{ij} and M^s_{ij} are positive quantities, we see from Eq. (2a) that

$$\pi^+ > \pi^0, \tag{4}$$

a result previously noted by Gal and Scheck.[9] From the experimental fact that the K^0 is heavier than the K^+, Eq. (2b) tells us that ϵ is positive. Then, from Eq. (2e), we see that

$$D^+ > D^0, \quad D^+ - D^0 > K^0 - K^+. \tag{5}$$

Also, since the charmed quark probably has a large effective mass,[10] $M^s_{c\bar{u}}$ is probably quite small. If so, we obtain from Eq. (2f)

$$D^{*+} > D^{*0} . \qquad (6)$$

Without further assumptions, these are the only results we can obtain for mesons. However, following Gal and Scheck,[10] we can assume SU(6) invariance of the wave functions. Then for the uncharmed mesons, C^s_{ij} and M^s_{ij} become independent of the quarks and of spin. We then obtain the relation

$$\rho^+ - \rho^0 = \pi^+ - \pi^0 - \tfrac{3}{2}(K^{*0} - K^{*+}) + \tfrac{3}{2}(K^0 - K^+) \qquad (7)$$

as well as the inequalities

$$\pi^+ - \pi^0 > \rho^+ - \rho^0 ,$$

$$K^{*0} - K^{*+} > K^0 - K^+ . \qquad (8)$$

Equation (7) and the inequalities (8) were implied by the work of Gal and Scheck, although not explicitly written down by them.

Using the values of the mass differences from the latest tables of the Particle Data Group,[11]

$$\pi^+ - \pi^0 = 4.6 \text{ MeV}, \quad K^0 - K^+ = 4.0 \text{ MeV},$$

$$K^{*0} - K^{*+} = 6.1 \pm 1.5 \text{ MeV},$$

we obtain from Eq. (7)

$$(\rho^+ - \rho^0)_{\text{predicted}} = 1.4 \pm 2.3 \text{ MeV}.$$

This is to be compared with the experimental value

$$\rho^+ - \rho = -4.4 \pm 2.4 \text{ MeV}.$$

(In 1967, when Gal and Scheck wrote their paper, experiment indicated that the ρ^+ was heavier than the ρ^-, in agreement with the prediction.) However, one would not expect the SU(6) result to be good, because of the large fractional difference between the mass of the ρ and π.

Turning to the charmed mesons, according to the ideas of asymptotic freedom,[12] the difference in mass between the D and D^* ought to be small. If so, we can neglect the spin dependence in Eqs. (2e) and (2f), and obtain the relation

$$D^+ - D^0 > D^{*+} - D^{*0} . \qquad (9)$$

In view of our argument that $M^s_{c\bar{u}}$ is small, we would expect the inequality (9) to be very nearly an equality.

Assuming full SU(8) symmetry, Itoh *et al.*[5] have obtained equations relating the charmed mass splittings to the uncharmed ones. Our assumption that the magnetic moment interaction is much smaller for charmed quarks than for uncharmed ones seems to us to be much more plausible than the SU(8) assumption that the magnetic moments of the c and u quarks are equal.

Our results for charmed particles depend on the assignment $Q_c = \frac{2}{3}e$ for the charmed quark. There have been suggestions[13] that $Q_c = -\frac{1}{3}e$ or $-\frac{4}{3}e$. If so, Eqs. (2e) and (2f) will have to be replaced by other equations, which can be easily calculated, and different inequalities will be obtained.

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