Deep-inelastic neutrino scattering: A double spectral form viewpoint*

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The scattering of neutrinos on polarized nucleons, viewed as forward virtual WN scattering, is discussed in a model-independent way based upon the use of a double spectral representation for the invariant amplitudes. In this framework, many of the scaling results of deep-inelastic νN scattering emerge naturally: for example, the linear neutrino-energy dependence of the double differential cross section, and for unpolarized, average nucleon targets, its y dependence; the scaling behavior of the various structure functions; and the vanishing of the longitudinal cross section in the scaling limit. This approach is seen to be useful in organizing disparate data from the different scattering regimes. Of particular interest is the derivation of new sum rules for the scaling functions and for various WN total cross sections.

I. INTRODUCTION

There are now a number of experimental measurements of the inclusive scattering of high-energy neutrinos on nucleonic targets for large momentum transfer, the so-called deep-inelastic regime.[1-3] These experiments provide information about the structure of the nucleon that is complementary to the data obtained by using electrons (or muons) as probes.[4,5] Hence it appears to be an opportune time to ask whether the salient features of these results, such as scaling, can be understood from a rather general point of view as opposed to the more speculative microscopic approaches embodied, for example, in the parton models,[6-9] or the light-cone algebra.[6,10]

For eN scattering, viewed as virtual Compton scattering, Schwinger[11,12] has proposed such a framework in which many of the observed features appear in a natural way. The important dynamical assumption is that the forward Compton amplitude be represented as a double spectral form.[13] The approach is phenomenological in that no particular inner structure is assumed, and that deep-inelastic scattering is correlated with elastic scattering and photoproduction. Our aim here is to extend these ideas to study νN scattering, now viewed as virtual-W–nucleon scattering (where the intermediate vector boson W may be fictitious). We list the following as examples of results that emerge in this scheme: the scaling behavior of various form factors, which in turn implies the linear dependence of the double differential cross section on the energy of the neutrino, sum rules for the structure functions, and various relations among the scaling functions. All the experimental data available[1-3] are for targets of about equal numbers of protons and neutrons, so whenever we discuss or use the data we will assume an average nucleon target.

We begin our discussion of WN forward scattering by first considering the unpolarized target case (Sec. II). Here, by a suitable choice of basis tensors, we calculate the WN total cross sections for various W polarizations, and the double differential cross section for νN scattering; then we introduce the double spectral representation for the WN forward scattering invariant amplitudes, and obtain the scaling limits of the various structure functions and cross sections. A parallel development for the polarized target case follows (Sec. III). Quasielastic scattering is then discussed, together with its implications in the deep-inelastic region (Sec. IV). Sum rules, which follow from the spectral representation, from crossing symmetry, and from low-energy theorems, are then derived (Sec. V). Finally, we present a summary and some general remarks concerning this model-independent approach to the neutrino-scattering experiments (Sec. VI). The Appendix contains a real-field description of the scattering amplitude and the consequences of alternative choices for the basis tensors.

II. SCATTERING OF NEUTRINOS ON UNPOLARIZED NUCLEON TARGETS

A. Virtual forward WN scattering

The incoming neutrino serves (hypothetically) to produce a virtual W boson that, upon colliding with the nucleon, produces some hadronic state. The inclusive scattering process may, by the optical theorem, be conveniently discussed in terms of the WN forward scattering amplitude, which we may express as[14] ($e^2 = 2^{3/2} m_W{}^2 G$)

$$1 + 4ie^2 V d\omega_p W^{*\mu}(-q) W^\nu(q) \sum_{i=1}^{3} (T_i)_{\mu\nu} H_i \,, \tag{2.1}$$

where the first term refers to the situation of noninteraction, V is the interaction volume, p is the nucleon momentum, and q is the momentum of the incoming W. Here we consider a definite target nucleon (proton or neutron) and a definite charge for the W (W^+ or W^-); therefore, the H_i has implicit reference to these quantum numbers. There is a wide variety of basis tensors $T_i^{\mu\nu}$ that can be used to describe this process; a particular set of these tensors, which is a generalization of Schwinger's choice[11] for Compton scattering, is (where m is the nucleon mass)

$$T_1^{\mu\nu}=m^2(q^\mu q^\nu - q^2 g^{\mu\nu}), \tag{2.2}$$

$$T_2^{\mu\nu} = q^2 p^\mu p^\nu - (qp)(p^\mu q^\nu + p^\nu q^\mu) + (qp)^2 g^{\mu\nu} - T_1^{\mu\nu}, \tag{2.3}$$

$$T_3^{\mu\nu} = -q^2\, i\epsilon^{\mu\nu\lambda\sigma} q_\lambda p_\sigma, \tag{2.4}$$

where we have neglected terms that do not contribute to νN scattering when the lepton masses are neglected (such neglect also suggests the choice of a gauge-invariant basis), and have assumed the absence of second-class currents.[7,15] As we will see, the presence of the q^2 factor in T_3 is suggested by the experimental indication that, in the deep-inelastic region, $\sigma^\nu \neq \sigma^{\bar\nu}$. Implicitly, in Eq. (2.1), the invariant amplitude H_i represents one of the four possible WN scattering processes. However, charge symmetry implies the identity of the amplitudes for W^+p and W^-n (designated $H_i^{(+)}$), and for W^-p and W^+n (designated $H_i^{(-)}$). These two independent amplitudes, $H_i^{(\pm)}$, are characterized by the product of the sign of the W charge q' and the eigenvalue of the third component of isospin τ_3' of the nucleon such that $q'\tau_3' = \pm 1$. Since the amplitudes $H_i^{(\pm)} T_i^{\mu\nu}$ are invariant under the crossing operations $q \to -q$, $\mu \leftrightarrow \nu$, and the interchange of the W charge, and since the T_i are symmetric, we deduce that

$$H_i^{(\pm)}(q^2, -qp) = H_i^{(\mp)}(q^2, qp), \tag{2.5}$$

These amplitudes are related to the isoscalar (S) and isovector (V) combinations by

$$H_i^{(S,V)} = \tfrac{1}{2}(H_i^{(+)} \pm H_i^{(-)}), \tag{2.6}$$

which are symmetric for S, and antisymmetric for V, under crossing. An alternative approach to WN forward scattering, which builds in charge symmetry and CP invariance by using real-field representations for the W and nucleon, can be found in the Appendix.

We will now proceed to calculate the total cross sections for WN scattering for particular but unspecified choices of q' and τ_3' and for various choices of W polarizations. Longitudinal (S) polarization of the W is characterized by the field

$$W_S^\mu = \{q^2[m^2q^2 + (pq)^2]\}^{-1/2}[q^2p^\mu - (qp)q^\mu]W(q), \tag{2.7}$$

which then implies that the corresponding total cross section is

$$\sigma_S = \frac{8\pi\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2} m^2 q^2 \,\mathrm{Im} H_1, \tag{2.8}$$

where H_1 is either $H_1^{(+)}$ or $H_1^{(-)}$, depending on the eigenvalue of $\hat{q}\tau_3$, and we have used the flux definition

$$F = d\omega_p W(-q) W(q) 4m(q^2+\nu^2)^{1/2} \tag{2.9}$$

and the definition

$$qp = -m\nu. \tag{2.10}$$

For transverse polarizations of the W, with right- and left-handed circularly polarized states corresponding to helicity ±1, we find

$$\sigma_{R(L)} = \frac{8\pi\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2} m^2\left[-q^2\,\mathrm{Im}H_1 + (q^2+\nu^2)\,\mathrm{Im}H_2 \pm \frac{q^2}{m}(q^2+\nu^2)^{1/2}\,\mathrm{Im}H_3\right]. \tag{2.11}$$

B. Differential cross sections for νN scattering

Since the neutrino-lepton-W vertex is presumably understood, all the physical interest lies in the W-hadron interaction, which for inclusive reactions on unpolarized nucleons is fully described by the cross sections presented in the preceding subsection. However, because of the paucity of information on neutrino interactions, this unfolding process will not be completed for some time, particularly with regard to the dependence on the spin of the nucleon (see Sec. III); therefore, it is worthwhile to present explicit expressions for the νN differential cross section.

Starting from the probability of nonforward νN scattering,

$$2e^2 V d\omega_p (\nu_{L(R)})^* \gamma^0\gamma^\mu (1 \mp i\gamma_5)\mu\,\mu^*\gamma^0\gamma^\nu(1\mp i\gamma_5)\nu_{L(R)}\,\frac{1}{(q^2+m_W^2)^2}\sum_{i=1}^{3}(T_i)_{\mu\nu}\,\mathrm{Im}H_i\,, \tag{2.12}$$

we obtain immediately, in the lab frame $[q^2 = 2EE'(1-\cos\theta)]$,

$$\frac{d\sigma^{\nu,\bar\nu}}{dE'd\Omega}=\frac{2G^2}{\pi^3}m\frac{E'}{E}\left(\frac{m_W^2}{q^2+m_W^2}\right)^2 q^2\left[-q^2\,\mathrm{Im}H_1+\left(E^2+E'^2+\tfrac{1}{2}q^2\right)\mathrm{Im}H_2\mp\frac{q^2}{m}(E+E')\mathrm{Im}H_3\right], \tag{2.13}$$

where E is the energy of the incoming neutrino, E' is that of the outgoing muon, θ is the angle that the muon makes with the incoming neutrino direction; and the lepton mass has been neglected. It is to be noted that the $\pm$ sign corresponds to the sign of the charge of the exchanged W and that the H_i is $H_i^{(+)}$ for νp and $\bar\nu n$, and $H_i^{(-)}$ for νn and $\bar\nu p$. The usual local weak interaction is obtained in the limit $q^2 \ll m_W^2$. Since only this situation is probed in present experiments, we will neglect the factor $[m_W^2/(q^2+m_W^2)]^2$ in the following discussions.

C. Deep-inelastic νN scattering and scaling

To this point our discussion has been mainly kinematic (although, as we shall see, there is some dynamical content in our choice of basis). We now introduce the central dynamical hypothesis, that the invariant amplitudes H_i [with ($\pm$) superscript suppressed] may be represented as unsubtracted double spectral forms[11–13]:

$$H_i(q^2,qp)=\int\frac{dM_+^2}{M_+^2}\frac{dM_-^2}{M_-^2}\frac{2h_i(M_+^2,M_-^2)}{[(p+q)^2+M_+^2][(p-q)^2+M_-^2]}. \tag{2.14}$$

The imaginary part corresponding to an incoming W and nucleon is then

$$\frac{1}{\pi}\mathrm{Im}H_i=\int\frac{dM_+^2}{M_+^2}\frac{dM_-^2}{M_-^2}\frac{\delta(q^2-2m\nu-m^2+M_+^2)}{q^2+\frac{1}{2}(M_+^2+M_-^2)-m^2}\times h_i(M_+^2,M_-^2) \tag{2.15}$$

or, letting

$$\bar h_i\left(\zeta,\frac{m^2}{M_+^2}\right)=\pi\int\frac{dM_-^2}{M_-^2}h_i(M_+^2,M_-^2)\times\exp\left(-\frac{M_+^2+M_-^2-2m^2}{2M_+^2}\zeta\right), \tag{2.16}$$

we obtain

$$\mathrm{Im}H_i=\frac{1}{(M_+^2)^2}\int_0^\infty d\zeta\, e^{-(q^2/M_+^2)\zeta}\bar h_i\left(\zeta,\frac{m^2}{M_+^2}\right), \tag{2.17}$$

where now

$$M_+^2=m^2+2m\nu-q^2. \tag{2.18}$$

All of this is just as in the Compton scattering case.[11]

This representation is specifically adapted to the discussion of deep-inelastic scattering, where $2m\nu$ and q^2 are both large, but their ratio

$$\omega=\frac{2m\nu}{q^2} \tag{2.19}$$

is finite,

$$1\le\omega<\infty. \tag{2.20}$$

(This is the so-called Bjorken limit.) In this case

$$M_+^2\simeq q^2(\omega-1). \tag{2.21}$$

Since this last quantity is large compared to the nucleon mass, it is not unreasonable to make the hypothesis[16]

$$M_+^2\gg m^2:\ \ \bar h_i(\zeta,m^2/M_+^2)\simeq\bar h_i(\zeta). \tag{2.22}$$

If we now define structure functions in the Bjorken limit as

$$f_i(\omega)=\frac{\omega}{(\omega-1)^2}\int_0^\infty d\zeta\, e^{-\zeta/(\omega-1)}\bar h_i(\zeta), \tag{2.23}$$

and define

$$x=1/\omega,\quad y=\nu/E, \tag{2.24}$$

we can write the double differential cross sections [Eq. (2.13)] in the limit $\nu, q^2\to\infty$ as

$$\frac{d\sigma^{\nu,\bar\nu}}{dx\,dy}=\frac{2G^2}{\pi^2}mE[(2-2y+y^2)f_2(x)\mp2xy(2-y)f_3(x)]. \tag{2.25}$$

Thus, in our framework, the observed linear E dependence of the double differential cross sections and the scaling behavior of the structure functions emerge naturally.

It is also interesting to obtain the scaling limits of the total WN cross sections given in Sec. II A. We find, from Eq. (2.8), that

$$\sigma_S=0, \tag{2.26}$$

which conforms with the approximate vanishing of σ_S relative to σ_L (defined below) without invoking the "Callan-Gross relation for spin-$\frac{1}{2}$ constituents."[17]

In the same limit, the transverse cross sections [Eq. (2.11)] become

$$\sigma_{R(L)}=\frac{2\pi\alpha}{m\nu}\frac{1}{x}[f_2(x)\pm2xf_3(x)]\ge0, \tag{2.27}$$

and the integrated version of Eq. (2.25) is

$$\sigma^{\nu,\bar\nu}=\frac{8G^2}{3\pi^2}mE\int_0^1dx[f_2(x)\mp xf_3(x)]. \tag{2.28}$$

Recall that for the scattering of ν on p (n) the relevant structure function is $f_i^{(+)}$ ($f_i^{(-)}$), while for $\bar{\nu}$ it is $f_i^{(-)}$ ($f_i^{(+)}$). Therefore, if we consider an average nucleon target, the structure function for either ν or $\bar{\nu}$ becomes the same

$$f_i^{(s)} = \tfrac{1}{2}(f_i^{(+)} + f_i^{(-)}), \tag{2.29}$$

and the corresponding total cross section will be denoted by $\bar{\sigma}^{\nu,\bar{\nu}}$. The approximate experimental result[1,2]

$$\bar{\sigma}^{\bar{\nu}}/\bar{\sigma}^{\nu} \simeq \tfrac{1}{3} \tag{2.30}$$

implies, from the positivity requirement [Eq. (2.27)],

$$f_2^{(s)}(x) \simeq -2x f_3^{(s)}(x), \tag{2.31}$$

or equivalently [from Eq. (2.27)],

$$\bar{\sigma}_R \simeq 0. \tag{2.32}$$

One can reverse this argument to deduce Eq. (2.30) from Eq. (2.32). (Similar arguments are presented in Refs. 8 and 9.) Because there is now only one independent structure function, one infers the following definite y dependence of the double differential cross section for a spin-averaged, isoscalar target

$$\frac{d\bar{\sigma}^{\bar{\nu}}}{dx\,dy} = \frac{4G^2}{\pi^2} mE(1-y)^2 f_2^{(s)}(x), \tag{2.33}$$

$$\frac{d\bar{\sigma}^{\nu}}{dx\,dy} = \frac{4G^2}{\pi^2} mE f_2^{(s)}(x), \tag{2.34}$$

which also appear to be valid experimentally[1] (although the $\bar{\nu}$ cross section seems[3] to be more nearly constant in y at high energies, $E \geq 30$ GeV).

III. SCATTERING OF NEUTRINOS ON POLARIZED NUCLEON TARGETS

The preceding section has dealt with neutrino scattering on unpolarized targets. We will now extend this analysis to incorporate the effects of target polarization.

A. Virtual forward *WN* scattering

The WN forward scattering amplitude is still of the form of Eq. (2.1) but with the sum on i running from 1 to 8, since there are five additional basis tensors referring explicitly to the target polarization; that may be described by the pseudovector s^μ obeying

$$p^\mu s_\mu = 0\,. \tag{3.1}$$

As in Sec. II A, there is a wide variety of basis tensors $T_i^{\mu\nu}$ for $i = 4$–8 that can be used to describe this process, some of which will be discussed in the Appendix. A convenient set of these is[18]

$$T_4^{\mu\nu} = -2m^3 i\epsilon^{\mu\nu\lambda\sigma} q_\lambda s_\sigma\,, \tag{3.2}$$

$$T_5^{\mu\nu} = m(qs) i\epsilon^{\mu\nu\lambda\sigma} q_\lambda p_\sigma\,, \tag{3.3}$$

$$T_6^{\mu\nu} = \frac{1}{m}(qs) T_1^{\mu\nu}\,, \tag{3.4}$$

$$T_7^{\mu\nu} = \frac{1}{m}(qs) T_2^{\mu\nu} - \frac{(pq)}{2m^2} T_8^{\mu\nu}\,, \tag{3.5}$$

$$T_8^{\mu\nu} = mq^2(p^\mu s^\nu + p^\nu s^\mu) - m(qs)(p^\mu q^\nu + p^\nu q^\mu) - m(qp)(q^\mu s^\nu + q^\nu s^\mu) + 2m(qs)(qp) g^{\mu\nu}\,. \tag{3.6}$$

Again, we have neglected terms that do not contribute to νN scattering when the lepton masses are negligible; we have used a gauge-invariant set of tensors and have assumed the absence of second-class currents.[7,15] This basis is similar to, but distinct from, that of Dicus.[10] Apart from various factors[19] of q^2, the chief differences are the presence of T_6 and T_8 in T_7, and a less singular choice of T_8. Since the $T_i^{\mu\nu}$ for $i = 1, 2, 3, 4, 8$ are symmetric, and for $i = 5, 6, 7$ are antisymmetric, we deduce the following behavior under crossing [cf. Eq. (2.5)]:

$$H_i^{(\pm)}(q^2, -qp) = \epsilon_i H_i^{(\mp)}(q^2, qp)\,, \tag{3.7}$$

where

$$\epsilon_i = \begin{cases} 1, & i = 1\text{–}4, 8\,, \\ -1, & i = 5\text{–}7. \end{cases} \tag{3.8}$$

The isoscalar and isovector combinations [Eq. (2.6)] have definite symmetry under crossing as dictated by Eq. (3.7).

For the study of polarized WN total cross sections (with definite q' and τ_3'), we define ζ as the projection of s along the direction of q:

$$\zeta = (q^2 + \nu^2)^{-1/2}(qs)\,. \tag{3.9}$$

The spin dependence of the total cross section for longitudinal (S) polarization of the W [Eq. (2.7)] is given by

$$\tfrac{1}{2}(\sigma_{S,\zeta} - \sigma_{S,-\zeta}) = \frac{8\pi\alpha}{m\nu}\left(1 + \frac{q^2}{\nu^2}\right)^{-1/2} \zeta m q^2 (q^2 + \nu^2)^{1/2} \,\mathrm{Im} H_6\,. \tag{3.10}$$

For transverse polarizations of the W [cf. Eq. (2.11)], we find

$$\tfrac{1}{2}(\sigma_{R(L),\zeta}-\sigma_{R(L),-\zeta})=\frac{8\pi\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2}\zeta m\Big[\mp 2m\nu\,\mathrm{Im}H_4\mp(q^2+\nu^2)\,\mathrm{Im}H_5 - \frac{q^2}{m}(q^2+\nu^2)^{1/2}(\mathrm{Im}H_6-\mathrm{Im}H_7)-2\nu(q^2+\nu^2)^{1/2}\,\mathrm{Im}H_8\Big]. \qquad (3.11)$$

From Eqs. (3.10) and (3.11), we only have three independent cross sections. To obtain the two additional ones we must consider cross sections corresponding to mixtures of longitudinal and transverse W-boson polarizations. One possibility is to let

$$W^{\mu}_{\perp,\pm}=m^{-1}(q^2+\nu^2)^{-1/2}\{a(q^2)^{-1/2}[q^2p^{\mu}-(qp)q^{\mu}]\pm ib\epsilon^{\mu\alpha\beta\gamma}q_{\alpha}p_{\beta}s_{\gamma}\}W(q)\,, \qquad (3.12)$$

where the transverse polarization is orthogonal to s. If, further, $\zeta=0$, we find

$$\tfrac{1}{2}(\sigma_{\perp,+}-\sigma_{\perp,-})=\frac{8\pi\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2}2ab(q^2)^{1/2}2m^3\,\mathrm{Im}H_4\,. \qquad (3.13)$$

A second choice for the transverse polarization vector ϵ^{μ} satisfies

$$\epsilon p=\epsilon q=0\,,\quad \epsilon s=1\,, \qquad (3.14)$$

so that with

$$W^{\mu}_{\parallel,\pm}=m^{-1}(q^2+\nu^2)^{-1/2}\times\{a(q^2)^{-1/2}[q^2p^{\mu}-(qp)q^{\mu}]\pm m^2b\epsilon^{\mu}\}W(q) \qquad (3.15)$$

we have

$$\tfrac{1}{2}(\sigma_{\parallel,+}-\sigma_{\parallel,-})=-\frac{8\pi\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2}2ab(q^2)^{1/2}\times m^3\left(\mathrm{Im}H_8+\frac{\nu}{2m}\mathrm{Im}H_7\right). \qquad (3.16)$$

B. Differential cross-sections for νN scattering

We will here present an explicit expression for the νN differential cross section in the particular case $\vec{s}\parallel\vec{k}$ ($\vec{k}$ is the neutrino momentum), since the other possibility, $\vec{s}\perp\vec{k}$, gives a contribution that is one power of E smaller. Then, from Eq. (2.12), we obtain [cf. Eq. (2.13)], in terms of $\xi=\vec{s}\cdot\vec{k}/|\vec{k}|=\pm1$,

$$\frac{d}{dE'd\Omega}\,\frac{1}{2}(\sigma^{\nu,\bar\nu}_{\xi}-\sigma^{\nu,\bar\nu}_{-\xi}) =\xi\frac{2G^2}{\pi^3}m\frac{E'}{E}q^2\Big[\pm 2m(E+E'\cos\theta)\,\mathrm{Im}H_4\pm(E+E')(E-E'\cos\theta)\,\mathrm{Im}H_5-\frac{q^2}{m}(E-E'\cos\theta)\,\mathrm{Im}H_6 +\frac{q^2}{2m}(2E+E'-E'\cos\theta)\,\mathrm{Im}H_7-2(E^2+E'^2\cos\theta)\,\mathrm{Im}H_8\Big]. \qquad (3.17)$$

This double differential cross section coincides with that of Dicus[10] when corresponding changes in the basis are made.

C. Deep-inelastic νN scattering and scaling

By using the representations Eqs. (2.14)–(2.18), we now proceed to investigate the polarization effects in the deep-inelastic region. In terms of the structure functions $f_i(\omega)$ defined in Eq. (2.23), the spin-dependent double differential cross section of Eq. (3.17) in the Bjorken limit has the following scaling form, with the characteristic linear E dependence:

$$\frac{d}{dx\,dy}\,\frac{1}{2}(\sigma^{\nu,\bar\nu}_{\xi}-\sigma^{\nu,\bar\nu}_{-\xi})=\xi\frac{2G^2}{\pi^2}mE[\pm y(2-y)f_5(x) -2xy^2f_6(x)+2xyf_7(x) -2(2-2y+y^2)f_8(x)]. \qquad (3.18)$$

It is also interesting to obtain the scaling limits of the total WN cross sections. We find from Eq. (3.10) that

$$\tfrac{1}{2}(\sigma_{S,\zeta}-\sigma_{S,-\zeta})=\frac{4\pi\alpha}{m\nu}\zeta f_6(x)\,. \qquad (3.19)$$

If we suppose, as suggested by Eq. (2.26), that the longitudinal cross section vanishes for all ζ in the scaling region, we obtain

$$f_6(x)=0\,, \qquad (3.20)$$

which is equivalent to a relation derived by Dicus[10] on the basis of light-cone algebra. In the same limit, the transverse cross section [Eq. (3.11)] has the following spin dependence:

$$\tfrac{1}{2}(\sigma_{R(L),\zeta}-\sigma_{R(L),-\zeta}) =\zeta\frac{2\pi\alpha}{m\nu}\frac{1}{x}\{\mp f_5(x)-2x[f_6(x)-f_7(x)]-2f_8(x)\}\,. \qquad (3.21)$$

IV. SINGLE NUCLEON EXCHANGE

The quasielastic scattering situation, where the final hadronic state is a single nucleon, besides being interesting in its own right, can provide information about the scaling region. Again we view this process in terms of forward WN scattering, this time with a single particle exchanged. The scattering amplitude is in the form of Eq. (A1) with

$$\sum_i H_i^{(S,V)} T_i^{\mu\nu} = \tfrac{1}{2} m u^* \gamma^0 \left[\pm \Gamma^\mu(-q) \frac{1}{m+\gamma(p+q)} \Gamma^\nu(q) + \Gamma^\nu(q) \frac{1}{m+\gamma(p-q)} \Gamma^\mu(-q) \right] u \,, \tag{4.1}$$

where

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) - \frac{1}{2m} i\sigma^{\mu\nu} q_\nu F_2(q^2) - i\gamma_5 \gamma^\mu G_A(q^2) \,, \tag{4.2}$$

neglecting second-class currents[7,15] and terms which for νN scattering vanish with the lepton mass. The results of an elementary calculation are

$$H_i^{(S,V)} = \frac{\mathcal{G}_i}{q^2}\left(\pm \frac{1}{q^2 - 2m\nu} + \epsilon_i \frac{1}{q^2+2m\nu}\right) + C_i^{(S,V)} \,, \tag{4.3}$$

where ϵ_i is given by Eq. (3.8),

$$C_i^{(S)} = 0 \quad \text{for } i = 2,3,5,6,7,8 \,, \tag{4.4}$$

$$C_1^{(S)} = -\frac{(F_2)^2}{8m^4} \,, \tag{4.5}$$

$$C_4^{(S)} = \frac{F_1 F_2}{8m^4} + \frac{G_A^{\,2}}{2m^2 q^2} \,, \tag{4.6}$$

$$C_i^{(V)} = 0 \quad \text{for } i = 2,3,7,8 \,, \tag{4.7}$$

$$C_1^{(V)} = \frac{\nu}{mq^4}(F_1^{\,2} + G_A^{\,2}) \,, \tag{4.8}$$

$$C_4^{(V)} = \frac{\nu}{4m^3 q^2} F_1 F_2 \,, \tag{4.9}$$

$$C_5^{(V)} = -\frac{F_1 F_2}{2m^2 q^2} \,, \tag{4.10}$$

$$C_6^{(V)} = -\frac{2}{q^4} F_1 G_A \,, \tag{4.11}$$

and

$$\mathcal{G}_1 = G_E^{\,2} \,, \tag{4.12}$$

$$\mathcal{G}_2 = \frac{G_E^{\,2} + (q^2/4m^2) G_M^{\,2}}{1 + q^2/4m^2} + G_A^{\,2} \,, \tag{4.13}$$

$$\mathcal{G}_3 = G_A G_M \,, \tag{4.14}$$

$$\mathcal{G}_4 = \frac{q^2}{4m^2} G_E G_M \,, \tag{4.15}$$

$$\mathcal{G}_5 = \frac{q^2}{4m^2} \frac{G_M(G_M - G_E)}{1 + q^2/4m^2} + G_A^{\,2} \,, \tag{4.16}$$

$$\mathcal{G}_6 = 0 \,, \tag{4.17}$$

$$\mathcal{G}_7 = \frac{G_A(G_E - G_M)}{1 + q^2/4m^2} \,, \tag{4.18}$$

$$\mathcal{G}_8 = \frac{G_A[G_E + (q^2/4m^2) G_M]}{1 + q^2/4m^2} \,, \tag{4.19}$$

with

$$G_E = F_1 - \frac{q^2}{4m^2} F_2 \,, \tag{4.20}$$

$$G_M = F_1 + F_2 . \tag{4.21}$$

The relation of these form factors to the electromagnetic ones is provided by the conserved vector current hypothesis[7,15]

$$G_{E,M} = (G_{E,M}^{p} - G_{E,M}^{n})^{(\text{electromagnetic})} \tag{4.22}$$

which, at $q^2 = 0$, implies

$$G_E(0) = 1, \qquad \mu \equiv 1 + \mu_a \equiv G_M(0) = 4.7; \tag{4.23}$$

also, the axial-vector coupling constant is

$$g_A \equiv G_A(0) = -1.2 . \tag{4.24}$$

We will assume that G_E, G_M, and G_A are approximately described by the same dipole fit.[7,20]

The quasielastic total WN cross sections can be obtained from the imaginary parts of the H_i:

$$m^2 q^2 \frac{1}{\pi} \mathrm{Im} H_i^{(-)} = 2\delta\left(\frac{M_+^{\,2}}{m^2} - 1\right) \mathcal{G}_i \,, \tag{4.25}$$

$$\mathrm{Im} H_i^{(+)} = 0 . \tag{4.26}$$

Obviously, whenever quasielastic cross sections are mentioned, it is to be understood that they refer to either $W^- p$ or $W^+ n$ scattering. For the longitudinal polarization, we obtain [from Eqs. (2.8) and (3.10)]

$$\sigma_{S,\zeta} = \frac{16\pi^2 \alpha}{m\nu}\left(1 + \frac{q^2}{\nu^2}\right)^{-1/2} \delta\left(\frac{q^2 - 2m\nu}{m^2}\right) G_E^{\,2} \,, \tag{4.27}$$

which has no spin dependence, while for the transverse polarization we have [from Eqs. (2.11) and (3.11)]

$$\sigma_{R(L),\zeta}=\frac{16\pi^2\alpha}{m\nu}\left(1+\frac{q^2}{\nu^2}\right)^{-1/2}\delta\left(\frac{q^2-2m\nu}{m^2}\right)(1\mp\zeta)\left\{\frac{q^2}{4m^2}G_M{}^2+\left(1+\frac{q^2}{4m^2}\right)G_A{}^2\pm\frac{1}{m}\left[q^2\left(1+\frac{q^2}{4m^2}\right)\right]^{1/2}G_AG_M\right\}, \tag{4.28}$$

which shows a simple correlation between the W helicity and nucleon spin. If we define $\sigma_{\pm\zeta}$ as the cross sections when the spins are parallel and antiparallel, then in the quasielastic situation

$$\sigma_{+\zeta}=0. \tag{4.29}$$

[Note that this means in the analogous γN scattering case, where G_A is zero (so there is no residual ζ dependence), that the polarization asymmetry defined there[12] by

$$P=\frac{\sigma_+-\sigma_-}{\sigma_++\sigma_-} \tag{4.30}$$

in the quasielastic limit tends to -1; since this limit corresponds to the scaling variable $\omega=1$, this is some evidence in favor of a conjecture raised by Schwinger.[12]]

In this W situation we could define, for example, another asymmetry using the notation introduced immediately above:

$$A=\frac{\sigma_{--}-\sigma_{-+}}{\sigma_{--}+\sigma_{-+}}. \tag{4.31}$$

The quasielastic limit of this, for large q^2, is

$$A\to\frac{2G_AG_M}{G_A{}^2+G_M{}^2}. \tag{4.32}$$

Then, assuming a common dipole form for each of the asymptotic form factors appearing here, we have

$$A\to\frac{2g_A\mu}{g_A{}^2+\mu^2}\simeq-\tfrac{1}{2}. \tag{4.33}$$

We conjecture then that in the scaling region, as $\omega\to1$, A should tend to this value, which supplies a weak constraint on the structure functions.

Returning now to the representation (2.17) for the $\mathrm{Im}H_i$ specialized to the quasielastic case

$$h_i\left(\zeta,\frac{m^2}{M_+{}^2}\right)=2\pi\delta\left(\frac{M_+{}^2}{m^2}-1\right)h_i(\zeta), \tag{4.34}$$

we can present Eq. (4.25) in the form

$$\mathcal{G}_i=\frac{q^2}{m^2}\int_0^\infty d\zeta\, e^{-(q^2/m^2)\zeta}h_i(\zeta)$$
$$=\int_0^\infty d\zeta\, e^{-(q^2/m^2)\zeta}h_i'(\zeta). \tag{4.35}$$

Here we have used the fact that

$$h_i(\zeta=0)=0, \tag{4.36}$$

due to the behavior of the $\mathcal{G}_i$ for large q^2 (assuming dipole behavior):

$$q^2\to\infty:\quad \mathcal{G}_i\sim\left(\frac{1}{q^2}\right)^4,\quad i\neq4,6,7,$$
$$\mathcal{G}_4\sim(1/q^2)^3,$$
$$\mathcal{G}_7\sim(1/q^2)^5. \tag{4.37}$$

In fact, this asymptotic behavior implies that

$$\zeta\to0:\quad h_i'(\zeta)\sim\zeta^3,\quad i\neq4,6,7,$$
$$h_4'(\zeta)\sim\zeta^2,$$
$$h_7'(\zeta)\sim\zeta^4. \tag{4.38}$$

Equations (4.36) and (4.38) may also be expressed as superconvergence relations.[11] If we assume that this same behavior holds for the $\bar{h}_i'(\zeta)$, we expect [from Eq. (2.23)]

$$\omega\to1:\quad f_i(\omega)\sim(\omega-1)^3,\quad i\neq4,6,7,$$
$$f_4(\omega)\sim(\omega-1)^2,$$
$$f_7(\omega)\sim(\omega-1)^4. \tag{4.39}$$

On the basis of Eq. (4.26), we might expect the $f_i^{(+)}$ to go to zero (as $\omega\to1$) faster than $f_i^{(-)}$ [Eq. (4.39)].

The corresponding extrapolation between the elastic and deep-inelastic regions for electron scattering is in reasonable accord with experiment.[11] We would therefore expect this extrapolation to be valid here, but as yet there is no experimental confirmation.

V. SUM RULES

In our framework, where the invariant amplitudes are assumed to have double spectral representations, it is very natural to derive sum rules for various WN total cross sections. Since crossing symmetry will play an important role here, we find it is more convenient to use isoscalar and isovector amplitudes [Eq. (2.6)] which have definite behavior under crossing. Since $T_i^{\mu\nu}$ is symmetric for $i=1,2,3,4,8$, and is antisymmetric for $i=5,6,7$, we deduce immediately from the spectral representation [Eq. (2.14)] that both $h_i^{(S)}$, $i=5,6,7$, and $h_i^{(V)}$, $i=1,2,3,4,8$, are antisymmetric under the interchange of their arguments $M_+{}^2\leftrightarrow M_-{}^2$; this antisymmetry then implies the sum rules [see also Eq. (2.15)]

$$\int dM_+{}^2\frac{1}{\pi}\mathrm{Im}H_i^{(S)}=0,\quad i=5,6,7, \tag{5.1}$$

$$\int dM_+^2 \frac{1}{\pi} \mathrm{Im} H_i^{(V)} = 0, \quad i = 1, 2, 3, 4, 8, \qquad (5.2)$$

where M_+^2 is defined by Eq. (2.18). If we explicitly display the quasielastic contributions, these become

$$\int_{>m^2}^{\infty} dM_+^2 \frac{1}{\pi} \mathrm{Im} H_i^{(S)} = -\frac{1}{q^2} \mathcal{G}_i, \quad i = 5, 6, 7, \qquad (5.3)$$

$$\int_{>m^2}^{\infty} dM_+^2 \frac{1}{\pi} \mathrm{Im} H_i^{(V)} = \frac{1}{q^2} \mathcal{G}_i, \quad i = 1, 2, 3, 4, 8, \qquad (5.4)$$

where $\mathcal{G}_i$ are given by Eqs. (4.12)–(4.19). Because of the asymptotic behavior given in Eq. (4.37), we obtain the scaling version of the sum rules[21]

$$\int_1^{\infty} \frac{d\omega}{\omega} f_i^{(S)}(\omega) = 0, \quad i = 5, 6, 7, \qquad (5.5)$$

$$\int_1^{\infty} \frac{d\omega}{\omega} f_i^{(V)}(\omega) = 0, \quad i = 1, 2, 3, 4, 8. \qquad (5.6)$$

The $i = 5$ case of Eq. (5.3) and (5.5) is a generalization of a sum rule seen before in the photon situation.[12,22,23] In general, we may express $\mathrm{Im} H_i$ in terms of various WN total cross sections [Eqs. (2.8), (2.11), (3.10), (3.11), (3.13), and (3.16)]. Particularly interesting are the $i = 1$ and 3 cases of Eq. (5.4): in the limit $q^2 \to 0$, we obtain [see Eqs. (2.8) and (2.11)]

$$\int_{>0}^{\infty} d\nu' \, \nu' \sigma_S^{(V)}(\nu') = 4\pi^2 \alpha, \qquad (5.7a)$$

$$\int_{>0}^{\infty} d\nu' [\sigma_R(\nu') - \sigma_L(\nu')]^{(V)} = \frac{8\pi^2 \alpha}{m} g_A \mu. \qquad (5.7b)$$

Additional sum rules may be inferred from low-energy theorems. We assume, in analogy to the low-energy theorem in Compton scattering,[12,24] that when q^2, $m\nu \ll m^2$, the invariant amplitudes are dominated by the single nucleon exchange contribution. When this assumption is combined with the double spectral representation [Eq. (2.14)] expressed in the form

$$H_i^{(S,V)}(q^2, -m\nu) = \int d\nu' \left(\frac{1}{\nu' - \nu} \pm \epsilon_i \frac{1}{\nu' + \nu} \right) \frac{1}{\pi} \mathrm{Im} H_i^{(S,V)}(q^2, -m\nu'), \qquad (5.8)$$

with ϵ_i defined in Eq. (3.8), several sum rules may be obtained. In particular, Eqs. (5.8) and (5.1) imply

$$H_4^{(S)} + \frac{\nu}{2m} H_5^{(S)} = \int d\nu' \left(\frac{1}{\nu' - \nu} + \frac{1}{\nu' + \nu} \right) \frac{1}{\pi} \mathrm{Im} \left(H_4^{(S)} + \frac{\nu'}{2m} H_5^{(S)} \right). \qquad (5.9)$$

The quasielastic contribution to the imaginary parts, Eqs. (4.25) and (4.26), just reproduce the denominator structure of Eq. (4.3). However, there is an additional constant which, by the "low-energy theorem," must be reproduced by larger values of ν'. In terms of cross sections [from Eq. (3.11), as $q^2 \to 0$ with $\zeta = +1$]

$$(\sigma_{L+} + \sigma_{R-} - \sigma_{L-} - \sigma_{R+})^{(S)} = 64\pi\alpha m^2 \mathrm{Im} \left(H_4^{(S)} + \frac{\nu}{2m} H_5^{(S)} \right), \qquad (5.10)$$

we obtain

$$\int_0^{\infty} \frac{d\nu'}{\nu'} (\sigma_{L+} + \sigma_{R-} - \sigma_{L-} - \sigma_{R+})^{(S)}(q^2 = 0) = \frac{4\pi^2 \alpha}{m^2} \mu_a^2, \qquad (5.11)$$

which is an analog of the Drell-Hearn sum rule.[25]

VI. SUMMARY AND DISCUSSION

We have examined the general features of polarized νN scattering, in a model-independent way, by conjecturing a double spectral representation for the WN invariant amplitudes, corresponding to an appropriate choice of tensor basis. From this general framework, the following results emerge naturally (in general, an average nucleon target is assumed):

(i) the linear dependence of the double differential cross section on the neutrino energy [Eqs. (2.25) and (3.18)];

(ii) the scaling behavior of the various structure functions [Eq. (2.23)];

(iii) the vanishing of the longitudinal cross section in the scaling limit [Eq. (2.26)], which in turn suggests the vanishing of the scaling function $f_6(x)$ [Eq. (3.20)];

(iv) the relation between $\bar{\sigma}_R \simeq 0$ and $\bar{\sigma}^{\bar{\nu}}/\bar{\sigma}^{\nu} \simeq \frac{1}{3}$ [Eqs. (2.30)–(2.32)] and the y dependence of the double differential cross section in the scaling region [Eqs. (2.33)–(2.34)] for unpolarized targets;

(v) the behavior of the scaling functions near $\omega = 1$ as inferred from quasielastic scattering [Eq. (4.39)];

(vi) sum rules for the scaling functions [Eqs. (5.5)–(5.6)] and for various WN total cross sections [Eqs. (5.7) and (5.11)].

In obtaining the above results we have been guided by the experimental situation, which is still in an early stage. Data are available only on the three spin-independent structure functions, and then only for nucleon targets composed of approximately equal numbers of protons and neutrons. In the analysis of this data, we have chosen a basis suggested by simplicity, gauge invariance, and the conformity to experiment. In particular, the fact that $\bar{\sigma}^{\nu} \neq \bar{\sigma}^{\bar{\nu}}$ in the deep-inelastic region requires the presence of q^2 in T_3 [see Eq. (2.4)]. Then from our framework we obtain the results (i) through (iv) listed above.

Once we try to extend our analysis to situations not yet accessible to experiment, such as those employing polarized targets, our predictions become speculative. But, *faute de mieux,* we have continued to exploit consequences of simplicity and physical continuity (that certain features found in one domain persist in another) which, for example, lead to results (iii) and (v) above. It should be borne in mind, however, that even if these assumptions fail, the general framework provides a formalism for incorporating and correlating new experimental data as they become available.

There is only one central dynamical assumption, the particular form of the double spectral representation, Eq. (2.14). The applicability of this assumption is an experimental question, one that will not be answered in the near future. However, for now, it is extremely useful for characterizing and organizing the features of the phenomena. Of course, one of the central theoretical issues is to further investigate this double spectral form and try to determine what conditions are required for its validity.[26] In this way, we could hope to learn something of the hadronic interactions that the deep-inelastic experiments are probing.

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APPENDIX

A. Real field description of the scattering amplitude

In the text we have used the complex-field representation to describe the scattering amplitude [Eq. (2.1)]. Here we will present an alternative description using real fields, which builds in charge symmetry and CP invariance explicitly. In this representation, the scattering amplitude for WN forward scattering becomes

$$1+4ie^2 V d\omega_p \sum_{a,b=1}^{2} \sum_{i=1}^{8} W_a^{\mu}(-q)(\tilde{T}_i)_{\mu\nu} H_{iab} W_b^{\nu}(q), \tag{A1}$$

where $W_{1,2}$ is related to $W^{\pm}$ by

$$W^{\pm} = \frac{1}{\sqrt{2}}(W_1 \mp iW_2). \tag{A2}$$

The invariant amplitude has the general form

$$H_{iab} = \delta_{ab} H_i^{(S)} + \hat{q}_{ab}(\nu\tau_3)' H_i^{(V)}, \tag{A3}$$

in which $\tau_3'(\nu')$ is the eigenvalue of the third component of isospin (nucleonic charge), and $\hat{q}_{ab}$ is the charge matrix for the W boson. The basis tensors $\tilde{T}_i$ are

$$\tilde{T}_i = T_i \quad \text{for } i=1,2,4,5$$

and (A4)

$$\tilde{T}_i = \nu' T_i \quad \text{for } i=3,6,7,8,$$

where the T_i are given by Eqs. (2.2)–(2.4) and (3.2)–(3.6). It can now be easily seen that $(\tilde{T}_i)_{\mu\nu} H_{iab}$ is invariant under $a \leftrightarrow b$, $\tau_3' \to -\tau_3'$ (charge symmetry) and $\mu \leftrightarrow \nu$, $\nu' \to -\nu'$, $\hat{q} \to -\hat{q}$, $q,p \to -q,-p$ (effectively CP invariance). Crossing symmetry ($a \leftrightarrow b$, $\mu \leftrightarrow \nu$, $q \to -q$) implies the relation

$$H_i^{(S,V)}(q^2, pq) = \pm\epsilon_i H_i^{(S,V)}(q^2, -pq). \tag{A5}$$

For W^+p and W^-n scattering ($\nu'=+1$), Eq. (A3) becomes

$$H_i^{(+)} = H_i^{(S)} + H_i^{(V)}, \tag{A6}$$

while for W^+n and W^-p we have

$$H_i^{(-)} = H_i^{(S)} - H_i^{(V)}. \tag{A7}$$

B. Basis tensors

As mentioned in the text, a wide variety of basis tensors can be used. The choice made there is a natural extension of the electron situation.[11,12] Since the lepton current is approximately conserved (because the lepton mass can be neglected at high energy), we have used a gauge-invariant set of tensors. For the WN interaction, there is no strong compelling reason for such a choice. So one possible alternate is to abandon the gauge-invariance requirement. Likewise, the set in the text yields a cross section that grows linearly with the neutrino energy E, but this linear growth could be obtained in other ways, such as modifying the hypothesis of Eq. (2.22). So the basis in the text is hardly unique, but it does have many attractive features. Experimental data will be very helpful in choosing the basis tensors that most readily accommodate the data. Lacking such guidance, we will here mention a few further possible choices and some of the resulting consequences:

(1) $T_4' = T_4$,

$$T_5' = T_5 + \frac{(pq)}{2m^2} T_4. \tag{A8}$$

The important changes are

$$\begin{aligned}
&\text{in (3.11):}\quad \mp(q^2+\nu^2)\,\mathrm{Im}\,H_5 \to \mp q^2\,\mathrm{Im}\,H_5,\\
&\text{in (3.13):}\quad \mathrm{Im}\,H_4 \to \mathrm{Im}\,H_4-(\nu/2m)\,\mathrm{Im}\,H_5,\\
&\text{in (3.17):}\quad \pm(E+E')(E-E'\cos\theta)\,\mathrm{Im}\,H_5 \to \pm q^2\,\mathrm{Im}\,H_5,\\
&\text{in (3.18):}\quad \pm y(2-y)f_5(x)\to 0,\\
&\text{in (3.21):}\quad \mp f_5(x)\to 0,\\
&\text{in (4.6):}\quad C_4^{(S)}\to -F_2^2/8m^4,\\
&\text{in (4.9):}\quad C_4^{(V)}\to 0,\\
&\text{in (4.15):}\quad \mathcal{G}_4 \to \frac{q^2}{4m^2}\frac{G_M[G_E+(q^2/4m^2)G_M]}{1+q^2/4m^2}+\frac{q^2}{4m^2}G_A^2.
\end{aligned} \tag{A9}$$

Of these, the most striking changes are in Eqs. (3.18) and (3.21); neither H_4 nor H_5 contribute in the deep-inelastic region. In particular, for isoscalar targets, the effect of nucleon polarization is the same for ν and $\bar{\nu}$. Also, the sum rule Eq. (5.11) would now be a direct result of the low-energy theorem for $H_4^{(S)}$.

$$(2)\ T_7' = T_7 + \frac{(pq)}{2m^2}T_8. \tag{A10}$$

For this case, we will simply comment on one consequence. If we were to use the hypothesis Eq. (2.22) for the deep-inelastic region, the contribution of $\mathrm{Im}\,H_7$ would grow quadratically with the energy E. To retain the linear behavior, we would have to alter Eq. (2.22):

$$h_7(\zeta, m^2/M_+^2) \simeq \frac{m^2}{M_+^2}\bar{h}_7(\zeta). \tag{A11}$$

Certainly this is possible but it does seem to be somewhat artificial. A test of this basis is the implied simple y dependence [assuming Eq. (3.20)] for an isoscalar target:

$$\frac{d\sigma^\nu}{dx\,dy}+\frac{d\sigma^{\bar\nu}}{dx\,dy} \propto 2-2y+y^2. \tag{A12}$$

$$\begin{aligned}
(3)\ T_6' &= m^3(qs)g^{\mu\nu},\\
T_7' &= m(qs)p^\mu p^\nu,\\
T_8' &= m^3(s^\mu p^\nu + s^\nu p^\mu).
\end{aligned} \tag{A13}$$

This choice simply abandons gauge invariance for the last three tensors. Two of the consequences of this choice are:

(i) The only contribution in the deep-inelastic region comes from $\mathrm{Im}\,H_7$; the change in Eq. (3.18) is indicated by

$$-2xy^2f_6+2xyf_7-2(2-2y+y^2)f_8 \to \frac{1-y}{x}f_7 \tag{A14}$$

which, in turn, predicts a definite y behavior for the spin-dependent part of the average of the ν and $\bar{\nu}$ cross sections. However, the $1/x$ factor might be a source of concern.

(ii) The single-nucleon exchange results are

$$\begin{aligned}
\mathcal{G}_6' &= -\frac{q^2}{m^2}G_AG_M,\\
\mathcal{G}_7' &= \frac{q^2}{m^2}\frac{G_A(G_E-G_M)}{1+q^2/4m^2},\\
\mathcal{G}_8' &= \frac{q^2}{m^2}G_AG_E,\\
C'^{(S,V)}_{6-8} &= 0 \text{ except for } C'^{(S)}_8=\frac{F_2F_4}{2m^4}.
\end{aligned} \tag{A15}$$

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