Production of baryons with large transverse momentum

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The multiple scattering of constituent quarks provides a natural mechanism for fairly copious production of large-transverse-momentum baryons in nucleon-nucleon collisions. The predicted scaling law agrees well with available data, and the mechanism provides a qualitative explanation of nuclear-target effects. In comparison with previous parton models, correlations are predicted to be qualitatively different, and large-p_T baryon production by meson beams is relatively suppressed.

I. INTRODUCTION

While there are a number of theoretical models[1] proposed to explain the production of mesons at large transverse momentum, the corresponding production of protons has proved more difficult to understand. For this reason it seems desirable to explore further models for the production of large-p_T particles. In this paper we present a model, based on the multiple scattering of constituent quarks, which has some interesting new features and which may explain large-p_T proton production in pp collisions.

The Chicago-Princeton (CP) and British-Scandinavian (BS) collaborations[2,3] find that the production of large-p_T protons is surprisingly copious. Our model seems to be the most promising candidate for explaining this.

CP also find that if one writes the large-p_T inclusive cross section in the form

$$E\frac{d^3\sigma}{d^3p} = p_T^{-n} f(x_T, \theta), \quad x_T = \frac{2p_T}{\sqrt{s}} \tag{1}$$

with θ the center-of-mass angle of the outgoing proton, the effective value of n varies with x_T in the way shown in Fig. 1. Of particular interest is the fact that the value of n for proton production appears to become nearly constant at 14 for large x_T. Our model leads to an expression of type (1) where, at sufficiently high energy, $n=14$. Further, f remains constant as $x_T \to \sin\theta$, the upper end of its kinematically allowed range. This implies that at any fixed energy our mechanism dominates over all previously proposed mechanisms for sufficiently large x_T. At currently accessible energies the model gives a smaller effective value of n at small x_T, and if x_T is small enough presumably other mechanisms become important. Thus we have an understanding of Fig. 1, with the prediction that n will remain equal to 14 when data are obtained at larger values of x_T.

CP perform their experiments on nuclear targets and present interesting results on the A dependence of their data. At small p_T the cross section depends on the nuclear mass number roughly as $A^{2/3}$. This is expected, since due to absorption in initial- and final-state interactions the surface nucleons play the principal role in the process. However, at large p_T it is known[4] that initial- and final-state interaction effects cancel in the single-particle cross section, so that one can understand[5] how one might obtain a dependence like A in this régime. In fact, for $p_T \gtrsim 4$ GeV/c, CP find that $A^{1.1}$ gives the best fit for pion production, and $A^{1.3}$ for proton production. No model has been previously proposed that seems capable of producing an effect as dramatic as $A^{1.3}$, which seems clearly to require some multiple-scattering effect. Our model is a multiple-scattering model which does not depend in any way on the short-distance structure of the target, so that it is the first possible candidate for explaining the effect. Nuclear effects are, of course, too complicated for us to be able to say more than this.

One of us[6] has previously proposed a multiple-scattering model for pp elastic scattering at wide angle. No trace of this has been found in 90° scattering up to values of s of about 40 GeV^2. It is often assumed that there is a close connection between inclusive and exclusive processes. There is no fundamental reason why this should be the case, but certainly there is a close connection between the multiple-scattering models for inclusive and exclusive processes. With some plausible assumptions, we exploit this connection to deduce from the inclusive large-p_T proton production data what is the expected magnitude of the multiple-scattering contribution to pp elastic 90° scattering. We find that it is too small to be seen in the existing data, but that it should begin to dominate at SPS (super-proton synchrotron) (Fermilab) energies (100–400 GeV^2).

II. MULTIPLE-SCATTERING MODEL

The proposed mechanism is shown in Fig. 2. Each proton emits three virtual quarks of finite "mass" and with momenta closely aligned with the momentum of their parent. Each quark from one of the protons scatters on a quark from the other proton in such a way that there are three closely aligned quarks in the final state capable of recombining to produce the observed large-p_T proton. The other three quarks which have undergone a wide-angle scatter into the final state do not have to have their momenta aligned as they are not required to recombine into a single baryon. Their combined invariant mass is, in general, large so that they are expected to materialize as a system of hadrons— usually one baryon and a number of mesons.

If the three central high-energy wide-angle quark-quark scatterings are scale-free one obtains an expression of the form (1) with $n=14$. Details of the calculation are given in the Appendix. The form (1) is an asymptotic form which can only be expected to hold at sufficiently large s and p_T. As we show in the Appendix each central scatter has a momentum transfer squared which on average is about $\frac{2}{9}p_T^2$. As p_T decreases the quark-quark scattering will eventually no longer take place in a régime where the scale-free wide-angle behavior holds but instead the Regge régime for the quark-quark scattering will be approached. Accordingly in this subasymptotic régime, one expects that the effective n in (1) will decrease, providing a picture which is qualitatively suggestive of Fig. 1. Note that this effect is of much greater significance in the multiple-scattering model under discussion than in models which involve only a single hard scattering, for which the average momentum transfer squared in the hard scattering will be much larger (about $2p_T^2$).

The assumption of scale-free quark scattering is essential to obtain $n=14$ in (1). However, this assumption also leads to (1) with $n=4$ for *all* large-p_T inclusive reactions via the single-hard-scattering mechanism of Berman, Bjorken, and Kogut,[7] which is not in accord with present data. In the BBK process single quarks from each proton scatter and the large-p_T hadron is a fragment of one of the scattered quarks. In a large-p_T experiment the triggering conditions require the quark to fragment in a rather special way, giving most of its momentum to the trigger particle. This might well suppress the BBK contribution.[8] It may be significant that the mechanism of Fig. 2, in contrast with BBK, produces the observed final-state baryon without the need of final-state interactions to carry quantum numbers between the large-p_T system and the residual fragments of the parent protons. Perhaps this permits Fig. 2 to give a cross section which is numerically more significant than the BBK process at present energies.

An alternative suggestion has been made[9] that quark-quark scattering is only *asymptotically* scale-free, in the sense that the virtual quark masses have also to be large before the scale-free behavior is obtained. This has been linked with a dynamical scheme which realizes dimensional counting.[10] If this is the case, BBK and Fig. 2 are both suppressed. Eventually experiment must decide between these possibilities: a decision which requires that one pursue the theoretical predictions of both points of view.

The assumption of scale-free behavior for quark scattering with finite virtual masses also has consequences for elastic proton-proton scattering at high energy and wide angle. The differential cross section is predicted[6] to have the asymptotic form

$$\frac{d\sigma}{dt} \sim s^{-m} F(\cos\theta) \qquad (2)$$

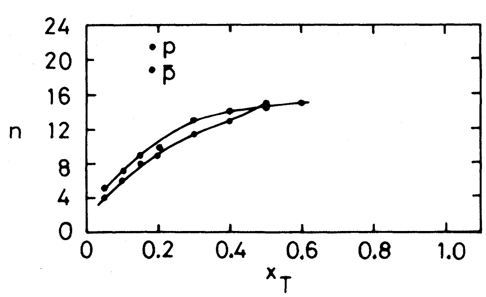

FIG. 1. The local value of n at different values of x_T for protons and antiprotons.

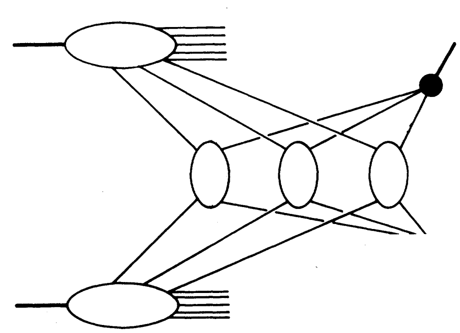

FIG. 2. The multiple-scattering mechanism for the production of large-p_T protons.

with $m = 8$ for the process of Fig. 3. An analysis[11] of large-t pp scattering at values of s up to about 40 GeV^2 shows that the data are in good agreement with the form (2) but with the dimensional counting[10] value of $m = 10$. There is, however, a suggestion[12] that the data at CERN ISR energies (1000–3500 GeV^2) support a value of $m = 9$ or less.

The notion of correspondence[13] enables a connection to be made between inelastic and elastic processes, since it assumes that a suitable average of the inelastic cross section at the edge of phase space will reproduce the elastic cross section. The requirement that this hold for the relation of Fig. 2 to Fig. 3 requires that $f(x_T, \theta)$ in (1) has the property that

$$f(x_T, \theta) \text{ remains finite as } x_T \to \sin\theta. \qquad (3)$$

In the Appendix we show that (3) is indeed true for Fig. 2. As $x_T \to \sin\theta$ the missing mass M becomes smaller so that the number of hadronic fragments emerging from the upper and lower bubbles of Fig. 2 is reduced, eventually becoming zero. That is, as $x_T \to \sin\theta$ Fig. 2 reduces to the case where the incident protons couple to just three quarks, just as in Fig. 3.

The result (3) has the important implication that Fig. 2 will dominate over all other known mechanisms at the edge of phase space, that is for x_T sufficiently large for any fixed s. Single-hard-scattering models give f's which vanish at least like $(\sin\theta - x_T)^3$ in this region.

Since the properties of Figs. 2 and 3 are consistent with the idea of correspondence, one can attempt to use the principle, together with data on inclusive cross sections, to estimate the size of the contribution of Fig. 3. Such an estimation is fraught with uncertainty, partly because exactly how to make the average in applying correspondence is not clearly defined, and partly because the extrapolation of the inclusive 90° data from $x_T \lesssim 0.6$ to $x_T \sim 1$ is not free from ambiguity. The best estimates we have been able to make suggest that the expected elastic contribution from Fig. 3 is smaller than the observed 90° cross section at $s \sim 40$ GeV^2 but that it would be dominant at SPS (Fermilab) energies. Clearly it will be of great interest to have data on wide-angle elastic scattering at these higher energies. In the meantime we are encouraged to think that the use of multiple-scattering mechanism to explain large-p_T baryon production is not in contradiction with the apparent absence of this multiple-scattering mechanism in available elastic cross sections.

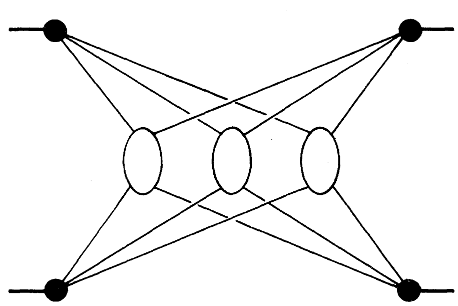

FIG. 3. The multiple-scattering mechanism for elastic pp scattering.

III. FURTHER PROPERTIES

In this section we discuss some further points of qualitative understanding which follow from the mechanism of Fig. 2.

A. Correlations

The fact that the quark constituents of each proton move approximately parallel to their parent hadron implies that the process of Fig. 2 is almost coplanar in the plane defined by the beams and the trigger, just as in hard-scattering models. However, each quark can have a small ($\lesssim \frac{1}{2}$ GeV/c, perhaps) component of momentum transverse to that of its parent. The cumulative effect of these is to allow a significant momentum $p\!\uparrow$ out of the beam-trigger plane for the three quarks whose fragments form the "away" system. Thus the process is in fact only approximately coplanar. Because of the larger number of participating quarks this effect should be more significant for the multiple-scattering mechanism of Fig. 2 than for single-hard-scattering models. Thus we expect the average $p\!\uparrow$ to be greater for proton triggers than for meson triggers.

The fragments of the three "away" side quarks balance the p_T of the trigger. We expect that the "away" side multiplicity will be larger for Fig. 2 than for other mechanisms, in particular larger than for the single-hard-scattering mechanisms thought to be responsible for the bulk of meson production. Two alternative lines of argument lead to this conclusion. If the three "away" quarks fragment essentially independently of each other, the very fact that there are three of them will tend to increase the associated multiplicity. If they fragment collaboratively by substantial interactions among themselves an enhanced multiplicity is again expected because the invariant mass of the "away" system is large (in contrast with single-hard-scattering systems where the "away" side system has finite invariant mass).

Because the longitudinal momenta of the initiating quarks are all different the "away" side particles are to be expected to have a fanlike distribution in longitudinal rapidity, *event by event*. The opening angle of the fan will depend on x_T and θ but not s. In single-hard-scattering models such a fanlike distribution is only expected in inclusive distributions formed by summing over events. In the other popular suggested mechanism for producing large-p_T protons—the leading-particle mechanism[1]—the fanlike distribution is wholly absent. The balancing-p_T-"away" side particles form a jet which, moreover, is kinematically constrained to emerge at small angle to the beams.

Finally, we may observe that though we have supposed in Fig. 2 that the three quarks on the "toward" side combine to form a single proton, there will also be processes in which they form additional particles, principally pions, for example, by the formation of resonances which decay into baryons and mesons.

B. Nuclear-target effects

We expect that predominantly the three quarks from a nuclear target will originate from the same nucleon. Because of the cancellation of initial- and final-state interactions[4] this could give a single-particle cross section proportional[5] to A. However there will be some small probability that the quarks originate from different nucleons, leading to terms proportional to A^2 and A^3.[14] We are unable to estimate this quantitatively, but since we are taking a picture in which multiple scattering is particularly important in baryon production, it is to be expected that these higher A-dependent effects will manifest themselves most clearly in proton cross sections, as is indeed the case experimentally.[2,3]

Moreover, the A dependence from a multiple-scattering mechanism may be expected to be more marked than from most other mechanisms. The mechanism does not depend in any way on the short-distance structure of the target, so that constituents of widely separated parts of the nucleus can freely participate in the reaction. Also, the hard scatterings of the constituents are not closely correlated in time, which is why the constituents all stay close to the mass shell. In contrast, if the same constituent were required to undergo more than one hard scattering, these scatterings would have to occur in rapid succession and so the constituent would go far off shell between the scatterings, giving a p_T dependence different from that for just one scattering and in all probability making the effect rather small. In the model that we have described, the p_T dependence is the same for all the terms:

$$E\frac{d^3\sigma}{d^3p}\sim p_T^{-14}[Af_1(x_T,\theta)+A^2f_2(x_T,\theta)+A^3f_3(x_T,\theta)],$$

with $f_1 \gg f_2 \gg f_3$. The observed A dependence for proton production only requires that f_2 be of the order of 1% of f_1.

C. Antiproton production

The picture[15] of the structure of the proton which is deduced from deep-inelastic electron and neutrino scattering is that it consists of three valence quarks with a sea of quark-antiquark pairs. The valence quarks predominate at large fractional longitudinal momentum x, but the sea becomes significant at small x.

For very large p_T in Fig. 2 one needs partons of large x, which are overwhelmingly likely to be valence quarks. Thus at large p_T one computes a large $p/\bar{p}$ ratio. At smaller p_T, however, some partons can have small x so that the participation of antiquarks becomes possible, and the $p/\bar{p}$ ratio is expected to decrease. We are unable to make a quantitative estimate of the effect. However, it seems certain that one could not explain the $p/\bar{p}$ ratio of ~1 found by BS at $p_T \sim 1.7$ GeV/c. We believe that this value of p_T is sufficiently small for there to be significant contamination of the cross section by small-p_T mechanisms. Support for this comes from the fact[16] that p and $\bar{p}$ production fall off at small p_T appreciably slower than the e^{-6p_T} found for pions. It is probable that one has to get to at least $p_T \sim 3$ GeV/c before one is in the large-p_T régime for baryons.

D. Other processes

Although the multiple-scattering mechanism is expected to be particularly significant in the production of baryons in pp collisions it will also, of course, operate in other processes. In contrast to most single-hard-scattering mechanisms, multiple scattering gives a value of n in (1) depending only on the large-p_T trigger particle and not on the incident beams. However, the latter do determine the behavior as $x_T \to \sin\theta$.

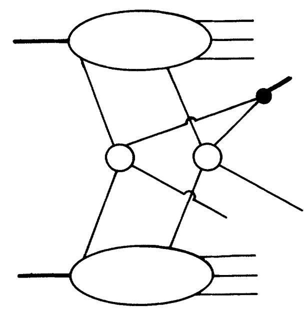

FIG. 4. The multiple-scattering mechanism for the production of large-p_T mesons.

The processes

$$N+N\to\begin{pmatrix}M\\ \gamma\end{pmatrix}+X, \tag{4}$$

for meson or photon (lepton pair) production can proceed via the double-scattering mechanism of Fig. 4. This leads to (1) with $n=9$. Because at least one nonvalence antiquark is involved the magnitude of the coefficient function f is expected to be small. Note that because multiple-scattering mechanisms do not probe hadronic wave functions at short distances these mechanisms give similar p_T dependences for meson and lepton-pair cross sections, in contrast to single-hard-scattering models.

If Fig. 2 is applied to the processes

$$\begin{pmatrix}\pi\\ \gamma\end{pmatrix}+N\to N+X, \tag{5}$$

at least one of the scattering partons must be nonvalence. Thus, although the cross section is again given by (1) with $n=14$, the numerical value of f is expected to be small and it will vanish as $x_T\to\sin\theta$. However, there is another possibility of the form of Fig. 4 in which the incident nucleon emits two constituents which are a quark and a diquark "core," respectively. Using dimensional counting rules for q-qq scattering one obtains (1) with $n=13$, and f in this case blows up like $(x_T-\sin\theta)^{-1/2}$ at the edge of phase space, as we explain in the Appendix. A related mechanism which also gives $n=13$ is shown in Fig. 5, where all the internal lines represent quarks and there is an intermediate propagating quark which is far off shell.

Such double-scattering mechanisms are, of course, also possible in NN scattering. However, they are expected to be less significant than the triple scattering mechanism discussed above. If both nucleons emit a q and a qq constituent one obtains (1) with $n=17$. If one nucleon emits a q and qq, and the other two q's, one finds that

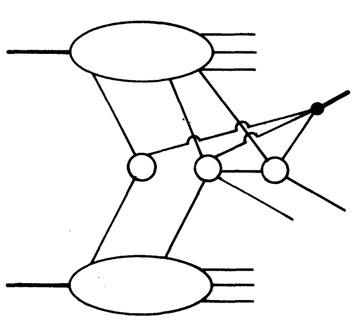

FIG. 5. A multiple-scattering mechanism involving a propagating off-shell quark.

$n=13$ but f then vanishes as $x_T\to\sin\theta$.

In the processes

$$\begin{pmatrix}\pi\\ \gamma\end{pmatrix}+N\to\begin{pmatrix}\pi\\ \gamma\end{pmatrix}+X, \tag{6}$$

all constituents can be valence in the mechanism of Fig. 4. One finds (1) with $n=9$ at large p_T (but effectively smaller at smaller p_T as before). One finds that f vanishes slowly as $x_T\to\sin\theta$, since the two quarks from the nucleon are in the limit forced to carry off all the longitudinal momentum of their parent. Of course, other mechanisms are expected to be important for pion[17] and photon[18] beams.

Finally one may note for completeness that in the unobservable process

$$\pi+\pi\to\pi+X \tag{7}$$

one again has $n=9$ for large p_T and f blows up as $x_T\to\sin\theta$.

IV. CONCLUSION

In this paper we have explored multiple-scattering mechanisms for the inclusive production of particles at large p_T. These mechanisms depend for their consequences on the short-distance nature of the fundamental quark hard scattering but not on the short-distance behavior of the wave functions of the participating hadrons. A consequence of this is that the value of n appearing in equation (1) depends only on the detected final-state particle and not on the initiating reaction. However, the behavior as $x_T\to\sin\theta$ at the edge of phase space does depend on the incident particles.

The work of this paper has assumed that quark-quark scattering is scale-free at finite virtual quark masses. We estimate that this is consistent both with the mechanism being the principal one for the production of large-p_T protons and also with the absence of multiple-scattering effects in elastic p-p $90°$ scattering at values of s up to 40 GeV^2. However, if the picture described here is correct, one must expect to see the dominance of multiple-scattering effects, giving $m=8$ in (2), at SPS (Fermilab) energies. It will clearly be of great interest to see if this proves to be the case.

The multiple-scattering mechanism seems likely to be of the greatest significance in the production of large-p_T baryons in proton-proton collisions. We believe that it is the most promising candidate for explaining the data. Important reasons for this view are

(a) that it gives (1) with $n=14$ (and an effectively lower n at lower x_T);

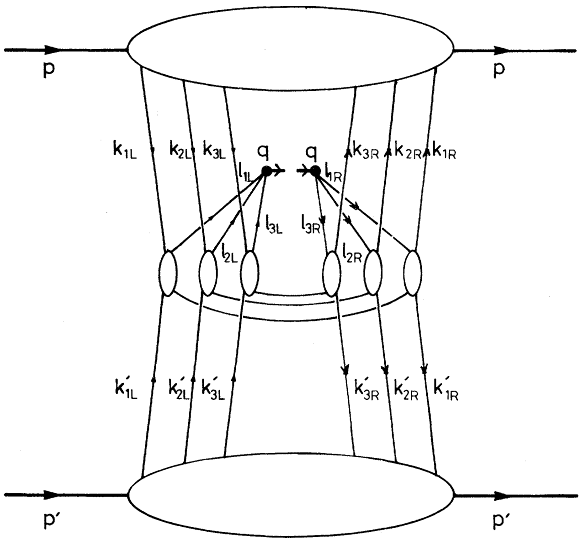

FIG. 6. The Mueller-Regge diagram for the process of Fig. 2.

(b) that the mechanism is the only known one which does not vanish towards the edge of phase space (where $x_T \to \sin\theta$) so that it must dominate at sufficiently large x_T for any fixed s;

(c) that it provides a natural explanation of the significant A-dependence effects observed by CP.

An interesting prediction of the mechanism is that one expects to see a fanlike structure for the "away" hadrons *event by event* and that there will be a large "away" multiplicity with the large p_T shared among a number of hadrons.

Another interesting suggestion arising from the mechanism is that there should be some production of large-p_T strange baryons. This requires one of the participating quarks to be a λ from the sea, but one does not have to pay too high a kinematic penalty for this single necessarily small-x parton. Our knowledge of distributions in the sea is not sufficient for us to make a quantitative estimate but certainly one would expect such production to be more copious than $\bar{p}$ production at large p_T.

ACKNOWLEDGMENT

We are grateful to Dr. S. D. Ellis for his vigorous and constructive criticism.

APPENDIX. CALCULATION OF MULTIPLE SCATTERING

For simplicity, we calculate with the spins of all the particles put equal to zero. Provided the quark scatterings are kept scale-free, spin is an inessential complication. For the contribution from Fig. 2, we need the Mueller diagram of Fig. 6, with a discontinuity taken in the missing mass, that is down the middle of the diagram.

In the center-of-mass frame we have, to leading order,

$$\begin{aligned} p &= \tfrac{1}{2}\sqrt{s}\,(1,0,0,1),\\ p' &= \tfrac{1}{2}\sqrt{s}\,(1,0,0,-1),\\ q &= (q_T/\sin\theta)R(\theta)\{(1,0,0,1)\}, \end{aligned} \tag{A1}$$

where $R(\theta)$ is a rotation through angle θ about the y axis:

$$R(\theta)\{(v_0, v_1, v_2, v_3)\} = (v_0,\, v_1\cos\theta + v_3\sin\theta,\, v_2,\, -v_1\sin\theta + v_3\cos\theta).$$

Our basic assumption is that, before they scatter, the quarks have small transverse momentum and are not too far off shell. Hence we parametrize their momenta as

$$\begin{aligned} k_{iL} &= (\tfrac{1}{2}\sqrt{s}\,x_{iL} + y_{iL}/\sqrt{s},\, \kappa_{1iL},\, \kappa_{2iL},\, \tfrac{1}{2}\sqrt{s}\,x_{iL}),\\ k'_{iL} &= (\tfrac{1}{2}\sqrt{s}\,x'_{iL} + y'_{iL}/\sqrt{s},\, \kappa'_{1iL},\, \kappa'_{2iL},\, -\tfrac{1}{2}\sqrt{s}\,x'_{iL}),\\ i &= 1,2,3, \end{aligned} \tag{A2}$$

with similar parametrizations for the right-hand variables. Then as $s\to\infty$ the variables $x_{iL}, x'_{iL}, y_{iL}, y'_{iL}$ and their right-hand counterparts remain bounded, as also do the κ, κ' variables. We may use these variables as integration variables:

$$\int d^4k_{iL} = \frac{1}{2}\int dx_{iL}\,dy_{iL}\,d^2\vec{\kappa}_{iL} \quad \text{etc.}$$

Similarly, we assume that the three quarks l_{iL} can only coalesce to form the baryon q if their momentum components transverse to $\vec{q}$ are small and if they are not too far off shell (otherwise, the wave function of the baryon is supposed very small). Hence we parametrize

$$l_{iL} = R(\theta)\left\{\left(\frac{q_T}{\sin\theta}X_{iL} + \frac{\sin\theta}{q_T}Y_{iL},\, \lambda_{1iL},\, \lambda_{2iL},\, \frac{q_T}{\sin\theta}X_{iL}\right)\right\},\quad i=1,2,3, \tag{A3}$$

with again similar parametrizations for the l_{iR}. Then also X_{iL}, Y_{iL}, and $\lambda_{i\lambda}$ remain bounded as $s\to\infty$ and

$$\int d^4 l_{iL} = \frac{1}{2}\int dX_{iL}\,dY_{iL}\,d^2\vec{\lambda}_{iL}.$$

Notice that, now that k^2 etc. are constrained to be finite, their actual values depend on next-to-leading order terms not explicitly exhibited in (A1).

The various momenta k, k', and l are not all independent. If we choose to integrate over all the variables, we must include the δ functions

$$\frac{64}{q_T{}^2 s^2}\prod_{i=1}^{3}[\,\delta(x_{iL}-x_{iR})\delta(x'_{iL}-x'_{iR})\delta(X_{iL}-X_{iR})\delta(\kappa_{2iL}+\kappa'_{2iL}-\lambda_{2iL}-\kappa_{2iR}-\kappa'_{2iR}+\lambda_{2iR})]$$

$$\times\delta\Big(\sum_{i=1}^{3}X_{iL}-1\Big)\delta\Big(\sum_{i=1}^{3}y_{iL}-\sum_{i=1}^{3}y_{iR}\Big)\delta\Big(\sum_{i=1}^{3}y'_{iL}-\sum_{i=1}^{3}y'_{iR}\Big)\prod_{\alpha=L,R}\delta\Big(\sum_{i=1}^{3}Y_{i\alpha}\Big)\delta^{(2)}\Big(\sum_{i=1}^{3}\vec{\kappa}_{iL}-\sum_{i=1}^{3}\vec{\kappa}_{iR}\Big)$$

$$\times\delta^{(2)}\Big(\sum_{i=1}^{3}\vec{\kappa}'_{iL}-\sum_{i=1}^{3}\vec{\kappa}'_{iR}\Big)\delta^{(2)}\Big(\sum_{i=1}^{3}\vec{\lambda}_{iL}\Big)\delta\Big(\sum_{i=1}^{3}\lambda_{1iR}\Big). \tag{A4}$$

Notice that these δ functions constrain the components of the two-dimensional momenta $\vec{\kappa}$, $\vec{\kappa}'$, and $\vec{\lambda}$ perpendicular to the beam-trigger plane in a different way from the component in the beam-trigger plane.

The squared momenta on the three quark lines that join the left- and right-hand sides of the diagrams are

$$\sigma_i \sim s\left[x_{iL}x'_{iL}-\frac{q_T}{\sqrt{s}}X_{iL}(x_{iL}\cot\tfrac{1}{2}\theta+x'_{iL}\tan\tfrac{1}{2}\theta)\right]+\cdots. \tag{A5}$$

Taking the required discontinuity of the diagram introduces the factor

$$\rho(\sigma_1)\rho(\sigma_2)\rho(\sigma_3),$$

where $\rho(\sigma)$ is the spectral function of the quark propagator, which, as is usual in parton model calculations, we take to satisfy

$$\int d\sigma\,\rho(\sigma)=1. \tag{A6}$$

It is convenient to introduce three new variables ξ_i in place of the X_{iL}:

$$X_{iL}=X^0_{iL}+\cdots+\frac{\xi_i}{q_T\sqrt{s}\,(x_{iL}\cot\frac{1}{2}\theta+x'_{iL}\tan\frac{1}{2}\theta)},$$
$$X^0_{iL}=\frac{\sqrt{s}\,x_{iL}x'_{iL}}{q_T(x_{iL}\cot\frac{1}{2}\theta+x'_{iL}\tan\frac{1}{2}\theta)}, \tag{A7}$$

where the omitted terms are of order $s^{-1/2}$ and q_T/s, and are such that when ξ_i are bounded the terms of order $s^{1/2}$ and q_T in σ_i are canceled:

$$\sigma_i=\xi_i+\text{finite}.$$

Because of the assumption that the center wide-angle scatterings are scale-free, the variables ξ_i appear to leading order in $\rho(\sigma_i)$ only, so that because of (A6) they can be trivially integrated, leaving

$$\delta\Big(\sum_{i=1}^{3}X_{iL}-1\Big)\sim\delta\Big(\sum_{i=1}^{3}X^0_{iL}-1\Big). \tag{A8}$$

We have now done enough to count the over-all power of q_T and s: $q_T{}^{-2}s^{-2}$ from (A4) and $q_T{}^{-3}s^{-3/2}$ from the three changes of variable in (A7). With a flux factor s^{-1}, the result is a form as in (1), with $n=14$.

The structure of the coefficient function $f(x_T,\theta)$ is complicated, mainly because of the last δ function in the square bracket in (A4). This couples the component of momentum transverse to both p, p', and q in the various parts of the diagram. If we assume that the quark wave function is such that this component is constrained to be very small, we can imagine that all the variables except the x, x', and X have been integrated, and write the result as

$$E\frac{d^3\sigma}{d^3p}\sim\frac{1}{q_T{}^2s^6}\int dx_1dx_2dx_3dx'_1dx'_2dx'_3dX_1dX_2dX_3\,T(x_1,x_2,x_3)\,T'(x'_1,x'_2,x'_3)G(X_1,X_2,X_3)$$
$$\times\delta\Big(\sum_{i=1}^{3}X_i-1\Big)\prod_{i=1}^{3}\delta\Big(\frac{x_ix'_i}{x_i\cot\frac{1}{2}\theta+x'_i\tan\frac{1}{2}\theta}-\frac{q_T}{\sqrt{s}}X_i\Big)\prod_{i=1}^{3}[A(x_i,x'_i,X_i)]^2. \tag{A9}$$

Here the amplitude T is associated with the upper bubble and is interpreted as the probability of finding three constituents, with fractional momenta x_1, x_2, and x_3 simultaneously within the nucleon. Similarly, T' is associated with the lower bubble. The function G is the square of the wave function for the nucleon being in its pure three-quark configuration (with no sea), the quarks having fractional momenta X_1, X_2, and X_3. The amplitudes A correspond to the central wide-angle scatterings. We repeat that, in writing (A9), we have made an approximation on the transverse momentum distribution within each nucleon.

In (A9) we have taken the central scatterings to be scale-free. The energy and momentum-transfer variables for these scatterings are

$$s_i\sim x_ix'_i\,s,$$
$$t_i\sim -x_iX_i\,q_T\sqrt{s}\cot\tfrac{1}{2}\theta, \tag{A10}$$
$$u_i\sim -x'_iX_i\,q_T\sqrt{s}\tan\tfrac{1}{2}\theta,$$

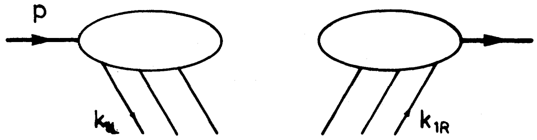

FIG. 7. Disconnected contributions to the top bubble of Fig. 6.

and only if all these are large is the scale-free assumption plausible.

To investigate this further, consider the case $\theta = 90°$. It is reasonable to expect that G is peaked about $X_1 = X_2 = X_3 = \frac{1}{3}$. The momentum transfers t_i and u_i are simultaneously greatest when $x_i = x_i' = 2q_T/3\sqrt{s}$, in which case $s_i \sim \frac{4}{9}q_T^2$ and $t_i \sim u_i \sim \frac{2}{9}q_T^2$. If the δ functions are solved with x_i different from x_i', s_i increases but either t_i or u_i decreases; that is, one approaches closer to the Regge region. So one is certainly not in the scale-free régime unless q_T is so large that when t_i or $u_i \sim \frac{2}{9}q_T^2$ one is well outside the Regge region. In the Regge region, A behaves as s^{α_i}, where α_i varies between 1 at zero momentum transfer, corresponding to Pomeron exchange, and 0 at large momentum transfer, where the Regge region merges into the scale-free region. Thus at moderate values of q_T the apparent value of n is locally smaller than the value obtained by assuming a scale-free A.

Finally we sketch the results of calculating the behavior of f as $x_T \to \sin\theta$. All subprocesses are forced to the edge of their phase space in this limit. The quark scatterings are already "at the edge" since they are elastic. Thus the effect is that the top and bottom bubbles of Fig. 6 are evaluated in the limit in which the emitted partons take all the available momentum. In the analogous situation in single-hard-scattering calculations this forces the relevant amplitudes to vanish, but in Fig. 6 there are disconnected contributions to the bubbles, of the form of Fig. 7, which remain nonzero in this limit since they are again already "at the edge." The invariant mass of the three "away" quarks also become finite and they must be capable of aligning to produce a single proton. Investigation shows that the conditions enforcing this are a smooth limit of the conditions enforcing finite quark masses away from the region $x_T \to \sin\theta$, so that this requirement produces no vanishing factors.

The contribution to $Ed^3\sigma/d^3p$ from the disconnected diagram is of the form of (A9), with extra δ functions

$$\delta\left(\sum_{i=1}^{3} x_i - 1\right)\delta\left(\sum_{i=1}^{3} x_i' - 1\right)$$

because in this diagram, the two initial protons each emit three (valence) quarks carrying all the parent's momentum. The approximations made in obtaining (A9) do not affect the following argument.

Define

$$\epsilon = \frac{M^2}{s} \simeq 1 - \frac{x_T}{\sin\theta}$$

When ϵ becomes small, the changes of variable which force the invariant mass of the three "away" quarks to become finite are

$$x_i' = x_i + \epsilon^{1/2}\overline{x}_i',$$

$$X_i = x_i + \epsilon^{1/2}\frac{1 - \frac{1}{2}x_T \tan\frac{1}{2}\theta}{x_T/\sin\theta}\overline{x}_i' + \epsilon\overline{X}_i.$$

It now follows immediately that the leading dependence on ϵ as $\epsilon \to 0$ is ϵ^0.

In the case of mechanisms involving only two partons in the "away" set, the corresponding conditions are actually more readily enforced at $x_T \sim \sin\theta$ than in the interior and it is this fact that produces singular behavior $(\sin\theta - x_T)^{-1/2}$, referred to in Sec. III. A similar conclusion can be reached by applying the correspondence principle.

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[14]*Note added in proof.* Professor A. Mueller (private communication) has suggested to us, on the basis of a configuration-space argument, that these terms are in fact proportional to $A^{4/3}$ and $A^{5/3}$. This would give a better phenomenological fit to the data; see J. H. Kuhn, SLAC Report No. SLAC-PUB-1633 (unpublished).
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