Nonleptonic weak decay and the Melosh transformation*

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The PCAC (partially conserved axial-vector current) hypothesis and the Melosh transformation are used with the soft-pion and infinite-momentum techniques to obtain expressions for the nonleptonic weak decay of amplitudes of the $SU(6)_W$ $\underline{56}$, $L=0$ baryons. The following results are obtained: (1) Modified Lee-Sugawara relations are exact for both the parity-violating and parity-conserving amplitudes. (2) Two additional sum rules involving Ω^- decay amplitudes are derived. (3) If the parity-violating part of the weak decay Hamiltonian transforms only as an $SU(6)_W$ $\underline{35}$, then $A(\Sigma_+^+)=0$ automatically. (4) Terms arising from the Melosh transformation are necessary to bring the decay width for $\Omega^-\to\Xi\pi$ into agreement with experiment. (5) All known amplitudes are fitted to within 10%.

I. INTRODUCTION

SU(6) symmetry first gained importance in particle physics in the 1960's as a classification scheme for hadrons[1] in which the SU(3) quarks q and antiquarks $\bar{q}$ carry spin $\frac{1}{2}$ and thus form the 6 and $\bar{6}$ representations of SU(6). The mesons are considered to be $q\bar{q}$ pairs and the baryons qqq states, which can be classified by the corresponding product representations of SU(6) and the relative orbital angular momentum L of the quark system. The possible SU(6) representations for the mesons and baryons are

$$\underline{6}\times\underline{\bar{6}}=\underline{1}+\underline{35}$$

and

$$\underline{6}\times\underline{6}\times\underline{6}=\underline{20}+\underline{56}+\underline{70}+\underline{70}',$$

respectively. It is found that the known mesons and baryons and their resonances do obey this scheme. The mesons fall into the $\underline{1}$ and $\underline{35}$ SU(6) multiplets with orbital angular momenta $L=0$, 1, 2, and possibly 3. The known baryons and baryon resonances fall into $\underline{56}$, $L=$ even and $\underline{70}$, $L=$ odd multiplets. The totally antisymmetric $\underline{20}$ representation is not seen.

In order to make SU(6) relativistically invariant along a single direction and thus suitable for the discussion of collinear decay amplitudes, ordinary spin was replaced by a relativistic generalization of spin, W spin.[2] W spin is defined by the generators

$$W_x=P_{\rm int}S_x\,,$$
$$W_y=P_{\rm int}S_y\,,$$
$$W_z=S_z\,,$$

where S is the ordinary spin and $P_{\rm int}$ is the intrinsic parity of the system. For a single quark these can be written in terms of Dirac matrices as

$$W_x=\tfrac{1}{2}\beta\sigma_x\,,$$
$$W_y=\tfrac{1}{2}\beta\sigma_y\,,$$
$$W_z=\tfrac{1}{2}\sigma_z\,.$$

The W-spin generators are invariant under boosts in the z direction and the W-spin charges have finite matrix elements among states in the limit $p_z\to\infty$. An $SU(6)_W$ algebra of currents is then constructed to apply to current-induced transitions among the hadrons. If the hadrons belong to simple representations of this $SU(6)_W$ algebra of currents, however, the theoretical predictions are in violent disagreement with the data. The axial-vector coupling constant g_A defined by the matrix element

$$\left\langle n\left|\frac{Q_5^{1-i2}}{\sqrt{2}}\right|p\right\rangle=-g_A\left(\frac{mm'}{EE'}\right)^{1/2}\bar{u}(p')\gamma_5u(p)$$

was predicted to be $\frac{5}{3}$ by this $SU(6)_W$ argument rather than the known experimental value 1.25. Although $SU(6)_W$ did strikingly predict that the ratio of magnetic moments of the proton and the neutron was[3]

$$\frac{\mu_T(\text{proton})}{\mu_T(\text{neutron})}=\frac{-3}{2}\,, \tag{1}$$

the theory also predicted that the anomalous magnetic moment of each individual nucleon must be zero. It also forbade an electromagnetic transition from the nucleon to the 3-3 resonance and decays such as $\omega\to\gamma\pi$. From these failures it was concluded that the hadron states could not be simple representations of the $SU(6)_W$ of currents, and elaborate, essentially *ad hoc* mixing schemes arose in an attempt to resolve these difficulties.

Recently, the representation structure of the hadron states has been clarified by Melosh.[4] Following the ideas of Gell-Mann, the currents are postulated to belong to irreducible representations of an $SU(6)_W$ of currents, while the particle states are classified by a different, constituent $SU(6)_W$. These two different $SU(6)_W$ symmetries are related by a unitary transformation, the Melosh transformation. Melosh explicitly constructed this transformation for the free quark model. The algebraic properties of currents transformed by the Melosh transformation have been extracted from this model and applied to physically relevant matrix elements through the use of the Wigner-Eckart theorem. The matrix element of an operator between two hadron states is the product of appropriate $SU(6)_W$ and angular momentum Clebsch-Gordan coefficients times a reduced matrix element. This reduced matrix element is left as a free parameter in the present applications but would, in principle, be determined from the true dynamical theory. This method removed the inconsistencies which had appeared in the old $SU(6)_W$ calculations. The axial-vector coupling constant g_A now has the form $g_A = \frac{5}{3}\eta$, where η is the value of an undetermined reduced matrix element. Thus g_A is no longer in contradiction with experiment. The famous ratio of magnetic moments of the nucleons remains unchanged, but the anomalous magnetic moment of each nucleon is again proportional to a reduced matrix element and thus need not be zero. Gilman, Kugler, Meshkov, and others used partially conserved axial-vector current (PCAC) in addition to the algebraic properties of the Melosh-transformed axial-vector current to satisfactorily predict pionic emission amplitudes for the decays of mesons and baryons.[5] Gilman, Karliner, and others also found that the application of the Melosh-transformation technique to real photon emissions from baryons and mesons is consistent with experiment.[6]

In the present paper the Melosh-transformation technique and PCAC are used to calculate the parity-violating (pv) and parity-conserving (pc) nonleptonic weak decay amplitudes. In Sec. II the general method of calculating nonleptonic weak decay amplitudes is discussed. The results for these amplitudes, two new sum rules, and predictions for the processes $\Omega^- \to \Xi^- + \pi^0$ and $\Omega^- \to \Xi^{*-} + \pi^0$ are derived in Sec. III. In the conclusion the basic assumptions and results of this work are again summarized.

II. GENERAL CONSIDERATIONS

Consider the nonleptonic weak decay

$$Y \to B + \pi^0$$

of a baryon Y to a baryon B and a pion. The standard Lehmann-Symanzik-Zimmermann (LSZ) reduction and application of PCAC[7] to this problem gives the following expression:

$$\sqrt{2q_0}\,\langle B(p), \pi^0(0)|H_{w(0)}^{\text{pv (pc)}}|Y(p)\rangle$$
$$= \frac{-i(m_\pi^2 - q_0^2)}{f_\pi m_\pi^2}\Bigg\{\langle B|[Q_5^3, H_w^{\text{pv (pc)}}]|Y\rangle - q_0 \sum_l \delta^3(\vec{p} - \vec{p}_l)\left[\frac{\langle B|A_0^3(0)|l\rangle\langle l|H_w^{\text{pv (pc)}}|Y\rangle}{q_0 + p_0^B - p_0^l + i\epsilon} - \frac{\langle B|H_w^{\text{pv (pc)}}|l\rangle\langle l|A_0^3(0)|Y\rangle}{q_0 + p_0^l - p_0^Y - i\epsilon}\right]\Bigg\} \quad (2.1)$$

for the decay matrix elements in the pion rest frame, with p the baryon momentum, q the pion momentum, m_π the pion mass, f_π the π-$\mu\nu$ decay constant, Q_5^3 the isospin third component of the axial charge, A_0^3 the corresponding zeroth component of the axial-vector current, $H_w^{\text{pv (pc)}}$ the parity-violating and parity-conserving hadronic parts, respectively, of the weak Hamiltonian, and $\sum_l$ a sum over all possible intermediate states. The left-hand side of Eq. (2.1) is expressed in terms of Lorentz-invariant amplitudes for each process. For the baryons belonging to the spin-$\frac{1}{2}$ SU(3) octet part of the 56, $L=0$ $SU(6)_W$ representation, i.e., N, Σ, Λ Ξ, the parity-violating s-wave amplitude A and parity-conserving p-wave amplitude B are defined by

$$\sqrt{2q_0}\,\langle B\pi^0|H_w|Y\rangle \equiv i\left(\frac{mm'}{p_0 p_0'}\right)^{1/2} \bar{u}(p')(A + B\gamma_5)u(p), \quad (2.2a)$$

where $u(p)$ and $\bar{u}(p')$ are plane-wave solutions of the Dirac equation for the initial and final baryon states, respectively. For the process $\Omega^- \to \Xi^- + \pi^0$ the parity-violating d-wave amplitude D and parity-conserving p-wave amplitude B are defined by

$$\sqrt{2q_0}\,\langle \Xi^-\pi^0|H_w|\Omega^-\rangle \equiv \left(\frac{mm'}{p_0 p_0'}\right)^{1/2} \frac{iq_\lambda}{m} \times \bar{u}(p')(B + D\gamma_5)u_\lambda(p), \quad (2.2b)$$

and

$$\sqrt{2q_0}\langle \Xi^{*-}\pi^0|H_w|\Omega^-\rangle \equiv i\left(\frac{mm'}{p_0p_0'}\right)^{1/2}\bar{u}_\lambda(p')\times\left[(A+B\gamma_5)\delta_{\lambda\nu}+\frac{q_\lambda q_\nu}{mm'}(D+F\gamma_5)\right]u_\nu(p) \tag{2.2c}$$

defines the parity-violating s- and d-wave amplitudes A and D, and the parity-conserving p- and f-wave amplitudes B and F for the process $\Omega^-\to\Xi^{*-}+\pi^0$. The wave functions u_λ and u_ν in Eqs. (2.2b) and (2.2c) are plane-wave solutions of the Rarita-Schwinger equation[8] with $\bar{u}_\lambda\equiv u_\lambda^\dagger\gamma_4$ for $\lambda=1,2,3$ and $\bar{u}_4\equiv -u_4^\dagger\gamma_4$ to ensure Lorentz invariance. For a current-current interaction the commutators of the weak Hamiltonian satisfy the relation

$$[Q_5^3, H_w^{\text{pv (pc)}}]=[Q^3, H_w^{\text{pc (pv)}}], \tag{2.3}$$

where Q^3 is the generator of the third component of isospin. This allows the first term on the right-hand side of Eq. (2.1) to be simplified to

$$\langle B|[Q_5^3,H_w^{\text{pv (pc)}}]\,|Y\rangle=\langle B|[Q^3,H_w^{\text{pc (pv)}}]\,|Y\rangle = (I_3^B-I_3^Y)\langle B|H_w^{\text{pc (pv)}}|Y\rangle \tag{2.4}$$

with I_3^B, and I_3^Y the eigenvalues of Q^3 for the baryon states B and Y, respectively. In order to simplify the momentum dependence of expression (2.1) the invariant amplitudes defined in Eqs. (2.2a)–(2.2c) are evaluated in the infinite-momentum limit for the baryons. This limit is taken simultaneously with the soft-pion limit required by PCAC. Thus Eq. (2.1) is evaluated in the limit $p\to\infty$; $q_0\to 0$ such that

$$\lim_{p\to\infty;\,q_0\to 0} p_0q_0=\tfrac{1}{2}(m_Y^2-m_B^2). \tag{2.5}$$

Using Eq. (2.4) and taking the above limit, Eq. (2.1) becomes[9]

$$\lim\sqrt{2q_0}\langle B(p),\pi^0(0)|H_w^{\text{pv (pc)}}(0)|Y(p)\rangle=\frac{i}{f_\pi}\left\{(I_3^Y-I_3^B)\langle B|H_w^{\text{pc (pv)}}|Y\rangle + (m_Y^2-m_B^2)\sum_l\left[\frac{\langle B|A_0^3|l\rangle\langle l|H_w^{\text{pv (pc)}}|Y\rangle}{m_Y^2-m_l^2}-\frac{\langle B|H_w^{\text{pv (pc)}}|l\rangle\langle l|A_0^3|Y\rangle}{m_l^2-m_B^2}\right]_{\vec{p}_l=\vec{p}}\right\}, \tag{2.6}$$

where all matrix elements on the right-hand side are assumed to be evaluated in the limit (2.5). The details of this limiting procedure are discussed at the end of this section. We furthermore assume that the sum over all intermediate states in Eq. (2.6) may be replaced by a sum over all physical baryon resonances. The matrix elements on the right-hand side of Eq. (2.6) are related by $SU(6)_W$ and Melosh-transformation considerations.

A. Algebraic properties

The contents of any irreducible representation R of $SU(6)_W$ can be expressed in terms of the subgroup classifications (A,σ) of $SU(3)\times SU(2)$, where A is the SU(3) representation label and $\sigma=2W+1$ is the W-spin multiplicity. The Melosh transformation commutes with the component J_z of the total angular momentum J but not with J itself so that the algebraic properties of a Melosh-transformed operator are specified by the notation

$$\{(A,\sigma)_{W_z}, L_z\}_R$$

with R the $SU(6)_W$ representation classification and W_z and L_z the z components of W spin and orbital angular momentum, respectively. Since the Melosh transformation V is defined in terms of light-cone variables which do not carry definite parity, all operators will be classified according to a generalization of parity, $\mathcal{R}$ parity, where[2,4]

$$\mathcal{R}=e^{-i\pi J_y}P, \tag{2.7}$$

with J_y the y component of angular momentum and P the ordinary parity operator. The Melosh transformation V itself is $\mathcal{R}$ parity even. Before the Melosh transformation is applied, the weak decay Hamiltonian has $J=0$. Thus,

$$\mathcal{R}H_w^{\text{pc (pv)}}\mathcal{R}^{-1}=e^{-i\pi J_y}PH_w^{\text{pc (pv)}}P^{-1}e^{i\pi J_y} = \pm e^{-i\pi J_y}H_w^{\text{pc (pv)}}e^{i\pi J_y} = \pm H_w^{\text{pc (pv)}} \tag{2.8}$$

so that the parity-conserving part of the weak decay Hamiltonian is $\mathcal{R}$ parity even and the parity-violating part is $\mathcal{R}$ parity odd.

Since the component A_0^3 of the axial-vector current is bilinear in quark fields, it belongs to the $\underline{35}$ representation of $SU(6)_W$. A_0^3 transforms like a π^0 and thus has the subgroup classification

$$A_0^3 \sim \{(8,3)_0, 0\}_{35}\,.$$

The corresponding Melosh-transformed operator $VA_0^3V^{-1}$ has $L_z=\pm1$ as well as $L_z=0$ parts. Using the fact that

$$e^{i\pi J_y}|J,J_z\rangle = (-1)^{J+J_z}|J,J_z\rangle\,,$$

the $\mathcal{R}$-parity property of any SU(3)×SU(2)×O(2) subgroup element is given by

$$\mathcal{R}\{(A,2W+1)_{W_z}, L_z\}_R\mathcal{R}^{-1} = (-1)^{W+W_z+L_z}\{(A,2W+1)_{-W_z}, -L_z\}_R\,. \qquad (2.9)$$

Thus the $L_z=\pm1$ parts of $VA_0^3V^{-1}$ transform under $\mathcal{R}$ as

$$\mathcal{R}\{(8,3)_1, -1\}_{35}\mathcal{R}^{-1} = -\{(8,3)_{-1}, 1\}_{35}\,.$$

Since $VA_0^3V^{-1}$ must be $\mathcal{R}$ parity odd, the appropriate algebraic combinations are

$$VA_0^3V^{-1} \sim \{(8,3)_0, 0\} \qquad (2.10a)$$

and

$$\{(8,3)_1, -1\}_{35} + \{(8,3)_{-1}, 1\}_{35}\,. \qquad (2.10b)$$

We assume that the $\Delta I=\frac{1}{2}$ rule is rigorous for the the hadronic part of the weak decay Hamiltonian. This implies that the Hamiltonian transforms like an SU(3) octet. The SU(3)×SU(2) contents of the $\underline{35}$ representation of $SU(6)_W$ are

$$\underline{35} = (8,3), (8,1), (1,3)\,, \qquad (2.11)$$

so that the simplest assumption which incorporates the $\Delta I=\frac{1}{2}$ rule is that the Hamiltonian transforms under $SU(6)_W$ like a $\underline{35}$. A simple model for which this is true, the quark-density model, is discussed in Appendix A. The quark-density model adequately describes the parity-violating decay amplitudes, but is inconsistent with the data for the parity-conserving amplitudes. A more realistic approach, dictated by the recent work on gauge theories, is that the Hamiltonian has a current-current form. Each current belongs to an $SU(6)_W$ $\underline{35}$ representation. Thus, higher representations corresponding to the product

$$\underline{35}\times\underline{35} = \underline{1} + \underline{35} + \underline{35}' + \underline{189} + \underline{280} + \underline{\overline{280}} + \underline{405} \qquad (2.12)$$

need to be considered.[10] Since we assume that the sum over all intermediate states can be replaced by a sum over physical baryon resonances, this requires a sum over all $SU(6)_W$ $\underline{56}$ and $\underline{70}$ representations for arbitrary orbital angular momentum. Besides the fact that no baryon resonances are found to belong to the $SU(6)_W$ $\underline{20}$ representation, the $\underline{20}$ representation cannot contribute to the sum of intermediate states in Eq. (2.6) because

$$\underline{35}\times\underline{56} = \underline{56} + \underline{70} + \underline{700} + \underline{1134}$$

does not contain $\underline{20}$, and thus $\underline{20}$ cannot couple to $\underline{56}$ through A_0^3. Thus the only representations in (2.12) for the weak interaction Hamiltonian which can contribute are those which are also contained in the products

$$\underline{\overline{56}}\times\underline{56} = \underline{1} + \underline{35} + \underline{405} + \underline{2695} \qquad (2.13)$$

and

$$\underline{\overline{70}}\times\underline{56} = \underline{35} + \underline{280} + \underline{405} + \underline{3200}\,. \qquad (2.14)$$

Furthermore, the antisymmetric $\underline{280}$ representation is eliminated since the current-current form implies a symmetric Hamiltonian. Thus, the weak decay Hamiltonian can only belong to the $SU(6)_W$ $\underline{35}$ and $\underline{405}$ representations. The SU(3)×SU(2) content of the $\underline{405}$ representation is

$$\underline{405} = (27,5), (27,3), (27,1), (10,3), (\overline{10},3), (8,5), (8_A,3), (8_B,3), (8,1), (1,5), (1,1)\,, \qquad (2.15)$$

where A and B label two different (8, 3) multiplets defined in Ref. 11. The $\Delta I=\frac{1}{2}$ rule implies that only the (8, 3) and (8, 1) contribute from the $\underline{35}$ representation and the (8, 5), $(8_A,3)$, $(8_B,3)$, and (8, 1) contribute from the $\underline{405}$. Before the Melosh transformation is applied the transformation properties of A_0^3 restrict the intermediate states in Eq. (2.6) to $L_z=0$ states. In this case only $W_z=0$, $L_z=0$ parts of the Hamiltonian given by

$$H_w^{pv} \sim \{(8,3)_0, 0\}_{35}, \{(8_A,3)_0, 0\}_{405}, \{(8_B,3)_0, 0\}_{405}\,, \qquad (2.16a)$$

$$H_w^{pc} \sim \{(8,1)_0, 0\}_{35}, \{(8,5)_0, 0\}_{405}, \{(8,1)_0, 0\}_{405} \qquad (2.16b)$$

can contribute. After the Melosh transformation is applied, the transformation properties of $VA_0^3V^{-1}$ given in Eqs. (2.10a) and (2.10b) again restrict the intermediate states to $L_z=0,\pm1$ values. Thus the Melosh-transformed weak Hamiltonian may have in addition to (2.15a) and (2.15b) only terms transforming like

$$\begin{aligned} H_w^{pv} \sim\, & \{(8,3)_1, -1\}_{35} + \{(8,3)_{-1}, 1\}_{35}, \\ & \{(8_A,3)_1, -1\}_{405} + \{(8_A,3)_{-1}, 1\}_{405}, \\ & \{(8_B,3)_1, -1\}_{405} + \{(8_B,3)_{-1}, 1\}_{405}, \\ & \{(8,5)_1, -1\}_{405} - \{(8,5)_{-1}, 1\}_{405}\,, \end{aligned} \qquad (2.17a)$$

$$\begin{aligned} H_w^{pc} \sim\, & \{(8,3)_1, -1\}_{35} - \{(8,3)_{-1}, 1\}_{35}, \\ & \{(8_A,3)_1, -1\}_{405} - \{(8_A,3)_{-1}, 1\}_{405}, \\ & \{(8_B,3)_1, -1\}_{405} - \{(8_B,3)_{-1}, 1\}_{405}, \\ & \{(8,5)_1, -1\}_{405} + \{(8,5)_{-1}, 1\}_{405}\,. \end{aligned} \qquad (2.17b)$$

The $\mathcal{R}$-parity properties of expressions (2.17a) and (2.17b) may be verified using Eq. (2.9).

The Wigner-Eckart theorem and the algebraic properties of the operators A_0^3, H_w^{pv}, and H_w^{pc} are used to calculate the matrix elements on the right-hand side of Eq. (2.6) in terms of the appropriate $SU(6)_W$ and angular momentum Clebsch-Gordan coefficients times undetermined reduced matrix elements. Thus, the matrix element of any irreducible tensor operator $\mathcal{O}$,

$$\langle RAYII_z LSJJ_z | \mathcal{O}(R''A''Y''I''I_z''W''W_z'') | R'A'Y'I'I_z'L'S'J'J_z' \rangle$$

$$= \sum_{S_z S_z'} (L'L_z'S'S_z', J'J_z')(LL_z SS_z, JJ_z) \left(\begin{matrix} R'' & R' \\ (A'',\sigma'') & (A',\sigma') \end{matrix} \middle| \begin{matrix} R \\ (A,\sigma) \end{matrix} \right)$$

quark angular momentum coupling — SU(6) isoscalar factor (Refs. 11 and 12)

$$\times \begin{pmatrix} A'' & A' & A \\ Y''I''I_z'' & Y'I'I_z', & YII_z \end{pmatrix} (W''W_z''W'W_z', WW_z) \langle RLL_z \| \mathcal{O} \| R'L'L_z' \rangle , \tag{2.18}$$

SU(3) Clebsch-Gordan coefficient (Ref. 13) — W-spin Clebsch-Gordan coefficients (Ref. 14) — reduced matrix element

where L, L_z, S, S_z, and J, J_z are the respective orbital angular momentum, spin, and total angular momentum labels for the initial and final states, and R, A, and W, W_z, and $\sigma = 2W+1$ are the respective $SU(6)_W$, SU(3), and W-spin multiplet labels. Consider the products of matrix elements which occur in Eq. (2.6). In the $SU(6)_W$ symmetry limit the masses of states within a given representation R are degenerate. Thus,

$$\sum_{l \in R} \frac{\langle B | A_0^3 | l \rangle \langle l | H_w^{pv\,(pc)} | Y \rangle}{m_B{}^2 - m_l{}^2} \to \frac{1}{m_B{}^2 - m_l{}^2} \sum_{l \in R} \langle B | A_0^3 | l \rangle \langle l | H_w^{pv\,(pc)} | Y \rangle . \tag{2.19}$$

Introducing the notation

l	$\sim R$	L	J	A	Y	I	I_z	W	W_z	σ
B	~ 56	0	J'	A'	Y'	I'	I_z'	W'	W_z'	σ'
Y	~ 56	0	$\tilde{J}$	$\tilde{A}$	$\tilde{Y}$	$\tilde{I}$	$\tilde{I}_z$	$\tilde{W}$	$\tilde{W}_z$	$\tilde{\sigma}$
A_0^3	~ 35			8	0	1	0	1	W_z''	3
$H_w^{pv\,(pc)}$	$\sim R'''$			8	Y'''	I'''	I_z'''	W'''	W_z'''	σ'''

the individual matrix elements in Eq. (2.19) can be written

$$\langle B | A_0^3 | l \rangle = (\langle l | A_0^3 | B \rangle)^*$$

$$= \sum_{L_z} (LL_z WW_z, J\lambda)(00W'W_z', J'\lambda)(1W_z''W'W_z', WW_z) \left(\begin{matrix} 35 & 56 \\ 8,3 & A',\sigma' \end{matrix} \middle| \begin{matrix} R \\ A,\sigma \end{matrix} \right) \begin{pmatrix} 8 & A' & A \\ 010 & Y'I'I_z', & YII_z \end{pmatrix} Q \tag{2.20a}$$

and

$$\langle l | H_w^{pv\,(pc)} | Y \rangle = \sum_{\bar{L}_z} (L\bar{L}_z WW_z, J\lambda)(00\tilde{W}\tilde{W}_z, \tilde{J}\lambda)(W'''W_z'''\tilde{W}\tilde{W}_z, WW_z) \left(\begin{matrix} R''' & 56 \\ 8,\sigma''' & \tilde{A},\tilde{\sigma} \end{matrix} \middle| \begin{matrix} R \\ A,\sigma \end{matrix} \right) \begin{pmatrix} 8 & \tilde{A} & A \\ Y'''I'''I_z''' & \tilde{Y}\tilde{I}\tilde{I}_z, & YII_z \end{pmatrix} S \tag{2.20b}$$

with Q and S the respective reduced matrix elements.

Using the orthogonality condition

$$\sum_{J,\lambda} (LL_z WW_z, J\lambda)(L\bar{L}_z W\bar{W}_z, J\lambda) = \delta_{L_z \bar{L}_z} \delta_{W \bar{W}_z}$$

for SU(2) Clebsch-Gordan coefficients, the product of equations (2.20a) and (2.20b) becomes

$$\sum_{l\in R}\langle B|A_0^3|\,l\rangle\langle\, l\,|H_w^{\text{pv (pc)}}|\,Y\rangle=\sum_{A_l W}(1W_z''W'W_z', WW_z)(W'''W_z'''\tilde{W}\tilde{W}_z, WW_z)$$
$$\times\begin{pmatrix}35 & 56 & R\\ 8,3 & A'\sigma' & A,\sigma\end{pmatrix}\begin{pmatrix}R''' & 56 & R\\ 8,\sigma''' & \tilde{A},\tilde{\sigma} & A,\sigma\end{pmatrix}\begin{pmatrix}8 & A' & A\\ 010 & Y'I'I_z', & YII_z\end{pmatrix}$$
$$\times\begin{pmatrix}8 & \tilde{A} & A\\ Y'''I'''I_z''' & \tilde{Y}\tilde{I}\tilde{I}_z, & YII_z\end{pmatrix}QS. \tag{2.21}$$

The only dependence on the orbital angular momentum L in expression (2.21) lies in the reduced matrix elements QS. It is thus possible to sum over all possible orbital angular momentum L for a given $SU(6)_W$ representation by simply replacing QS by the sum $\sum_L Q_L S_L$ of reduced matrix elements for each L. This sum again acts as a single undetermined parameter in this approach so that the form of (2.21) remains unchanged. Thus the sum over all physical baryon resonances reduces to a sum over the $SU(6)_W$ representations 56 and 70 for each set of algebraic transformation properties of the operators A_0^3 and $H_w^{\text{pv (pc)}}$ given in Eqs. (2.10a) and (2.10b), (2.16a) and (2.16b), and (2.17a) and (2.17b). The second product of matrix elements in Eq. (2.6) is treated analogously to Eq. (2.19).

B. Infinite-momentum limit

Using Table I, which lists the infinite-momentum limit of appropriate Dirac and Rarita-Schwinger bilinear forms, Eq. (2.6) can be evaluated in the limit (2.5). The appropriate bilinear forms for the left-hand side of Eq. (2.6) are given in expressions (2.2a)–(2.2c). The right-hand side is evaluated using the bilinear forms

$$\langle p'|A_0^3|p\rangle\sim\left(\frac{mm'}{p_0p_0'}\right)^{1/2}\bar{u}(p')\gamma_4\gamma_5u(p), \tag{2.22a}$$

$$\langle p'|H_w^{\text{pv}}|p\rangle\sim\left(\frac{mm'}{p_0p_0'}\right)^{1/2}(p_0-p_0')\bar{u}(p')\gamma_4\gamma_5u(p), \tag{2.22b}$$

TABLE I. Infinite-momentum limit of Dirac and Rarita-Schwinger bilinear forms. $k=p_1-p_2$; $\xi\equiv$ Pauli spinor; $\hat{p}$ is a unit vector in the direction of momentum.

Bilinear form	$S_z=\pm\frac{3}{2}$	$S_z=\pm\frac{1}{2}$
$2(m_1m_2)^{1/2}\bar{u}_2u_1$		$(m_1+m_2)\xi_2^\dagger\xi_1$
$2(m_1m_2)^{1/2}\bar{u}_2\gamma_4u_1$		$2\|p\|\,\xi_2^\dagger\xi_1$
$2(m_1m_2)^{1/2}\bar{u}_2\gamma_5u_1$		$(m_1-m_2)\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_1$
$2(m_1m_2)^{1/2}\bar{u}_2\gamma_4\gamma_5u_1$		$-2\|p\|\,\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_2$
$2(m_1m_2)^{1/2}k_\lambda\bar{u}_2u_{1\lambda}$	0	$(\frac{2}{3})^{1/2}\frac{(m_1+m_2)^2}{2m_1}(m_1-m_2)\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_1$
$2(m_1m_2)^{1/2}k_\lambda\bar{u}_2\gamma_5u_{1\lambda}$	0	$(\frac{2}{3})^{1/2}\frac{(m_1-m_2)^2}{2m_1}(m_1+m_2)\xi_2^\dagger\xi_1$
$2(m_1m_2)^{1/2}\bar{u}_{2\lambda}u_{1\lambda}$	$(m_1+m_2)\xi_2^\dagger\xi_1$	$\frac{1}{3}\frac{m_1^2+m_2^2+m_1m_2}{m_1m_2}(m_1+m_2)\xi_2^\dagger\xi_1$
$2(m_1m_2)^{1/2}\bar{u}_{2\lambda}\gamma_5u_{1\lambda}$	$(m_1-m_2)\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_1$	$\frac{1}{3}\frac{m_1^2+m_2^2-m_1m_2}{m_1m_2}(m_1-m_2)\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_1$
$2(m_1m_2)^{1/2}k_\lambda k_\nu\bar{u}_{2\lambda}u_{1\nu}$	0	$\frac{1}{3}\frac{(m_1^2-m_2^2)^2}{2m_1m_2}(m_1+m_2)\xi_2^\dagger\xi_1$
$2(m_1m_2)^{1/2}k_\lambda k_\nu\bar{u}_{2\lambda}\gamma_5u_{1\nu}$	0	$\frac{1}{3}\frac{(m_1^2-m_2^2)^2}{2m_1m_2}(m_1-m_2)\xi_2^\dagger\sigma\cdot\hat{p}\,\xi_1$

and

$$\langle p'|H_w^{\rm pc}|p\rangle \sim \left(\frac{mm'}{p_0 p_0'}\right)^{1/2}(p_0 - p_0')\bar{u}(p')\gamma_4 u(p)\,. \qquad (2.22c)$$

The effective bilinear forms (2.22b) and (2.22c) for the current-current Hamiltonian are derived in Appendix B. Since the reduced matrix element is the same throughout a given SU(6) representation, it is necessary to treat only the Dirac spinor case to extract the explicit mass dependence of these matrix elements in the infinite-momentum limit. The final expressions for the Lorentz-invariant amplitudes of the helicity $J_z = \frac{1}{2}$ state of the decaying baryon are

$$A(Y\to B\pi^0) = (m_Y - m_B)\left\{(I_Y^3 - I_B^3)(\langle B|H_w^{\rm pc}|Y\rangle)_{\rm eff} + \left[\sum_l (\langle B|A_0^3|l\rangle\langle l|H_w^{\rm pv}|Y\rangle)_{\rm eff}\right]_-\right\}, \qquad (2.23a)$$

$$B(Y\to B\pi^0) = (m_Y + m_B)\left\{(I_Y^3 - I_B^3)(\langle B|H_w^{\rm pv}|Y\rangle)_{\rm eff} + \left[\sum_l (\langle B|A_0^3|l\rangle\langle l|H_w^{\rm pc}|Y\rangle)_{\rm eff}\right]_-\right\} \qquad (2.23b)$$

for Σ, Λ, and Ξ decays and

$$D(\Omega^-\to \Xi^-\pi^0) = \frac{\sqrt{6}\, m_\Omega^{\,2}}{(m_\Omega - m_{\Xi^-})}\left\{(I_\Omega^3 - I_{\Xi^-}^3)(\langle \Xi^-|H_w^{\rm pc}|\Omega\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^-|A_0^3|l\rangle\langle l|H_w^{\rm pv}|\Omega\rangle)_{\rm eff}\right]_-\right\}, \qquad (2.23c)$$

$$B(\Omega^-\to \Xi^-\pi^0) = \frac{\sqrt{6}\, m_\Omega^{\,2}}{(m_\Omega + m_{\Xi^-})}\left\{(I_\Omega^3 - I_{\Xi^-}^3)(\langle \Xi^-|H_w^{\rm pv}|\Omega^-\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^-|A_0^3|l\rangle\langle l|H_w^{\rm pc}|\Omega^-\rangle)_{\rm eff}\right]_-\right\}, \qquad (2.23d)$$

$$\frac{(m_\Omega^{\,2} + m_{\Xi^*}^{\,2} + m_\Omega m_{\Xi^*})}{m_\Omega m_{\Xi^*}(m_\Omega - m_{\Xi^*})}A(\Omega^-\to \Xi^{*-}\pi^0) + \frac{(m_\Omega - m_{\Xi^*})(m_\Omega + m_{\Xi^*})^2}{2m_\Omega^{\,2} m_{\Xi^*}^{\,2}}D(\Omega^-\to \Xi^{*-}\pi^0)$$

$$= 3\left\{(I_\Omega^3 - I_{\Xi^{*-}}^3)(\langle \Xi^{*-}|H_w^{\rm pc}|\Omega\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^{*-}|A_0^3|l\rangle\langle l|H_w^{\rm pv}|\Omega^-\rangle)_{\rm eff}\right]_-\right\}, \qquad (2.23e)$$

$$\frac{(m_\Omega^{\,2} + m_{\Xi^*}^{\,2} - m_\Omega m_{\Xi^*})}{m_\Omega m_{\Xi^*}(m_\Omega + m_{\Xi^*})}B(\Omega^-\to \Xi^{*-}\pi^0) + \frac{(m_\Omega - m_{\Xi^*})^2(m_\Omega + m_{\Xi^*})}{2m_\Omega^{\,2} m_{\Xi^*}^{\,2}}F(\Omega^-\to \Xi^{*-}\pi^0)$$

$$= 3\left\{(I_\Omega^3 - I_{\Xi^{*-}}^3)(\langle \Xi^{*-}|H_w^{\rm pv}|\Omega^-\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^{*-}|A_0^3|l\rangle\langle l|H_w^{\rm pc}|\Omega^-\rangle)_{\rm eff}\right]_-\right\} \qquad (2.23f)$$

for Ω^- decays. Expressions for the helicity state $J_z = \frac{3}{2}$ of the $\Omega^-\to \Xi^{*-}\pi^0$ decay are

$$A(\Omega^-\to \Xi^{*-}\pi^0) = (m_\Omega - m_{\Xi^*})\left\{(I_\Omega^3 - I_{\Xi^{*-}}^3)(\langle \Xi^{*-}|H_w^{\rm pc}|\Omega^-\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^{*-}|A_0^3|l\rangle\langle l|H_w^{\rm pv}|\Omega^-\rangle)_{\rm eff}\right]_-\right\} \qquad (2.23g)$$

and

$$B(\Omega^-\to \Xi^{*-}\pi^0) = (m_\Omega + m_{\Xi^{*-}})\left\{(I_\Omega^3 - I_{\Xi^{*-}}^3)(\langle \Xi^{*-}|H_w^{\rm pv}|\Omega^-\rangle)_{\rm eff} + \left[\sum_l (\langle \Xi^{*-}|A_0^3|l\rangle\langle l|H_w^{\rm pc}|\Omega^-\rangle)_{\rm eff}\right]_-\right\}. \qquad (2.23h)$$

The notation $(\langle a|\mathcal{O}|b\rangle)_{\rm eff}$ represents the product of appropriate Clebsch-Gordon coefficients times the undetermined reduced matrix element after the mass dependence dictated by the infinite-momentum limit has been removed and

$$\left[\sum_l \langle a|A|l\rangle\langle l|H|b\rangle\right]_- \equiv \frac{1}{f_\pi}\left[\sum_l \langle a|A|l\rangle\langle l|H|b\rangle - \sum_l \langle a|H|l\rangle\langle l|A|b\rangle\right].$$

III. RESULTS

Equations (2.23a)–(2.23h) are used to calculate Lorentz-invariant amplitudes for the decays

$$
\begin{aligned}
&\Sigma_0^+:\ \Sigma^+ \to p + \pi^0,\\
&\Sigma_0^0:\ \Sigma^0 \to n + \pi^0,\\
&\Lambda_0^0:\ \Lambda^0 \to n + \pi^0,\\
&\Xi_0^0:\ \Xi^0 \to \Lambda + \pi^0,\\
&\Omega_0^-:\ \Omega^- \to \Xi^- + \pi^0,\\
&\Omega_0^{-*}:\ \Omega^- \to \Xi^{*-} + \pi^0.
\end{aligned}
\tag{3.1}
$$

The more easily observed decays involving charged pions are

$$
\begin{aligned}
&\Sigma_+^+:\ \Sigma^+ \to n + \pi^+,\\
&\Sigma_-^-:\ \Sigma^- \to n + \pi^-,\\
&\Lambda_-^0:\ \Lambda^0 \to p + \pi^-,\\
&\Xi_-^-:\ \Xi^- \to \Lambda + \pi^-.
\end{aligned}
\tag{3.2}
$$

The amplitudes in (3.2) are related to those in (3.1) by the $\Delta I = \frac{1}{2}$ rule. Our present phase convention is chosen such that

$$
\begin{aligned}
\Sigma_+^+ &= \frac{1}{\sqrt{2}}(\sqrt{2}\,\Sigma_0^0 - \Sigma_0^+),\\
\Sigma_-^- &= \frac{1}{\sqrt{2}}(\sqrt{2}\,\Sigma_0^0 + \Sigma_0^+),\\
\Lambda_-^0 &= -\sqrt{2}\,\Lambda_0^0,\\
\Xi_-^- &= \sqrt{2}\,\Xi_0^0.
\end{aligned}
\tag{3.3}
$$

The decay $\Omega^- \to \Lambda K^-$ is also discussed at the end of this section. Let the notation $a^R_{(A,\sigma)}$ and $X^{R',L_z}_{R,(A,\sigma)}$ denote the reduced matrix element arising from the equal time commutation term and the product term $[\sum_{l\in R'} (\langle B|A_0^3|l\rangle\langle l|H_w^{pv}|Y\rangle)_{\text{eff}}]_-$, respectively, in Eqs. (2.23a), (2.23c), (2.23e), and (2.23g) where R' is the $SU(6)_W$ representation of the intermediate states and R, A, σ, and L_z specify the $SU(6)_W$, SU(3), W-spin, and orbital angular momentum of the parity-violating part of the weak decay Hamiltonian H_w^{pv}. Let $b^R_{(A,\sigma)}$ and $Y^{R',L_z}_{R,(A,\sigma)}$ be the corresponding notations for the reduced matrix elements appearing in equations (2.23b), (2.23d), (2.23f), and (2.23h) for the parity-conserving amplitudes.

A. Parity-violating amplitudes

The parity-violating amplitudes for nonleptonic weak decay can be expressed in a simple form by defining new variables

$$X^{35} = \frac{1}{6\sqrt{5}} a^{35}_{(8,1)} - \frac{1}{45\sqrt{2}} (X^{56,0}_{35,(8,3)} + 2X^{56,\pm 1}_{35,(8,3)}) - \frac{1}{24\sqrt{2}} (X^{70,0}_{35,(8,3)} + 2X^{70,\pm 1}_{35,(8,3)}),$$

$$X^{405} = \frac{\sqrt{7}}{90\sqrt{15}} a^{405}_{(8,1)} - \frac{\sqrt{7}}{(45)^2\sqrt{6}} (X^{56,0}_{405,(8_B,3)} + 2X^{56,\pm 1}_{405,(8_B,3)}) - \frac{1}{648\sqrt{10}} (X^{70,0}_{405,(8_B,3)} + 2X^{70,\pm 1}_{405,(8_B,3)}),$$

$$X^{8,5} = \frac{\sqrt{7}}{45\sqrt{5}} a^{405}_{(8,5)} + \frac{3\sqrt{42}}{(45)^2} X^{56,\pm 1}_{405,(8,5)} + \frac{1}{36\sqrt{10}} X^{70,\pm 1}_{405,(8,5)}\,.$$

Then the parity-violating amplitudes are

$$A(\Sigma_0^+) = (m_\Sigma - m_p)\left[X^{35} + 11X^{405} - \frac{4\sqrt{7}}{405\sqrt{15}} (X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm 1}_{405,(8_A,3)}) - \frac{1}{162} (X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm 1}_{405,(8_A,3)})\right], \tag{3.4a}$$

$$\sqrt{2}\,A(\Sigma_0^0) = (m_\Sigma - m_n)\left[X^{35} + 11X^{405} - \frac{16\sqrt{7}}{405\sqrt{15}} (X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm 1}_{405,(8_A,3)}) + \frac{5}{162} (X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm 1}_{405,(8_A,3)})\right], \tag{3.4b}$$

$$A(\Sigma_+^+) = \frac{1}{\sqrt{2}} (m_\Sigma - m_N)\left[\frac{-12\sqrt{7}}{405\sqrt{15}} (X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm 1}_{405,(8_A,3)}) + \frac{1}{27} (X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm 1}_{405,(8_A,3)})\right], \tag{3.4c}$$

$$A(\Sigma_-^-) = \frac{1}{\sqrt{2}} (m_\Sigma - m_N)\left[2X^{35} + 22X^{405} - \frac{20\sqrt{7}}{405\sqrt{15}} (X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm 1}_{405,(8_A,3)}) + \frac{2}{81} (X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm 1}_{405,(8_A,3)})\right], \tag{3.4d}$$

$$A(\Lambda_-^0) = \sqrt{3}\,(m_\Lambda - m_n)\left[X^{35} + 3X^{405} + \frac{4\sqrt{7}}{405\sqrt{15}} (X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm 1}_{405,(8_A,3)}) - \frac{1}{81} (X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm 1}_{405,(8_A,3)})\right], \tag{3.4e}$$

$$A(\Xi_-^-)=\sqrt{3}\,(m_\Xi-m_\Lambda)\left[X^{35}+7X^{405}-\frac{1}{108}(X^{70,0}_{405,(8_A,3)}+2X^{70,\pm1}_{405,(8_A,3)})\right]\,. \quad (3.4f)$$

Since each column, apart from the mass factor, satisfies the Lee-Sugawara relation[15]

$$\sqrt{3}\,A(\Sigma_0^+)+A(\Lambda_-^0)=2A(\Xi_-^-)\,, \quad (3.5)$$

the modified Lee-Sugawara relation

$$\frac{\sqrt{3}A(\Sigma_0^+)}{m_\Sigma-m_p}+\frac{A(\Lambda_-^0)}{m_\Lambda-m_n}=\frac{2A(\Xi_-^-)}{m_\Xi-m_\Lambda} \quad (3.6)$$

is satisfied exactly. This modified form of the Lee-Sugawara relation for parity-violating amplitudes is satisfied experimentally to 10%. By use of the Gell-Mann-Okubo mass formula, however, the contribution of X^{35} itself satisfies the original form (3.5) of the Lee-Sugawara relation exactly.

Expressions for the parity-violating Ω^- decay amplitudes are found using (2.23c), (2.23e), and (2.23g) to be

$$(m_\Omega-m_\Xi)D(\Omega_0^-)=\sqrt{6}m_\Omega^{\,2}\left[X^{8,5}+\frac{2\sqrt{7}}{405\sqrt{10}}(X^{56,0}_{405,(8_A,3)}-X^{56,\pm1}_{405,(8_A,3)})+\frac{1}{108\sqrt{6}}(X^{70,0}_{405,(8_A,3)}-X^{70,\pm1}_{405,(8_A,3)})\right.$$
$$\left.-\frac{4\sqrt{7}}{(45)^2}(X^{56,0}_{405,(8_B,3)}-X^{56,\pm1}_{405,(8_B,3)})-\frac{1}{54\sqrt{15}}(X^{70,0}_{405,(8_B,3)}-X^{70,\pm1}_{405,(8_B,3)})\right], \quad (3.7a)$$

$$\frac{(m_\Omega^{\,2}+m_{\Xi^*}^{\,2}+m_\Omega m_{\Xi^*})}{m_\Omega m_{\Xi^*}(m_\Omega-m_{\Xi^*})}A(\Omega_0^-{}^*)+\frac{(m_\Omega-m_{\Xi^*})(m_\Omega+m_{\Xi^*})^2}{2m_\Omega^{\,2}m_{\Xi^*}^{\,2}}D(\Omega_0^-{}^*)$$
$$=3\left[-\sqrt{2}X^{8,5}-\sqrt{3}X^{35}+\frac{\sqrt{7}}{90\sqrt{5}}a^{405}_{(8,1)}-\frac{2\sqrt{7}}{405\sqrt{5}}(2X^{56,0}_{405,(8_A,3)}+X^{56,\pm1}_{405,(8_A,3)})-\frac{1}{108\sqrt{3}}(2X^{70,0}_{405,(8_A,3)}+X^{70,\pm1}_{405,(8_A,3)})\right.$$
$$\left.+\frac{\sqrt{7}}{(45)^2\sqrt{2}}(7X^{56,0}_{405,(8_B,3)}-10X^{56,\pm1}_{405,(8_B,3)})+\frac{1}{216\sqrt{30}}(7X^{70,0}_{405,(8_B,3)}-10X^{70,\pm1}_{405,(8_B,3)})\right], \quad (3.7b)$$

and

$$A(\Omega_0^-{}^*)=(m_\Omega-m_{\Xi^*})\left[-\sqrt{3}X^{35}+\sqrt{2}X^{8,5}+\frac{\sqrt{7}}{90\sqrt{5}}a^{405}_{(8,1)}-\frac{2\sqrt{7}}{135\sqrt{5}}X^{56,\pm1}_{405,(8_A,3)}\right.$$
$$\left.-\frac{1}{36\sqrt{3}}X^{70,\pm1}_{405,(8_A,3)}+\frac{3\sqrt{7}}{(45)^2\sqrt{2}}(-3X^{56,0}_{405,(8_B,3)}+2X^{56,\pm1}_{405,(8_B,3)})+\frac{1}{72\sqrt{30}}(-3X^{70,0}_{405,(8_B,3)}+2X^{70,\pm1}_{405,(8_B,3)})\right]. \quad (3.7c)$$

Combining expressions (3.7) with (3.4), it is easy to derive the exact sum rule:

$$\frac{(m_\Omega-m_\Xi)D(\Omega_0^-)}{\sqrt{3}m_\Omega^{\,2}}+\frac{(m_\Omega^{\,2}+m_{\Xi^*}^{\,2}+m_\Omega m_{\Xi^*})}{3m_\Omega m_{\Xi^*}(m_\Omega-m_{\Xi^*})}A(\Omega_0^-{}^*)+\frac{(m_\Omega-m_{\Xi^*})(m_\Omega+m_{\Xi^*})^2}{6m_\Omega^{\,2}m_{\Xi^*}^{\,2}}D(\Omega_0^-{}^*)$$
$$=\frac{\sqrt{6}}{4}\left[\frac{A(\Sigma_-^-)-3A(\Sigma_+^+)}{m_\Sigma-m_N}\right]-\frac{3A(\Lambda_-^0)}{2(m_\Lambda-m_N)}\,. \quad (3.8)$$

We assume that the amplitudes are well behaved, i.e., do not become infinite, in the $SU(6)_W$ symmetry limit. This demands for consistency that the right-hand side of Eq. (3.7a) be zero. Similarly, combining Eqs. (3.7b) and (3.7c), one finds the relation

$$(m_\Omega-m_{\Xi^*})\left[\frac{A(\Omega_0^-{}^*)}{m_\Omega m_{\Xi^*}}+\frac{(m_\Omega+m_{\Xi^*})^2}{2m_\Omega^{\,2}m_{\Xi^*}^{\,2}}D(\Omega_0^-{}^*)\right]$$
$$=3\left[-2\sqrt{2}X^{8,5}-\frac{4\sqrt{7}}{405\sqrt{5}}(X^{56,0}_{405,(8_A,3)}-X^{56,\pm1}_{405,(8_A,3)})-\frac{1}{54\sqrt{3}}(X^{70,0}_{405,(8_A,3)}-X^{70,\pm1}_{405,(8_A,3)})\right.$$
$$\left.+\frac{16\sqrt{7}}{(45)^2\sqrt{2}}(X^{56,0}_{405,(8_B,3)}-X^{56,\pm1}_{405,(8_B,3)})+\frac{2}{27\sqrt{30}}(X^{70,0}_{405,(8_B,3)}-X^{70,\pm1}_{405,(8_B,3)})\right] \quad (3.9)$$

which must by the same argument vanish. Equations (3.7a) and (3.9) give the same constraint on the reduced matrix elements which is

$$\sqrt{2}X^{8,5}+\frac{2\sqrt{7}}{405\sqrt{5}}\left(X^{56,0}_{405,(8_A,3)}-X^{56,\pm1}_{405,(8_A,3)}\right)+\frac{1}{108\sqrt{3}}\left(X^{70,0}_{405,(8_A,3)}-X^{70,\pm1}_{405,(8_A,3)}\right)$$
$$-\frac{8\sqrt{7}}{(45)^2\sqrt{2}}\left(X^{56,0}_{405,(8_B,3)}-X^{56,\pm1}_{405,(8_B,3)}\right)-\frac{1}{27\sqrt{30}}\left(X^{70,0}_{405,(8_B,3)}-X^{70,\pm1}_{405,(8_B,3)}\right)=0\,. \qquad (3.10)$$

A sufficient, but not necessary, condition for Eq. (3.10) to be satisfied is that the parity-violating part of the weak decay Hamiltonian transforms only like a $SU(6)_W$ $\underline{35}$ representation. Under this simple assumption, one automatically obtains

$$A(\Sigma_+^+)=0 \qquad (3.11)$$

as well as the Lee-Sugawara relation (3.5). The sum rule given in Eq. (3.8) reduces to the simple form

$$\frac{A(\Omega_0^{-*})}{(m_\Omega-m_{\Xi^*})}=\frac{\sqrt{6}A(\Sigma_-^-)}{4(m_\Sigma-m_N)}-\frac{3A(\Lambda_-^0)}{2(m_\Lambda-m_n)}\,. \qquad (3.12)$$

The parity-violating amplitudes are all expressed in terms of a single parameter X^{35} in the following way:

$$A(\Sigma_-^-)=\frac{1}{\sqrt{2}}(m_\Sigma-m_N)2X^{35}\,, \qquad (3.13a)$$

$$A(\Sigma_0^+)=(m_\Sigma-m_p)X^{35}\,, \qquad (3.13b)$$

$$A(\Lambda_-^0)=\sqrt{3}(m_\Lambda-m_n)X^{35}\,, \qquad (3.13c)$$

$$A(\Xi_-^-)=\sqrt{3}(m_\Xi-m_\Lambda)X^{35}\,, \qquad (3.13d)$$

$$A(\Omega_0^{-*})=-\sqrt{3}(m_\Omega-m_{\Xi^*})X^{35}\,. \qquad (3.13e)$$

The results of a χ^2 fit to the amplitudes (3.13a)-(3.13d) expressed in units of $G\mu_c^2$, where $G=10^{-5}$ m_p^{-2} and μ_c is the mass of the charged pion,[14] is given in Table II and determines the parameter X^{35} to be

$$X^{35}=5.281\ \text{GeV}^{-1}\,.$$

The fit is good to 10%. This again indicates that $\underline{405}$ contributions, if any, must be small. Using the above value of X^{35} gives the prediction

$$A(\Omega_0^{-*})=-1.253\,. \qquad (3.14)$$

We notice that the results (3.11) and (3.13a)–(3.13d) are of the same form as those from a current algebra calculation, except for the appearance of mass factors which significantly improve the agreement with experiment. The same mass factors also occur in the K^* dominance calculation.[16] If corrections to expressions (3.7a) and (3.9) are of at least second order in the mass differences, one may also conclude that

$$D(\Omega_0^-)=0 \qquad (3.15a)$$

and

$$D(\Omega_0^{-*})=\frac{-2m_\Omega m_{\Xi^*}}{(m_\Omega+m_{\Xi^*})^2}A(\Omega_0^{-*})=0.625\,. \qquad (3.15b)$$

Since there is, however, no *a priori* reason why first-order corrections may not occur, Eqs. (3.15a) and (3.15b) need not be true.

B. Parity-conserving amplitudes

The parity-conserving amplitudes for nonleptonic weak decay can be simplified by defining new variables

$$Y_1=\frac{1}{18\sqrt{5}}b^{35}_{(8,3)}-\frac{\sqrt{7}}{90\sqrt{15}}b^{405}_{(8_B,3)}-\frac{1}{135\sqrt{2}}Y^{56,0}_{35,(8,1)}-\frac{1}{72\sqrt{2}}Y^{70,0}_{35,(8,1)}+\frac{9\sqrt{7}}{(45)^2\sqrt{6}}Y^{56,0}_{405,(8,1)}$$
$$+\frac{1}{72\sqrt{10}}Y^{70,0}_{405,(8,1)}+\frac{2}{45\sqrt{2}}Y^{56,\pm1}_{35,(8,3)}+\frac{7\sqrt{14}}{(45)^2\sqrt{3}}Y^{56,\pm1}_{405,(8_B,3)}-\frac{2}{81\sqrt{10}}Y^{70,\pm1}_{405,(8_B,3)}\,, \qquad (3.16a)$$

$$Y_2=\frac{-2\sqrt{7}}{45\sqrt{15}}b^{405}_{(8_B,3)}+\frac{6\sqrt{7}}{(45)^2\sqrt{3}}\left(Y^{56,0}_{405,(8,5)}+\sqrt{3}Y^{56,\pm1}_{405,(8,5)}\right)+\frac{1}{108\sqrt{5}}\left(Y^{70,0}_{405,(8,5)}+\sqrt{3}\,Y^{70,\pm1}_{405,(8,5)}\right)$$
$$+\frac{12\sqrt{7}}{(45)^2\sqrt{6}}Y^{56,0}_{405,(8,1)}+\frac{1}{54\sqrt{10}}Y^{70,0}_{405,(8,1)}-\frac{2\sqrt{7}}{135\sqrt{15}}Y^{56,\pm1}_{405,(8_A,3)}-\frac{1}{108}Y^{70,\pm1}_{405,(8_A,3)}\,, \qquad (3.16b)$$

$$Y_3=\frac{2\sqrt{7}}{(45)^2\sqrt{3}}\left(Y^{56,0}_{405,(8,5)}+\sqrt{3}Y^{56,\pm1}_{405,(8,5)}\right)+\frac{1}{324\sqrt{5}}\left(Y^{70,0}_{405,(8,5)}+\sqrt{3}Y^{70,\pm1}_{405,(8,5)}\right)-\frac{8\sqrt{7}}{(45)^2\sqrt{6}}Y^{56,0}_{405,(8,1)}$$
$$-\frac{1}{81\sqrt{10}}Y^{70,0}_{405,(8,1)}-\frac{4}{135\sqrt{2}}Y^{56,\pm1}_{35,(8,3)}+\frac{Y^{70,\pm1}_{35,(8,3)}}{36\sqrt{2}}-\frac{2\sqrt{7}}{405\sqrt{15}}Y^{56,\pm1}_{405,(8_A,3)}-\frac{Y^{70,\pm1}_{405,(8_A,3)}}{324}$$
$$-\frac{6\sqrt{14}}{(45)^2\sqrt{3}}Y^{56,\pm1}_{405,(8_B,3)}+\frac{1}{36\sqrt{10}}Y^{70,\pm1}_{405,(8_B,3)}\,, \qquad (3.16c)$$

$$Y_M=\frac{\sqrt{7}}{45\sqrt{6}}\,b^{405}_{(8_A,3)}+\frac{2}{45\sqrt{2}}\,Y^{56,\pm1}_{35,(8,3)}-\frac{1}{24\sqrt{2}}\,Y^{70,\pm1}_{35,(8,3)}+\frac{2\sqrt{7}}{135\sqrt{15}}\,Y^{56,\pm1}_{405,(8_A,3)}$$
$$+\frac{1}{108}\,Y^{70,\pm1}_{405,(8_A,3)}+\frac{\sqrt{14}}{135\sqrt{3}}\,Y^{56,\pm1}_{405,(8_B,3)}-\frac{5}{216\sqrt{10}}\,Y^{70,\pm1}_{405,(8_B,3)}\,. \qquad (3.16\text{d})$$

Notice that the variable Y_M defined in Eq. (3.16d) depends solely on terms which appear only as the result of the Melosh transformation. Expressed in terms of the above parameters (3.16a)–(3.16d) the parity-conserving amplitudes have the forms:

$$\sqrt{2}\,B(\Sigma_+^+)=(m_\Sigma+m_N)(6Y_3)\,, \qquad (3.17\text{a})$$
$$\sqrt{2}\,B(\Sigma_-^-)=(m_\Sigma+m_N)(-2Y_1+2Y_2+4Y_3)\,, \qquad (3.17\text{b})$$
$$B(\Sigma_0^+)=(m_\Sigma+m_p)(-Y_1+Y_2-Y_3)\,, \qquad (3.17\text{c})$$
$$B(\Lambda_-^0)=\sqrt{3}\,(m_\Lambda+m_n)(3Y_1-Y_2+Y_3)\,, \qquad (3.17\text{d})$$
$$B(\Xi_-^-)=\sqrt{3}\,(m_\Xi+m_\Lambda)(Y_1)\,. \qquad (3.17\text{e})$$

Apart from the mass factors, it is easy to see that each contribution to (3.17c)–(3.17e) satisfies the Lee-Sugawara relation for B amplitudes[15]

$$\sqrt{3}\;B(\Sigma_0^+)+B(\Lambda_-^0)=2B(\Xi_-^-)\,. \qquad (3.18)$$

Thus a modified Lee-Sugawara relation

$$\frac{\sqrt{3}B(\Sigma_0^+)}{(m_\Sigma+m_p)}+\frac{B(\Lambda_-^0)}{(m_\Lambda+m_n)}=\frac{2B(\Xi_-^-)}{m_{\Xi^0}+m_\Lambda} \qquad (3.19)$$

is an exact relation for B amplitudes. This modified Lee-Sugawara relation is experimentally good to 7%.

Expressions for the parity-conserving Ω^- decay amplitudes are

$$B(\Omega_0^-)=\frac{6m_\Omega^2}{(m_\Omega+m_\Xi)}\,(-2Y_1+Y_2-\tfrac{1}{2}Y_3+Y_M)\,, \qquad (3.20\text{a})$$
$$B(\Omega_0^-{}^*)=3\sqrt{3}\,(m_\Omega+m_{\Xi^*})(-Y_1-Y_3), \qquad (3.20\text{b})$$
$$\frac{(m_\Omega^2+m_{\Xi^*}^2-m_\Omega m_{\Xi^*})}{m_\Omega m_{\Xi^*}}\,B(\Omega_0^-{}^*)+\frac{(m_\Omega-m_{\Xi^*})^2(m_\Omega+m_{\Xi^*})}{2m_\Omega^2 m_{\Xi^*}^2}\,F(\Omega_0^-{}^*)=3\sqrt{3}\,(m_\Omega+m_{\Xi^*})(-Y_1-Y_3)\,. \qquad (3.20\text{c})$$

Comparing Eqs. (3.20b) with (3.17a) and (3.17e) one obtains the additional exact sum rule

$$\frac{B(\Omega_0^-{}^*)}{3(m_\Omega+m_{\Xi^*})}+\frac{B(\Xi_-^-)}{m_\Xi+m_\Lambda}+\frac{B(\Sigma_+^+)}{\sqrt{6}\,(m_\Sigma+m_N)}=0\,. \qquad (3.21)$$

By combining Eqs. (3.20b) and (3.20c) one obtains a possible constraint equation analogous to Eq. (3.9) which has the form

$$(m_\Omega-m_{\Xi^*})^2\left[\frac{B(\Omega_0^-{}^*)}{m_\Omega m_{\Xi^*}(m_\Omega+m_{\Xi^*})}+\frac{(m_\Omega+m_{\Xi^*})}{2m_\Omega^2 m_{\Xi^*}^2}\,F(\Omega_0^-{}^*)\right]=0\,. \qquad (3.22)$$

This equation vanishes identically for $\Omega_0^-{}^*$ amplitudes.

Table III shows the results of a χ^2 fit to the amplitudes given in Eqs. (3.17b)–(3.17e). The fit is good to 10% with the parameters

$$Y_1=-1.536\ \text{GeV}^{-1}\,, \qquad (3.23\text{a})$$
$$Y_2=-5.549\ \text{GeV}^{-1}\,, \qquad (3.23\text{b})$$
$$Y_3=1.899\ \text{GeV}^{-1}\,. \qquad (3.23\text{c})$$

Deviations from the experimental values reflect the experimental deviations from the $\Delta I=\frac{1}{2}$ rule and the modified Lee-Sugawara relation given in Eq. (3.19). The fit is consistent with the vanishing

TABLE II. Parity-violating amplitudes with H_ω^{pv} in SU(6)_W 35.

Amplitude	Theory	Experiment (Ref. 14)	Deviation (%)
$A(\Sigma_0^+)$	1.33	1.48 ± 0.05	10
$A(\Sigma_-^-)$	1.87	1.93 ± 0.01	3
$A(\Lambda_-^0)$	1.61	1.48 ± 0.01	9
$A(\Xi_-^-)$	1.87	2.04 ± 0.02	8

TABLE III. Parity-conserving nonleptonic weak decay amplitudes.

Amplitude	Theory	Experiment (Ref. 14)	Deviation (%)
$B(\Sigma_+^+)$	17.18	19.05 ± 0.16	10
$B(\Sigma_-^-)$	−0.65	−0.65 ± 0.08	...
$B(\Sigma_0^+)$	−12.58	−12.04 ± 0.59	4
$B(\Lambda_-^0)$	10.11	10.17 ± 0.24	1
$B(\Xi_-^-)$	−6.46	−6.73 ± 0.41	4

of $B(\Sigma_-^-)$, but the present approach does not predict this automatically. Some higher symmetry or other dynamical constraint on the reduced matrix elements is still needed to prove that result. Using the parameters given in Eqs. (3.23a)–(3.23c), the prediction for the $B(\Omega_0^-{}^*)$ amplitude, in units of $G\mu_c^2$ is

$$B(\Omega_0^-{}^*) = -6.049\,. \tag{3.24}$$

If corrections to Eq. (3.21) begin in third or higher order in the mass difference, one may also conclude that

$$F(\Omega_0^-{}^*) = \frac{-2m_\Omega m_{\Xi^*}}{(m_\Omega + m_{\Xi^*})^2} B(\Omega_0^-{}^*) = 3.019\,. \tag{3.25}$$

The $B(\Omega_0^-)$ amplitude given in Eq. (3.20a) provides an interesting test of the importance of the Melosh transformation in nonleptonic weak decay. If the Melosh transformation is not present, the term Y_M will not appear in Eq. (3.20a). Thus without the Melosh transformation, one obtains the prediction

$$B(\Omega_0^-) = -19.21\,. \tag{3.26}$$

Significant deviations from this value would indicate that terms arising from the Melosh transformation must be present.

C. Partial decay width foı Ω^-

The effective Lagrangian for $\Omega^- \to \Xi^- \pi^0$ decay is

$$\mathcal{L}_{\text{eff}} = G\mu_c^2 \overline{\psi}_\lambda (B + D\gamma_5)\psi \frac{\partial_\lambda \varphi}{m_\Omega}\,, \tag{3.27a}$$

where the dimensionless constant

$$\frac{(G\mu_c^2)^2}{8\pi} = 1.9488 \times 10^{-15}$$

is used to conform with the conventions in the April 1974 Review of Particle Properties, and $\overline{\psi}_\lambda$, ψ, and φ are Rarita-Schwinger, Dirac, and pion fields, respectively. B and D are the p- and d-wave Lorentz-invariant amplitudes for this decay. The corresponding effective Lagrangian for $\Omega^- \to \Xi^* \pi^0$ decay is

$$\mathcal{L}_{\text{eff}} = i G\mu_c^2 \left[\overline{\psi}_\lambda (A + B\gamma_5)\psi_\lambda \varphi - \overline{\psi}_\lambda (D + F\gamma_5)\psi_\nu \frac{\partial_\lambda \partial_\nu \varphi}{m_\Omega m_{\Xi^*}} \right], \tag{3.27b}$$

where A, B, D, and F are the s-, p-, d-, and f-wave amplitudes, respectively. Using the projection operators

$$\sum_{\text{spin}} u(k)\overline{u}(k) = \frac{-i\gamma\cdot k + m}{2m} \tag{3.28a}$$

for Dirac spinors and

$$\sum_{\text{spin}} u_\lambda(k)\overline{u}_\nu(k) = \frac{-i\gamma\cdot k + m}{2m}\left[\delta_{\lambda\nu} + \frac{k_\lambda k_\nu}{m^2} - \frac{1}{3}\left(\delta_{\lambda\rho} + \frac{k_\lambda k_\rho}{m^2}\right)\left(\delta_{\nu\sigma} + \frac{k_\nu k_\sigma}{m^2}\right)\gamma^\rho\gamma^\sigma\right] \tag{3.28b}$$

for Rarita-Schwinger spinors[8] with the above effective Lagrangians, decay widths for these processes can be derived. They are

$$\Gamma = \frac{(G\mu_c^2)^2}{24\pi}\,\frac{q^3}{m_\Omega^4}\{|B|^2[(m_\Omega + m_\Xi)^2 - \mu^2] + |D|^2[(m_\Omega - m_\Xi)^2 - \mu^2]\} \tag{3.29a}$$

for $\Omega^- \to \Xi^- \pi^0$ and

$$\Gamma = \frac{(G\mu_c^2)^2}{8\pi}\,\frac{q}{m_\Omega^2}\,\frac{1}{9}\left[5 + \frac{(m_\Omega^2 + m_{\Xi^*}^2 - \mu^2)^2}{m_\Omega^2 m_{\Xi^*}^2}\right]\{|A|^2[(m_\Omega + m_{\Xi^*})^2 - \mu^2] + |B|^2[(m_\Omega - m_{\Xi^*})^2 - \mu^2] + \cdots\} \tag{3.29b}$$

for $\Omega^- \to \Xi^* \pi^0$, where μ is the mass of the pion and

$$q = \frac{\{[(m_1 + m_2)^2 - \mu^2][(m_1 - m_2)^2 - \mu^2]\}^{1/2}}{2m_1} \tag{3.30a}$$

is the center-of-mass momentum of the decay products. For $\Omega^- \to \Xi^{*-}\pi^0$ the center-of-mass momentum available to the decay products is

$$q = 0.0229\ \text{GeV}/c\,, \tag{3.30b}$$

which is small. Since the d- and f-wave amplitudes always appear with higher powers of q, their contribution to Eq. (3.29b) has been neglected. Similarly the q value for $\Omega^- \to \Xi^- \pi^0$ is

$$q = 0.290\ \text{GeV}/c\,. \tag{3.30c}$$

Since for consistency $(m_\Omega - m_\Xi)D = 0$ in the symmetry limit, the D amplitude can be neglected in Eq. (3.29a). The partial decay width for $\Omega^- \to \Xi^- \pi^0$ is calculated to be

$$\Gamma(\Omega_0^-) = 6.681 \times 10^{-15}\ \text{GeV} = 10.15 \times 10^9\ \text{sec}^{-1} \tag{3.31}$$

when the Melosh term Y_M is neglected. The total mean life for Ω^- is, however, 1.3×10^{-10} s which implies a total decay width of only[14]

$$\Gamma_{\text{total}} = (7.7 \pm 1.8) \times 10^9\ \text{s}^{-1}. \tag{3.32}$$

Thus, Eq. (3.31) is in clear disagreement with experiment. This indicates that the term Y_M arising from the Melosh transformation is essential in a consistent treatment of nonleptonic weak decay. Since the principal decay modes of Ω^- are $\Xi\pi$ and $K\Lambda$ and since the q value for these decays are nearly equal, it is reasonable to guess[17] that the branching ratio of each should be about 50%. The $\Delta I=\frac{1}{2}$ rule indicates that the contribution of $\Xi^0\pi^-$ to the decay width is twice that of $\Xi^-\pi^0$ so that the decay width to all $\Xi\pi$ modes should be

$$\Gamma_{\Xi\pi}=3\Gamma(\Omega_0^-)\,. \tag{3.33}$$

Using Eqs. (3.33) and (3.32), a 50% branching ratio into $\Xi\pi$ would require

$$B(\Omega_0^-)\sim-6.83$$

which indicates a value

$$Y_M\sim 2.21\ \text{GeV}^{-1}\,. \tag{3.34}$$

The estimate of Y_M in Eq. (3.34) is of the same order of magnitude as the other parameters in this approach, and therefore is not unreasonable.

The partial decay width $\Gamma(\Omega_0^-{}^*)$ for the process $\Omega^-\to\Xi^{*-}\pi^0$ is predicted to be

$$\Gamma(\Omega_0^-{}^*)=2.57\times10^{-16}\ \text{GeV}=0.391\times10^{9}\ \text{s}^{-1} \tag{3.35}$$

which gives a 5% branching ratio of $\Omega^-\to\Xi^{*-}\pi^0$. The $\Delta I=\frac{1}{2}$ rule again indicates that the branching ratio to all $\Xi^*\pi$ modes is then

$$\frac{\Gamma_{\Xi^*\pi}}{\Gamma_{\text{total}}}=\frac{3\Gamma(\Omega_0^-{}^*)}{\Gamma_{\text{total}}}=15\%\,. \tag{3.36}$$

In the present treatment Ξ^* is treated as though it is a stable particle. In fact, it has a width of 10 MeV. Since the $\Xi^*\pi$ system is close in mass to the Ω^-, only roughly half of the resonance is allowed kinematically to participate in the decay. Clearly Eq. (3.36) can only be considered an upper bound on the branching ratio of $\Omega^-\to\Xi^*\pi$. If one naively assumed that the actual width is proportional to the kinematically allowed area of the resonance, a branching ratio of 7–8% might be expected for these decay modes. The experimental situation itself is unclear concerning $\Xi^*\pi$ decay modes. A more detailed theoretical treatment which fully takes into account the resonant nature of Ξ^* is necessary before any definite conclusion can be reached concerning $\Omega^-\to\Xi^*\pi$ decays.

D. $\Omega^-\to K^-\Lambda$ decay

Using the method described in Sec. II with kaon rather than pion PCAC, it is possible to calculate explicit expressions for the Lorentz-invariant amplitudes of $\Omega^-\to K^-\Lambda$ decay. Since, however, the kaon mass is no longer small, these expressions can be expected to give at best an order-of-magnitude estimate for the amplitudes. Following the notation of Sec. II, the appropriate matrix element in the infinite-momentum limit (2.5) is

$$\lim\sqrt{2q_0}\,\langle\Lambda(p),K^-(0)|H_w^{\text{pv (pc)}}|\Omega^-(p)\rangle=\frac{i}{f_K}\Bigg\{-\langle\Lambda|[(\tfrac{1}{2})^{1/2}Q_5^{4+i5},H_w^{\text{pv (pc)}}]|\Omega^-\rangle + (m_\Omega{}^2-m_\Lambda{}^2)\sum_l\left(\frac{\langle\Lambda|(\tfrac{1}{2})^{1/2}A_0^{4+i5}|l\rangle\langle l|H_w^{\text{pv (pc)}}|\Omega\rangle}{m_\Omega{}^2-m_l{}^2} - \frac{\langle\Lambda|H_w^{\text{pv (pc)}}|l\rangle\langle l|(\tfrac{1}{2})^{1/2}A_0^{4+i5}|\Omega\rangle}{m_l{}^2-m_\Lambda{}^2}\right)_{\vec p_l=\vec p}\Bigg\}\,. \tag{3.37}$$

The equal-time-commutator term can be simplified for a current-current interaction by using the fact that

$$[(\tfrac{1}{2})^{1/2}Q_5^{4+i5},H_w^{\text{pv (pc)}}]=[(\tfrac{1}{2})^{1/2}Q^{4+i5},H_w^{\text{pc (pv)}}]\,. \tag{3.38}$$

Thus

$$\begin{aligned}\langle\Lambda|[(\tfrac{1}{2})^{1/2}Q_5^{4+i5},H_w^{\text{pv (pc)}}]|\Omega\rangle&=\langle\Lambda|[(\tfrac{1}{2})^{1/2}Q^{4+i5},H_w^{\text{pc (pv)}}]|\Omega\rangle\\&=\tfrac{1}{2}\sqrt{3}\,\langle\Xi^-|H_w^{\text{pc (pv)}}|\Omega\rangle\\&\quad-\sqrt{\tfrac{3}{2}}\langle\Lambda|H_w^{\text{pc (pv)}}|\Xi^{*0}\rangle\,.\end{aligned} \tag{3.39}$$

This term vanishes identically for each algebraic classification of the Hamiltonian given in (2.16a) and (2.16b). The infinite momentum limit of Eq. (3.37) is then taken analogously to that for $\Omega^-\to\Xi^-\pi^0$ decays, yielding the expressions

$$D(\Omega^-\to K^-\Lambda)=\frac{\sqrt6\,m_\Omega{}^2}{(m_\Omega-m_\Lambda)}\frac{f_\pi}{f_K}\times\left[\sum_l\left(\langle\Lambda|(\tfrac{1}{2})^{1/2}A_0^{4+i5}|l\rangle\langle l|H_w^{\text{pv}}|\Omega\rangle\right)_{\text{eff}}\right]_- \tag{3.40a}$$

and

$$B(\Omega^-\to K^-\Lambda)=\frac{\sqrt6\,m_\Omega{}^2f_\pi}{(m_\Omega+m_\Lambda)f_K}\times\left[\sum_l\left(\langle\Lambda|(\tfrac{1}{2})^{1/2}A_0^{4+i5}|l\rangle\langle l|H_w^{\text{pc}}|\Omega\rangle\right)_{\text{eff}}\right]_- \tag{3.40b}$$

for the parity-violating d-wave and parity-conserving p-wave amplitudes. The reduced matrix elements associated with

$$\left[\sum_l \left(\langle\Lambda|(\tfrac{1}{2})^{1/2}A_0^{4+i5}|l\rangle\langle l|H_w^{\text{pv (pc)}}|\Omega\rangle\right)_{\text{eff}}\right]_-$$

are the same as those for decays involving pions and f_K is the K-$\mu\nu$ decay constant. The explicit forms of the amplitudes are

$$(m_\Omega - m_\Lambda)D(\Omega^- \to K^-\Lambda) = \sqrt{6}\, m_\Omega{}^2 \frac{f_\pi}{f_K}\left[-\frac{8\sqrt{21}}{(45)^2}\left(X^{56,0}_{405,(8_B,3)} - X^{56,\pm1}_{405,(8_B,3)}\right) + \frac{2}{27\sqrt{5}}\left(X^{70,0}_{405,(8_B,3)} - X^{70,\pm1}_{405,(8_B,3)}\right)\right] \tag{3.41}$$

and

$$B(\Omega^- \to K^-\Lambda) = \frac{\sqrt{6}\, m_\Omega{}^2 f_\pi}{(m_\Omega + m_\Lambda)f_K}\left[\frac{4}{45}Y^{56,0}_{35,(8,1)} - \frac{1}{12}Y^{70,0}_{35,(8,1)} + \frac{12\sqrt{21}}{(45)^2}Y^{56,0}_{405,(8,1)} + \frac{1}{12\sqrt{5}}Y^{70,0}_{405,(8,1)} + \frac{2\sqrt{14}}{45\sqrt{15}}Y^{56,\pm1}_{405,(8_A,3)} + \frac{6\sqrt{42}}{(45)^2}\left(Y^{56,0}_{405,(8,5)} + \sqrt{3}\,Y^{56,\pm1}_{405,(8,5)}\right) + \frac{16\sqrt{21}}{(45)^2}Y^{56,\pm1}_{405,(8_B,3)} + \frac{2}{27\sqrt{5}}Y^{70,\pm1}_{405,(8_B,3)}\right]. \tag{3.41b}$$

The combination of reduced matrix elements on the right-hand side of Eq. (3.41b) is linearly independent of Y_1, Y_2, Y_3, and Y_M. Thus no numerical prediction can be made for the amplitude $B(\Omega^- \to K^-\Lambda)$ without further information. As in Eq. (3.7a) for $\Omega^- \to \Xi^-\pi^0$ decay consistency demands that in the symmetry limit

$$(m_\Omega - m_\Lambda)D(\Omega^- \to K^-\Lambda) \to 0.$$

Thus the right-hand side of Eq. (3.41a) must vanish giving the constraint equation

$$-\frac{4\sqrt{21}}{(45)^2}\left(X^{56,0}_{405,(8_B,3)} - X^{56,\pm1}_{405,(8_B,3)}\right) + \frac{1}{27\sqrt{5}}\left(X^{70,0}_{405,(8_B,3)} - X^{70,\pm1}_{405,(8_B,3)}\right) = 0. \tag{3.42}$$

This equation is linearly independent of Eq. (3.10) and is again consistent with the assumption that the 405 representation does not contribute to matrix elements of the parity-violating part of the weak decay Hamiltonian.

E. Additional consistency constraints

Equations (3.10) for the process $\Omega^- \to \Xi^-\pi^0$, Eqs. (3.10) and (3.22) for the process $\Omega^- \to \Xi^{*-}\pi^0$, and Eq. (3.42) for the process $\Omega^- \to K^-\Lambda$ are examples of constraint equations which occur because the Lorentz-invariant nonleptonic weak decay amplitudes are required to remain finite in the $SU(6)_W$ symmetric limit. Additional consistency constraints of this type must also exist for the other unobservable, but theoretically possible, nonleptonic weak decays of baryons in the 56, $L=0$ decuplet (10, 4). Decays of the form

$$Y(10,4) \to B(8,2)\pi, \tag{3.43a}$$

such as

$$Y^{*+} \to p\pi^0 \quad \text{or} \quad \Delta^+ \to \Sigma^+\pi^0,$$

and

$$Y(10,4) \to B(8,2)K, \tag{3.43b}$$

such as

$$\Xi^{*-} \to K^-n \quad \text{or} \quad \Delta^0 \to K^-p,$$

provide 28 processes which must satisfy the constraint condition

$$\langle B(8,2)\,\pi \quad \text{or} \quad K|H_w^{\text{pv}}|Y(10,4)\rangle = O((m_Y - m_B)^2) \tag{3.44}$$

For the W-spin 1 part of H_w^{pv}, the product of W-spin Clebsch-Gordan coefficients $C(L_z=0)$ in Eq. (2.21) for the $L_z=0$ contribution to the matrix elements is just

$$C(L_z=0) = -C(L_z=\pm1),$$

where $C(L_z=\pm1)$ is the corresponding W-spin factor for the $L_z=\pm1$ contributions, so that the $L_z=0$ and $L_z=\pm1$ terms will always appear in the combination

$$X^{R',0}_{R,(8,3)} - X^{R',\pm1}_{R,(8,3)} \equiv X^{R'}_{R,(8,3)}.$$

Thus the contraint Eqs. (3.44) have the general form

$$C_1 a^{405}_{8,5} + C_2 X^{56}_{35,(8,3)} + C_3 X^{70}_{35,(8,3)} + C_4 X^{56}_{405,(8_A,3)} + C_5 X^{70}_{405,(8_A,3)} + C_6 X^{56}_{405,(8_B,3)} + C_7 X^{70}_{405,(8_B,3)} + C_8 X^{56,\pm1}_{405,(8,5)} + C_9 X^{70,\pm1}_{405,(8,5)} = 0. \tag{3.45}$$

For all of the decays (3.43a) and (3.43b), the 35 contribution to the constraint (3.45) vanishes identically, i.e., $c_2 = c_3 = 0$. The nine decays of the forms

$$Y(10,4) \to B(10,4)\pi , \tag{3.46a}$$

such as

$$Y^{*+} \to \Delta^{+}\pi^{0} ,$$

and

$$Y(10,4) \to B(10,4)K , \tag{3.46b}$$

such as

$$\Omega^{-} \to Y^{*0}K^{-} ,$$

must satisfy the condition

$$\langle B(10,4)\,\pi \text{ or } K|H_w^{\text{pv}}|Y(10,4)J_z=\tfrac{3}{2}\rangle - \langle B(10,4)\,\pi \text{ or } K|H_w^{\text{pv}}|Y(10,4)J_z=\tfrac{1}{2}\rangle = O((m_Y - m_B)^2). \tag{3.47}$$

All of the decays (3.46a) give the same constraint equation (3.10). The decays (3.46b) involving kaons give a single additional constraint of the form (3.45) which is

$$\frac{-8\sqrt{3}}{135\sqrt{2}} X^{56}_{35,(8,3)} + \frac{\sqrt{3}}{18\sqrt{2}} X^{70}_{35,(8,3)} + \frac{4\sqrt{7}}{405\sqrt{5}} X^{56}_{405,(8_A,3)} + \frac{1}{54\sqrt{3}} X^{70}_{405,(8_A,3)} + \frac{4\sqrt{14}}{(45)^2} X^{56}_{405,(8_B,3)} - \frac{7}{54\sqrt{30}} X^{70}_{405,(8_B,3)} = 0 . \tag{3.48}$$

Thus the unobservable nonleptonic weak decays provide rather weak theoretical constraints on the reduced matrix elements of H_w^{pv}. The constraints are consistent with the assumption that the parity-violating part of the weak decay Hamiltonian transforms only as an $\text{SU}(6)_W$ 35, but are not alone sufficient to prove this assumption. The decays of the forms (3.46a) and (3.46b) also must satisfy the constraint condition

$$\langle B(10,4)\,\pi \text{ or } K|H_w^{\text{pc}}|Y(10,4)J_z=\tfrac{3}{2}\rangle - 3\langle B(10,4)\,\pi \text{ or } K|H_w^{\text{pc}}|Y(10,4)J_z=\tfrac{1}{2}\rangle = 0((m_Y - m_B)^2) \tag{3.49}$$

for the parity-conserving part of the weak decay Hamiltonian. All of the decays (3.46a) satisfy Eq. (3.49) identically. The decays (3.46b) involving kaons give one additional constraint which is

$$\frac{\sqrt{21}}{(45)^2}\left(Y^{56,0}_{405,(8,5)} - \frac{2}{\sqrt{3}} Y^{56,\pm1}_{405,(8,5)}\right) - \frac{1}{108\sqrt{5}}\left(Y^{70,0}_{405,(8,5)} - \frac{2}{\sqrt{3}} Y^{70,\pm1}_{405,(8,5)}\right) = 0 . \tag{3.50}$$

Again this is a very weak constraint which has no effect on the present results.

IV. CONCLUSION

In this paper the standard LSZ reduction technique and PCAC are used to obtain expressions for the nonleptonic weak decay amplitudes for baryons belonging to the $\text{SU}(6)_W$ 56, $L=0$ representation. These expressions are simplified by determining the Lorentz-invariant amplitudes in the infinite-momentum limit $p \to \infty$ for the baryons. This infinite-momentum limit and the soft-pion limit $q_0 \to 0$ required by PCAC are taken simultaneously such that

$$\lim_{p\to\infty;\,q_0\to 0} p_0 q_0 = \tfrac{1}{2}(m_Y{}^2 - m_B{}^2) ,$$

where Y and B are the initial and final baryons, respectively. We make the following general assumptions:

(a) The sum over all intermediate states in the expressions for the amplitudes may be replaced by a sum over the physical baryon resonances. Algebraically, this requires a sum over all $\text{SU}(6)_W$ 56 and 70 representations with arbitrary orbital angular momentum L.

(b) The hadronic weak decay interaction Hamiltonian is of the current-current type consistent with recent gauge models. This implies that the interaction Hamiltonian for these processes transforms like the $\text{SU}(6)_W$ 35 and 405 representations.

(c) The $\Delta I = \frac{1}{2}$ rule is rigorous for this Hamiltonian. Thus, the Hamiltonian must transform as an SU(3) octet.

(d) The Melosh transformation is applicable to the axial-vector current and Hamiltonian operators appearing in these expressions. The general algebraic properties expected for such Melosh-transformed operators are used with the Wigner-Eckart theorem to express the decay amplitudes in terms of Clebsch-Gordan coefficients times undetermined reduced matrix elements.

For the parity-violating decay amplitudes, we obtain a Lee-Sugawara–type relation which is satisfied experimentally to 10% and an additional sum rule involving the parity-violating Ω^- decay amplitudes. If we further assume that

(e) the parity-violating part of the Hamiltonian H_w^{pv} transforms only as an $SU(6)_W$ $\underline{35}$, we obtain

(1) $A(\Sigma_+^+)=0$.

(2) The Lee-Sugawara relation is exact in its original form

$$\sqrt{3}\,A(\Sigma_0^+)+A(\Lambda_-^0)-2A(\Xi_-^-)=0\,.$$

(3) The sum rule for Ω amplitudes reduces to

$$\frac{A(\Omega_0^{-*})}{(m_\Omega-m_{\Xi^*})}=\frac{\sqrt{6}\,A(\Sigma_-^-)}{4(m_\Sigma-m_N)}-\frac{3A(\Lambda_-^0)}{2(m_\Lambda-m_n)}\,.$$

(4) A single-parameter fit to the parity-violating amplitudes $A(\Sigma_0^+)$, $A(\Lambda_-^0)$, $A(\Sigma_-^-)$, and $A(\Xi_-^-)$ is good to 10%.

(5) Constraint conditions on the reduced matrix elements which arise from the requirement that the amplitudes remain finite in the $SU(6)_W$ symmetry limit are consistent with, but not sufficient to prove assumption (e).

The $\underline{405}$ contribution to the parity-conserving Hamiltonian is important. Thus under assumptions (a)–(d) we obtain the following results:

(1) A modified Lee-Sugawara relation

$$\frac{\sqrt{3}\,B(\Sigma_0^+)}{m_\Sigma+m_p}+\frac{B(\Lambda_-^0)}{(m_\Lambda+m_n)}=\frac{2B(\Xi_-^-)}{m_\Xi+m_\Lambda}$$

is exact. This is experimentally good to 7%.

(2) An exact sum rule involving Ω decay

$$\frac{B(\Omega_0^{-*})}{3(m_\Omega+m_{\Xi^*})}+\frac{B(\Xi_-^-)}{m_\Xi+m_\Lambda}+\frac{B(\Sigma_+^+)}{\sqrt{6}\,(m_\Sigma+m_N)}=0$$

is obtained.

(3) A three-parameter fit to the known octet amplitudes is good to 10%.

(4) Terms arising from the Melosh transformation are necessary to bring the decay width for $\Omega^-\to\Xi\pi$ into agreement with experiment. This indicates that the Melosh transformation is important in nonleptonic weak decay.

(5) An upper limit on the branching ratio of the decay mode $\Omega^-\to\Xi^*\pi$ is 15%. The treatment of this decay mode is somewhat ambiguous due to the width of Ξ^* and the closeness in mass of the $\Xi^*\pi$ system to the Ω^-. An estimate of this effect reduces the branching ratio by about half. The present work provides a consistent algebraic treatment of nonleptonic weak decay. It allows for the smallness of $B(\Sigma_-^-)$ and the apparent $\underline{35}$ dominance of H_w^{pv}, but it cannot predict them automatically. Some higher symmetry or dynamical mechanism which is outside of the scope of the present work is needed to completely explain these two points.

APPENDIX A: QUARK-DENSITY MODEL

A simple model from which the $\Delta I=\frac{1}{2}$ rule follows automatically is the quark-density model. In the quark-density model the Hamiltonian is bilinear in the quark fields and thus belongs only to the $SU(6)_W$ $\underline{35}$ representation with

$$H_w^{pv}\sim\bar{\psi}\gamma_5\lambda^6\psi \tag{A1a}$$

and

$$H_w^{pc}\sim i\bar{\psi}\lambda^7\psi\,. \tag{A1b}$$

The matrices λ^6 and λ^7 are 3×3 matrix representations of the sixth and seventh SU(3) generators. The algebraic classification of the Melosh-transformed Hamiltonian is then

$$H_w^{pv}\sim\{(8,3)_0,0\}_{35},\{(8,3)_1,-1\}_{35}+\{(8,3)_{-1},1\}_{35} \tag{A2a}$$

and

$$H_w^{pc}\sim\{(8,1)_0,0\}_{35},\{(8,3)_1,-1\}_{35}-\{(8,3)_{-1},1\}_{35}\,. \tag{A2b}$$

Using (A2a) and (A2b) to evaluate Clebsch-Gordan coefficients and the forms (A1a) and (A1b) to evaluate the infinite-momentum limit in Eq. (2.6), expressions are obtained for the Lorentz-invariant weak decay amplitudes. The parity-violating amplitudes can be written

$$A(\Sigma_0^+)=\frac{1}{6\sqrt{5}}\,a_{8,1}^{35}+(m_{\Sigma^+}-m_p)X'\,, \tag{A3a}$$

$$\sqrt{2}\,A(\Sigma_0^0)=\frac{1}{6\sqrt{5}}\,a_{8,1}^{35}+(m_{\Sigma^0}-m_n)X'\,, \tag{A3b}$$

$$A(\Lambda_-^0)=\sqrt{3}\left[\frac{1}{6\sqrt{5}}\,a_{8,1}^{35}+(m_\Lambda-m_n)X'\right], \tag{A3c}$$

$$A(\Xi_-^-)=\sqrt{3}\left[\frac{1}{6\sqrt{5}}\,a_{8,1}^{35}+(m_\Xi-m_\Lambda)X'\right] \tag{A3d}$$

with

$$X'=-\frac{X_{35,(8,3)}^{56,0}}{45\sqrt{2}}-\frac{2X_{35,(8,3)}^{56,\pm1}}{45\sqrt{2}}-\frac{X_{35,(8,3)}^{70,0}}{24\sqrt{2}}-\frac{X_{35,(8,3)}^{70,\pm1}}{12\sqrt{2}}\,.$$

Using the phase conventions (3.3), it is clear that

$$A(\Sigma_+^+)=0 \tag{A4}$$

automatically. Similarly, using the Gell-Mann–Okubo mass formula, one can see that the Lee-Sugawara relation

$$\sqrt{3}\,A(\Sigma_0^+)+A(\Lambda_-^0)-2B(\Xi_-^-)=0 \tag{A5}$$

is satisfied exactly. In this model, the parity-conserving amplitudes are

$$B(\Sigma_0^+)=\frac{-b^{35}_{8,3}}{18\sqrt{5}}-\frac{(m_\Sigma+m_p)}{(m_\Sigma-m_p)}\,g_A\,\frac{a^{35}_{8,1}}{30\sqrt{5}}+(m_\Sigma+m_p)[5Y-Y'], \tag{A6a}$$

$$\sqrt{2}\,B(\Sigma_+^+)=\frac{g_A a^{35}_{8,1}}{30\sqrt{5}}\left[5\frac{(m_\Sigma+m_N)}{(m_\Sigma-m_N)}+\frac{(m_\Sigma+m_p)}{(m_\Sigma-m_p)}-6\frac{(m_\Sigma+m_N)}{(m_\Lambda-m_n)}\right]-10(m_\Sigma+m_N)Y, \tag{A6b}$$

$$\sqrt{2}\,B(\Sigma_-^-)=\frac{-b^{35}_{8,3}}{9\sqrt{5}}+\frac{g_A a^{35}_{8,1}}{30\sqrt{5}}\left[5\frac{(m_\Sigma+m_N)}{(m_\Sigma-m_N)}-\frac{(m_\Sigma+m_p)}{(m_\Sigma-m_p)}-6\frac{(m_\Sigma+m_N)}{(m_\Lambda-m_n)}\right]-2(m_\Sigma+m_N)Y', \tag{A6c}$$

$$B(\Lambda_-^0)=\sqrt{3}\left\{\frac{b^{35}_{8,3}}{6\sqrt{5}}+\frac{g_A a^{35}_{8,1}}{30\sqrt{5}}\left[5\frac{(m_\Lambda+m_n)}{(m_\Lambda-m_n)}-2\frac{(m_\Lambda+m_n)}{(m_\Sigma-m_n)}\right]+(m_\Lambda+m_n)(-5Y+3Y')\right\}, \tag{A6d}$$

$$B(\Xi_-^-)=\sqrt{3}\left\{\frac{b^{35}_{8,3}}{18\sqrt{5}}+\frac{g_A a^{35}_{8,1}}{30\sqrt{5}}\left[2\frac{(m_\Lambda+m_\Xi)}{(m_\Xi-m_\Sigma)}-\frac{(m_\Xi+m_\Lambda)}{(m_\Xi-m_\Lambda)}\right]+(m_\Xi+m_\Lambda)(-2Y+Y')\right\} \tag{A6e}$$

where $g_A=1.25$ is the axial-vector coupling constant. The reduced matrix elements for $L>0$ intermediate states are combined into the parameters Y and Y', where

$$Y=\frac{2Y^{56,0}_{35,(8,1)}}{135\sqrt{2}}-\frac{Y^{70,0}_{35,(8,1)}}{72\sqrt{2}}$$

and

$$Y'=\frac{Y^{56,0}_{35,(8,1)}}{135\sqrt{2}}-\frac{Y^{70,0}_{35,(8,1)}}{36\sqrt{2}}+\frac{2Y^{56,\pm1}_{35,(8,3)}}{135\sqrt{2}}+\frac{Y^{70,\pm1}_{35,(8,3)}}{36\sqrt{2}}.$$

The vanishing of $B(\Sigma_-^-)$ is not explicit in the form (A6c), a Lee-Sugawara-type relation for the B amplitudes does not follow automatically, and a combined fit shown in Table IV to minimize the percentage error in the A and B amplitudes using this parametrization is good to only 40%. Thus, although the model works very well for the parity-violating amplitudes, the addition of the Melosh transformation is not sufficient to bring the B amplitudes into agreement with experiment.

TABLE IV. Quark-density model fit to A and B amplitudes.

Amplitude	Theory	Experiment (Ref. 14)	Deviation (%)
$A(\Sigma_-^-)$	1.75	1.93±0.01	12
$A(\Xi_-^-)$	1.70	2.04±0.02	8
$A(\Lambda_-^0)$	1.87	1.48±0.01	18
$B(\Sigma_+^+)$	14.61	19.05±0.16	22
$B(\Sigma_-^-)$	0.54	−0.65±0.08	
$B(\Xi_-^-)$	−5.11	−6.73±0.41	31
$B(\Lambda_-^0)$	13.98	10.17±0.24	38

APPENDIX B: EFFECTIVE BILINEAR FORM FOR CURRENT-CURRENT HAMILTONIAN

If, as indicated by recent work on gauge theories, the weak interaction is mediated by heavy vector bosons, the nonleptonic weak decay of a baryon is a second-order process in the weak interaction for which the S matrix is given by

$$S^{(2)}=\frac{(-i)^2}{2!}\int\int d^4x\,d^4y\;T(H(x)H(y)), \tag{B1}$$

where $H(x)$ is the interaction Hamiltonian in gauge field models. Pictorially, the matrix element of the S matrix between two quarks q and q' is given in Fig. 1 where the shaded area represents a contraction over the vector boson and all strong interaction corrections. More explicitly,

$$S^{(2)}_{fi}=\frac{-g^2}{2}\int\int d^4x\,d^4y\;\bar\psi(x)\gamma_\mu(1+\gamma_5)$$
$$\times\qquad\qquad\gamma_5)\psi(y) \tag{B2}$$

where ψ is the quark field and $G_{\mu\nu}(x,y)$ is a general function representing the contractions and strong interaction corrections indicated schematically in Fig. 1. The SU(3) indices have been suppressed for simplicity. Defining new variables

$$X=\tfrac{1}{2}(x+y)$$

and

$$r=x-y,$$

Eq. (B2) is rewritten as

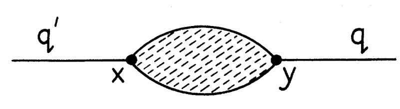

FIG. 1. The diagram for the second-order S-matrix element of the weak interaction between two quarks.

$$S_{fi}^{(2)}=\frac{-g^2}{2}\int\int d^4r\, d^4X\, \bar\psi(X+\tfrac{1}{2}r)\gamma_\mu(1+\gamma_5)G_{\mu\nu}(X,r)\gamma_\nu(1+\gamma_5)\,\psi(X-\tfrac{1}{2}r)$$
$$\equiv -i\int d^4r\, H_{\rm eff}(r) \tag{B3}$$

which defines an effective interaction Hamiltonian density. $H_{\rm eff}(0)$ then has the form

$$H_{\rm eff}(0)=\frac{ig^2}{2}\int d^4X\,\bar\psi(X)\gamma_\mu(1+\gamma_5)G_{\mu\nu}(X,0)\gamma_\nu(1+\gamma_5)\,\psi(X). \tag{B4}$$

In terms of Lorentz invariants, $G_{\mu\nu}(X,0)$ has the general form

$$G_{\mu\nu}(X,0)=\delta_{\mu\nu}\gamma_\rho\frac{\partial}{\partial X_\rho}G_1(X^2)+\gamma_\mu\frac{\partial}{\partial X_\nu}G_2(X^2)+\gamma_\nu\frac{\partial}{\partial X_\mu}G_3(X^2), \tag{B5}$$

where terms which are not linear in γ matrices are dropped since they vanish in the expression

$\gamma_\mu(1+\gamma_5)G_{\mu\nu}(X,0)\gamma_\nu(1+\gamma_5)$.

Using Eq. (B5) one obtains

$$\gamma_\mu(1+\gamma_5)G_{\mu\nu}(X,0)\gamma_\nu(1+\gamma_5)=2\gamma_\mu G_{\mu\nu}(X,0)\gamma_\nu(1+\gamma_5)$$
$$=4\gamma_\rho\frac{\partial}{\partial X_\rho}[-G_1(X^2)+2G_2(X^2)+2G_3(X^2)](1+\gamma_5) \tag{B6}$$

so that $H_{\rm eff}(0)$ becomes

$$H_{\rm eff}(0)=2ig^2\int d^4X\,\bar\psi(X)\gamma_\mu(1+\gamma_5)\psi(X)\frac{\partial}{\partial X_\mu}[-G_1(X^2)+2G_2(X^2)+2G_3(X^2)]. \tag{B7}$$

Assuming that the quark field operators act on quarks within the baryons, the weak decay matrix element of interest between baryon states Y and B becomes

$$\langle B(p_2)|H_{\rm eff}(0)|Y(p_1)\rangle=2ig^2\left(\frac{m_1m_2}{E_1E_2}\right)^{1/2}\bar u(p_2)\gamma_\mu(1+\gamma_5)u(p_1)$$
$$\times\int d^4X\,e^{-i(p_2-p_1)X}\frac{\partial}{\partial X_\mu}[-G_1(X^2)+2G_2(X^2)+2G_3(X^2)]. \tag{B8}$$

Integrating (B8) by parts gives the result

$$\langle B(p_2)|H_{\rm eff}(0)|Y(p_1)\rangle=-2g^2\left(\frac{m_1m_2}{E_1E_2}\right)^{1/2}(p_2-p_1)_\mu\bar u(p_2)\gamma_\mu(1+\gamma_5)u(p_1)F((p_2-p_1)^2), \tag{B9}$$

where $F((p_2-p_1)^2)$ is the Fourier transform of the function

$[-G_1(X^2)+2G_2(X^2)+2G_3(X^2)]$.

Thus the effective bilinear form for a matrix element between two baryon states arising from a current-current weak interaction is

$$\langle B(p_2)|H_{\rm eff}(0)|Y(p_1)\rangle\sim\left(\frac{m_1m_2}{E_1E_2}\right)^{1/2}(p_2-p_1)_\mu\bar u(p_2)\gamma_\mu(1+\gamma_5)u(p_1). \tag{B10}$$

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