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Scattering of polarized electrons in the quark model and in the quark-diquark model

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The inelastic scattering of polarized electrons from polarized nucleons in the resonance region is studied in the quark model and in the quark-diquark model. It is suggested that by the measurement of the polarized cross section in the resonance region information about masses of the constituent particles (quark or quark and diquark) will be obtained.

I. INTRODUCTION

In the nonrelativistic quark model it has been known that if we use a harmonic wave function, which is indicated by level spacing, the elastic form factor is predicted to be a Gaussian.[1,2] This form factor drops much more rapidly with increasing momentum transfer than does the observed dipole form factor. If a $1/r$ potential is used, then the elastic form factors are improved, but it predicts too small values for the higher-resonance production processes.[3]

Recently, the author has calculated[4,5] the form factors, using the modified Woods-Saxon potential (MWP),

$$V(r) = -V_0 \frac{r/b+1}{r/b+\exp[(r-R)/a]}\,. \qquad (1)$$

This potential has merits of both the harmonic-oscillator potential (HOP) and the $1/r$ potential, and gives good agreement with the experimental data.[4,5] This potential also predicts desirable energy levels.[4]

Similar results have been obtained in the quark-diquark model[6] proposed by the author[7] and Lichtenberg[8].

In the calculations made by Thornber for the HOP and for the $1/r$ potential, the nonrelativistic form factors were obtained in the N^* rest frame. After that, an improvement of the nonrelativistic approximation was proposed.[9-11] First one calculates the S-matrix element relativistically and separates out the kinematical factors. Thus the Lorentz-invariant matrix elements are extracted. If the relativistic calculation of this matrix element were possible, this matrix element should have a constant value for any frame. However, since we need to calculate it nonrelativistically this quantity depends on the frame chosen. If we choose the frame in which the nonrelativistic approximation is good, i.e., the least velocity frame[9] (LVF), then we can obtain the best approximation.

In this note we compute the asymmetry parameter of the polarized electron scattering using this approximation. This experiment is supposed to be made in the near future. Several years ago experimental data for the unpolarized electroproduction processes were obtained by the SLAC-MIT group.[12-14] In this experiment three prominent bumps in the inelastic electron-proton scattering cross section were observed. The lowest peak is at $W=1236$ MeV. This corresponds to $\Delta_{33}(1236)$. The next bump is at $W=1525$ MeV (S_{11}, D_{13}), and the last at $W=1690$ MeV $(D_{15}, D_{13}, F_{15}, S_{11}, S_{31}, D_{33})$. We compute the asymmetry parameter, assuming that in the polarized experiments, which will be done in the near future, these three bumps will also be observed and the asymmetry parameters will be measured not for each resonance but for each bump.

The unpolarized cross section contains only the combined forms of

$$\sum_J |(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)|^2$$

and

$$\sum_J [|(J_f||\hat{T}_J^{\text{el}}||J_i)|^2+|(J_f||\hat{T}_J^{\text{mag}}||J_i)|^2]$$

of the reduced matrix elements. On the contrary, the polarized cross section involves the magnitudes and the relative phases of the quantities $(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)$, $(J_f||\hat{T}_J^{\text{el}}||J_i)$, and $(J_f||\hat{T}_J^{\text{mag}}||J_i)$. Therefore, these relative phases will be obtained if we make a polarized experiment.

In Sec. II we have shown how to make the nonrelativistic approximation in the cross section. In Sec. III the polarized cross section is expressed in terms of the reduced matrix elements. In Sec. IV and in Sec. V, the asymmetry parameters are obtained using the quark model and quark-diquark model, respectively. In Sec. VI we present several concluding remarks.

II. CROSS SECTION

The scattering matrix in the Born approximation is given by

$$S=1-i\int d^4x\hat{H}_1(x)$$
$$=1-\frac{\alpha}{q^2}\int d^4x j^e_\mu(x)J_\mu(x)\,, \qquad (2)$$
$$\alpha=1/137.036$$

where $j^e_\mu(x)$ is the electron current, $J_\mu(x)$ is the nucleon current, and q^2 is the square of the four-momentum transfer.

Denoting the matrix element of the interaction operator between initial wave functions and final wave functions by M_{fi}, the transition matrix element is given by

$$S_{fi}=\delta_{fi}-i(2\pi)^4\delta^4(p_e'+P'-p_e-P)M_{fi} \qquad (3)$$

and the cross section by

$$d\sigma=\frac{E\epsilon}{P\cdot p_e}|M_{fi}|^2\frac{d\vec{p}_e'}{(2\pi)^3}\frac{d\vec{P}'}{(2\pi)^3}(2\pi)^4 \times\delta^4(p_e'+P'-p_e-P)\,, \qquad (4)$$

$$=\frac{1}{4\pi^2}(\epsilon\epsilon'EE'|M_{fi}|^2)\frac{d\vec{p}_e'}{\epsilon'}\frac{d\vec{P}'}{E'}\frac{1}{P\cdot p_e} \times\delta^4(p_e'+P'-p_e-P)\,, \qquad (5)$$

where the notations of momenta and energies are the same as those of Refs. 4–6 and are shown in Fig. 1. Since this expression of the scattering cross section is in Lorentz-invariant form, the factor $\epsilon\epsilon' EE'|M_{fi}|^2$ should be the same in all frames. We choose to calculate it in LVF (the frame in which the proton and the resonance have equal and opposite velocities), because in this frame the velocities of the proton and the resonance become minimum.[9-11] We will mention the frame dependence of the obtained results later.

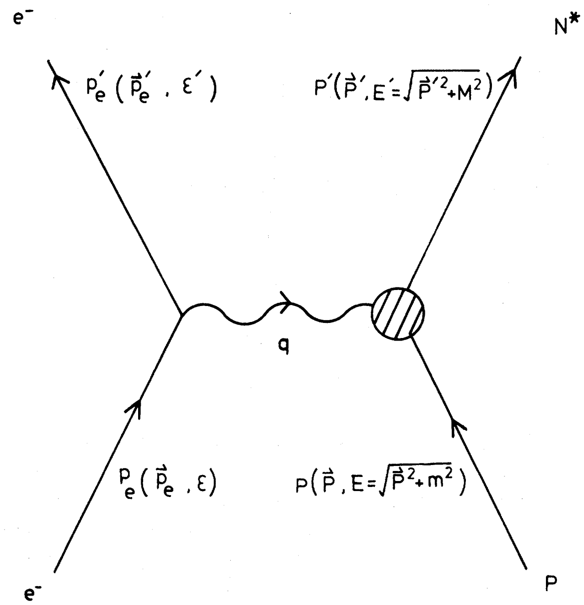

FIG. 1. Isobar electroproduction in the one-photon exchange approximation.

If the factor $\epsilon\epsilon' EE'|M_{fi}|^2$ is calculated in the laboratory frame, one gets

$$\left.\frac{d\sigma_0}{d\Omega}\right|_{\rm lab}=2\pi\left(\frac{d\sigma}{d\Omega}\right)_{NS}\left\{\frac{q^4}{\vec{q}^4}\sum_{J=0}^{\infty}|(J_f\|\hat{M}_J^{\rm Coulomb}(|\vec{q}|)\|J_i)|^2 +\left(\frac{q^2}{2\vec{q}^2}+\tan^2\frac{\theta}{2}\right)\sum_{J=1}^{\infty}[|(J_f\|\hat{T}_J^{\rm el}(|\vec{q}|)\|J_i)|^2+|(J_f\|\hat{T}_J^{\rm mag}(|\vec{q}|)\|J_i)|^2]\right\}\,, \qquad (6)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{NS}=\frac{\alpha^2\cos^2(\theta/2)}{4\epsilon^2\sin^4(\theta/2)}\frac{1}{1+(2\epsilon/m)\sin^2(\theta/2)}\,, \qquad (7)$$

where

$$\hat{M}_{JM}^{\rm Coulomb}(|\vec{q}|)=\int d^3x j_J(|\vec{q}|x)Y_{JM}(\Omega_x)\rho(\vec{x})\,,$$
$$\hat{T}_{JM}^{\rm el}(|\vec{q}|)=\frac{1}{|\vec{q}|}\int d^3x[\vec{\nabla}\times j_J(|\vec{q}|x)\vec{Y}^M_{JJ1}(\Omega_x)]\cdot\vec{J}(\vec{x})\,, \qquad (8)$$
$$\hat{T}_{JM}^{\rm mag}(|\vec{q}|)=\int d^3x[j_J(|\vec{q}|x)\vec{Y}^M_{JJ1}(\Omega_x)]\cdot\vec{J}(\vec{x})\,,$$

where $\vec{J}(\vec{x})$ and $\rho(\vec{x})$ are the nucleon-currents operators to be calculated in various models. $\vec{Y}^M_{JJ1}(\Omega_x)$ is a spherical vector harmonic. θ is the scattering angle. Equation (6) is the expression of the unpolarized cross section obtained by de Forest and Walecka.[15]

III. POLARIZATION EFFECT

The most general calculation, in the framework of the first Born approximation, of the effects of electron polarization and target orientation which has arbitrary spin on nuclear electron scattering was performed by Weigert and Rose.[16] The form factors were calculated in the laboratory frame.

The differential cross section for the electroexcitation of the oriented nucleon by polarized incident electrons is composed of two parts:

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_0}{d\Omega}+\frac{d\sigma_p}{d\Omega}, \tag{9}$$

where $d\sigma_0/d\Omega$ is the differential cross section of unpolarized electrons from unoriented nucleon target. The remaining contribution $d\sigma_p/d\Omega$ comes from the polarization of the incident beam and the orientation of the nucleon target. The contribution from the orientation of the target with unpolarized electrons does not arise, because in our case the spin of the target is $\frac{1}{2}$. This term contributes if the spin of the target is more than 1.

The polarized cross section differs from the unpolarized cross section only if both the target nucleon and the incident electron are polarized.

Calculating the invariant quantities $\epsilon\epsilon' EE'|M_{fi}|^2$ in LVF, we obtain the unpolarized cross section

$$\left.\frac{d\sigma_0}{d\Omega}\right|_{\text{lab}}=\frac{2\pi(d\sigma/d\Omega)_{NS}}{\cos^2(\theta/2)\epsilon\epsilon' EE'}\left(\epsilon\epsilon' EE'\cos^2(\theta/2)\left\{\frac{q^4}{\vec{q}^4}\sum_{J=0}^{\infty}|(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)|^2 + \left(\frac{q^2}{\vec{q}^2}+\tan^2\frac{\theta}{2}\right)\sum_{J=1}^{\infty}[|(J_f||\hat{T}_J^{\text{el}}||J_i)|^2+|(J_f||\hat{T}_J^{\text{mag}}||J_i)|^2]\right\}\right)_F\frac{E'}{m}, \tag{10}$$

where the quantities in parentheses or brackets with the subscript F should be calculated in LVF and the other quantities in the laboratory frame. $d\sigma_0/d\Omega|_{\text{lab}}$ is the cross section measured in the laboratory frame. The last factor E'/m was neglected in Ref. 15.

The scattering cross section of polarized electrons from the oriented nucleon target is

$$\left.\frac{d\sigma_p}{d\Omega}\right|_{\text{lab}}=\frac{2\pi(d\sigma/d\Omega)_{NS}}{\cos^2(\theta/2)\,\epsilon\epsilon' EE'}(EE'A)_F\,\frac{E'}{m}, \tag{11}$$

$$\begin{aligned}A=\alpha_1\sum_{LL'}(i)^{L-L'}\Big\{&\vec{V}_T\cdot\vec{P}_e P_1(\cos\theta_N)\,F_1^{(1)}(LL'J_fJ_i)\\&\times[(J_f||\hat{M}_L^{\text{Coulomb}}||J_i)(J_f||\hat{M}_{L'}^{\text{Coulomb}}||J_i)^*+(J_f||\hat{T}_L^{\text{mag}}||J_i)(J_f||\hat{T}_{L'}^{\text{mag}}||J_i)^*\\&+2(J_f||\hat{T}_L^{\text{el}}||J_i)(J_f||\hat{T}_{L'}^{\text{el}}||J_i)^*]\\&+\frac{\sqrt{2}\,q^2\vec{V}_I\cdot\vec{P}_e}{\vec{q}^2}P_1^1(\cos\theta_N)\,F_1^{(01)}(LL'J_fJ_i)(J_f||\hat{M}_L^{\text{Coulomb}}||J_i)\,[(J_f||\hat{T}_{L'}^{\text{el}}||J_i)-(J_f||\hat{T}_{L'}^{\text{mag}}||J_i)]\Big\},\end{aligned} \tag{12}$$

$$F_\nu^{(1)}(LL'J_fJ_i)=(-1)^{J_f-J_i+1}[(2L+1)(2L'+1)(2J_i+1)]^{1/2}\,W(J_iJ_iLL';\nu J_f)(L1L'-1|\nu 0), \tag{13}$$

$$F_\nu^{(01)}(LL'J_fJ_i)=(-1)^{J_f-J_i+1}[(2L+1)(2L'+1)(2J_i+1)]^{1/2}\,W(J_iJ_iLL';\nu J_f)(L0L'1|\nu 1), \tag{14}$$

where α_1 is the first-rank statistical tensor[17] for the target nucleon. If the target nucleon is oriented parallel to the z axis, $\alpha_0=\alpha_1=\frac{1}{2}$, and if the target nucleon is oriented antiparallel to the z axis, $\alpha_0=\frac{1}{2}$, $\alpha_1=-\frac{1}{2}$. J_i and J_f are the spins of the target nucleon and the final resonance, respectively. $W(J_iJ_iLL';\nu J_f)$ are Racah coefficients. $\vec{P}_e$ is the polarization vector of the incident electron. The vectors $\vec{V}_T$ and $\vec{V}_I$ are described by their components along the three directions $\hat{p}_e$ (the direction of the incident beam), $\hat{s}\equiv\vec{p}_e\times\vec{p}_e'/|\vec{p}_e\times\vec{p}_e'|$ (the unit normal to the scattering plane), and $\hat{a}\equiv\hat{s}\times\hat{p}_e$ (lying in the scattering plane).

$$\begin{aligned}&\vec{V}_T\cdot\hat{p}_e=\epsilon|\vec{q}|-\epsilon q_0\hat{q}\cdot\hat{p}_e,\\&\vec{V}_T\cdot\hat{a}=O(m_e)\approx 0,\\&\vec{V}_T\cdot\hat{s}=0,\\&\vec{V}_I\cdot\hat{p}_e=-\epsilon\epsilon'\sin\theta\cos\phi_N,\\&\vec{V}_I\cdot\hat{a}=O(m_e)\approx 0,\\&\vec{V}_I\cdot\hat{s}=O(m_e)\approx 0,\end{aligned} \tag{15}$$

where the angle θ_N and ϕ_N describe the direction of the target orientation $\hat{n}$ in the $(\hat{s}\times\hat{q})$-$\hat{s}$-$\hat{q}$-system (see Fig. 2).

In this paper we neglect the mass of electron

m_e. Therefore, $\vec{V}_T$ and $\vec{V}_I$ are parallel to the initial electron momentum and the target orientation perpendicular to the incident electron momentum gives no contributions to the asymmetry parameter.

In the quark model the final resonance state and the initial proton state are composed of three quarks. Since the spin of the quark is $\frac{1}{2}$, the total spin (S_f) of the three quarks in the final resonance state is $\frac{1}{2}$ or $\frac{3}{2}$. This spin angular momentum S_f couples with the orbital angular momentum L_f. Therefore, the total angular momentum of the final resonance state J_f becomes $J_f = L_f \pm \frac{1}{2}, \pm\frac{3}{2}$. In the multipole expansion for the N^*-N-γ vertex, matrix elements for the Coulomb transition $(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)$ and for the electric transition $(J_f||\hat{T}_J^{\text{el}}||J_i)$ are not zero only for $J = L_f$ and the matrix element for the magnetic transition $(J_f||\hat{M}_J^{\text{mag}}||J_i)$ is not zero only for $J = L_f + P_f$, where

$$\begin{aligned} &P_f = 1 \text{ for } J_f = L_f + \tfrac{1}{2} \text{ or } J_f = L_f + \tfrac{3}{2}, \\ &P_f = -1 \text{ for } J_f = L_f - \tfrac{1}{2} \text{ or } J_f = L_f - \tfrac{3}{2}. \end{aligned} \tag{16}$$

The values of J for the nonzero reduced matrix elements in the quark-diquark model are the same as those in the quark model, assuming that S_f and L_f are the total spin of the quark and the diquark and the orbital angular momentum between the quark and the diquark, respectively.

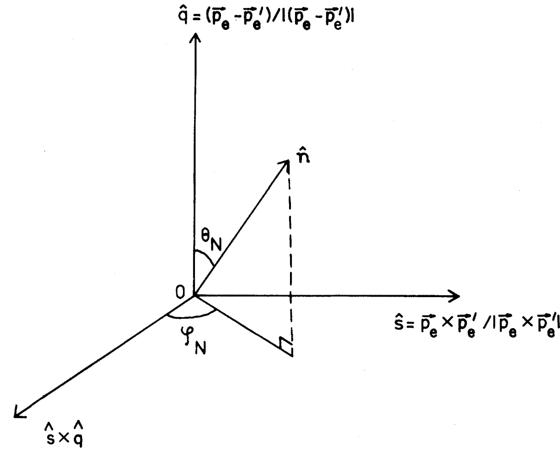

FIG. 2. The angles θ_N and ϕ_N describing the direction of target orientation $\hat{n}$ in the $(\hat{s}\times\hat{q})$-$\hat{s}$-$\hat{q}$ system.

Now we can obtain the asymmetry parameters[18]

$$\Delta_\perp \equiv \frac{(d\sigma/d\Omega)(\uparrow\leftarrow) - (d\sigma/d\Omega)(\uparrow\rightarrow)}{(d\sigma/d\Omega)(\uparrow\leftarrow) + (d\sigma/d\Omega)(\uparrow\rightarrow)} = O(m_e) \approx 0, \tag{17}$$

$$\Delta_\parallel \equiv \frac{(d\sigma/d\Omega)(\uparrow\downarrow) - (d\sigma/d\Omega)(\uparrow\uparrow)}{(d\sigma/d\Omega)(\uparrow\downarrow) + (d\sigma/d\Omega)(\uparrow\uparrow)} = \left(\frac{N}{U}\right)_F, \tag{18}$$

We have defined

$$U = 2\epsilon\epsilon' \cos^2\frac{\theta}{2} \left\{ \frac{q^4}{\vec{q}^4} |(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)|^2 + \left(\frac{q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right) [|(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)|^2 + |(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)|^2] \right\}, \tag{19}$$

$$\begin{aligned} N = &-\vec{V}_T \cdot \hat{p}_e \cos\theta_q \{ F_1^{(1)}(L_f L_f J_f \tfrac{1}{2}) |(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)|^2 + F_1^{(1)}(L_f + P_f, L_f + P_f, J_f, \tfrac{1}{2}) |(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)|^2 \\ &\qquad + 2F_1^{(1)}(L_f, L_f + P_f, J_f, \tfrac{1}{2}) (J_f||\hat{T}_{L_f}^{\text{el}}||J_i)[-i(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)]^* (-1)^{(P_f+1)/2} \} \\ &+ \frac{q^2}{\vec{q}^2} \sqrt{2}\, \vec{V}_I \cdot \hat{p}_e \sin\theta_q \{ F_1^{(01)}(L_f, L_f, J_f, \tfrac{1}{2}) (J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)^* \\ &\qquad - F_1^{(01)}(L_f, L_f + P_f, J_f, \tfrac{1}{2}) (J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)[-i(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)]^* (-1)^{(P_f+1)/2} \}. \end{aligned} \tag{20}$$

$(d\sigma/d\Omega)(\uparrow\downarrow)$, for example, is the cross section when spins of the electron and the nucleon are antiparallel. θ_q is the angle between $\vec{q}$ and $\vec{p}_e$. The values of $F_1^{(1)}(LL'J_f\frac{1}{2})$ and $F_1^{(01)}(LL'J_f\frac{1}{2})$ for each resonance are listed in Table I.

IV QUARK MODEL RESULTS

In the first place we calculate the asymmetry parameter in the quark model. Assuming that quarks are point particles, electric densities and currents are

$$\begin{aligned} \rho(\vec{x}) &= \sum_{i=1}^{3} \hat{Q}(i)\delta(\vec{x} - \vec{r}_i), \\ \vec{J}(\vec{x}) &= \sum_{j=1}^{3} \frac{\hat{Q}(j)}{2im_q}[\delta(\vec{x} - \vec{r}_j)\vec{\nabla} + \vec{\nabla}\delta(\vec{x} - \vec{r}_j)] \\ &\quad + \vec{\nabla} \times \sum_{i=1}^{3} \hat{Q}(i)\mu_q \delta(\vec{x} - \vec{r}_i)\vec{\sigma}(i). \end{aligned} \tag{21}$$

Using these expressions, the absolute values of the reduced matrixes $(J_f|\,|\hat{M}^{\text{Coulomb}}|\,|J_i)$, $(J_f|\,|\hat{T}^{\text{el}}|\,|J_i)$, and $(J_f|\,|\hat{T}^{\text{mag}}|\,|J_i)$ are already

TABLE I. The quantities $F_1^{(1)}(LL'J_f\frac{1}{2})$ and $F_1^{(01)}(LL'J_f\frac{1}{2})$, the phases of the reduced matrix elements, and the asymmetry parameter $\Delta_\parallel$, for each resonance. 1. $F_1^{(1)}(L_fL_fJ_f\frac{1}{2})$. 2. $F_1^{(1)}(L_f+P_f, L_f+P_f, J_f, \frac{1}{2})$. 3. $F_1^{(1)}(L_f, L_f+P_f, J_f, \frac{1}{2})$. 4. $F_1^{(01)}(L_fL_fJ_f\frac{1}{2})$. 5. $F_1^{(01)}(L_f, L_f+P_f, J_f, \frac{1}{2})$. 6. $(J_f\|\hat{M}_{L_f}^{\mathrm{Coulomb}}\|J_i)/|(J_f\|\hat{M}_{L_f}^{\mathrm{Coulomb}}\|J_i)|$ in the quark model (QM). 7. $(J_f\|\hat{T}_{L_f+P_f}^{\mathrm{mag}}\|J_i)/|(J_f\|\hat{T}_{L_f+P_f}^{\mathrm{mag}}\|J_i)|$ in the QM. 8. $(J_f\|\hat{T}_{L_f}^{\mathrm{el}}\|J_i)/|(J_f\|\hat{T}_{L_f}^{\mathrm{el}}\|J_i)|$ in the QM. 9. $\lim_{q^2\to 0}\Delta_\parallel$ in the QM. 10. $\lim_{q^2\to 0}\Delta_\parallel$ in the quark-diquark model.

	P_f	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
P_{11} (940)	1	0	−1	0	0	−1	1	i	···	0	0
P_{11}(1470)	1	0	−1	0	0	−1	1	i	···	1	1
D_{13}(1520)	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	1	i	−1	$-\frac{1}{2}$	$-\frac{1}{2}$
S_{11}(1535)	−1	−1	0	0	1	0	1	···	1	1	1
F_{15}(1690)	1	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2\sqrt{2}}{3}$	$-\frac{1}{\sqrt{3}}$	$-(\frac{2}{3})^{1/2}$	1	i	−1	$-\frac{7}{9}$	$-\frac{7}{9}$
P_{11}(1751)	1	0	−1	0	0	−1	1	i	···	1	1
P_{13}(1861)	−1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	1	i	1	$-\frac{1}{2}$	···
G_{17}(2190)	1	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{\sqrt{15}}{4}$	$-(\frac{3}{8})^{1/2}$	$-(\frac{5}{8})^{1/2}$	1	i	−1	$-\frac{7}{8}$	···
P_{33}(1236)	1	0	$\frac{1}{2}$	0	0	0	···	$-i$	···	$-\frac{1}{2}$	$-\frac{1}{2}$
S_{31}(1640)	−1	−1	0	0	1	0	1	···	−1	1	1
D_{33}(1691)	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	1	$-i$	1	$-\frac{1}{2}$	$\frac{1}{7}$
F_{35}(1913)	1	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2\sqrt{2}}{3}$	$-\frac{1}{\sqrt{3}}$	$-(\frac{2}{3})^{1/2}$	···	$-i$	−1	$\frac{71}{171}$	···
P_{31}(1934)	−1	0	−1	0	0	0	···	i	···	1	···
F_{37}(1950)	1	0	$\frac{1}{4}$	0	0	0	···	$-i$	···	$-\frac{1}{4}$	$-\frac{1}{4}$

obtained and listed in Ref. 4 in the quark model. In order to compute the polarized cross sections, we must also know the relative phases among these reduced matrixes. These are listed in Table I. These phases coincide with those in Ref. 19. We must mention that the phases listed in Ref. 19 are obtained in the limit of $\vec{q}^2 \to 0$, but our results are calculated in the limit of $\vec{q}^2 \to \infty$.

If the masses of the quarks m_q are infinity, reduced matrix elements $(J_f|\,|\hat{M}^{\mathrm{Coulomb}}|\,|J_i)$, $(J_f|\,|\hat{T}^{\mathrm{el}}|\,|J_i)$, and $(J_f|\,|\hat{T}^{\mathrm{mag}}|\,|J_i)$ are proportional to the overlap integral I_{fi}:

$$I_{fi}(|\vec{q}|) = \int r^2 R_{L_f}(r) j_{L_f}(|\vec{q}|r) R_S(r)\,dr\,. \qquad (22)$$

From Eqs. (18)–(20) we see that $\Delta_\parallel$ for each resonance does not depend on the form of the binding potential as long as we calculate it separately for each resonance. Since the second peak and the third peak are composed of several resonances, $\Delta_\parallel$ for these peaks still depends on the form of the potential.

A. Elastic scattering

The elastic scattering cross sections depend neither on the quark mass nor on the form of the binding potential. From Eqs. (18)–(20) it follows

$$\Delta_\parallel(\text{elastic}) = \left(\frac{\mu(|\vec{q}|^3\mu\cos\theta_q - |\vec{q}|\,\epsilon'\sin\theta\sin\theta_q)}{\epsilon'\cos^2(\theta/2)\{1+[1+2\tan^2(\theta/2)]\vec{q}^2\mu^2\}}\right)_F, \qquad (23)$$

Since for elastic scattering $q^2 = \vec{q}^2$ holds,

$$\lim_{q^2\to 0} \Delta_\parallel(\text{elastic}) = 0\,. \qquad (24)$$

The asymmetry parameter $\Delta_\parallel$(elastic) predicted in the quark model is shown in Fig. 3 for various scattering angles θ. $\Delta_\parallel$ is negative and $|\Delta_\parallel|$ is small ($\lesssim 0.1$) for the small scattering angles ($\theta \lesssim 30°$). $\Delta_\parallel$ becomes large and positive (~1) for the large scattering angles ($\theta \gtrsim 135°$).

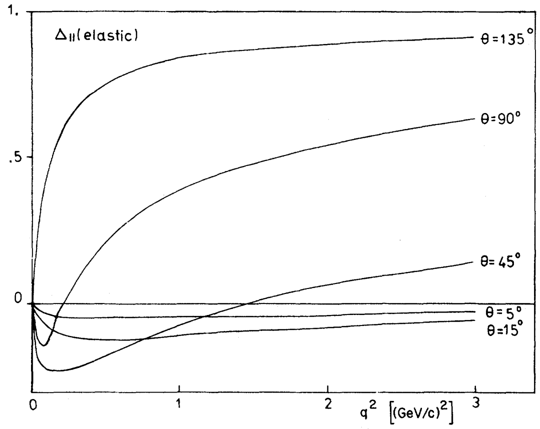

FIG. 3. The q^2 dependence of $\Delta_{\parallel}$ (elastic) for various scattering angles θ obtained in the quark model (QM). $\Delta_{\parallel}$ (elastic) depends neither on m_q nor on the form of the potential. The quark-diquark model (QDM) predicts the same $\Delta_{\parallel}$ as that of the QM.

B. First peak: $\Delta_{33}(1236)$

$\Delta_{\parallel}$ for this peak also depends neither on the quark mass nor on the form of the potential. The asymmetry parameter $\Delta_{\parallel}$ is

$$\Delta_{\parallel}(\text{first peak}) = -\left\{\frac{(|\vec{q}| - q_0\cos\theta_q)\cos\theta_q}{2\epsilon'\cos^2(\theta/2)[q^2/\vec{q}^2 + 2\tan^2(\theta/2)]}\right\}_F, \tag{25}$$

and

$$\lim_{q^2\to 0}\Delta_{\parallel}(\text{first peak}) = -\tfrac{1}{2}. \tag{26}$$

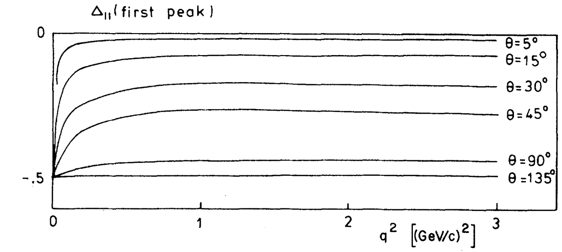

FIG. 4. Same as Fig. 3 but with $\Delta_{\parallel}$ (first peak).

The predictions of this model are shown in Fig. 4.

Except for small q^2 [$q^2 \lesssim 0.6$ (GeV/c)^2] $\Delta_{\parallel}$ does not sensitively depend on q^2. $\Delta_{\parallel}$ is negative for all q^2 and θ. $|\Delta_{\parallel}|$ becomes large as θ becomes large and it approaches $|\Delta_{\parallel}| \sim 0.5$.

C. Second peak

The second peak is composed of $D_{13}(1520)$ and $S_{11}(1535)$. For the unpolarized scattering $D_{13}(1520)$ dominates this peak in the quark model.[4] The predicted asymmetry parameter for the polarized scattering sensitively depends on the quark mass m_q.

In the first place let us assume that masses of quarks are infinity ($m_q = \infty$). Neglecting the mass difference between $D_{13}(1520)$ and $S_{11}(1535)$, one gets

$$\Delta_{\parallel}(D_{13}) = -\left(\frac{q^2\mu\sin\theta\sin\theta_q + (|\vec{q}|/\epsilon')(|\vec{q}| - q_0\cos\theta_q)\cos\theta_q\vec{q}^2\mu^2}{2|\vec{q}|\cos^2(\theta/2)\{q^4/\vec{q}^4 + \vec{q}^2\mu^2[q^2/\vec{q}^2 + 2\tan^2(\theta/2)]\}}\right)_F = -\tfrac{1}{2}\Delta_{\parallel}(S_{11}), \tag{27}$$

$$\lim_{q^2\to 0}\Delta_{\parallel}(D_{13}) = -\tfrac{1}{2}, \quad \lim_{q^2\to 0}\Delta_{\parallel}(S_{11}) = 1. \tag{28}$$

Adding the contributions of D_{13} and S_{11}, we obtain

$$\Delta_{\parallel}(D_{13} + S_{11}) = \Delta_{\parallel}(\text{second peak}) = 0. \tag{29}$$

For the finite quark mass, this accidental cancellation disappears and $\Delta_{\parallel}$(second peak) depends on A_1 defined in Ref. 4,

$$A_1 = \int r^2 R_P(r)[j_2(|\vec{q}|r) + j_0(|\vec{q}|r)]\frac{\partial}{\partial r}R_S(r)dr, \tag{30}$$

where R_S and R_P are radial wave functions for the S state and P state, respectively. Assuming $m_q = m_p/2.793$, i.e., $g_q = 1$, and using MWP with the set of parameters[4]

$$R = 1.1\text{ fm}, \quad a = 0.03R, \quad b = 0.07R, \quad V_0 = 23.5/(m_q R^2), \tag{31}$$

we obtain $\Delta_{\parallel}$(second peak). The results are shown in Fig. 5. The predicted $\Delta_{\parallel}$ is positive. The larger θ becomes, the larger becomes the value of $\Delta_{\parallel}$.

D. Third peak

This peak is composed of $D_{15}(1680)$, $D_{13}(1675)$, $F_{15}(1690)$, $S_{11}(1710)$, $S_{31}(1640)$, and $D_{33}(1691)$. Owing to the Moorhouse selection rule[20] the matrix elements of $D_{15}(1680)$, $D_{13}(1675)$, and $S_{11}(1710)$ vanish in the quark model. Among the remaining resonances, $F_{15}(1690)$ dominates this peak for the polarized scattering cross section as well as the unpolarized cross section.[4] Assuming $m_q=\infty$, $\Delta_{\parallel}$ for this peak is

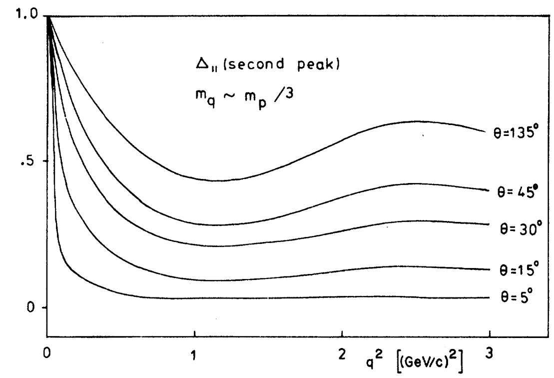

FIG. 5. The q^2 dependence of $\Delta_{\parallel}$ (second peak) obtained in QM using MWP with $m_q=m_p/2.793$.

$$\Delta_{\parallel}(F_{15}) = -\left(\frac{\frac{7}{9}\vec{q}^{\,2}\mu^2(|\vec{q}| - q_0\cos\theta_q)\cos\theta_q + (q^2/3|\vec{q}|)\epsilon'\mu\sin\theta\sin\theta_q}{\epsilon'\cos^2(\theta/2)\{q^4/\vec{q}^{\,4} + [q^2/\vec{q}^{\,2} + 2\tan^2(\theta/2)]\vec{q}^{\,2}\mu^2\}}\right)_F,$$

$$\lim_{q^2\to 0}\Delta_{\parallel}(F_{15}) = -\tfrac{7}{9}. \tag{32}$$

The summed asymmetry parameter for the third peak, $\Delta_{\parallel}$(third peak), depends both on m_q and on the form of the potential. The predicted $\Delta_{\parallel}$(third peak) with $m_q=\infty$ is shown in Fig. 6 for MWP with the parameter (31) and in Fig. 7 for HOP.

As is expected, $\Delta_{\parallel}$ does not depend on the form of the potential sensitively.

The predicted $\Delta_{\parallel}$(third peak) is negative. For large scattering angle ($45° \lesssim \theta \lesssim 135°$), $\Delta_{\parallel}$(third peak) is around -0.6 and $\lim_{q^2\to 0}\Delta_{\parallel} \sim -0.6$. For small θ, $|\Delta_{\parallel}|$ becomes small.

Although $\Delta_{\parallel}$ does not sensitively depend on the potential, it depends on m_q considerably. Assuming $m_q=m_p/2.793$, and using MWP with parameters (28), we obtain $\Delta_{\parallel}$(third peak). The results are shown in Fig. 8. $\Delta_{\parallel}$ still has negative values, but $|\Delta_{\parallel}|$ is considerably smaller than that of $m_q=\infty$, and $\lim_{q^2\to 0}\Delta_{\parallel} \sim -0.08$.

V. QUARK-DIQUARK MODEL RESULTS

In this section we examine the results of the quark-diquark model. We use the resonance assignment in this model shown in Ref. 7. As was shown,[21] the relations among magnetic moments of members of the lowest $\frac{1}{2}^+$ baryon octet predicted in our model are completely the same as those predicted in the quark model if a relation $g_q=4g_d$ holds (g_q and g_d are the gyromagnetic ratios of the quark and diquark, respectively). We will investigate the asymmetry parameter using the g_q and g_d obtained from the magnetic moments of the proton and neutron.[21] This is the same assumption as in Ref. 6.

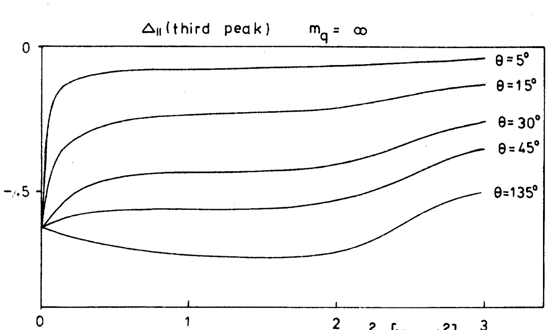

FIG. 6. The q^2 dependence of $\Delta_{\parallel}$ (third peak) obtained in QM using MWP with $m_q=\infty$.

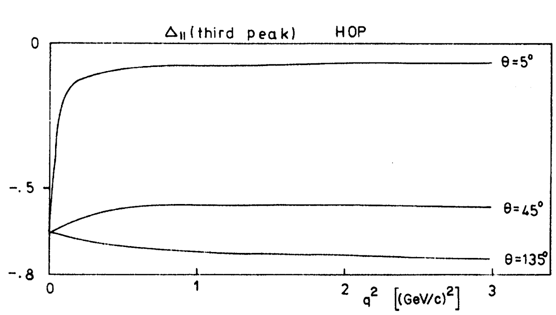

FIG. 7. Same as Fig. 6 but using harmonic-oscillator potential (HOP).

The predicted asymmetry parameter in the quark-diquark model has similar properties as in the quark model. Assuming that quarks and diquarks are point particles and that masses of quarks and diquarks are infinite, the electric density and the current are[6]

$$\rho(\vec{x}) = \hat{Q}_q \delta(\vec{x} - \vec{r}_q) + \hat{Q}_d \delta(\vec{x} - \vec{r}_d), \tag{33}$$

$$\vec{J}(\vec{x}) = \frac{\hat{Q}_q}{2iM_q}[\delta(\vec{x} - \vec{r}_q)\vec{\nabla}]_{\text{sym}} + \frac{\hat{Q}_d}{2iM_d}[\delta(\vec{x} - \vec{r}_d)\vec{\nabla}]_{\text{sym}} + \delta(\vec{x} - \vec{r}_q)\hat{\mu}_q \vec{\nabla} \times \frac{\vec{\sigma}_q}{2} + \delta(\vec{x} - \vec{r}_d)\hat{\mu}_d \vec{\nabla} \times \vec{J}_d, \tag{34}$$

where

$$\hat{\mu}_q = \frac{g_q}{2m}\hat{Q}_q \text{ and } \hat{\mu}_d = \frac{g_d}{2m}\hat{Q}_d.$$

M_q and M_d are the masses of a quark and diquark, respectively. From the experimental values of the magnetic moments of a proton and neutron, we obtain[21] $g_q = 16.8$ and $g_d = 4.2$.

For $M_q = \infty$, $M_d = \infty$, reduced matrix elements $(J_f \| \hat{M}^{\text{Coulomb}} \| J_i)$, $(J_f \| \hat{T}^{\text{el}} \| J_i)$ and $(J_f \| \hat{T}^{\text{mag}} \| J_i)$ are proportional to the overlap integral $J_{fi}(|\vec{q}|)$:

$$J_{fi}(|\vec{q}|) \equiv \int r^2 R_{L_f}(r) j_{L_f}(|\vec{q}| r/2) R_S(r) dr, \tag{35}$$

where $R_{L_f}(r)$ is the radial wave function between the quark and the diquark in the final state.

Comparing with Eq. (22), one gets

$$J_{fi}(|\vec{q}|) = I_{fi}(|\vec{q}|/2). \tag{36}$$

The factor 2 comes from the removal of the center-of-mass motion. For the elastic scattering and for the first resonance production processes, the predicted $\Delta_{\parallel}$ in the quark-diquark model is completely the same as that in the quark model.

For the second peak the relations

$$\Delta_{\parallel}(D_{13}) = -\tfrac{1}{2}\Delta_{\parallel}(S_{11}), \tag{37}$$

$$\lim_{q^2 \to 0} \Delta_{\parallel}(D_{13}) = -\tfrac{1}{2}, \quad \lim_{q^2 \to 0} \Delta_{\parallel}(S_{11}) = 1, \tag{38}$$

$$\Delta_{\parallel}(D_{13} + S_{11}) = \Delta_{\parallel}(\text{second peak}) = 0, \tag{39}$$

hold in the quark-diquark model as well as in the quark model.

The only difference between the quark model and the quark-diquark model comes from the third peak composed of[6] $D_{15}(^4P_{5/2})$, $F_{15}(^2D_{5/2})$, $S_{11}(^4P_{1/2})$, $S_{31}(^4P_{1/2})$, and $D_{33}(^4P_{3/2})$. Since the Moohrhause selection rule[20] does not hold in the quark-diquark model, none of contributions of these resonances vanishes. $F_{15}(^2D_{5/2})$ gives the largest contribution to the asymmetry parameter $\Delta_{\parallel}$, but the contributions from the other resonances are not negligible.

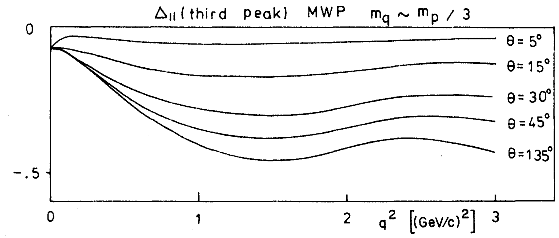

FIG. 8. Same as Fig. 6 but with $m_q = m_p/2.793$.

The results which are calculated using MWP between the quark and diquark with parameters[6]

$$R = 2.8 \text{ fm}, \quad a = 0.05R, \quad b = 0.15R,$$
$$V_0 = 2 \times 9.38/(\mu R^2), \quad \mu = \tfrac{1}{2}m_q, \tag{40}$$

are shown in Fig. 9.

The predicted Δ is negative and $|\Delta_{\parallel}|$ is substantially smaller than those in the quark model. Since in the quark-diquark model the Moorhouse selection rule does not hold, many resonances contribute to this peak. In the numerator of $\Delta_{\parallel}$ they cancel with each other, but in the denominator of $|\Delta_{\parallel}|$ they contribute positively. Therefore, the absolute values of $\Delta_{\parallel}$ in the quark-diquark model become smaller than those in the quark model.

VI. COMMENTS AND SUMMARY

Till now we have shown the results in the case where nonrelativistic form factors are calculated in LVF. The larger the difference between the mass of the initial target (proton) and that of the final resonance becomes, the more $\Delta_{\parallel}$ depends on the frame. Therefore, in order to see the frame dependence of our results we have shown the third resonance predictions for LVF and for Breit frame in Fig. 10. As can be seen from this figure, $\Delta_{\parallel}$ does not depend on the frame sensitively. This is also true for the other resonance production processes which are considered here.

As is shown in Sec. V, except for the third peak the quark model and quark-diquark model predict nearly the same values for $\Delta_{\parallel}$. From Table I we see that the predicted values of $\lim_{q^2 \to 0} \Delta_{\parallel}$ in both models are also nearly identical. In Sec. IV we

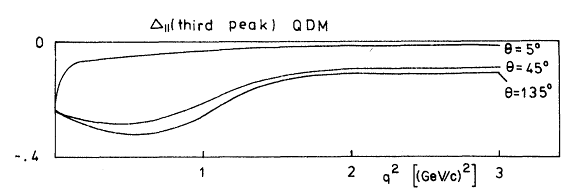

FIG. 9. The q^2 dependence of $\Delta_{\parallel}$ (third peak) obtained in QDM using MWP.

have already mentioned that $\Delta_{\parallel}$ does not sensitively depend on the form of the binding potential.

On the contrary, $\Delta_{\parallel}$ depends on the masses of the quark and/or the diquark sensitively for the second peak and for the third peak. Especially for the second peak, if the mass of the quark in the quark model is infinity, the asymmetry parameter is nearly zero. If the mass of the quark is, for example, $m_q/2.793$, $\Delta_{\parallel}$ becomes large positive. For the third peak $|\Delta_{\parallel}|$ decreases rapidly if the mass of the quark is changed from infinity to $m_q/2.793$. Similar results are obtained in the quark-diquark model.

Therefore, we can obtain information about masses of constituent particles (quark or diquark) by measuring $\Delta_{\parallel}$ in the energy region of the second and third peak.

From the unpolarized cross section, the q^2 dependence of the combinations $|(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)|^2$ and $|(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)|^2+|(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)|^2$ were obtained and from them the form of the binding potential was determined.[4-6] The relative phases between reduced matrixes $(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)$, $(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)$, and $(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)$ can also be determined from the data of the polarized cross sections.

(i) Elastic scattering. Assuming the scaling law $\mu_p G_E/G_M=1$ and $(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)=0$, which hold in the quark model and in the quark-diquark model, we get

$$\frac{|(J_f|\hat{M}_{L_f}^{\text{Coulomb}}||J_i)|}{|(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)|}=\frac{\sqrt{2}\,m_p}{\mu_p|\vec{q}|}=\frac{1}{\sqrt{2}\,|\vec{q}|\mu},\qquad \mu_p=2.793,\quad \mu=\frac{\mu_p}{2m_p},\tag{41}$$

and

$$\Delta_{\parallel}(\text{elastic})=\left(\frac{\mu^2|\vec{q}|^3\cos\theta_q\,[1-(-1)^{\Pi}(\epsilon'/\vec{q}^{\,2}\mu)\sin\theta\tan\theta_q]}{\epsilon'\cos^2(\theta/2)\{1+[1+2\tan^2(\theta/2)]\vec{q}^{\,2}\mu^2\}}\right)_F,\tag{42}$$

where

$$(-)^{\Pi}=\frac{(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)/|(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)|}{-i(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)/|(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)|}\tag{43}$$

In the quark model and in the quark-diquark model, $(-)^{\Pi}=1$. In this way, the relative phase between $(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)$ and $(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)$ can be determined by measuring $\Delta_{\parallel}$.

(ii) First peak: $\Delta_{33}(1236)$. For the decay process emitting a real photon ($\Delta_{33}\to N\gamma$), the E_2 amplitude vanishes in the quark model[22] and in the quark-diquark model[21] and agrees with the experimental data.[23] For the electroproduction process $(J_f||\hat{T}_{L_f}^{\text{el}}||J_i)$ and $(J_f||\hat{M}_{L_f}^{\text{Coulomb}}||J_i)$ are zero in both the models. Only one reduced matrix element $(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)$ remains.

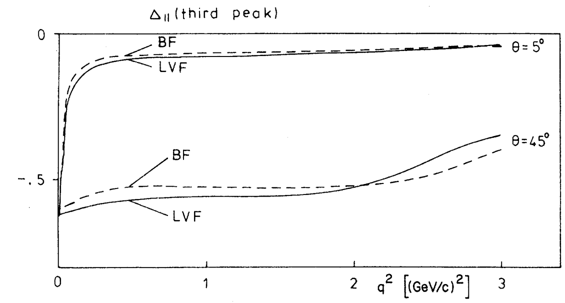

FIG. 10. Same as Fig. 6 but nonrelativistic form factors are calculated in LVF and in the Breit frame (BF). From Fig. 3 to Fig. 9 form factors are calculated in LVF.

The absolute phase of $(J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)$ has no physical meaning and no new information about reduced matrix elements will be obtained by the polarized experiment for the first peak.

(iii) Second peak and third peak. Theoretically, it is possible to determine the relative phases of the resonances composing these peaks from the polarized scattering data. However, these determinations are very complicated and we do not mention them in this paper.

Finally we will briefly mention the results of the neutron scattering, especially for the elastic $en\to en$ scattering and for $en\to e\Delta_{33}^0$ scattering. The reduced matrix elements and the asymmetry parameter for the $en\to en$ scattering in the quark model are

$$(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)=(J_f||\hat{T}_J^{\text{el}}||J_i)=0,\qquad (J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)=-\frac{2}{3}\frac{1}{\sqrt{\pi}}|\vec{q}|i\mu I_{SS},\tag{44}$$

$$\Delta_{\parallel}(en\to en)=\left\{\frac{|\vec{q}|\cos\theta_q}{\epsilon'[1+\sin^2(\theta/2)]}\right\}_F.\tag{45}$$

$\Delta_{\parallel}(en\to en)$ is independent both of the quark mass and of the form of the binding potential. The quark-diquark model predicts the same $\Delta_{\parallel}(en\to en)$. We have shown the results in Fig. 11. The predicted $\Delta_{\parallel}(en\to en)$ is positive. For the small values of the scattering angle θ ($\theta\lesssim 5°$), $\Delta_{\parallel}(en\to en)$ becomes very small ($\Delta_{\parallel}\lesssim 0.007$).

For the process $en\to e\Delta_{33}^0(1236)$ one gets

$$(J_f||\hat{M}_J^{\text{Coulomb}}||J_i)=(J_f||\hat{T}_J^{\text{el}}||J_i)=0,\qquad (J_f||\hat{T}_{L_f+P_f}^{\text{mag}}||J_i)=-\frac{4}{\sqrt{3\pi}}|\vec{q}|i\mu I_{SS},\tag{46}$$

$$\Delta_{\parallel}(en\to e\Delta^0_{33}) = -\left\{\frac{(|\vec{q}|-q_0\cos\theta_q)\cos\theta_q}{2\epsilon'\cos^2(\theta/2)[q^2/\vec{q}^2+2\tan^2(\theta/2)]}\right\}_F, \tag{47}$$

in the quark model. Therefore,

$$\Delta_{\parallel}(en\to e\Delta^0_{33}) = \Delta_{\parallel}(ep\to e\Delta^+_{33}). \tag{48}$$

$\Delta_{\parallel}(en\to e\Delta^0_{33})$ predicted in the quark-diquark model is the same as that in the quark model.

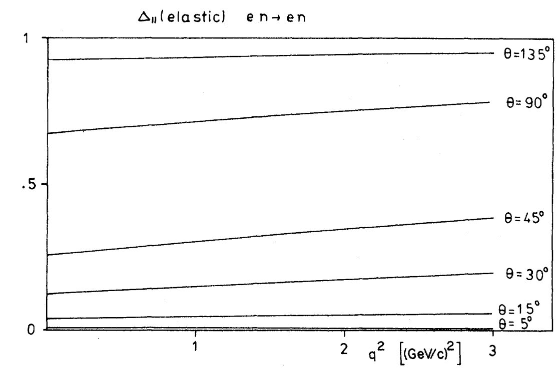

FIG. 11. Same as Fig. 3 but with $\Delta_{\parallel}(en\to en)$. $\Delta_{\parallel}(en\to en)$ depends neither on m_q nor on the form of the potential. QDM predicts the same $\Delta_{\parallel}$ as that of QM.

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