Hadron-nucleus collisions at high energies*

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We discuss high-energy hadron-nucleus scattering in terms of models in which the basic hadron-hadron interaction is due to the exchange of Regge poles and cuts. Much of the paper uses a configuration-space description in which the time scales of interactions are of critical importance, although equivalent results may be obtained in momentum space. We study in detail the general behavior of elastic, total, and inclusive cross sections, the relevance of the Glauber expansion, and the influence of Regge cuts and interactions. Most of the discussion is based on the softened-field-theory model of Reggeons, although we also consider the contrasting planar dual string model and show how one may experimentally distinguish between models of these two types.

I. INTRODUCTION

In this paper we shall discuss some of those aspects of high-energy hadron-nucleus collisions which are not strongly dependent on the detailed dynamics of the nucleus. We shall treat only large nuclei in that effects due to the edge of the nucleus will be neglected in comparison to area effects. In particular we shall discuss in detail total cross sections, elastic cross sections, and the distribution of produced particles of low transverse momentum. Most of our discussion will be based on the softened-field-theory model of Regge poles and cuts.

The first question which naturally arises is whether one can obtain any information from hadron-nucleus collisions which is not already available from hadron-hadron collisions. The answer to this seems to be in the affirmative. Crudely speaking, hadron-nucleus collisions allow one to obtain information on the time scale and longitudinal distances, involved in hadron-hadron interactions. (That something of this sort should be the case has been emphasized by Gottfried[1] for some time.) We shall attempt to clarify this point in some detail later. The only other types of experiments which seem to give such direct information as to the time scales involved in almost-on-shell hadron interactions are the inelastic electron experiments at SLAC, although the information from these is somewhat different. Before going further into the development of hadron-nucleus collisions, let us recall what the SLAC experiments have to say about hadronic time scales.[2]

The imaginary part of the virtual Compton amplitude, measured at SLAC, is

$$\int d^4x\, e^{-iq\cdot x}(P|j_\nu(x)j_\mu(0)|P)\propto P_\mu P_\nu W_2(\nu,q^2)+\cdots ,$$

where $\nu=2P\cdot q$. $|P)$ is a proton state and spin averaging is assumed. The total photodisintegration cross section is proportional to

$$\lim_{q^2\to 0}\frac{1}{q^2}W_2(\nu,q^2).$$

The deeply inelastic electron scattering experiments tell us the strengths of the light-cone singularities of $(P|j_\nu(x)j_\mu(0)|P)$. If we now fix q^2 (even $q^2=0$ is allowed) and let $\nu\to\infty$,

$$\frac{P_\mu P_\nu}{q^2}W_2(\nu,q^2)\propto\frac{1}{q^2}\int d^4x\exp\left[-i\frac{\nu}{2m}(x_0-x_3)-i\frac{mq^2}{\nu}x_3\right](P|j_\nu(x)j_\mu(0)|P)-\cdots ,$$

where we have written

$$q_\mu=\left(\frac{\nu}{2m},0,0,\left[\left(\frac{\nu}{2m}\right)^2-q^2\right]^{1/2}\right)$$

in the laboratory frame of the proton. The singularity at $x^2=0$ is not strong enough to maintain the observed $1/\nu$ behavior of $(1/q^2)W_2(\nu,q^2)$. Such a high-energy behavior must come from coordinate regions $x_0,x_3=O(\nu)$ with $x^2=O(1/q^2)$. Thus, long distances and times are involved in the *elastic* Compton amplitude. This is, of course, well known, and we only emphasize it here because any reasonable hadron theory which does not have a sharp distinction between scattering of *real photons* and hadrons must involve long time scales for the elastic hadron-hadron amplitude. We should also say that the long time scale which we will mostly be referring to in this paper is the long time scale of an elastic amplitude.[3] We shall discuss in some detail a string model without loops[4] in which the time scale for the elastic amplitude is short although there may well be a long time scale involved in the formation of a multiparticle final state in an inelastic collision. The string model without loops has a time scale incompatible with SLAC results. This means that either (i) hadron-hadron scattering is very different from on-shell photon-hadron scattering or (ii) the nonloop model is not a realistic model of high-energy elastic hadron-hadron scattering and loop corrections are significant.

The theory which we shall deal with in most detail in this paper is the Regge pole model which comes from a soft field theory, a particular realization of which is the multiperipheral model.[5] This model naturally has the long time scales which the SLAC results seem to demand. Also, the general features of high-energy hadron-hadron collisions are nicely accounted for once one stipulates, but does not derive, that Regge cuts are small compared to Regge poles. The detailed structure of the field theory is not important for these general considerations of space-time structure, only that momentum transfers are not large and that elementary vector mesons are not present. In the scattering of a fast particle of momentum p off a target at rest the important interactions begin already at a time proportional to p *before* the incoming projectile wave packet reaches the target.[3,6] When the projectile reaches the target its state vector, $\Omega^{(+)}|p)$, has various types of components well formed which determine whether one or more Regge poles can be exchanged in the elastic amplitude. Only the low-momentum parts of these components interact with the target. If the target is a sphere of nuclear matter of radius R, then only those parts of $\Omega^{(+)}|p)$ having momentum $\lesssim 2R$, when the projectile reaches the front of the nucleus, can actually interact with the target. The difference between different targets is just that the relative coupling of the Regge pole and the Regge cuts may vary considerably from target to target.

These space-time arguments are given in detail in Secs. II and III, where it is shown that for a trajectory which has $\alpha(0)=\alpha\leq 1$ the single-Regge-pole contribution to high-energy hadron-nucleus cross sections is proportional to $\pi R^2(p/R)^{\alpha-1}$ and n-Regge-pole exchange is proportional to $\pi R^2(p/R)^{n(\alpha-1)}$. These results are rederived from somewhat more formal arguments in Sec. V. A curious feature is present which at first sight seems paradoxical. When $\alpha<1$ the mean free path, as conventionally defined, is proportional to $p^{1-\alpha}$. If the radius, R, of the target nucleus is much greater than $p^{1-\alpha}$ one might expect that the hadron-nucleus cross section would be $2\pi R^2$. However, our formula above says that the cross section is more like $\pi R^2(p/R)^{\alpha-1}$, which may still be very small if p is in the region

$$\frac{R}{p}\ll 1\ll \frac{R}{p^{1-\alpha}}\,.$$

The breakdown of the arguments involving mean-free-path concepts and the accompanying breakdown of the Glauber expansion[7] are described in Sec. IV. Crudely, the mean free path is not a useful concept because a fast-moving hadron can exist in many possible configurations. Fluctuations between one of these configurations and another occur on a time scale proportional to p. When a hadron reaches a target a reaction will take place if the hadron is in a configuration with low-momentum components, but no reaction will take place if there are no components of the wave function $\lesssim 2R$. Only when the dimension of the target is much greater than the momentum is a reaction guaranteed to take place. Thus, in the context of the multiperipheral model and soft field theory there is no compelling reason why total cross sections should be $2\pi R^2$ for hadron scattering off large nuclei at high energies. All one can say is that the cross section should be *proportional* to πR^2.[8]

In Sec. VI a rather strong bound on the value of the inclusive cross section in the central region of a hadron-nucleus collision is derived. To explain this bound define $r=\sigma_{DD}/\sigma_{tot}$, where σ_{DD} is the diffractive part, including the elastic hadron-nucleus cross section, of the hadron-nucleus cross section, and σ_{in} is the truly inelastic part of the cross section. Also define

$$R_c=\left(\frac{(1/\sigma_{in})(d\sigma_{in}/dy)}{(1/\sigma)(d\sigma/dy)}\right)_{\text{central region}}\,,$$

where σ is the hadron-hadron cross section, and $d\sigma_{in}/dy$ and $d\sigma/dy$ are the inclusive cross sections in the central region for hadron-nucleus and hadron-hadron collisions, respectively. If n is an integer such that

$$\frac{1}{2}\,\frac{1-2^{2-n}}{1-2^{1-n}}\leq r\leq \frac{1}{2}\,\frac{1-2^{1-n}}{1-2^{-n}}\,,$$

then the minimum value of R_c is $n+1+2^{n-1}(2r-1)/(1-r)$. Thus, for example, if $r=\frac{1}{3}$, then $R_c\geq 2$. If $r=\frac{3}{7}$, then $R_c\geq 3$. This extremely strong bound means that one cannot have a large elastic cross section unless the inclusive cross section in the central region is also quite large. This inequality, which is amenable to experimental confrontation, is a direct consequence of the Abramovskii-Kancheli-Gribov[9] (AKG) cutting rules along with the assumption that Reggeon interactions are not very important at moderate energies. Since the coupling of a Regge pole to the nucleus is finite without any necessity of Reggeon interactions, Glauber corrections, or rescatterings outside the nucleus fragmentation region, there should be a region in energy where few-Reggeon exchange does not necessitate Reggeon interactions and provides a reasonable description of hadron-nucleus collisions. Even without a detailed computation the increase of the inclusive cross section as the elastic cross section increases is clear. This is because a large elastic cross section means that several-Regge-pole exchange must be important. Any soft field theory which allows several Regge poles to be exchanged must allow discontinuities through these Regge poles. These discontinuities will give large inclusive production. If it happened that $R_c \approx 1$ and $r \approx \frac{1}{2}$ for heavy nuclei we would find it difficult to entertain a soft field theory, or the multiperipheral model, as a reasonable theory of high-energy hadron-hadron collisions.

In Sec. VIII we discuss a possible alternative model of high-energy scattering. This is the planar dual string model without loops.[4] As mentioned earlier this model has the severe drawback that its time scale for interaction is such that it cannot accommodate the high-energy scattering of low-mass currents in a way compatible with SLAC experiments. Nevertheless, we feel that *if* $r \approx \frac{1}{2}$ and *if* $R_c \approx 1$ then something like the dual model, or for that matter Low's[10] development of the bag model into a model for high-energy collisions, must be seriously considered. So long as loops are not allowed a planar string model gives $R_c = 1$. In fact, $(1/\sigma_{\rm in})(d\sigma_{\rm in}/dy)$ is exactly as in hadron-hadron scattering in all regions of y. As discussed in Sec. VIII, the nucleus fragmentation region will certainly be modified by loops. What we cannot ascertain is how strong the modification of the central region and the projectile fragmentation region will be when loops are added. The essential difference between the multiperipheral model and the string model without loops is that the elastic scattering of two strings, one of which is a particle bound in a nucleus, occurs over a very short period of time. This short time scale for interaction modifies the AKG cutting rules.[11] Of course loops introduce a long time scale, but we have not been able to estimate how important this long time scale is and what the final cutting rules are.

One of the most difficult regions in which to discuss particle production is the fragmentation region of the nucleus. We know, at least in soft field theories and the multiperipheral model, that the size of the fragmentation region is $\ln Rm$.[12] However, the exact mass scale, m, is not known. In Sec. VII a simple illustrative model (a modification of a model by Schwimmer[13]) for the fragmentation region of the nucleus is solved. This model, a realization of the "coalescence of combs" discussed by Kancheli,[12] is an idealized version of the actual dynamics of the framentation region.

As far as the spirit of the soft-field-theory part of this paper is concerned we agree with and are strongly influenced by the presentation given by Kancheli, although we believe his diagrammatic realization to be too idealistic. Conclusions quite similar to Kancheli's have been reached by Lehman and Winbow.[14] The important paper of Gottfried[1] is much more in the spirit of a string model. We feel that there is a good possibility that high-energy hadron-nucleus collisions may tell us which of these types of models is close to the correct theory of hadron-hadron collisions.

II. SPACE-TIME DESCRIPTION OF HADRON-HADRON SCATTERING IN SOFTENED FIELD THEORY

As a prelude to hadron-nucleus scattering, we will first transform the momentum-space description of hadronic interactions into configuration space. Aside from its obvious intuitive relevance for large nuclei, a space-time formulation of hadron-hadron scattering is an interesting exercise in itself because the results are partly at variance with the naive expectation that a fast hadron Lorentz-contracts. The discussion that follows is based on the ϕ^3 ladder model of Reggeons[5]; this is the simplest of models leading to Regge behavior (in perturbation theory), and it will become clear that our general conclusions would follow in any soft field theory without vector mesons.

A. Time-ordered perturbation theory

The most convenient method of introducing coordinate space is an on-mass-shell description starting from old fashioned (time-ordered) perturbation theory. This approach has the advantage of a formal similarity to potential scattering and is closest to one's experience in nonrelativistic quantum mechanics. An alternative off-shell formulation is discussed in Sec. II E.

Consider the elastic scattering of two spinless particles of momenta p and P at large values of $s = (p+P)^2$. We find it convenient to work in the laboratory frame, where[15] $P = (m, \underline{0}, 0)$ and $p = (p + m^2/2p, \underline{0}, p)$. We suppose at first that the amplitude is given by single-Regge-pole exchange, which in the ϕ^3 model corresponds to the sum of covariant ladder graphs shown in Fig. 1. Each such graph may be expressed as a sum of time-ordered graphs, one for each possible ordering of the interaction times. However, in this model almost all of the internal lines carry a large z

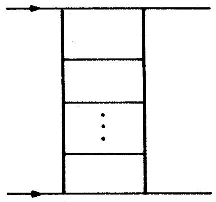

FIG. 1. Feynman diagram model of a Reggeon in ϕ^3 perturbation theory.

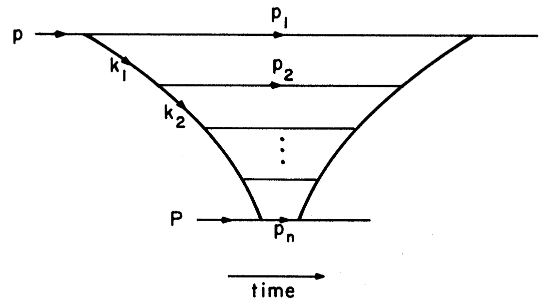

FIG. 2. A Reggeon in old-fashioned (time-ordered) perturbation theory. The typical time intervals are $O(p/m^2)$ at the top of a graph and $O(1/m)$ at the bottom.

momentum, and so the ordering in which all lines move forward dominates the others by powers of the large momentum.[16] (We shall verify these properties of the ϕ^3 model presently.) Therefore only the ordering shown in Fig. 2 survives. Strictly speaking, the internal lines at the bottom of the graph do not carry very large momentum and the other time orderings are required there, but we neglect this nicety since the Regge behavior arises from the sum over n and not from the low-momentum rungs. The signature of the Reggeon should be taken account of by adding to Fig. 1 a "twisted" ladder graph, with a left-hand cut. However, it is easy to see that both twisted and untwisted ladders correspond to the same time-ordered graph.

We use the normalization

$$\sigma_{\text{tot}} = \frac{2(s-2m^2)(2\pi)^6}{[s(s-4m^4)]^{1/2}}\,\text{Im}A(s,0)\,, \tag{2.1}$$

$$\text{Im}A = \tfrac{1}{2}(2\pi)^4\int d^3p_1\cdots d^3p_n\,\delta^4\Big(p+P-\sum_{i=1}^{n}p_i\Big)\times|A_n(s;p_1\cdots p_n)|^2\,. \tag{2.2}$$

The choice of factors in (2.1) leads to convenient unitarity bounds, while the noncovariant phase-space element in (2.2) facilitates the formal analogy to potential scattering. The production amplitude is given by

$$A_n = \frac{(ig)^n}{[2\omega_p(2\pi)^3 2m(2\pi)^3 2\omega_{p_1}\cdots 2\omega_{p_n}(2\pi)^3]^{1/2}}\times\prod_{i=1}^{n-1}\frac{i}{k_i^2-m^2+i\epsilon}\,, \tag{2.3}$$

where the labeling of momenta is shown in Fig. 2. The dominant region of integration, that leading to Regge behavior of ImA, is such that $p_{iz}=\lambda_i p$ with $0<\lambda_i<1$ and $|\underline{p}_i|=O(m)$, for all but a fixed number of p_i. These conditions are to ensure that k_i^2 is finite, as required by the softness of the theory, and that there be no large subenergy across an elementary propagator. We then have $\omega_{p_i}\approx p_i+(m^2+\underline{p}_i^2)/2p_i$ and so

$$\begin{aligned}
k_i^2-m^2 &= (p-p_1-\cdots-p_i)^2-m^2\\
&\approx\Big(p+\frac{m^2}{2p}-p_1-\frac{m^2+\underline{p}_1^2}{2p_1}-\cdots-p_i-\frac{m^2+\underline{p}_i^2}{2p_i}\Big)^2-(\underline{p}_1+\cdots+\underline{p}_i)^2-(p-p_1-\cdots-p_i)^2-m^2\\
&\approx 2(p-p_1-\cdots-p_i)\Big(\frac{m^2}{2p}-\frac{m^2+\underline{p}_1^2}{2p_1}-\cdots-\frac{m^2+\underline{p}_i^2}{2p_i}\Big)-(\underline{p}_1-\cdots-\underline{p}_i)^2-m^2\\
&\approx 2\Big[p-p_1-\cdots-p_i+\frac{m^2+(\underline{p}_1+\cdots+\underline{p}_i)^2}{2(p-p_1-\cdots-p_i)}\Big]\\
&\quad\times\Big[p+\frac{m^2}{2p}-p_1-\frac{m^2+\underline{p}_1^2}{2p_1}-\cdots-p_i-\frac{m^2+\underline{p}_i^2}{2p_i}-(p-p_1-\cdots-p_i)-\frac{m^2+(\underline{p}_1+\cdots+\underline{p}_i)^2}{2(p-p_1-\cdots-p_i)}\Big]\\
&\approx 2\omega_{k_i}(E_0-E_i)\,,
\end{aligned} \tag{2.4}$$

where

$$\omega_{k_i}\equiv[m^2+(\underline{p}_1+\cdots+\underline{p}_i)^2+(p-p_1-\cdots-p_i)^2]^{1/2}$$

$E_0\equiv\omega_p+m$, and $E_i\equiv\omega_{p_1}+\cdots+\omega_{p_i}+\omega_{k_i}+m$. The factor E_0-E_i is just the energy denominator associated with the ith intermediate state, and thus (2.3) is approximately equal to the time-ordered amplitude corresponding to Fig. 2. Again, the approximations leading to (2.4) break down at the bottom of the ladder and there is no unique time ordering there.

To estimate the time intervals involved in the scattering process, note that the lifetime of the ith intermediate state is on the order of

$$\tau_i = \frac{1}{E_0 - E_i}\,. \tag{2.5}$$

This follows formally from the fact that in the derivation of the old-fashioned perturbation expansion one encounters expressions of the form

$$\int_0^\infty d\tau_i\, e^{i\tau_i(E_0-E_i)}\langle i+1|H_I(0)|i\rangle\langle i|H_I(0)|i-1\rangle$$

and oscillations of the exponential wash out contributions with $|\tau_i(E_0 - E_i)| \gg 1$. Now in some of the intermediate states in Fig. 2 there are only secondaries with $p_i = O(p)$, and from (2.4) and (2.5) the corresponding $\tau_i = O(p/m^2)$.[17] Since high-momentum secondaries have velocities $\approx c$, the total distance traversed in such intermediate states is $\approx\tau_i$. Therefore the total time and distance intervals over which the interaction occurs are $O(p/m^2)$.[3] The physical origin of this result is clear. We have assumed that a high-energy scattering event involves a sequence of basic momentum-nearest-neighbor interactions, some of which have been boosted to momenta on the order of the projectile momentum. Each "basic" interaction requires a time interval $O(1/m)$ in its rest system (for lack of another scale), and in a high velocity frame this basic time interval is dilated by a factor $O(p/m)$.

From the point of view just described, factorization is an obvious property: The projectile itself interacts a long time and hence a long distance from the target and then propagates freely. The target only enters into the scattering at (nearly) the last stage after most of the real or virtual secondaries have been emitted. Formally, we can write

$$\mathrm{Im}A = \frac{1}{2(2\pi)^2}\langle p|\Omega^{(+)\dagger}Va_P\delta(E_0 - H_0)a_P^\dagger V\Omega^{(+)}|p\rangle\,, \tag{2.6}$$

where V is the ϕ^3 interaction, $a_P^\dagger$ and a_P are creation and annihilation operators for the target, $|p\rangle$ is the state of the projectile, and $\Omega^{(+)}$ is the Møller scattering operator, with

$$\Omega^{(+)} = 1 + \frac{1}{E_0 - H_0 + i\epsilon}V\Omega^{(+)}\,, \tag{2.7}$$

and we take only the single time ordering shown in Fig. 2. Thus we see that the scattering of p on P is essentially determined by

$$|\psi_p\rangle = \Omega^{(+)}|p\rangle\,, \tag{2.8}$$

which we can call the state vector of p, a quantity which is totally ignorant of the target.

In the ϕ^3 model, graphs more complicated than simple ladders generate a Regge pole,[18] and when we speak of a "one-Reggeon state" an average of such graphs is implied. At high energy, where the fluctuations about this average vanish as a power of p, we can speak of a well-defined rapidity density of particles for each of the virtual states which build the Reggeon. A "two-Reggeon state," which can always be distinguished in practice by the nature of its associated J-plane singularity, would have on the average twice the rapidity density of each virtual state. Hereafter we shall speak simply of the "particle density associated with a Reggeon," but this averaging should be borne in mind.

It is easy to see that the general features of this discussion remain valid in more general theories provided internal lines are damped at large masses and the basic interaction has short range in rapidity. One could also treat the case of a composite-particle projectile (say) by considering a two-elementary-particle incoming state and analytically continuing to the bound-state pole. For convenience we shall continue to work with elementary external particles and the ϕ^3 model.

B. Expansion of the wave function

We have seen that essentially all of the particle production in a scattering reaction is associated with the incident state $|\psi_p\rangle$, which suggests that we expand the state in a Fock-space basis. The normalization of the state will then imply a stringent unitarity bound on the particle production. We write

$$|\psi_p\rangle = \sum_n \frac{1}{(n!)^{1/2}}\int d^3p_1\cdots d^3p_n\,\delta^3(p - p_1 - \cdots - p_n)\psi_n(p;p_1\cdots p_n)|p_1\cdots p_n\rangle\,, \tag{2.9}$$

where the n-particle basis vectors have the normalization

$$\langle p_1'\cdots p_n'|p_1\cdots p_n\rangle = \sum_{\text{permutations}}\delta^3(p_1 - p_{i_1}')\cdots\delta^3(p_n - p_{i_n}')\,.$$

Then

$$\langle\psi_{p'}|\psi_p\rangle = \delta^3(p - p')\sum_n\int d^3p_1\cdots d^3p_n\,\delta^3(p - p_1 - \cdots - p_n)|\psi_n(p;p_1\cdots p_n)|^2\,.$$

The wave operator $\Omega^{(+)}$ is isometric, so $|\psi_p\rangle$ has the same norm as $|p\rangle$. Therefore

$$\sum_n \int d^3p_1 \cdots d^3p_n \delta^3(p - p_1 - \cdots - p_n)|\psi_n(p; p_1 \cdots p_n)|^2 = 1 . \tag{2.10}$$

We can now relate the distribution of momenta in the state to the Regge behavior of the model. Let $P(p,q)$ be the probability that the state $|\psi_p\rangle$ contains a momentum component q, where $q_z \ll p$. Clearly, the following is the expectation value of the number operator for q in the state:

$$\delta^3(p'-p)P(p,q) = \langle \psi_{p'}|a_q^\dagger a_q|\psi_p\rangle$$

$$= \delta^3(p'-p)\sum_n \int d^3p_1 \cdots d^3p_{n-1}\delta^3(p - p_1 - \cdots - p_{n-1} - q)|\psi_n(p; p_1 \cdots p_{n-1}q)|^2.$$

But this is *almost* the scattering amplitude for particles of momenta p and q; the only difference is that the factors V in (2.6) which couple the incoming state to the "target" are absent, and the low-momentum components are not treated properly. Since the latter are irrelevant for the Regge behavior of the amplitude,

$$P(p,q) \propto (p^\mu q_\mu)^{\alpha-1} \propto (p/q_z)^{\alpha-1} . \tag{2.11}$$

Here $\alpha \equiv \alpha(0)$ is the intercept of the leading Regge trajectory in the model, and the power is $\alpha-1$ rather than α because of our normalization in (2.1). In particular, the absorptive part of the scattering amplitude for p off a target at rest is essentially the probability $(p/m)^{\alpha-1}$ that the state $|\psi_p\rangle$ contains low-momentum components $q_z \sim m$. Similarly, the probability of finding a quantum of momentum q in a two-Reggeon state is essentially the amplitude corresponding to a two-Reggeon cut and is proportional to $(p/q_z)^{2\alpha-2}$. Strictly speaking, since a probability cannot exceed one, a trajectory with $\alpha(0)>1$ is forbidden, although in such a case the model can still be sensible over a range of p if the coefficient of $p^{\alpha-1}$ is small enough. Rather than commit ourselves to a detailed model, where numerical coefficients are calculable, we shall assume $\alpha(0)<1$ for convenience in discussion.[19] Note that since

$$\langle \psi_{p'}|\psi_p\rangle = \delta^3(p'-p)$$

exactly, it should be the case that the $p^{\alpha-1}$ probability of a low-momentum component is canceled by other regions of phase space. It is straightforward to verify that no $p^{\alpha-1}$ term appears in the (sum over n of) completely integrated graphs shown in Fig. 3, although such a term is present if the internal integration is restricted to the region of phase space where p_n is small.

C. Impact-space description

In many of our subsequent arguments it will be most convenient to consider amplitudes and state vectors as a function of transverse position coordinates and longitudinal momenta. This is useful because we expect elastic scattering to be diagonal in impact parameter at high energy and because a nucleus has a large and fairly well-defined transverse profile. Formally, we write an impact-parameter free particle state as

$$|p\underline{x}\rangle = \frac{1}{2\pi}\int d^2\underline{p}\, e^{i\underline{p}\cdot\underline{x}}|p\underline{p}\rangle$$

with normalization

$$\langle p'\underline{x}'|p\underline{x}\rangle = \delta(p-p')\delta^2(\underline{x}-\underline{x}') .$$

Similarly, for the incident state we write

$$|\psi_{p\underline{x}}\rangle = \frac{1}{2\pi}\int d^2\underline{p}\, e^{i\underline{p}\cdot\underline{x}}|\psi_{p\underline{p}}\rangle . \tag{2.12}$$

Introducing the Fock-space expansion (2.9) we have

$$|\psi_{p\underline{x}}\rangle = \sum_n \frac{1}{(n!)^{1/2}}\int dp_1 \cdots dp_n\, d^2x_1 \cdots d^2x_n\, \delta(p-p_1-\cdots-p_n)\psi_n(p\underline{x}; p_1\underline{x}_1 \cdots p_n\underline{x}_n)|p_1\underline{x}_1 \cdots p_n\underline{x}_n\rangle , \tag{2.13}$$

$$\psi_n(p\underline{x}; p_1\underline{x}_1 \cdots p_n\underline{x}_n) = \frac{1}{(2\pi)^{n+1}}\int d^2\underline{p}\, d^2\underline{p}_1 \cdots d^2\underline{p}_n\, e^{i(\underline{p}\cdot\underline{x}-\underline{p}_1\cdot\underline{x}_1-\cdots-\underline{p}_n\cdot\underline{x}_n)}\psi_n(p\underline{p}; p_1\underline{p}_1 \cdots p_n\underline{p}_n) . \tag{2.14}$$

Since $|\psi_{p\underline{x}}\rangle$ is obtained from $|p\underline{x}\rangle$ by an isometric transformation,

$$\langle \psi_{p'\underline{x}'}|\psi_{p\underline{x}}\rangle = \delta(p-p')\delta^2(\underline{x}-\underline{x}'),$$

which implies

$$\delta^2(\underline{x}-\underline{x}') = \sum_n \int dp_1\cdots dp_n\,\delta\left(p-\sum_1^n p_i\right) d^2x_1\cdots d^2x_n\,\psi_n^*(p\underline{x}';p_1\underline{x}_1\cdots p_n\underline{x}_n)\psi_n(p\underline{x};p_1\underline{x}_1\cdots p_n\underline{x}_n). \qquad (2.15)$$

As above, the probability that $|\psi_{p\underline{x}}\rangle$ has a low-momentum component is proportional to $p^{\alpha-1}$; explicitly

$$\langle \psi_{p'\underline{x}'}|a^\dagger_{q\underline{y}}a_{q\underline{y}}|\psi_{p\underline{x}}\rangle \propto \delta(p-p')\delta^2(\underline{x}-\underline{x}')(p/q)^{\alpha-1}f(\underline{x},\underline{y}). \qquad (2.16)$$

The general features of the transverse coordinate dependence of the wave functions $\psi_n(p\underline{x};p_1\underline{x}_1\cdots p_n\underline{x}_n)$ can be deduced easily.[20] ψ_n describes that part of the state $|\psi_{p\underline{x}}\rangle$ with n quanta, and in the ladder model (Fig. 4) we have

$$\psi_n(p\underline{p};p_1\underline{p}_1\cdots p_n\underline{p}_n) = g^{n-1}\{[2\omega_p(2\pi)^3 2\omega_{p_1}\cdots(2\pi)^3 2\omega_{p_n}]^{1/2} 2\omega_{k_1}\cdots 2\omega_{k_{n-2}}(E_0-E_1)\cdots(E_0-E_{n-1})\}^{-1}.$$

If we Fourier-transform according to (2.14) and rewrite the result in terms of $\underline{k}_i$ integrals,

$$\psi_n(p\underline{x};p_1\underline{x}_1\cdots p_n\underline{x}_n) = \frac{g^{n-1}}{(2\pi)^{5n/2+1}}\int d^2p\,d^2k_1\cdots d^2k_{n-2}\,d^2p_n \times \frac{\exp[i\underline{p}\cdot(\underline{x}-\underline{x}_1)+i\underline{k}_1\cdot(\underline{x}_1-\underline{x}_2)+\cdots+i\underline{p}_n\cdot(\underline{x}_n-\underline{x}_{n-1})]}{(2\omega_p 2\omega_{p_1}\cdots 2\omega_{p_n})^{1/2}2\omega_{k_1}\cdots 2\omega_{k_{n-2}}(E_0-E_1)\cdots(E_0-E_{n-1})}.$$

Now except for the low-momentum components at the bottom of the chain, each ω_i depends only on longitudinal momenta and may be taken outside the integral. To obtain a very rough estimate of the remaining integral we can make the "strong-ordering" approximation $p_i \gg k_i$. In this case, from (2.4),

$$E_0-E_i \approx -\frac{m^2+\underline{k}_i^{\,2}}{2k_i} \quad (i=1,2,\ldots,n-2),$$

$$E_0-E_{n-1} \approx -\frac{m^2+\underline{p}_n^{\,2}}{2P_n},$$

and then

$$\psi_n \propto \int d^2p\,d^2k_1\cdots d^2p_n\,\frac{\exp[i\underline{p}\cdot(\underline{x}-\underline{x}_1)+i\underline{k}_1\cdot(\underline{x}_1-\underline{x})+\cdots+\underline{p}_n\cdot(\underline{x}_n-\underline{x}_{n-1})]}{(m^2+\underline{k}_1^{\,2})\cdots(m^2+\underline{p}_n^{\,2})}.$$

Since

$$\int d^2p\,\frac{e^{i\underline{p}\cdot\underline{x}}}{\underline{p}^2+m^2} = \pi\int_0^\infty dr^2\,\frac{J_0(r|\underline{x}|)}{r^2+m^2}$$

is proportional to $e^{-m|\underline{x}|}$ at large $|\underline{x}|$, we see that ψ_n is negligible unless $|\underline{x}_i-\underline{x}_{i+1}| \lesssim 1/m$, and the $\underline{x}_i$ have, crudely, a random walk distribution.[21]

Finally we would like to derive two useful properties of the elastic scattering amplitude: Its diagonality

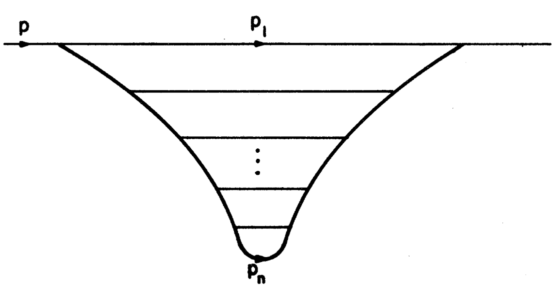

FIG. 3. Part of the normalization of the incident state $|\psi_p\rangle$.

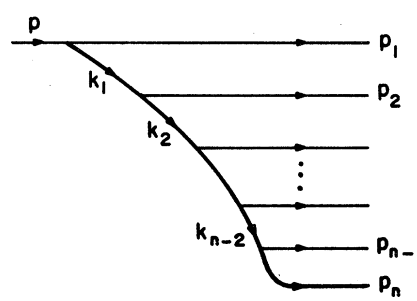

FIG. 4. A one-Reggeon component of the state $|\psi_p\rangle$.

in $\underline{x}$ and the corresponding unitarity bound. For the reaction $p+P\to p'+P'$, define

$$\langle p'\underline{x}'P'\underline{X}'|T|p\underline{x}P\underline{X}\rangle=\frac{1}{(2\pi)^4}\int d^2p'\,d^2P'\,d^2p\,d^2P\exp[-i(\underline{p}'\cdot\underline{x}'+\underline{P}'\cdot\underline{X}'-\underline{p}\cdot\underline{x}-\underline{P}\cdot\underline{X})]\times\{(2\pi)^4\delta^4(p'+P'-p-P)A(p'P';pP)\}, \tag{2.17}$$

where the quantity in braces is the matrix element of T between (noncovariant) momentum eigenstates in our normalization. Then

$$\langle p'\underline{x}'P'\underline{X}'|T|p\underline{x}P\underline{X}\rangle=\delta(\omega_{p'}+\omega_{P'}-\omega_p-\omega_P)\delta(p'+P'-p-P)\times\int d^2p\,d^2p'\,d^2P'\exp\{i[\underline{p}\cdot(\underline{x}-\underline{X})-\underline{P}'\cdot(\underline{X}'-\underline{X})-\underline{p}'\cdot(\underline{x}'-\underline{X})]\}A(p',P';p,p'+P'-p).$$

Now A is a function of the Lorentz invariants $(p+P)^2$ and $(p-p')^2$ alone, and at high energy in the lab frame these reduce to $2mp$ and $-(\underline{p}-\underline{p}')^2$, respectively, and we can write

$$\langle p'\underline{x}'P'\underline{X}|T|p\underline{x}P\underline{X}\rangle=\delta(\omega_{p'}+\omega_{P'}-\omega_p-\omega_P)\delta(p'+P'-p-P)\delta^2(\underline{X}'-\underline{X})\delta^2(\underline{x}'-\underline{x})\frac{p}{\omega_p}\tilde{A}(p,\underline{x}-\underline{X}), \tag{2.18}$$

which is diagonal in impact parameter and where we have defined

$$\tilde{A}(p,\underline{x})\equiv\frac{\omega_p}{p}(2\pi)^4\int d^2(p-p')\,e^{i(\underline{p}-\underline{p}')\cdot\underline{x}}A(p,(\underline{p}-\underline{p}')^2). \tag{2.19}$$

Similarly, for a $2\to n$ amplitude we define

$$\begin{aligned}\langle p_1\underline{x}_1\cdots p_n\underline{x}_n|T|p\underline{x}P\underline{X}\rangle&=\frac{1}{(2\pi)^{n-1}}\int d^2p\,d^2P\,d^2p_1\cdots d^2p_n\\&\quad\times\exp[-i(\textstyle\sum\underline{p}_i\underline{x}_i-\underline{p}\cdot\underline{x}-\underline{P}\cdot\underline{X})](2\pi)^4\delta^4(p+P-\sum p_i)A(p_i;pP)\\&=\frac{1}{(2\pi)^{n-2}}\delta(\textstyle\sum\omega_{p_i}-\omega_p-\omega_P)\delta(\sum p_i-p-P)\tilde{A}_n(p_i',\underline{x}_i-\underline{X};p,\underline{x}-\underline{X}).\end{aligned} \tag{2.20}$$

Note that, without loss of generality, we can set $\underline{X}=0$ everywhere.

Now consider the unitarity equation; if $S=1+iT$ then

$$\frac{1}{i}(T-T^\dagger)=T^\dagger T. \tag{2.21}$$

We take the expectation value of this equation in the impact-parameter state $|p\underline{x},P\underline{0}\rangle$, and insert

$$1=\sum_n\int\frac{1}{n!}dp_1\cdots dp_n\,d^2x_1\cdots d^2x_n\,|p_1\underline{x}_1\cdots p_n\underline{x}_n\rangle\langle p_1\underline{x}_1\cdots p_n\underline{x}_n|$$

between T and $T^\dagger$ on the right-hand side of (2.21). Then after some straightforward manipulation

$$\mathrm{Im}\tilde{A}(p\underline{x})=\tfrac{1}{2}|\tilde{A}(p\underline{x})|^2+\mathrm{Im}\tilde{A}_{\mathrm{inel}}(p\underline{x}). \tag{2.22}$$

[By $\mathrm{Im}\tilde{A}(p\underline{x})$ we mean "take the imaginary part of the elastic amplitude, as a function of momenta, and then take the Fourier transform."] The inelastic contribution can easily be shown to be positive definite, and we have the unitarity condition

$$\mathrm{Im}\tilde{A}(p\underline{x})=1-\{1-[\mathrm{Re}\tilde{A}(p\underline{x})]^2-2\,\mathrm{Im}\tilde{A}_{\mathrm{inel}}(p\underline{x})\}^{1/2}$$

together with the unitarity bound

$$\mathrm{Im}\tilde{A}(p\underline{x})\leq 1. \tag{2.23}$$

Saturation of (2.23) corresponds to scattering from a black disk. In the normalization of this paper, at high energy,

$$\begin{aligned}\sigma_{\mathrm{tot}}&=2\int d^2\underline{x}\,\mathrm{Im}\tilde{A}(p\underline{x}),\\ \sigma_{\mathrm{el}}&=\int d^2\underline{x}\,|\tilde{A}(p\underline{x})|^2,\end{aligned} \tag{2.24}$$

as one can easily check from the definitions above.

D. Longitudinal coordinate description at fixed time

We can also parametrize the incident state as a function of longitudinal coordinate by simply Fourier-transforming the longitudinal components of all momenta. Thus we can define (see Fig. 4)

$$\psi_n(p\underline{x};z_1\underline{x}_1\cdots z_n\underline{x}_n)=\frac{1}{(2\pi)^{n/2}}\int dp_1\cdots dp_n\,\delta(p-\textstyle\sum p_i)\exp[-i(p_1z_1+\cdots+p_nz_n)]\,\psi_n(p\underline{x};\,p_1\underline{x}_1\cdots p_n\underline{x}_n)$$

$$=\frac{e^{-ipz_1}}{(2\pi)^{n/2}}\int dk_1\cdots dk_{n-2}\,dp_n\exp[ik_1(z_1-z_2)+\cdots+ip_n(z_{n-1}-z_n)]\,\psi_n(p\underline{x};p_1\underline{x}_1\cdots p_n\underline{x}_n)\,. \tag{2.25}$$

(We do not transform p because we wish to consider scattering at a fixed energy.) At the top of the ladder we have $k_i=O(p)$, so $|z_i-z_{i+1}|=O(1/p)$; thus the high-momentum components of the state are Lorentz-contracted. If there are low-momentum components at the bottom of the ladder, then $k_i=O(m)$ there and the corresponding $|z_i-z_{i+1}|=O(1/m)$, so there is no over-all contraction. In a scattering event, in particular, the latter must be the case. Notice, however, that this is the longitudinal coordinate distribution at a *fixed time*,[22] and has nothing at all to do with the over-all distance and time involved in a scattering event (alternatively, the distance and time required to form the state) which we have earlier estimated to be on the order of the incident momentum.

E. Covariant space-time description

We can also discuss the length and time scales involved in a scattering reaction by simply examining the covariant Feynman diagrams in coordinate space, where the interaction vertices refer to space-time points. While the physical interpretation is not as clear as in the Fock-space expansion, the arithmetic is simpler and the result furnishes a check on our previous statements.

Consider the covariant ladder graph in Fig. 5, where the x_i label vertex positions in space-time. In coordinate space the graph is proportional to

$$\int\prod_1^n(d^4p_i\,e^{ip_ix_i})\prod_1^{n-1}\frac{1}{k_i^2-m^2+i\epsilon}\,\delta^4(p+P-\textstyle\sum p_i)$$

$$=\int\prod_1^{n-1}\left(d^4k_i\,e^{ik_i\xi_i}\frac{1}{k_i^2-m^2+i\epsilon}\right), \tag{2.26}$$

where $\xi_i=x_{i+1}-x_i$ is a coordinate difference, and we again do not transform P or p in order to continue to discuss scattering at a fixed energy. Since

$$k\cdot\xi=\tfrac{1}{2}(k_0+k_3)(\xi_0-\xi_3)+\tfrac{1}{2}(k_0-k_3)(\xi_0+\xi_3)-\underline{k}\cdot\underline{\xi}$$

the significant values of ξ_i satisfy

$$|\xi_{i0}+\xi_{i3}|\lesssim\left|\frac{1}{k_{i0}-k_{i3}}\right|,$$
$$|\xi_{i0}-\xi_{i3}|\lesssim\left|\frac{1}{k_{i0}+k_{i3}}\right|, \tag{2.27}$$

and

$$|\underline{\xi}_i|\lesssim\frac{1}{|\underline{k}_i|}\,.$$

At the top of the graph we have $k_{i0}+k_{i3}=O(p)$, $|k_{i0}-k_{i3}|=O(m^2/p)$ and $|\underline{k}_i|=O(m)$, and hence the time and longitudinal coordinate differences are $O(p/m^2)$ there, while the transverse coordinate differences exhibit the same rough random walk behavior we found previously. Similarly, at the bottom of the graph, all components of the coordinate differences are $O(1/m)$. The over-all coordinate displacement is

$$X=x_n-x_1=\xi_1+\xi_2+\cdots+\xi_{n-1}$$

and we have

$$X_0+X_3=O(p/m^2)\,,$$
$$X_0-X_3=O(1/m)\,,$$
$$|\underline{X}|=O((1/m)\log p/m)\,,$$

and thus $X^2=O(p/m^3)$. These results are similar

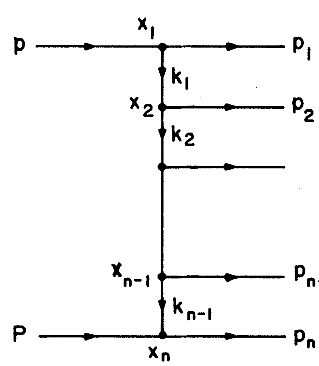

FIG. 5. Feynman diagram for a production amplitude.

to the coordinate space behavior found previously in the on-shell description.

F. Regge cuts

The discussion of softened field theory in configuration space is an amusing curiosity for single ladders and hence pure Regge poles, but becomes a valuable technique in the analysis of Regge cuts. We shall see that many of the standard results which are obtained only with great labor in momentum space are almost obvious in configuration space.

Consider first the AFS graph[5] in elastic scattering. The covariant amplitude is shown in Fig. 6, and the corresponding time-ordered graph is shown in Fig. 7 (ignore the vertical lines for the moment). As above, all other orderings of the vertices are suppressed by powers of a large momentum except at the bottom of the graph, where a sum over orderings is implied. The physical description of this process is as follows: An incoming fast hadron emits quanta and a time $O(p)$ later the state has developed low-momentum components which interact with the target. After the latter interaction the quanta begin to recombine and after a further time interval $O(p)$ only the original two particles remain. The incident particle has meanwhile propagated freely with a large z momentum while the target has had little momentum transferred to it; hence at this intermediate stage the projectile is a distance $O(p)$ past the target. It now begins to emit quanta again and these, according to the graph, are supposed to interact with the target a further time and distance $O(p)$ downstream. However, the target is far behind and the amplitude must be zero. One can restate this by observing that the lifetime of a low-momentum off-shell particle must be $O(1/m)$ while the graph alleges that the target persists a time $O(p)$ between interactions. A nonzero amplitude can only result from "bad" time orderings in which quanta propagate backwards, and these are suppressed by powers of p.

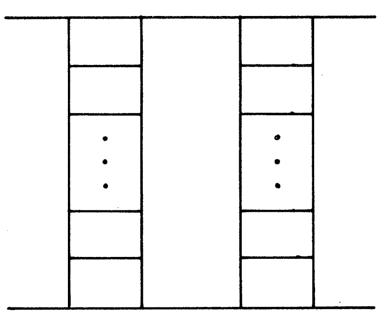

FIG. 6. Feynman diagram model of the AFS cut.

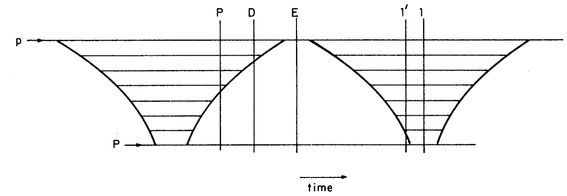

FIG. 7. The AFS cut in old-fashioned perturbation theory, together with some discontinuities.

It is also easy to deduce how the cancellations occur among the various contributions to the absorptive part. Consider the various possible discontinuities in Fig. 7. If a Reggeon is entirely cut, as in the discontinuity labeled 1, then the amplitude to the left of the cut line vanishes for the same reason the full amplitude does. Discontinuity 1′ is really the same as 1 because there is no unique ordering at the bottom of the graph. The elastic discontinuity is nonzero; the "physical" reason being that the target is on shell after the first scattering and can propagate indefinitely. A discontinuity such as D, which leads to a low-mass diffractively produced state is similar to the elastic intermediate state and is not zero for the same reason. Cuts partly through a Reggeon, like P, require that there be a large mass on an internal line and are negligible because of the softness of the theory.[9] The only nonvanishing discontinuities are of types E and D, and hence the elastic and diffractive intermediate states must cancel among themselves.[23]

We have seen that the AFS graph vanishes because each Reggeon interaction requires a long time interval and the emission of the second Reggeon does not begin until the projectile is far past the target. A graph with a nonvanishing two-Reggeon cut must then begin to emit both Reggeons long before the target is reached. The simplest possibility is shown in Fig. 8—the Mandelstam graph.[23] (The figure is somewhat schematic; the orderings of emission of each chain and the secondaries on each chain can be varied, although the over-all time scales are as shown.) The fact that

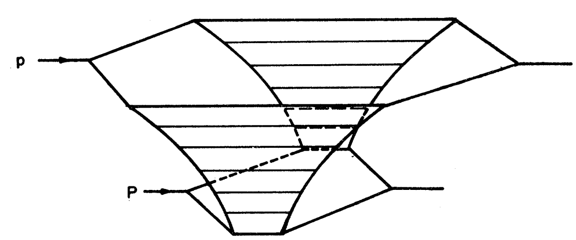

FIG. 8. The Mandelstam diagram in old-fashioned perturbation theory.

the structure at the bottom of the graph must be as shown can be verified by considering the scattering in the projectile rest frame. More general structures at the Reggeon "vertices" are possible, and it is easy to see that a general requirement for both Reggeons to be emitted at early times is that the two-Reggeon, two-particle vertex function possess a nonvanishing third double spectral function. By similar arguments one can verify that a nonvanishing triple-Reggeon vertex also requires such "overlapping" time orderings. One cannot obtain any useful information on the discontinuity content of the Mandelstam graph, at this stage, because the amplitude as well as several possible discontinuities are nonzero.

There is a possible loophole in the argument that the AFS graph vanishes which will come to have some importance for scattering on nuclei. Suppose we consider the "AFS-type" graph in which a fast hadron scatters successively from *two* targets, but with an elastic intermediate state (Fig. 9). If the separation between targets is small, $O(1/m)$, then the two hadrons will interact with each other and essentially make up a single composite state which will behave like a single target as far as the projectile is concerned. However, if the separation is large enough, e.g., macroscopic, then the graph is surely not zero. This is just the statement that a fast hadron can scatter elastically in a bubble chamber and then

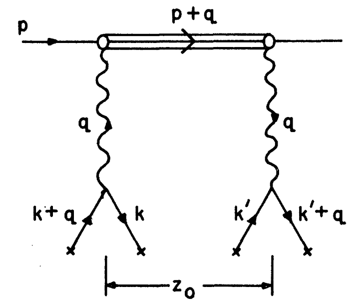

FIG. 9. Scattering from two hadrons separated by a distance z_0.

scatter again in the beam dump. We can obtain a rough criterion for the nonvanishing of planar or AFS-type rescattering by requiring that the projectile should not have passed the second target before the second Reggeon can develop low-momentum components; if z_0 is the separation between targets then the requirement is $p/m^2 \lesssim O(z_0)$.

To check the qualitative argument just given we shall reformulate it in momentum space. Suppose we describe the interaction with the two hadrons as occurring through approximately localized [up to $O(1/m)$] time-independent external sources J and J' with $J(\underline{x}, z) = J'(\underline{x}, z - z_0)$.[24,25] Then the scattering amplitude corresponding to Fig. 9 satisfies

$$A \propto \int d^3k\, d^3k'\, d^3q\, \tilde{J}(k+q)\tilde{J}(-k)\tilde{J}'(k')\tilde{J}'(-k'-q)\, \beta((k+q)^2, k^2; q^2)\, \beta(k'^2, (k'+q)^2; q^2)(p/m)^{2\alpha(q^2)-2} f((p+q)^2, q^2), \tag{2.28}$$

where α is the Regge trajectory and β its residue function, k and q are three-vectors,[26] and

$$\tilde{J}(k) = \frac{1}{(2\pi)^3}\int d^3x\, e^{-i(\underline{k}\cdot\underline{x} + k_z z)} J(\underline{x}, z).$$

Because of the relation between J and J' we have $\tilde{J}'(k) = e^{-ik_z z_0}\tilde{J}(k)$. The quantity f is the two-particle, two-Reggeon vertex function which for an elastic intermediate state has the form

$$f((p+q)^2, q^2) = [\beta(m^2, (p+q)^2; q^2)]^2 \frac{1}{(p+q)^2 - m^2 + i\epsilon};$$

plus a similar term with $(p+q)$ replaced by $(p-q)$. We are interested only in that part of (2.28) which depends on q_z, and if we suppress irrelevant variables this is

$$A \propto \int_{-\infty}^{\infty} dq_z\, e^{iq_z z_0} f((p+q)^2 + i\epsilon, q^2) F(q_z, \ldots). \tag{2.29}$$

We may suppose that $\tilde{J}$ and f are nonsingular functions which vanish at large values of their arguments, in which case F will have these properties as well. The contour of integration and the singularities of the integrand are shown in Fig. 10 for the case where f has intermediate states in $(p+q)^2$. [The $(p-q)^2$ terms in f are identically

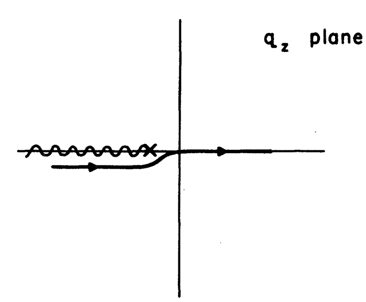

FIG. 10. Singularities in q_z plane in Eq. (2.28).

zero, from (2.29).] If $z_0=0$, we may close the contour below and obtain zero; this is the momentum space argument for the vanishing of the ordinary AFS cut.[27] Suppose $0<z_0\ll p/m^2$; then one can have $|pq_z|$ and hence $(p+q)^2$ much greater than typical values of m^2 in f, while $|q_z z_0|<1$. Now β and F fall rapidly in this limit so the contour distortion argument can be repeated (with a large but finite contour) and $A\approx 0$. However, if $z_0 \gtrsim p/m^2$ then large values of $(p+q)^2$ require large values of $|q_z z_0|$ and the exponential does not allow the contour to be closed below and the argument fails. This confirms the heuristic reasoning given above.

The same technique of using spatially separated targets can be applied to graphs with nonplanar Reggeon vertices, and one can derive the Abramovskii-Kancheli-Gribov cutting rules.[9] The details are given elsewhere[28] but we would like to review a few salient features of the argument here. In the derivation we let a hadron of momentum p scatter successively from two sources J and J' separated by a distance $z_0\ll p/m^2$, and considered different time sequences of interaction with the sources. For example, if p interacts with J before J' then the situation is as shown in Fig. 11 and we see that the only possible intermediate states (possible cuts through the diagram) are a diffractive and a one-Reggeon state. The analyticity properties of the amplitude allow one to obtain a constraint on the corresponding discontinuities of the two-particle, two-Reggeon vertex function. By considering other time orderings of interactions with the sources one can isolate other intermediate states and obtain further constraint equations. The key to the AKG rules is that the Reggeon vertex involves only high-momentum components, and its typical time scales are much larger than the time scales relating to the direct interactions with the sources. Hence the time sequences of interactions with the sources are irrelevant for the Reggeon vertex itself, and the various constraint equations refer to different discontinuities of the same function. If the Reggeon vertex is not controlled by long time scales, as is the case in the planar dual string model (see Sec. VIII), the AKG cutting rules are not correct.[11] See note added in proof.

III. SPACE-TIME DESCRIPTION OF HADRON-NUCLEUS SCATTERING

In this section we shall use our understanding of the space-time properties of hadronic interactions to give a qualitative discussion of hadron-nucleus scattering.[12] The general aspects of the problem are simple and obvious in this approach, although in a subsequent section we will revert to momentum space for more precise arguments.

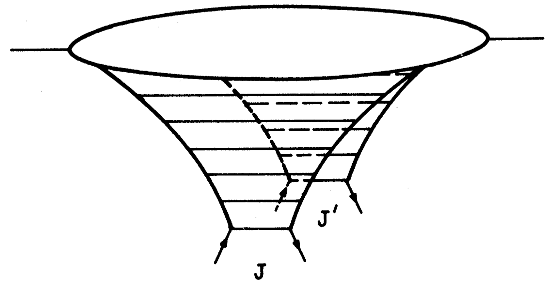

FIG. 11. Scattering from two separated sources in old-fashioned perturbation theory.

For simplicity we shall approximate the nucleus by a weakly time-dependent external source $J(t,\underline{x},z)$, coupled to the ϕ field through a term

$$\int d^4x\, J(x)\phi(x)=\int d^4k\, \tilde{J}(k)\tilde{\phi}(-k)$$

in the action. In order that the source resemble a real nucleus we assume J is nonvanishing only for $(\underline{x}^2+z^2)^{1/2}\lesssim R$ for all times. In principle we can allow local variations in space and time on a scale $O(1/m)$ although we shall neglect these fluctuations in practice. In consequence, the principle Fourier components of $\tilde{J}(k)$ satisfy $|\underline{k}|$, $k_z \lesssim O(1/R)$, $k_0\approx m$. Since these values of much less than typical momenta enter into hadron-hadron interactions, there will be little transverse momentum transferred to a fast incident hadron, and it is a reasonable approximation to assume that all scattering occurs at a single impact parameter.[29]

The scattering amplitude of momentum $p\gg R$ off the source J can be written

$$\langle p'|S|p\rangle_J=\langle p'|U_J(\infty,-\infty)|p\rangle\,, \tag{3.1}$$

where $U_J(t_2,t_1)$ is the time-evolution operator in the presence of the source $J(x)$:

$$U_J(t_2,t_1)=T\exp\left(i\int_{t_1}^{t_2}dt\int d^3x[\mathcal{L}_I(x)+J(x)\phi(x)]\right). \tag{3.2}$$

If we assume the first interaction with the source occurs at $t=0$ then

$$\langle p'|S|p\rangle_J=\langle p'|U_J(\infty,0)U(0,-\infty)|p\rangle\,,$$

where U is the usual time-evolution operator, given by (2.2) with $J=0$. Since $U((0,-\infty)=\Omega^{(+)}$, the wave operator, we can use (2.8) to write the scattering amplitude in terms of the "incident state vector" as

$$\langle p'|S|p\rangle_J=\langle p'|U_J(\infty,0)|\psi_p\rangle\,.$$

Now $|\psi_p\rangle$ contains quanta with longitudinal momenta ranging from zero to $O(p)$; the latter have lifetimes $O(p)$ and if $p \gg R$ they will have passed through the nucleus before they can interact with it. Any quanta which are going to recombine with these high-momentum components must have passed through the nucleus with a time $O(1/m)$ of $2R$. Therefore, we can choose a time $T \gtrsim 2R + O(1/m)$ such that no interactions with the source can occur at times later than T, for the class of amplitudes we consider. If we write $U_J(\infty, 0) = U_J(\infty, T)U_J(T, 0)$, then in the last equation we can replace $U_J(\infty, T)$ by $U(\infty, T) = U(\infty, 0)U(0, T)$ and thus[30]

$$\begin{aligned}\langle p'|U_J(\infty, -\infty)|p\rangle &= \langle p'|U(\infty, 0)[U(0, T)U_J(T, 0)]U(0, -\infty)|p\rangle \\ &= \langle \psi_{p'}|U(0, T)U_J(T, 0)|\psi_p\rangle . \end{aligned} \tag{3.3}$$

Now in the time interval T, an interaction with the nucleus can only occur if the state $|\psi_p\rangle$ contains particles with momenta $\lesssim R$. This follows from the fact that in a softened theory, a quantum of momentum k requires a time interval $O(k)$ to develop low-momentum components which can interact with the source J. If we work in the impact parameter basis of Sec. II C, then the probability that $|\psi_{p\underline{x}}\rangle$ has a quantum of momentum $\lesssim R$ at impact parameter $\underline{y}$ is proportional to $(p/R)^{\alpha-1}f(\underline{x}, \underline{y})$. Here the function f is of order unity, and from our earlier remarks we can take $\underline{y} \approx \underline{x}$. If we integrate this probability over impact parameter, then from (3.3) we have

$$\sigma_{\text{tot}} \propto \pi R^2 (p/R)^{\alpha-1} . \tag{3.4}$$

Furthermore, since the components of $|\psi_p\rangle$ with $p \gtrsim R$ are unaffected by the nucleus we expect that this part of the particle spectrum is not greatly different than the particle spectrum for a single hadron target.[12] The upper spectrum would be exactly the same if single-Reggeon states dominated, but this need not be the case (see Secs. V and VI). At very high energies, when there is a range of momenta k such that $p \gg k \gg R$, we can speak of a central region in which the AKG cutting rules imply that the shape (but not the normalization) of the inclusive cross section is the same as that on a hadron target.

The cross section in (3.4) is not the result that would obtain in a simple mean free path argument. Since the hadron-hadron total cross section is proportional to $p^{\alpha-1}$, the mean free path is

$$l = \frac{1}{\rho\sigma} \propto p^{1-\alpha}$$

[ρ is the density, which is $O(m^3)$]. A naive argument would give $\sigma_{\text{tot}} = 2\pi R^2$ once $R/l \sim Rp^{\alpha-1} \gg 1$, whereas our result is smaller. The mean free path reasoning is invalid because the high-momentum components of $|\psi_p\rangle$ are completely transparent to the nucleus.

So far in this section we have assumed $\alpha(0) < 1$ so that single-Reggeon states dominate, for momentum components $\gtrsim R$ at least, and a discussion in terms of probabilities is quite clear. If $\alpha(0) = 1$ [or $\alpha(0) > 1$ for a limited range of p] then all that can be said at this point is that $\sigma_{\text{tot}} \lesssim 2\pi R^2$; this bound follows from the fact that there can still be no interaction unless $|\psi_{p\underline{x}}\rangle$ contains momentum components $\lesssim R$, and the corresponding probability cannot exceed one at each value of $\underline{x}$. Our remarks on the high-momentum part of the particle spectrum depend only on the softness of the field theory model, and remain valid even if $\alpha(0) \gtrsim 1$.

IV. THE MULTIPLE-SCATTERING EXPANSION

The standard approach to quantum scattering from composite systems is an expansion in the number of interactions between the projectile and the individual constituents of the target—the well-known Glauber series.[7] An immediate consequence of this approach is that for an absorptive elementary interaction the cross section should approach a black disc form when the extent of the target becomes much larger than the mean free path of the projectile. In Sec. III we have shown that this result fails in the softened-field-theory description of hadron-nucleus scattering, and we wish to discuss the physical origin of the discrepancy.

Our first observation is that an essential assumption in the derivation of the multiple-scattering expansion is invalid in a softened-field-theory model. In order to relate a double-scattering event like that of Fig. 12 to the scattering amplitude for the projectile on a single constitutent, one assumes that the state or wave packet incident on the second constituent has the same scattering amplitude as an incoming free particle state. In other words, the time scales of the elementary interactions are presumed sufficiently short that the projectile after its first scattering has already evolved into a particlelike state before it begins to interact again. When the time scales involved in the elementary interactions grow with the incident momentum, however, then at high energy the state only becomes particlelike a long distance

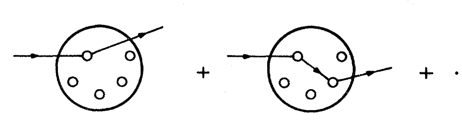

FIG. 12. The Glauber expansion.

beyond the first scattering, and the Glauber series cannot be derived.[31] One might hope that the elegance of the result would somehow transcend the limitations of its derivation, but this is not the case. We shall show that in the softened-field-theory model of Reggeons the multiple scattering series is explicitly nonunitary.

Consider first, for orientation, scattering off a large nucleus via an optical potential. (This can be thought of as an interaction between the incident hadron and the nuclear constituents through the exchange of vector mesons, but with an imaginary coupling to simulate an absorptive interaction.) The first term in the Glauber series is just the elementary interaction times the number of scatterers at the projectiles impact parameter, and is proportional to R.[32] At large R this violates the unitarity bounds on the state vector or on the scattering amplitude. Double scattering is proportional to the probability of finding two scatterers at impact parameter $\underline{x}$ and is proportional to R^2. Each successive term violates the unitarity bound even more strongly. However, the sum of the Glauber series corresponds to absorption of the incident wave, and hence the full series is perfectly consistent with elastic unitarity. The superficial violation of unitarity in the individual terms of the series presents no difficulty because different numbers of elementary scatterings do not lead to distinguishable final states and it is only the full, summed, amplitude that must be unitary. (Alternatively, any number of vector-meson exchanges is coherent with any other number and we need not expect a unitary answer until the series is summed.) This is physically sensible, since the statement that the single-scattering term violates unitarity means that single-scattering alone is a bad approximation.

The generalization of the Glauber series to scattering on nuclei in the relativistic case has been given by Gribov.[25] The result is formally quite similar to the nonrelativistic case: At each impact parameter one has a power series in $RA(p\underline{x})$ where A is the hadron-hadron scattering amplitude, proportional to $p^{\alpha-1}$ for Regge pole exchange. Again the separate terms in the series violate the unitarity bound while the sum does not. Unlike the potential scattering or vector-meson exchange case, however, the individual terms are not coherent. For example, a general two-Reggeon exchange diagram can have a discontinuity through both Reggeons leading to a final state orthogonal to any final state associated with one-Reggeon exchange. Thus, in any relativistic theory, where the interaction with a constituent of the nucleus can lead to particle production, different numbers of scatterings are not coherent, and the unitarity requirements must be satisfied for each incoherent component separately. In a sense, this difficulty is a failure of another of the assumptions leading to the Glauber series. It is customary to consider only the multiple scattering of the incident particle and possibly its low-mass excitations, but not the rescattering of an arbitrary multiparticle intermediate state which can be generated by an elementary interaction. It is just the latter intermediate states which complicate the unitarity question, and a calculation which does not treat them properly is somewhat suspect.

In the softened-field-theory model, each term in the multiple-scattering expansion corresponds to a different component of the incident state $|\psi_{p\underline{x}}\rangle$. For example, the single-scattering or single-Reggeon term comes from the part of the wave function shown (in old-fashioned perturbation theory) in Fig. 13, where $p_n \lesssim R$ in order that an interaction can take place before the projectile crosses the nucleus. Double scattering or two-Reggeon exchange requires a component of the form shown in Fig. 14, where $p_n, k_m \lesssim R$. Both Reggeons must be "formed" at the time the projectile crosses the front of the nucleus. The generalization to three or more Reggeon exchange is obvious. The point is that the higher-momentum components ($p_i \gtrsim R$) of these various states do not interact with the nucleus and remain orthogonal as the hadron crosses the nucleus. Therefore, the wave functions appropriate to the various terms in the multiple-scattering series are orthogonal and interference effects cannot cancel the unitarity violations in the individual terms.

We can make these remarks more precise by the following argument. We expand the incident state as

$$|\psi_{p\underline{x}}\rangle = \Omega^{(+)}|p\underline{x}\rangle = \lambda_0|\psi^{(0)}_{p\underline{x}}\rangle + \lambda_1|\psi^{(1)}_{p\underline{x}}\rangle + \cdots , \qquad (4.1)$$

where $|\lambda_0|^2 + |\lambda_1|^2 + \cdots = 1$. Here $|\psi^{(0)}_{p\underline{x}}\rangle$ is defined to have no quanta with momenta $\lesssim R$ (i.e., if we expand $|\psi^{(0)}_{p\underline{x}}\rangle$ in the Fock-space basis (2.13), then $\psi_n^{(0)}(p\underline{x}; p_1\underline{x}_1 \cdots p_n\underline{x}_n)$ vanishes when any $p_i \lesssim R$), while for $i = 1, 2, \ldots$ the $|\psi^{(i)}_{p\underline{x}}\rangle$ represent the i-Reggeon components of the state. Thus $|\psi^{(1)}_{p\underline{x}}\rangle$ has quanta with momenta $\lesssim R$ with probability proportional to $(p/R)^{\alpha-1}$, $|\psi^{(2)}_{p\underline{x}}\rangle$ has quanta with momenta $\lesssim R$ with probability $(p/R)^{2\alpha-2}$, and so on.[33] Substituting (4.1) into Eq. (3.3) for the scattering amplitude off a nucleus,

$$\langle p'\underline{x}'|S|p\underline{x}\rangle_J = \langle\psi_{p'\underline{x}'}|U(0,T)U_J(T,0)|\psi_{p\underline{x}}\rangle = \sum_{ij} \lambda_i^* \lambda_i \langle\psi^{(i)}_{p'\underline{x}'}|U(0,T)U_J(t,0)|\psi^{(j)}_{p\underline{x}}\rangle .$$

The matrix element is zero if $i \neq j$ because an interaction over a time $T \sim R$ cannot alter the density of quanta with momenta $>R$. This is the statement of incoherence of the exchanges of different numbers of Reggeons. Thus

$$\langle p'\underline{x}'|S|p\underline{x}\rangle = \sum_i |\lambda_i|^2 \langle \psi^{(i)}_{p'\underline{x}'}| U(0,T)U_J(T,0)|\psi^{(i)}_{p\underline{x}}\rangle . \quad (4.2)$$

Since U and U_J are unitary operators, if we write

$$\langle \psi^{(i)}_{p'\underline{x}'}| U(0,T)U_J(T,0)|\psi^{(i)}_{p\underline{x}}\rangle = \delta(p-p')\delta^2(\underline{x}-\underline{x}')\overline{\psi}_i(p\underline{x}),$$

then the Schwarz inequality implies $|\overline{\psi}_i(p\underline{x})| \le 1$. Thus (at fixed $\underline{x}$) the contribution of n-fold multiple scattering is bounded by a constant, in contrast to the result proportional to $[p^{\alpha-1}R]^n$ expected from a mean free path argument.

V. FORMAL ARGUMENTS FOR REGGEON-NUCLEUS COUPLINGS

In Sec. III we saw that the coupling of a Regge pole to a large nucleus varies as $R^2R^{1-\alpha}$. Our arguments there were based in detail on the wave function of a rapidly moving object. These arguments, especially as discussed in Sec. IV, are very field-theory–dependent. Indeed, the discussion of Sec. IV talks about the details of the Fock-space representation of a hadron. Perhaps the field-theory basis could be eliminated by defining the wave function by an analytic continuation in the energy. (Recall that for the deuteron it is possible, in potential theory, to obtain the wave function of the deuteron by an analytic continuation in energy of the proton-neutron scattering states.) We have not attempted to do this. Rather, we shall present some alternative arguments which are not so *intrinsically* tied to field theory. We shall still talk of Feynman diagram and space-time structure, but here a multiperipheral advocate can obtain the same conclusion without using the underlying field theory in any great detail. The reader will recall that, as in Sec. II E, it is really not necessary to use the detailed structure of the field theory in order to obtain a space-time description of the multiperipheral model. We begin our discussion by treating the case when the momentum of the incoming particle is on the order of the radius of the nucleus. The results obtained in this case will then be used in obtaining the form of the cross section when $p/R \gg 1$.

A. Scattering off a nucleus; p on the order of R

Consider the amplitude, $A(P',p',\underline{p}';P,p,\underline{p})$, for the elastic scattering of a high-momentum particle of momentum $p_\mu = ((p_\mu{}^2+p^2+\underline{p}^2)^{1/2}, \underline{p}, p)$ into a particle of momentum $p'_\mu = ((p'^2_\mu + p'^2 + \underline{p}'^2)^{1/2}, \underline{p}', p')$ off a large nucleus whose size is on the order of p. The problem simplifies in impact-parameter space where the elastic scattering is diagonal. Equivalently we may just consider a one-dimensional problem and at the end of the calculation multiply all amplitudes by πR^2 to get cross sections. We shall follow this latter procedure so that we are now dealing with a one-dimensional nucleus which, for simplicity, we shall take to be without correlations between the various nucleons.

In order to get a lower bound on the elastic scattering amplitude $A(P,p;Pp)$ we shall solve a modified problem. For our uncorrelated one-dimensional nucleus the probability density that the N nucleons are at positions $z_1, z_2, \ldots, z_N$ is given by

$$\mathcal{P}(z_1, z_2, \ldots, z_N) = |\psi(z_1)\psi(z_2)\cdots\psi(z_N)|^2,$$

where $\psi(z)$ is the wave function for a single nucleon in the nucleus. The probability density for finding a nucleon at z is simply

$$\mathcal{P}_1(z) = N|\psi(z)|^2,$$

while the probability density for finding a nucleon at z_1 and another nucleon at z_2 is

$$\mathcal{P}_2(z_1,z_2) = \frac{N(N-1)}{2}|\psi(z_1)\psi(z_2)|^2.$$

Higher probability functions are similarly given. The scattering of a high-momentum particle off the one-dimensional nucleus is given in terms of the $\mathcal{P}_i$'s once the hadron dynamics is known. What we shall do is solve the problem for the modified probabilities $\mathcal{P}'_i$ where

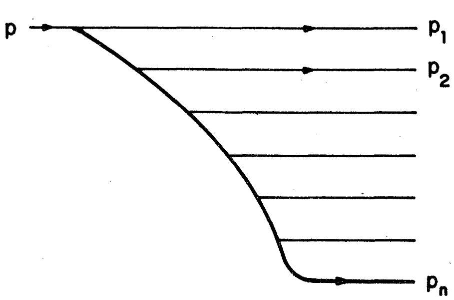

FIG. 13. A one-Reggeon state.

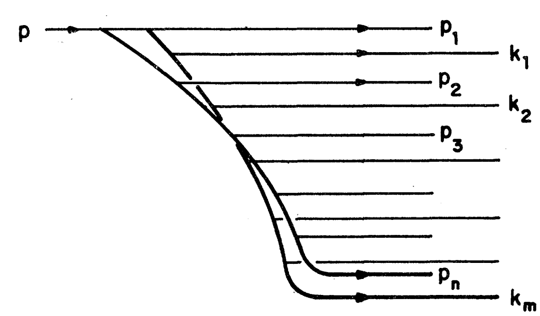

FIG. 14. A two-Reggeon state.

$$\mathcal{P}_1'(z)=\begin{cases}\mathcal{P}_1(z) \text{ if } n\Delta-\delta/2<z<n\Delta+\delta/2\\ 0 \text{ otherwise,}\end{cases}$$

$$\mathcal{P}_2'(z_1,z_2)=\begin{cases}\mathcal{P}_2(z_1,z_2) \text{ if } n_1\Delta-\delta/2<z_1<n_1\Delta+\delta/2 \text{ and } n_2\Delta-\delta/2<z_2<n_2\Delta+\delta/2\\ 0 \text{ otherwise,}\end{cases}$$

etc., where n, n_1, and $n_2=0, \pm1, \pm2, \ldots$. The probabilities $\mathcal{P}_i'$ correspond to a subset of the nucleus in which nucleons lying outside bands of widths δ, each separated by a distance Δ, are neglected. The reason for introducing this device is to avoid the serious multiple-counting problems which arise in a direct summation of hadron-nucleon interactions. We suppose that δ is on the order of the intranuclear spacing and we shall specify Δ later, although $\Delta \gg \delta$ will hold always. To obtain $\mathcal{P}_i'$ from $\mathcal{P}_i$ one simply rejects any measurement which does not occur in one of the small intervals $n\Delta-\delta/2<z<n\Delta+\delta/2$ for $n=0, \pm1, \pm2, \ldots$.

If the basic hadron-hadron interaction is predominately inelastic *with slow secondaries produced,* which is certainly the realistic situation, then the probability of an inelastic reaction using the $\mathcal{P}_i'$'s cannot be significantly greater than the situation when the $\mathcal{P}_i$'s are used. This is perhaps clear since going from $\mathcal{P}_i$ to $\mathcal{P}_i'$ means that only interactions are counted which occur in certain small intervals. If the basic hadron-hadron interaction did not produce slow secondaries the problem would be more complicated since then there could be interferences between interactions with various nucleons. If slow-moving secondaries are produced, of momentum on the order of m, in each hadron-hadron collision then an interaction of the incoming particle with a nucleon of the nucleus already produces physical particles before any subsequent interaction with a nucleon further along the path of the incoming particle takes place. Thus, eliminating matter (as going from $\mathcal{P}_i$ to $\mathcal{P}_i'$ does) can only decrease the probability of an inelastic interaction.

The calculation of $A'(p)$, the interaction of a fast particle (in one dimension) with the nuclear source defined by the $\mathcal{P}_i'$, can be done in terms of Regge poles, at least for certain regions of Δ. This result when multiplied by πR^2 should furnish a reasonable lower bound for the interaction of a high-momentum hadron with a real nucleus. For single-Regge-pole exchange, as illustrated in Fig. 15,

$$A_1'(p)\propto\sum_{i=1}^{2n}p^{\alpha-1}\beta_\gamma=(p/R)^{\alpha-1}(R^\alpha/\Delta)\beta_\gamma\,,$$

where $n=R/\Delta$ and R is one-half the length of the one-dimensional nucleus, β is the coupling of the Regge pole to the fast hadron, and γ is the coupling of the Regge pole to the small interval of matter defined by $\mathcal{P}_1(z)$. We assume $p \gtrsim R$.

Two-Reggeon exchange can be similarly calculated. For brevity we give only the triple-Reggeon part of the coupling although the form of the result does not depend on this simplifying assumption. Then, as illustrated in Fig. 16,

$$A_2'(p)\propto\frac{1}{p}\sum_{j=2}^{2R/\Delta}\sum_{i=1}^{j-1}\int dq_3\, e^{iq_3(z_j-z_i)}\gamma^2\beta\beta_3 \times[p^\mu q_\mu]^\alpha[p/p^\mu q_\mu]^{2\alpha-1}\,,$$

where β_3 is the triple-Reggeon coupling, and the q integral and exponential factors arise just as in our discussion of spatially separated hadrons in Sec. II F. Since $z_j-z_i=\Delta(j-i)$, we may set $q_3\sim[\Delta(j-i)]^{-1}$ and thus

$$A_2'(p)\propto\frac{1}{p}\sum_{j=2}^{2R/\Delta}\sum_{i=1}^{j-1}\gamma^2\beta\beta_3\left[\frac{p}{\Delta(j-i)}\right]^\alpha[\Delta(j-i)]^{2\alpha-1} \propto(p/R)^{\alpha-1}(R^\alpha/\Delta)^2\,.$$

Three-Reggeon exchange, as shown in Fig. 17 for a particular configuration of indices, is given by

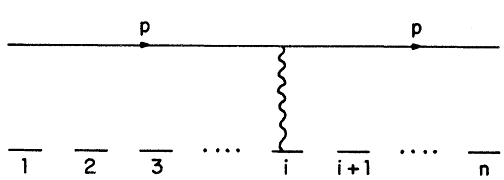

FIG. 15. One-Reggeon interaction with the "reduced" nucleus described in the text.

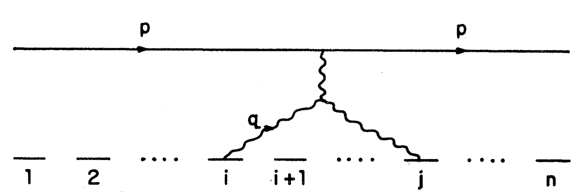

FIG. 16. Two-Reggeon interaction with the "reduced" nucleus.

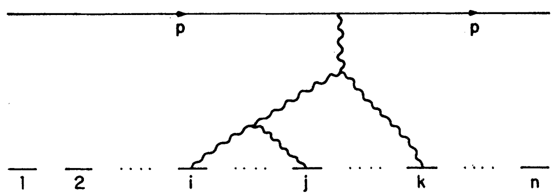

FIG. 17. Three-Reggeon interaction with the "reduced" nucleus.

$$A_3' \propto \frac{1}{p}\sum_{k=3}^{2R/\Delta}\sum_{j=2}^{k-1}\sum_{i=1}^{j-1}\gamma^3\beta_3{}^2\beta\left[\frac{p}{\Delta(k-i)}\right]^\alpha[(j-i)\Delta]^{2\alpha-1} \times\left[\frac{k-i}{j-i}\right]^\alpha[(k-i)\Delta]^{\alpha-1},$$

where again only triple-Regge couplings have been kept for simplicity. Now, one obtains

$$A_3' \propto (p/R)^{\alpha-1}\left(\frac{R^\alpha}{\Delta}\right)^3 .$$

The pattern for higher numbers of Reggeon exchange is clear. If $R^\alpha/\Delta \lesssim 1$ we have taken into account all relevant interactions and find that, as far as the total cross section is concerned, single-Reggeon exchange is not altered in any radical way. Thus $\sigma \gtrsim c\pi R^2(p/R)^{\alpha-1}$, where c is a constant of order one, should be a correct lower bound for the scattering of a high-energy hadron off a realistic nucleus. If $R^\alpha/\Delta \gg 1$ we again run into the multiple counting problems of Sec. IV and we can arrive at no result from direct Reggeon summations. In the case of $\alpha=1$, we may obtain the lower bound simply by taking a single band.

If p/R is on the order of one then our bound becomes $\sigma \gtrsim c\pi R^2$ and for an on-mass-shell particle the unitarity bound (2.23) implies that the cross section is in fact proportional to πR^2. It is not hard to show that going off the mass shell by an amount proportional to m^2 does not radically alter the cross section so long as $p \gtrsim R$. To see this we again go to one dimension although an impact-parameter representation could equally well be used. The integral equation

$$A(p', p) = V(p', p) + \frac{i}{(2\pi)^2}\int \frac{d^2p''}{p''^2 - m^2 + i\epsilon} \times V(p', p'')A(p'', p)$$

describes scattering off a one-dimensional nucleus. Here $V(p', p)$ is one-particle irreducible as far as the fast incoming hadron is concerned and variables expressing the intermediate nuclear states are suppressed. The equation is illustrated in Fig. 18. Now $V(p', p)$ is not strongly dependent on whether or not the particles are on their mass shell or not, and this is true irrespective of the relative sizes of R and p. (Recall $2R$ is the length of the one-dimensional nucleus.) The lack of dependence on mass-shell conditions comes from the elimination of the one-particle intermediate state. When $p/R \ll 1$, V is very large. In fact, V must be proportional to R/p since the interactions with the nucleus must all take place within a distance proportional to p and translation invariance demands that this region be placed arbitrarily in the nucleus. The second term on the right-hand side in the above integral equation can depend critically on whether or not the external particles are on their mass shells. Suppose $p/R \ll 1$; then if the incoming and outgoing particles are on their mass shells, A must be a constant independent of R for large R. This means that the second term on the right-hand side of the integral equation must cancel the first term, at least as far as any term which grows with R. However, for off-shell particles only a few iterations of the integral equation can be important. (In general, the number of iterations which are important will be proportional to $m^2 \times \max[(p^2 - m^2)^{-1}, (p'^2 - m^2)^{-1}]$, but less than R/p.) A few iterations cannot cancel V and the result must be proportional to R/p. If $p/R \approx$ constant the number of iterations of the potential, V, is always finite and again on- and off-mass-shell behavior cannot be very different and A must be independent of R for fixed p/R. Thus, we arrive at the result that for p/R on order of one the scattering of an on-shell particle on a realistic nucleus must have cross section proportional to πR^2 which is the magnitude of the single-Regge-pole residue for nuclear coupling.

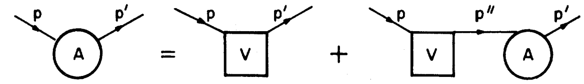

FIG. 18. Bethe-Salpeter equation for hadron-nucleus scattering.

B. Scattering off a nucleus; $p/R \gg 1$

Once we know the scattering amplitude for a particle of momentum p on the order of R off a nucleus it is not difficult to find the amplitude when $p/R \gg 1$. To begin, define $Y_R = \ln Rm$ and $Y = \ln p/m$. Then in the one-dimensional nucleus, or for a fixed impact parameter, write down all the old-fashioned perturbation-theory graphs which are relevant for the high-energy scattering. Now take the imaginary part of the amplitude, $A(p)$, and keep only those intermediate states, in the sense of old-fashioned perturbation theory, for which the number of particles in the intermediate state having momentum greater than Y_R is *exactly* equal to $\bar{n}(Y - Y_R)$, where $\bar{n}(Y)$ is the number of particles produced in a hadron-hadron collision of relative rapidity Y. Call the amplitude for all such events $\tilde{A}(p)$. It is clear that for all internal momenta k greater than $e^{Y_R}\tilde{A}(p)$ has a simple one-Regge-pole structure. The connection to momenta less than e^{Y_R} can be made, approximately, by hooking two off-shell particles of momentum less than or equal to e^{Y_R} to the nucleus as given below and illustrated in Fig. 19:

$$\tilde{A}(p) \propto \int' \frac{d^2k}{[k^2 - m^2]^2} \tilde{A}_{\bar{n}}(p,k)\,\mathrm{Im}A(k)\theta(R - |k_z|).$$

The region of integration is $|k_z| < R$, and $(1/k_z) \times A(k)$ is the scattering amplitude for an off-shell particle on the nucleus. $\tilde{A}_{\bar{n}}(p',k)$ is the imaginary part of the hadron-hadron amplitude which has intermediate states of exactly $\bar{n}(Y - Y_R)$ particles with rapidity greater than Y_R. Now in the above integral the region $k_z/R \ll 1$ is suppressed in a ϕ^3 theory, so that the dominant region is $k_z/R \approx$ constant, which gives, using the results of the previous section,

$$\tilde{A}(p) \propto \tilde{A}_{\bar{n}}(p_1 k)\,\mathrm{Im}A(k)|_{k_z \propto R}.$$

Now

$$\tilde{A}_{\bar{n}}(p'k)|_{k_z \propto R} \propto \frac{1}{[\bar{n}(Y - Y_R)]^{1/2}} \frac{1}{p} (p/R)^{\alpha},$$

while

$$\mathrm{Im}A(k)|_{k \propto R} \propto R;$$

thus

$$\tilde{A}(p) \propto \frac{1}{[\bar{n}(Y - Y_R)]^{1/2}} (p/R)^{\alpha - 1}.$$

If we drop the $[n(y - y_R)]^{1/2}$ we can identify $\tilde{A}(P)$ as the one-Regge-pole contribution to the imaginary part of hadron-nucleus scattering in one dimension (fixed impact parameter). (The $\sqrt{\bar{n}}$ comes from the fact that we are taking only one particular type of event and the width of the multiplicity distribution of particles produced is proportional to $\sqrt{\bar{n}}$ for one-Regge-pole exchange.) If we put in all impact parameters or go from the one-dimensional case we obtain

$$A(p) \propto \pi R^2 \times (p/R)^{\alpha - 1}$$

for one-Regge-pole exchange in hadron-nucleus scattering in agreement with the arguments given in Sec. III. If Regge cuts or interactions are present for rapidities greater than Y_R the procedure is similar, and a general rule is that any number of Reggeons couple to a nucleus with a strength proportional to πR^2 and the scale parameter of a Regge expansion is proportional to R.

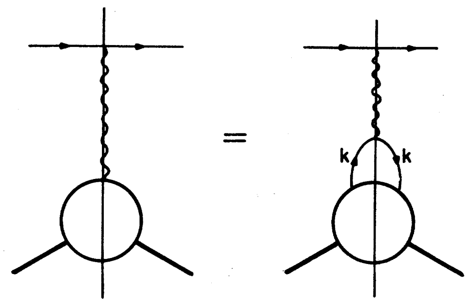

FIG. 19. Coupling one Reggeon to the nucleus.

VI. INCLUSIVE CROSS SECTIONS AND REGGE CUTS

We have seen in previous sections that for $\alpha = 1$ the Regge pole residue is proportional to πR^2, and it is also clear that the residue function for the exchange of higher numbers of Reggeons is proportional to πR^2 as well. [The distinction between pole and cut terms in nuclear scattering is most easily made when $\alpha(0) < 1$, and once the separation is made the limit $\alpha(0) \to 1$ can be taken.] Regge cuts need not be as negligible as in hadron-hadron scattering, and in this section we shall attempt to qualitatively discuss their influence on elastic and inclusive cross sections. We first describe what we feel is the "natural" behavior of softened field theory in this respect, and then go on to derive some rather strong constraints on the possible values of elastic and inclusive cross sections. We shall work in terms of the ratios

$$r = \left(\frac{\sigma_{DD}}{\sigma_{tot}}\right)_A \qquad (6.1)$$

and

$$R_c = \frac{[(1/\sigma_{inel})(d\sigma/dy)]_A}{[(1/\sigma_{inel})(d\sigma/dy)]_{had}}\bigg|_{\text{central region}}. \qquad (6.2)$$

Here the subscripts had and A refer to hadron and nuclear targets, respectively, σ_{DD} is the diffraction dissociation cross section *including* the elastic cross section, σ_{inel} is the inelastic cross section with diffractive excitations omitted; and $d\sigma/dy$ is the single-particle inclusive cross section for an unspecified final-state hadron.

We have seen in Secs. II and III that a high-momentum hadron incident on any target of size $\ll p$ "forms" most of its state long before reaching the target. Since one-Reggeon states dominate hadron-hadron scattering, we infer that the probability that a many-Reggeon component is present in the incident state is small. When the target is a nucleus rather than a hadron, the coupling of many-Reggeon states to the target may be larger but since the probability of any component of the state interacting is at most one, the target cannot greatly enhance the small probability of many-Reggeon states being present in the first place. One's natural expectation, then, is that exchange of at most a few Reggeons will dominate. Thus one expects an inclusive cross section of the form shown in Fig. 20, with the usual three regions:

(1) Nucleus fragmentation region: particles with momenta $\leq O(R)$ can rescatter before passing through the nucleus and we expect additional particle production here. An exact calculation of the particle density is impractical, but it is easy to see that there is an upper bound proportional to $A^{1/3}\sim R$. This follows from the fact that each of the few Reggeons deposits an energy $O(R)$ into this region and the particle density is maximum when all of this energy is converted to slow-particle production.

(2) Hadron fragmentation region: a detailed calculation is again impossible, but we can observe that the shape is likely to differ here because of the cut contribution. $d\sigma/dy$ cannot vanish because it receives an appreciable contribution from single-Reggeon exchange, which is nonvanishing in this region in hadron-hadron scattering.

(3) Central region: the AKB cutting rules imply that the inclusive cross section is given by the discontinuity of the single-Reggeon exchange 6-point function of Fig. 21, but the cuts do contribute to σ_{inel} and R_c need not exactly equal one. To the extent that cuts are nondominant, we expect the ratio r of (6.1) is not greatly different from its value in hadron-hadron scattering (≈ 0.2). The last statement may not be true in nature, where r may approach $\frac{1}{2}$ for large nuclei, and we now investigate what such a behavior would entail. (We remind the reader that all of the discussion in this section presupposes that p is sufficiently larger than R that a genuine central region exists.)

If r tends to $\frac{1}{2}$ then the elastic cross section approaches the inelastic and, naively at least, one imagines a black disc situation where a projectile certainly interacts with the target. This is unnatural in a Regge model (cf. Sec. III) but not obviously impossible. In such a situation the number of elementary interactions would be large, and since secondaries can be produced at each stage the inclusive cross section is enhanced, and thus we expect that increasing r also increases R_c. We shall now show that, in the absence of Reggeon interactions (see below for further discussion of this assumption), as r increases there is an increasingly strong lower bound on R_c and if $r\to\frac{1}{2}$ then $R_c\to\infty$. This result depends critically on the use of the AKG cutting rules, and so is restricted to softened-field-theory models of Reggeons. A model such as the planar dual string model, in which the time scales for an elastic interaction remain finite as the energy increases, would not satisfy the constraint equations we derive here. See note added in proof.

We begin with a simple example: Suppose $\alpha(0)=1$ and only one- and two-Reggeon exchange is allowed. The total cross section is then

$$\sigma_{\text{tot}}=A_1-A_2\ ,$$

where A_1 (A_2) is the one- (two-) Reggeon contribution, with A_1 and A_2 positive, and the AKG rules state that the partial cross sections for diffractive (including elastic), one-Reggeon, and two-Reggeon final states are, respectively,

$$\sigma_{\text{DD}}=A_2\ ,$$

$$\sigma_1=A_1-4A_2\ ,$$

$$\sigma_2=2A_2\ .$$

To maximize $r\equiv\sigma_{\text{DD}}/\sigma_{\text{tot}}$, we maximize A_2 but must respect the constraint that each partial cross section is non-negative. The maximum value occurs when $\sigma_1=0$ and $A_2=\frac{1}{4}A_1$, in which case $\sigma_{\text{tot}}=\frac{3}{4}A_1$, $\sigma_{\text{DD}}=\frac{1}{4}A_1$, $\sigma_2=\frac{1}{2}A_1$, and we find $r=\frac{1}{3}$ and $R_c=2$. Similarly, if one-, two-, and three-Reggeon exchange is allowed it is easy to show that the maximum value of r is $\frac{3}{7}$, in which case $R_c=3$. A large diffractive cross section is only obtained at the expense of a large inclusive cross section.

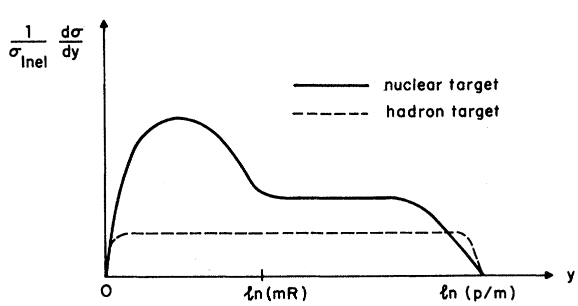

FIG. 20. Inclusive particle densities in hadron-hadron and hadron-nucleus collisions.

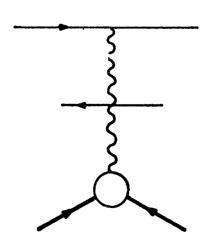

FIG. 21. Reggeon diagram for the inclusive cross section in a hadron-nucleus collision.

The diffractive cross section in hadron-hadron scattering has always been troublesome for Regge models, but the constraints of the AKG rules are particularly awkward. For example, the smallest value of r compatible with existing pp experiments is $r \approx \frac{1}{4}$ at Fermilab energies.[34] If we suppose that one- and two-Reggeon exchanges are to account for this, then arithmetic similar to that just performed yields $A_2 = \sigma_{DD} = \sigma_1 = \frac{1}{5}A_1$, $\sigma_2 = \frac{2}{5}A_1$, and $\sigma_{tot} = \frac{4}{5}A_1$. The triple-Pomeron contribution to the total and all partial cross sections is phenomenologically negligible here. The two-Reggeon term is "small" in the sense that the magnitude of its imaginary part is only $\frac{1}{5}$ that of the one-Reggeon term, but we see that it has a very large effect on the partial cross sections. The two-Reggeon cut is negligible for the total and single-particle inclusive cross sections, but can be expected to have a strong long-range effect on the two-particle correlation function. (One can easily show that allowing a three-Reggeon coupling as well does not alter this conclusion.) The Pisa-Stony Brook data[35] has no indication of long-range correlations, but one can presumably appeal to corrections from secondary trajectories and the absence of a sufficiently long rapidity interval.

Returning to hadron-nucleus scattering, we now consider the case where any number of Regge cuts (but still no Reggeon interactions) may be present. We write $\sigma_t = 2\pi R^2\gamma$, $\sigma_{DD} = 2\pi R^2 \gamma r$, and $\sigma_{in} = 2\pi R^2\gamma(1-r)$ where σ_t is the total cross section. γ and r are positive and less than one while R is the radius of the large nucleus. Further, write[9] $\gamma = \sum_{\nu=1}^{\infty}(-1)^{\nu+1}a_\nu$ where a_ν is the contribution to σ_t of ν Reggeon exchanges. We are taking $\alpha(0)=1$ so all of the $a_\nu \geq 0$. Define σ_k to be the cross section corresponding to exactly k cut Reggeons. Clearly

$$\sigma_t = \sigma_{DD} + \sum_{k=1}^{\infty} \sigma_k .$$

Define $\sigma_k = 2\pi R^2 \tilde{\sigma}_k$. Then, the AKG rules give

$$\gamma r = \sum_{\nu=2}^{\infty} (-1)^\nu a_\nu (2^{\nu-1}-1) \tag{6.3}$$

and

$$\tilde{\sigma}_k = \sum_{\nu=k}^{\infty} (-1)^{\nu-k}\binom{\nu}{k} 2^{\nu-1} a_\nu . \tag{6.4}$$

It will prove useful to express all quantities in terms of the $\tilde{\sigma}_k$. Equation (6.4) can be inverted to yield

$$a_\nu = 2^{1-\nu}\sum_{k=\nu}^{\infty}\binom{k}{\nu}\tilde{\sigma}_k .$$

Substituting this expression into Eq. (6.3) gives

$$\gamma r = \sum_{\nu=2}^{\infty}(-1)^\nu (2^{\nu-1}-1)2^{1-\nu}\sum_{k=\nu}^{\infty}\binom{k}{\nu}\tilde{\sigma}_k .$$

or (6.5)

$$\gamma r = \sum_{k=2}^{\infty}\tilde{\sigma}_k[1-2^{1-k}] .$$

The other relation between γr, γ, and the $\tilde{\sigma}_k$ is

$$\gamma = \gamma r + \sum_{k=1}^{\infty}\tilde{\sigma}_k \tag{6.6}$$

which simply expresses the total cross section in terms of the various types of events present. Now from the definition of R we can write

$$R_c = \frac{\sum_{k=1}^{\infty} k\tilde{\sigma}_k}{\gamma(1-r)} . \tag{6.7}$$

What we shall do now is to solve (6.5) and (6.6) for $\tilde{\sigma}_{n-1}$ and $\tilde{\sigma}_n$ in terms of r and γ. Then we shall minimize R_c over the remaining $\tilde{\sigma}_i$, $i \neq n, n-1$, holding r and γ fixed. This will give us the minimum value, R_{min}, which R_c can take for given γ and r. That is, this will give the minimum possible value of R_c for fixed value of σ_t, σ_{DD}, and σ_{in}.

Now (6.5) and (6.6) give

$$\tilde{\sigma}_n = 2\gamma(1-r) + 2^{n-1}\gamma(2r-1) - \sum_{k=1}^{\infty}{}' \tilde{\sigma}_k(2^{2-n}-2^{1-k}) ,$$

where the prime means that the $k=n-1$ and $k=n$ terms are not included in the sum. From this expression and (6.6) we obtain for (6.7)

$$R_c = n+1+\frac{2^{n-1}(2r-1)}{1-r} + \sum_{k=1}^{\infty}{}' \frac{\tilde{\sigma}_k(k-n-1+2^{n-k})}{\gamma(1-r)} . \tag{6.8}$$

We can now try to fix γ and r and minimize over the $\tilde{\sigma}_k$. If the $\tilde{\sigma}_k$ can take the value zero, this will certainly be the minimum value of R_c. The $\tilde{\sigma}_k$ can be zero only if

$$\gamma r = \tilde{\sigma}_{n-1}(1-2^{2-n}) + \tilde{\sigma}_n(1-1^{1-n})$$

and

$$\gamma(1-r) = \tilde{\sigma}_{n-1} + \tilde{\sigma}_n$$

is possible. This is possible if

$$\frac{1}{2}\,\frac{1-2^{2-n}}{1-2^{1-n}} < r < \frac{1}{2}\,\frac{1-2^{1-n}}{1-2^{-n}} . \tag{6.9}$$

Thus if r lies in the region (6.9) for some n then the minimum value of R_c is obtained by setting $\tilde{\sigma}_k = 0$, $k \neq n, n-1$, and this minimum value of R_c is

$$R_{min} = n+1+\frac{2^{n-1}(2r-1)}{1-r} . \tag{6.10}$$

Recalling that $\sigma_{DD}/\sigma_{in} = r/1-r$ it is clear that

(6.10) means that if σ_{DD} is at all comparable to σ_{in}, then R_c must be rather large.

If we attempt to understand a large value of r by introducing many strongly coupled Regge cuts, then the neglect of Reggeon interactions requires further thought. If only a few Regge cuts are present, then we could argue that the phenomenology of hadron-hadron scattering indicates that Reggeon couplings are small, and hence Reggeon interactions could only be very important in nuclei if these couplings were somehow enhanced. Such an enhancement would occur at very high energy, in the form of powers of the total rapidity interval arising from integrating over the position of the vertex, but this is of no interest for any feasible hadron-nucleus experiment. The only other possibility is a multiplication arising from a combinatoric factor reflecting the possibility of coupling to different nucleons in the nucleus, but we have seen that such enhancements are forbidden by unitarity. This we can safely neglect Reggeon interactions when only a few Reggeons are present. However, if we allow many-Reggeon cuts to appear, the situation changes. Recall that in configuration space a many-Reggeon state has a high density of particles at each rapidity and impact parameter, and it would be unreasonable to assume that the particles do not interact among themselves. This argument provides a loophole of sorts for evading the bounds (6.9) and (6.10): if $r\to\frac{1}{2}$ then many Reggeons are present and perhaps their interactions become critical. Physically, Reggeon interactions correspond to rescattering and/or reabsorption of produced particles, as in Fig. 22, and the statement that Reggeon interactions are important would imply that only a very complicated calculation of rescattering effects would suffice to describe hadron-nucleus and also hadron-hadron scattering, and a simple Regge expansion would have no value. In such a case it is possible that R_c could be considerably smaller than required by (6.10), but it is unlikely that it would be close to one.

VII. AN ILLUSTRATIVE MODEL OF THE NUCLEUS FRAGMENTATION REGION

Our previous arguments have not directly addressed the detailed dynamics of how a Reggeon couples to a nucleus. In Sec. III we simply assumed that the quanta in a Reggeon ladder with momenta $k_z \lesssim O(R)$ can and in general will interact with a nucleus of radius R, and in Sec. V we have shown that in fact the probability of such quanta interacting is bounded from above and below by a constant at each impact parameter. The reason for this circuitous procedure is that all slow

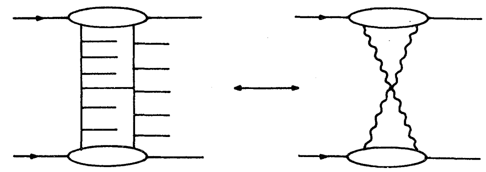

FIG. 22. Correspondence between some production amplitudes and Reggeon interactions.

$k \lesssim O(R)$ quanta can interact and rescatter repeatedly and a direct calculation is quite impractical. Of course, something more can be said if enough approximations are made, and in this section we will present a simple although quantitatively unreliable calculation of the nucleus fragmentation region, showing how various very complicated interactions can yield a result of the necessary form. The discussion in this section only depends on Sec. II, and the motivation for the approximations is clearest if we take a didactic point of view.

Suppose one assumes that high-energy hadron-nucleus scattering should be controlled by Regge poles and sets out to compute the residue function of the nucleus. Naively, one might say that since the basic interactions are of short range in momentum space, only the slowest particles in the Reggeon ladder can interact directly. Hence the elastic hadron-nucleus amplitude satisfies

$$A(p\underline{x}) \sim \beta_h (p/m)^{\alpha-1}\beta_R \tag{7.1}$$

for $|\underline{x}| \lesssim R$, where the nucleus residue function β_R is proportional to the cross section for slow ($k_z \sim m$), slightly off-shell ($k^2 \sim -m^2$) particles on a nucleus of radius R. Now an off-shell particle can only propagate a finite distance from its first interaction as its lifetime is $O(k_z/m^2)$, and the first interaction can occur anywhere inside the nucleus, so β_R must be proportional to[36]

$$2z_0(\underline{x}) \equiv 2(R^2 - \underline{x}^2)^{1/2}, \tag{7.2}$$

the width of the nucleus at impact parameter $\underline{x}$. Therefore the unitarity bound is exceeded for large-enough R.

The calculation just described only leads to embarrassment when applied to a nuclear target, which suggests looking for corrections which are small in other cases. An obvious candidate is the (time-ordered) graph shown in Fig. 23, which corresponds to rescattering of the virtual particle k. If the target were a single hadron the graph would be suppressed just as the AFS cut is suppressed: The lifetime of k is $O(k_z/m^2)$ and the second ladder only develops slow momentum components when it is far past the target. As in Sec. II F, such effects

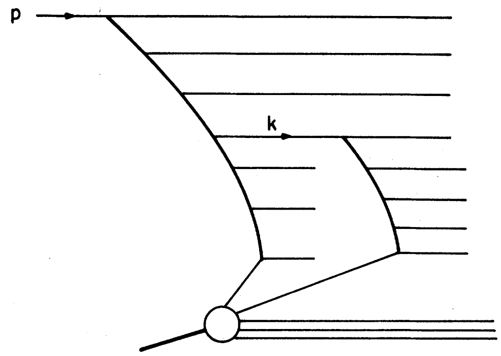

FIG. 23. Inelastic rescattering off a nucleus.

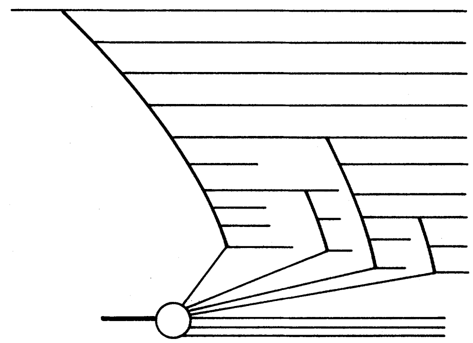

FIG. 25. A more complicated inelastic rescattering.

are appreciable in a nucleus when the lifetime of k is less than the nucleus' size, or $k_z \leq O(R)$. Of course, interactions much more complicated than that shown in the Fig. 23 are possible, since *all* virtual particles with $k_z \leq O(R)$ can rescatter. This means that a single Regge pole has no physical relevance for a scattering reaction with $p \leq O(R)$, although if $p \gg R$ there is no difficulty for the higher momentum components.

We would like to believe that such nuclear rescattering effects restore a sensible (i.e., unitary) Reggeon-nucleus residue function. An accurate calculation is clearly infeasible, but we shall present a simple illustrative calculation in which rescattering is included in an approximate way, and we trust that the functional dependence of the result is reliable even if its details are not. The calculation to follow was motivated by related work by Schwimmer,[13] although we would disagree with this reference in part. The graph shown in Fig. 23 is a particular discontinuity of Fig. 24, while the more complicated rescattering in Fig. 25 is a discontinuity of Fig. 26. As a simple model of nuclear rescattering, we shall consider the sum of all graphs of the form of Fig. 26 where the momentum at the uppermost triple-Regge vertex is $\leq O(R)$. (A vertex at larger momentum corresponds to an ordinary hadronic triple-Regge effect, which is small.) We are making a great idealization since all of the rescattering occurs over a rather limited momentum interval and the ladders do not contain a sufficient number of "rungs" to be accurately represented by a Regge pole. Furthermore, we neglect processes in which a recombination occurs, as in Fig. 27. This is a mathematical convenience; Fig. 27 corresponds to a certain discontinuity of Fig. 28, which is a closed-loop graph and requires a much more difficult analysis.

Notice that the graphs of Figs. 23 and 25, when Lorentz-transformed to the rest frame of the projectile, exactly describe the "coalescence of the combs" in Kancheli's[12] terminology. Unlike Kancheli, however, we do not in general assume that the "fusion" is complete—that there is only a one-Reggeon state at momenta greater than $O(R)$.

The summation of rescattering graphs can be carried out in either of two ways. The first is a straightforward graphical summation, leading to an integral equation. It is easy to convince oneself that all graphs of the type shown in Fig. 29 (we omit drawing the coupling to the nucleons) are generated by the recursion relation illustrated in Fig. 30. If we work at impact parameter $\underline{x}$ and in terms of rapidities, with $Y \equiv \ln(Rm)$, then Fig. 29 corresponds to the series

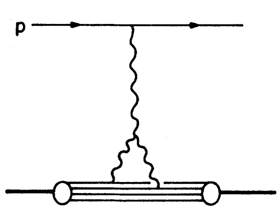

FIG. 24. Reggeon diagram corresponding to inelastic rescattering.

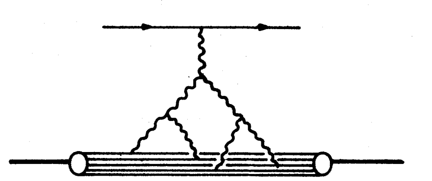

FIG. 26. Reggeon diagram corresponding to Fig. 25.

$$A(Y,\underline{x}) = i\lambda z_0(\underline{x})e^{-Y}\Big[e^{\alpha Y} + \int_0^Y dy\, e^{\alpha(Y-y)} gC(y)\, e^{(2\alpha-1)y}$$
$$+ \int_0^Y dy\, e^{\alpha(Y-y)} gC(y)\, e^{(\alpha-1)y} \int_0^y dy_1\, e^{\alpha(y-y_1)} gC(y_1)\, e^{(2\alpha-1)y_1} + \cdots\Big]\,. \tag{7.3}$$

The over-all factor of $\lambda z_0(\underline{x})$, where z_0 is defined in (7.2), occurs because each diagram contains at least one Reggeon whose coupling to the nucleus is proportional to z_0. The coupling of the other Reggeons to the nucleus, $C(y)$, is in general smaller because they originate at virtual particles whose momentum may not be sufficient for a typical lifetime to extend fully across the nucleus. Since the number of nucleons accessible to a Reggeon is proportional to its highest momentum component, we make the simple approximation $C(y) \propto e^y$. The constant g is a triple-Regge vertex strength; because the important time orderings in these graphs are not those which are significant in ordinary hadron-hadron scattering, g has no simple relation to couplings phenomenologically determined in inclusive reactions.[37] Since the rescattering here is an absorptive effect, $g<0$.

If we convert Fig. 30 into an equation, we have

$$A(Y,\underline{x}) = i\lambda z_0(\underline{x})\, e^{-Y}\left\{e^{\alpha Y} + \int_0^Y dy\, e^{\alpha(Y-y)} gC(y)\, e^{-y}[A(y,\underline{x})/i\lambda z_0(\underline{x})\, e^{-y}]^2\right\}. \tag{7.4}$$

As a check, note that this equation reproduces (7.3) upon iteration. Defining

$$B(Y,\underline{x}) = e^{-\alpha Y}\frac{A(Y,\underline{x})}{i\lambda z_0(\underline{x})\, e^{-Y}}\,,$$

and taking $C(y)=e^y$, with the appropriate constant of proportionality shifted into g, (7.4) becomes

$$B(Y,\underline{x}) = 1 + \int_0^Y dy\, g e^{\alpha y}[B(y,\underline{x})]^2\,.$$

Differentiating,

$$\frac{d}{dY}B(Y,\underline{x}) = g e^{\alpha Y}[B(Y,\underline{x})]^2\,,$$

and then integrating with the "boundary condition" $B=1$ when $g=0$,

$$B(Y,\underline{x}) = \frac{1}{1-(g/\alpha)e^{\alpha Y}}\,.$$

Thus

$$A(Y,\underline{x}) = \frac{i\lambda z_0(\underline{x})e^{(\alpha-1)Y}}{1-(g/\alpha)e^{\alpha Y}}\,. \tag{7.5}$$

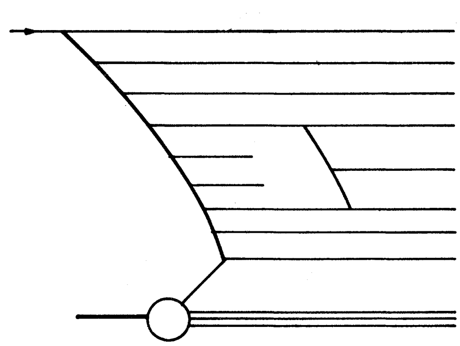

FIG. 27. Inelastic rescattering and reabsorption.

For nuclear rescattering we consider $Y = \ln z_0(x) \gg 1$, and hence

$$A(Y,\underline{x})|_{Y=\ln z_0(x)} \sim \frac{-i\lambda\alpha}{g}\,, \tag{7.6}$$

which is a positive imaginary constant. This is equivalent to the statement that the cross section for a particle with $p=O(R)$ on a nucleus is proportional to πR^2.

The summation of graphs can be carried out by a simpler method[13] if we notice that Figs. 24 and 26 are *tree* graphs, and make use of the well-known result that the sum of all tree graphs is the solution of the corresponding classical field theory.[38] The field theory in question is that of the Reggeon calculus, described here by the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\phi^\dagger \frac{\overleftrightarrow{\partial}}{\partial y}\phi + \alpha'\left|\frac{\partial\phi}{\partial \underline{x}}\right|^2$$
$$+ (1-\alpha(0))\,\phi^\dagger\phi + ige^y\phi^\dagger\phi^2\,,$$

where we have adopted the convention that ϕ creates Reggeons. The term in $\partial\phi/\partial\underline{x}$ is negligible to the extent that we ignore local variations on a scale $O(1/m)$ over the nuclear impact profile. The corresponding field equation is

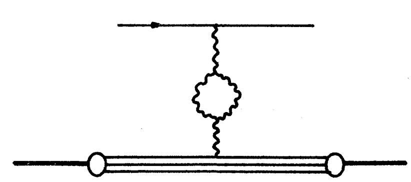

FIG. 28. Reggeon diagram corresponding to Fig. 27.

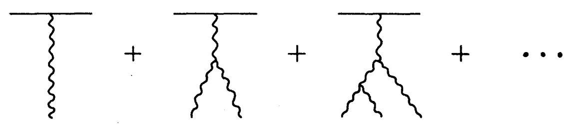

FIG. 29. Model of inelastic rescattering.

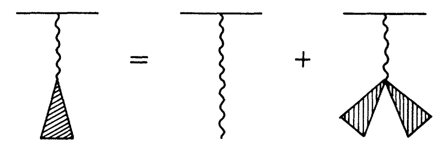

FIG. 30. Integral equation for inelastic rescattering.

$$0=\frac{\partial\phi}{\partial y}+(1-\alpha)\phi+ige^{y}\phi^{2},$$

which is just the differential equation satisfied by $[ie^{(\alpha-1)Y}B(Y,\underline{x})]$.

VIII. PLANAR STRING MODEL WITHOUT LOOPS

The planar string model without loops[4] presents a very different picture of Regge behavior than does a field theory or multiperipheral model. We shall see that in the string model a Regge pole is formed in a finite time in contrast to ϕ^3-type models. The consequence of this short time scale is that the AKG cutting rules are changed[11] and it is impossible to cut through more than one Reggeon at a time. Thus when a string scatters off a large nucleus, and dual loops are neglected, in order to calculate the elastic scattering amplitude one must calculate a sequential set of scatterings as the initial string moves through the nucleus. The procedure for calculating the elastic amplitude is much like the Glauber series with inelastic corrections absent. Between scatterings the string moves freely from one scatterer to the next. Let us begin by calculating the scattering of a string off a localized on-mass-shell external source in order to find the time during which interactions can take place.

Suppose we have a classical external source which emits and absorbs on-mass-shell ground states of the string. At a given time, say $t=0$, we can localize the source to be in the region $(x_1^2+x_2^2+x_3^2)^{1/2}\lesssim 1/\mu$ where μ is less than or on the order of the mass, $|m|$ characterizing the mass scale in the string model. To second order in the source, J, the amplitude for scattering of a string off the source is

$$A\propto\sum_{n=0}^{\infty}\int d^3k\int_{-\infty}^{0}d\tau\,\exp\{-i\tau[(p_0-p_3)_i+(k_0-k_3)-(P_0-P_3)_n]\}$$
$$\times V_{fn}V_{ni}\tilde{J}(k)\tilde{J}(k')\delta([(p_i+k-p_f)^2+m^2]^{1/2}-(p_i-p_f)_0-(k^2+m^2)^{1/2}),$$

where the process is illustrated in Fig. 31. Here p_i and p_f are the initial and final momenta of the string in the i and f states, respectively. V is the dual resonance vertex function and k' is determined by overall energy-momentum conservation. Also,

$$k_1^2+k_2^2+k_3^2+m^2=k_0^2;\quad k_1'^2+k_2'^2+k_3'^2+m^2=k_0'^2.$$

Now $(p_{1,2})_i+k_{1,2}=(P_{1,2})_n$ and $(p_0+p_3)_i+k_0+k_3=(P_0+P_3)_n$ so

$$(P_0-P_3)_n=\frac{M_n^2+\underline{P}_n^2}{(P_0+P_3)_n}\approx\frac{M_n^2}{2(p_3)_i},$$

where M_n is the mass of the state n and $\underline{P}=(P_1,P_2)$. [We now assume that i and f are low-lying excitations so that $(p_0+p_3)\approx 2(p_3)_i$ for a fast-moving string.] Thus

$$A\propto\sum_{n}\int d^3k\,\delta([(p_i+k-p_f)^2+m^2]^{1/2}-(p_i-p_f)_0-(k^2+m^2)^{1/2})$$
$$\times\int_{-\infty}^{0}d\tau\,\exp\left\{-i\tau\left[(k^2+m^2)^{1/2}-k_3-\frac{M_n^2}{2(p_3)_i}+i\epsilon\right]\right\}V_{fn}V_{ni}\tilde{J}(k)\tilde{J}(k').$$

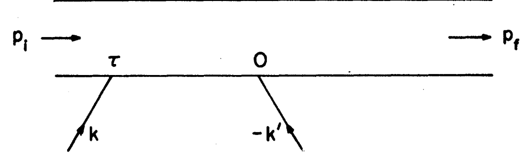

FIG. 31. Elastic scattering in the planar dual model.

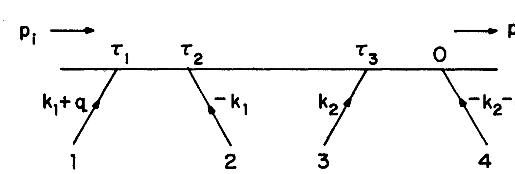

FIG. 32. Scattering from two targets in the string model.

Now for a fixed p_i and p_f

$$\sum_{\substack{n\\ M^2=M_n^2}} V_{fn}V_{ni}$$

varies very slowly with M since that sum gives the residue of the pole in the four-point amplitude at $(p+k)^2=M^2$. The scale of the M dependence of

$$\sum_{\substack{n\\ M^2=M_n^2}} V_{fn}V_{ni}$$

must be the order of M^2 itself. Thus we may replace

$$\sum_{\substack{n\\ M^2=M_n^2}} V_{fn}V_{ni}$$

by $\rho_{fi}(M^2)$ and write

$$A^\pm \propto \int_{-\infty}^0 d\tau \int d^3k\, \delta([(p_i+k-p_f)^2+m^2]^{1/2}-(p_i-p_f)_0-(k^2+m^2)^{1/2})\rho_{fi}(M^2)\,dM^2$$
$$\times \exp\left\{-i\tau\left[\pm(k^2+m^2)^{1/2}-k_3-\frac{M^2}{2(p_3)_i}+i\epsilon\right]\right\}\tilde{J}(k)\tilde{J}(k').$$

If m^2 is positive the argument is trivial. Since the scale of variation of M^2 is on the order of $2(p_3)_i m$, the average value of M^2, the values of τ must be less than or on the order of $1/m$. Since $\tau=x_0+x_3$, the time during which interactions can take place in an elastic or quasielastic scattering must be on the order of $1/m$. In a "realistic" string model $m^2=-1$ and so the above argument is not so simple. However, we can simply take $\tilde{J}(k)$ so that $|\underline{k}|$ is greater than, say, $\sqrt{2}\,m$. Then the argument goes through as before.

Thus we see that an elastic scattering in the string model takes place over a very short time interval which clearly means that a Regge-pole exchange occurs over a very small time interval. This does not mean that an inelastic process of many particle production takes place over a small time interval. It may, in fact, take a very long time for the highly excited states of the dual resonance model to decay into the "stable" ground states of the string. The short time for the elastic scattering just reflects the presumed equivalent completeness of states of the many-particle ground states of the string and the highly excited narrow resonances. There should be no difficulty in extending the above arguments to external sources consisting of excited states of the string model.

Now consider a string scattering off two well-separated classical sources, J and J'. For simplicity we shall take the classical sources to emit and absorb on-mass-shell ground-state excitations of the string, but we shall assume that the mass of the ground-state excitation, m, is real. The process is illustrated in Fig. 32 where 1 and 2 refer to scatterings off J and 3 and 4 refer to scatterings off J'. J and J' are identical except that J' is displaced a distance z_0 further in the $+z$ direction than J. Then the amplitude for forward elastic scattering is

$$A \propto \sum_{n_1,n_2,n_3} \int d^3k_1\, d^3k_2\, d^3q\, \delta([(k_2+q)^2+m^2]^{1/2}-[(k_1+q)^2+m^2]^{1/2}+(k_1^2+m^2)^{1/2}-(k_2^2+m^2)^{1/2})$$
$$\times \int_{-\infty}^0 d\tau_3 \int_{-\infty}^{\tau_3} d\tau_2 \int_{-\infty}^{\tau_2} d\tau_1 \exp\left\{+i(\tau_2-\tau_1)\left[[(k_1+q)^2]^{1/2}-(k_1+q)_3-\frac{M_{n_1}^2}{2p}+i\epsilon\right]\right\}$$
$$\times \exp\left\{+i(\tau_3-\tau_2)\left[[(k_1+q)^2+m^2]^{1/2}-(k_1^2+m^2)^{1/2}-q_3-\frac{M_{n_2}^2}{2p}+i\epsilon\right]\right\}$$
$$\times \exp\left\{-i\tau_3\left[+[(k_2+q)^2+m^2]^{1/2}-(k_2+q)_3-\frac{M_{n_3}^2}{2p}+i\epsilon\right]\right\}$$
$$\times V_{fn_3}V_{n_3n_2}V_{n_2n_1}V_{n_1i}\tilde{J}(k_1+q)\tilde{J}(-k_1)\tilde{J}(k_2)\tilde{J}(-k_2-q)\,e^{iq_3z_0},$$

where $k_{10}=(k_1^2+m^2)^{1/2}$, $(k_1+q)_0=[(k_1+q)^2+m^2]^{1/2}$, $k_{20}=(k_2^2+m^2)^{1/2}$. We have taken only a particular τ ordering where emission occurs after absorption of particles of the source. The other orderings are trivially added when τ_1 and τ_2 are exchanged and τ_3 and $\tau_4=0$ are exchanged. It is clear from the sequential ordering of the scatterings that a discontinuity involves only one Reggeon at a time.

Thus the final states produced by taking discontinuities of this amplitude will be exactly as in particle-particle scattering at least so long as $q^2 \ll m^2$ and triple-Reggeon couplings can be neglected.

Now, as is well known, the discontinuities of those time orderings which give the dominant contribution to the elastic amplitude at high energy do not necessarily give the correct final states.[39] Thus, in the case considered here, there is the time ordering where $\tau_1 < \tau_3 < \tau_4 < \tau_2 = 0$. Because of the arrangement of the sources, the elastic amplitude at high energies is zero for this τ ordering, illustrated in Fig. 33, as are the individual discontinuities for well-separated sources. Clearly, adding more scatters is not going to change the situation as far as the final states are concerned. It appears that so long as one does not include dual loops the final-state spectrum of inelastic events in hadron-nucleus collisions is exactly as in hadron-hadron collisions.

The above picture of inelastic events will be modified when loops or nonplanar graphs are included. Since elastic high-energy scattering can take place over a short period of time in the laboratory system it is clear that in passing through the nucleus an excited state of the string can emit additional low-momentum hadrons which will modify the nucleus fragmentation region. These are parts of dual loop contributions and they presumably will not be small. An example of such a contribution is shown in Fig. 34. At τ_5 a low-momentum string is emitted and reabsorbed at τ_6. (This amplitude is zero but its discontinuities are not.) We know that the emission of low-momentum hadrons as the string passes through the nucleus must be important. The question which we are unable to answer is whether the probability of emission of high-momentum excited states is also large. If this were the case the central region and hadron fragmentation regions would also be modified from the nonloop case. The question is very much the same as how an excited string decays into the many-particle ground-state excitations of the string. In addition to such modifications of the nonloop situation there are, of course, those loops where the string breaks up before it reaches the nucleus. These latter modifications presumably also include terms with long time scales of interaction. We are unable to give any indication of the relative importance of these various contributions. Finally, we do not mean to propose that the planar string model without loops can be a truly realistic model of hadron-hadron scattering as it stands. In particular, such a model requires that all cross sections be equal, which is very unrealistic.[40] Its particular interest lies in the fact that elastic cross sections can be large and still not require large multiplicities, because of the short time scale of interaction.

IX. NUCLEUS-NUCLEUS SCATTERING

The high-energy scattering of two nuclei is of less immediate interest than hadron-nucleus scattering because detailed high-energy experiments are not expected in the near future, but the subject nevertheless deserves mention because it helps clarify certain basic issues. We shall only consider the softened-field-theory model for the scattering of two large nuclei.

It is convenient to first restate our analysis of hadron-nucleus scattering. In the rest frame of the nucleus an incident high-energy hadron begins to emit quanta in the form of one or more ladders and after a long time has developed quanta with momenta $O(m^2R)$ or less. The corresponding probability is proportional to $(p/m^2R)^{\alpha-1}$ for a single ladder, and so on for other configurations. These slow quanta then interact with the nucleus with constant probability (nonzero and independent of p and R) and integrating over impact parameter we obtain

$$\sigma_{\text{tot}}^{hR} \sim \pi R^2 (p/m^2R)^{\alpha-1} + \cdots . \tag{3.4}$$

Let us now view this process in the rest frame of the hadron[12]: An incident nucleus with momentum p per nucleon begins to emit quanta which may interact with each other and with various nucleons in a complicated way, and after a time $O(p/m^2)$ there appear, with constant probability at each impact parameter, one or more quanta with mo-

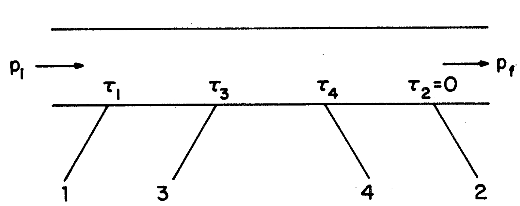

FIG. 33. A different time ordering for double scattering.

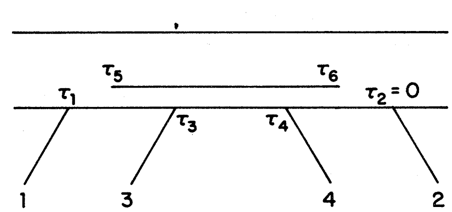

FIG. 34. A graph with a discontinuity contributing to the nucleus fragmentation region in the string model.

mentum $O(p/mR)$. The latter continue to emit sequentially and independently, forming one or more ladders which eventually develop slow quanta, which finally interact with the hadron at rest. We oversimplify in saying that the entire interaction occurs at a single impact parameter independently of the rest of the nucleus, but the form of the cross section remains valid if we allow a finite transverse correlation.

Now if we consider a nucleus of radius R_1 and momentum *per nucleon* p incident on another nucleus of radius R_2 at rest, the total cross section is the product of four factors: (1) a constant probability for the incident nucleus to develop quanta of momentum $O(p/mR_1)$, (2) a probability $(p/m^3R_1R_2)^{\alpha-1}$, say, for a particle of momentum $O(p/mR_1)$ to evolve through a one-ladder state and develop quanta of momentum $O(m^2R_2)$, (3) a constant probability for the latter quanta to interact with the second nucleus, and (4) a factor $\pi(R_1+R_2)^2$ for summing over impact parameters. Note that the last factor arises because the basic hadron-hadron interaction occurs effectively at a single impact parameter (compared to the transverse extent of a large nucleus) and the nuclei only interact if they "overlap." Thus

$$\sigma_{\text{tot}}^{R_1R_2} \sim \pi(R_1+R_2)^2\left(\frac{p}{m^3R_1R_2}\right)^{\alpha-1}+\cdots . \qquad (9.1)$$

Comparing (3.4) and (9.1) we see that even if a single ladder dominates the intermediate momentum region the resulting total cross sections do not factorize and we do not have Regge poles in the usual sense. (This probably remains true in a model with instantaneous interactions.) If we could integrate *separately* over the transverse coordinates of the nucleons which emit and absorb the ladders the result would be a factorized form proportional to $\pi R_1{}^2\times\pi R_2{}^2$, but this result[8] only obtains when the Regge radius is comparable to the nuclear radius, $(\alpha'\ln p/m)^{1/2}\sim R$, and requires astronomical energies. Indeed, the factorized form would violate the unitarity bound (2.23) unless the energy is high enough for a Reggeon to be fairly "transparent" in impact parameter.

The single-particle inclusive cross section will display two nuclear fragmentation regions, of rapidity intervals $\ln(mR_1)$ and $\ln(mR_2)$, respectively, together with a central region if $p \gg m^3R_1R_2$. The height of the central plateau is now

$$\left.\frac{d\sigma}{dy}\right|_{\text{central region}} \propto (\pi R_1{}^2)(\pi R_2{}^2)\,; \qquad (9.2)$$

the reason is that we know (from the AKG rules) $d\sigma/dy$ arises from the discontinuity of a one-ladder exchange graph and the ladder in question may be coupled, so to speak, in $\pi R_1{}^2$ ways to the projectile and independently in $\pi R_2{}^2$ ways to the target. In this crude approximation the inclusive particle density does factorize although the total cross section does not. All this is simply a restatement of what Kancheli has already said.[12]

Note added in proof

(i) The arguments given in Sec. IV showing that a simple Glauber expansion does not follow in a softened field theory also show that a "Regge-eikonal" expression

$$A(S,\underline{x})=-i\,(l^{i\chi(x,S)}-1)\,,$$

with $\chi(\underline{x},S)$ given by single-Reggeon exchange, lacks motivation in such theories. If χ is large, this model will be unitarity-violating for the same reasons as discussed in Sec. IV.

(ii) Our statements on the validity of the AKG Reggeon cutting rules are, perhaps, in need of elaboration. Consider the scattering of a string off two well-separated and large hadronic sources. The planar process shown in Fig. 32 has only elastic and one-Reggeon discontinuities so long as q^2 is near zero, and these discontinuities have relative weights $+1$ and -2, respectively, as shown by DeGrand and DeTar.[11] It is true that in the string model a planar two-Reggeon exchange corresponds to a single Regge pole rather than a Regge cut, but this is of course irrelevant for our purposes. If different ends of the string scatter off the two sources J and J', then this nonplanar graph has the usual AKG cutting rules, as McLerran and Weis[41] have shown. In order to get discontinuities through more than two Reggeons it is necessary to attach dual Pomerons to the internal region of the string. Such connections, unfortunately, violate the Froissart bound and would have to be modified considerably before they could be used in a realistic situation. Thus for all planar scatterings $\sigma=2\pi R^2$ and $R_c=1$, while if planar and nonplanar interactions are allowed $\sigma=2\pi R^2$ and $R_c=2$ for the scattering of a string off a large nucleus.

(iii) A simple example of hadron-nucleus scattering not discussed in the text of this paper is hadron-deuteron scattering. On this matter we are in complete agreement with the result obtained by Gribov.[42] For a discussion on the validity of the Glauber expansion in this case see Landshoff.[43]

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[11]T. A. DeGrand and C. E. DeTar, Nucl. Phys. B90, 189 (1975).

[12]O. V. Kancheli, JETP Lett. 18, 274 (1973).

[13]A. Schwimmer, Nucl. Phys. B94, 445 (1975).

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[15]Our convention for four-vectors is $p_\mu = (p_0, \underline{p}, p_z)$ with $\underline{p} = (p_x, p_y)$. For an on-shell particle we write $p_0 = \omega_p = (m^2 + \underline{p}^2 + p_z{}^2)^{1/2}$.

[16]S. Weinberg, Phys. Rev. 150, 1313 (1966).

[17]"m" really represents an average transverse mass, $(m^2 + \langle p^2 \rangle)^{1/2}$, and in a more realistic multiperipheral model should be interpreted as an average transverse *cluster* mass. Numerically we expect $m \sim 1$ GeV.

[18]Some examples are given in R. J. Eden, *et al.*, *The Analytic S Matrix* (Cambridge Univ. Press, New York, 1966).

[19]Since α really represents an effective trajectory, and the energies available in hadron-nucleus scattering are not so high, $\alpha < 1$ is even phenomenologically reasonable.

[20]J. Kogut and L. Susskind, Ref. 3.

[21]Detailed studies indicate that the random walk result is only numerically reliable for weak coupling; see B. R. Webber, Nucl. Phys. B87, 269 (1975).

[22]To be precise, $t = 0$ since $\Omega^{(+)} = U(-\infty, 0)$ where U is the time evolution operator. Since $|\psi_p\rangle$ is an eigenstate of the full Hamiltonian, the state of any other time is identical to $|\psi_p\rangle$, up to a phase.

[23]S. Mandelstam, Nuovo Cimento 30, 1113 (1963); 30, 1127 (1963); 30 1148 (1963).

[24]An equivalent formulation would be to describe the targets by a two-particle wave function and introduce a momentum vertex function for the two-particle state. This technique is used by Gribov (see Ref. 25) to derive the Glauber expansion in hadron-nucleus scattering. The crux of the argument to follow is the presence of the factor $e^{iq_z z_0}$ in Eq. (2.29), and this factor is basically a consequence of translation invariance.

[25]V. N. Gribov, Zh. Eksp. Teor. Fiz. 57, 1306 (1969). [Sov. Phys.–JETP 30, 709 (1970)]. For an early discussion see E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin Nuovo Cimento 42, 365 (1965).

[26]A time-independent source transfers no energy.

[27]V. N. Gribov, JETP 26, 414 (1968).

[28]J. Koplik and A. H. Mueller, Phys. Lett. 58B, 166 (1975).

[29]In impact parameter, this is the statement that the transverse coordinate structure of hadron-hadron scattering has a scale $O(1/m)$, which is much smaller than the transverse profile of the entire nucleus and can be neglected. Notice also that the longitudinal momentum transfer can enter through a term $k_z p_z$ in $k \cdot p$, and is not negligible. See also Ref. 25.

[30]Strictly speaking, the last matrix element in this equation should be between the asymptotic incoming and outgoing states $\langle \psi_p^{(-)}|$ and $|\psi_p^{(+)}\rangle$, respectively, since $U(\infty, 0) = \Omega^{(-)\dagger}$. However, the stability of one-particle states implies there is no difference between the two.

[31]K. Gottfried, proceedings of the 1972 CERN Summer School (unpublished).

[32]To be precise, it is proportional to $(R^2 - \underline{x}^2)^{1/2}$, where the projectile has impact parameter $\underline{x}$. When integrated over $\underline{x}$ the result is proportional to R^3, which violates the unitarity bound by a single power of R. This same remark qualifies the later statements in this section.

[33]In the case $\alpha(0) = 1$, we can define $|\psi_{px}^{(1)}\rangle$ as the component of the state having i times the single Reggeon rapidity density of particles. The case $\alpha(0) = 1$, however, appears to have many subtleties and the above arguments are most clear when $\alpha(0) < 1$.

[34]R. D. Schamberger, Jr. *et al.*, Phys. Rev. Lett. 34, 1121 (1975).

[35]See L. Foà, Proceedings of the 2nd International Conference on Elementary Particles, Aix en Provence, 1973 [J. Phys. (Paris) Suppl. 34, CI-317 (1973)].

[36]An alternative argument is that particle production is small at low incident momentum, and the Glauber series should suffice. The result for β_R is thus derived by Gribov in Ref. 25. Contrary to some suggestions in the literature, the result is *not* simply $2z_0(x)\beta_h$ which would give $\sigma_{hA}^{\text{tot}} = A\sigma_{hh}^{\text{tot}}$.

[37]Schwimmer, Ref. 13, considers such triple-Regge rescattering at all rapidities, without distinguishing the nucleus fragmentation region. In that case g *is* an ordinary triple-Regge coupling, but Schwimmer in effect asserts that hadron-nucleus scattering depends critically on the existence of large-mass diffractive dissociation.

[38]Y. Nambu, Phys. Lett. 26B, 626 (1968).

[39]A nice example is high-energy hadron-hadron scattering in a ϕ^3 model where the leading trajectory satisfies $\alpha(0) < 1$. The AFS graph, which is zero in the amplitude, gives the leading contribution to the two-particle intermediate state.

[40]R. C. Brower and J. H. Weis, Rev. Mod. Phys. 47, 605 (1975).

[41]L. D. McLerran and J. H. Weis, Seattle report (unpublished).

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