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Vector model of the weak interactions*

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We search for a renormalizable gauge theory incorporating our modified weak current. The right-handed quarks participate to give a natural explanation for the nonleptonic $\Delta I = 1/2$ rule. We argue that extra quarks and/or extra leptons are necessary. We focus on a vector-like model involving six different quark "flavors," each in a color triplet: 18 quarks all told. The model also involves a third charged lepton and several massive neutral leptons. We compare the predictions of our model with the conventional (12 quarks) model. Present experimental data are insufficient to discriminate between the two models. We give detailed analyses and predictions concerning charged-current and neutral-current phenomena, including inclusive scattering, scattering of neutrinos by electrons, single-pion production, and neutrino or antineutrino elastic scattering on protons.

I. INTRODUCTION

We have suggested[1] a modification of the conventional charged weak-interaction current involving unorthodox $V+A$ couplings of the charmed quark. The modified current provides a novel and simple explanation for the observed enhancement of nonleptonic weak processes satisfying the $\Delta I = \frac{1}{2}$ selection rule, as we recall in Sec. II. However, the changed charmed current is theoretically unacceptable because it upsets the cancellation of anomalies of the conventional model.[2] Further changes of the current must be made if the underlying field theory is to be renormalizable.[2,3] We sketch in Sec. III some of the many ways these anomalies could be made to cancel, all of which imply the existence of new fundamental fermions besides the conventional 12 quarks and the known leptons. In Sec. IV we focus on one of these possibilities, which we call the vector theory. In Sec. V we demonstrate that present experimental data are unable to discriminate between it and the conventional theory. Predictions concerning neutral-current effects are vastly different for the two theories, so that this failure merely reflects the inadequacy of presently available experimental data. We discuss those experiments which can be done in the near future to determine which (if either) of the two possibilities is correct. In Sec. VI we show how our model may be incorporated into a truly unified theory wherein strong, weak, and electromagnetic interactions find their common origin. Section VII contains our conclusions.

II. THE CHARGED WEAK CURRENT

In a previous paper[1] we suggested an explanation for the nonleptonic $\Delta I = \frac{1}{2}$ rule within the context of a gauge theory of weak and electromagnetic interactions based on the group $SU(2)\times U(1)$.[4] In the conventional theory, the left-handed quark fields transform under the SU(2) gauge subgroup as doublets,

$$\begin{pmatrix}\mathcal{P}\\ \mathcal{N}_\theta\end{pmatrix}_L, \quad \begin{pmatrix}\mathcal{P}'\\ \lambda_\theta\end{pmatrix}_L, \tag{1}$$

where $\mathcal{N}_\theta = \mathcal{N}\cos\theta + \lambda\sin\theta$ and $\lambda_\theta = \lambda\cos\theta - \mathcal{N}\sin\theta$, and θ is the Cabibbo angle. The right-handed quarks are assumed to be singlets. The hadronic charged weak current in this theory is the conventional $V-A$ current[5]

$$J^\alpha = \bar{\mathcal{P}}\gamma^\alpha(1+\gamma_5)\mathcal{N}_\theta + \bar{\mathcal{P}}'\gamma^\alpha(1+\gamma_5)\lambda_\theta\,. \tag{2}$$

We proposed an alternative theory in which the right-handed $\mathcal{P}'$ and $\mathcal{N}$ quarks transform as a doublet,

$$\begin{pmatrix}\mathcal{P}'\\ \mathcal{N}\end{pmatrix}_R \tag{3}$$

rather than as singlets. In our theory there is a $V+A$ term in the weak current

$$J'^\alpha = \bar{\mathcal{P}}'\gamma^\alpha(1-\gamma_5)\mathcal{N}\,. \tag{4}$$

The dominant contribution to $\Delta I = \frac{1}{2}$ strangeness-changing nonleptonic decays comes from a term in the effective action associated with the one-loop diagram shown in Fig. 1. The logarithmical-

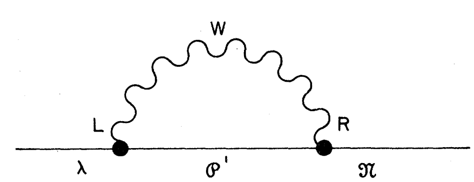

FIG. 1. Feynman diagram dominating $\Delta I = \frac{1}{2}$ nonleptonic processes in our model. L (R) denotes the coupling of W to left- (right-) handed quark fields.

ly divergent part of this diagram is removed by mass and wave-function renormalizations. The leading remaining finite expression

$$\frac{G m_{\mathcal{P}'} \cos\theta}{4\sqrt{2}\,\pi^2}\,\bar{\lambda}(i\partial\!\!\!/ - m_\lambda)(1-\gamma_5)(i\partial\!\!\!/ - m_{\mathcal{P}})\mathfrak{N} + \text{H.c.} \qquad (5)$$

contributes solely to $\Delta I = \frac{1}{2}$ nonleptonic decays. The only contribution to $\Delta I = \frac{3}{2}$ nonleptonic strangeness-violating decays comes from the conventional current-current interaction,

$$\frac{G}{\sqrt{2}}\,J^\alpha J_\alpha^+, \qquad (6)$$

which, of course, also contributes to $\Delta I = \frac{1}{2}$ processes. Because the new term (5) is of order $G m_{\mathcal{P}'} q \cos\theta$ (where q is a characteristic decay energy, say m_K), the $\Delta I = \frac{1}{2}$ processes are evidently enhanced relative to $\Delta I = \frac{3}{2}$. There are two mechanisms for enhancement: There is no Cabibbo suppression in our new contribution to $\Delta I = \frac{1}{2}$, and the mass of the $\mathcal{P}'$ quark ($m_{\mathcal{P}'} \cong 1.5$ GeV) is large compared to momenta encountered in weak decays. More quantitative comparisons are impossible without a detailed understanding of strong-interaction dynamics.

Other experimental implications of our modified charged current associated with the production of charmed particles by neutrinos are discussed briefly in Sec. V A and elsewhere.[1,6,7]

Since we have departed from the GIM[5] coupling scheme for the charmed quark, we must check that the suppression of strangeness-changing neutral-current amplitudes in our theory is consistent with experiment. The neutral intermediate vector boson, Z, couples to a $\Delta S = 0$ current, so there is no problem in order G. Likewise, radiative corrections of order αG are $\Delta S = 0$. There is a potential difficulty in order G^2. Consider, for example, the following $\Delta S = 2$ term in the effective action[8]:

$$\Delta = \frac{-1}{16\pi^2}\,G^2 m_{\mathcal{P}'}^2 \cos^2\theta \left(\ln\frac{M_W^2}{m_{\mathcal{P}'}^2} - \frac{3}{2}\right) \times [\bar{\lambda}\gamma^\mu\gamma^\nu(1-\gamma_5)\mathfrak{N}][\bar{\lambda}\gamma_\nu\gamma_\mu(1-\gamma_5)\mathfrak{N}] + \text{H.c.} \qquad (7)$$

This term is associated with the diagram shown in Fig. 2. It is larger than the analogous diagram in the conventional theory[9] because of the absence of Cabibbo suppression (no $\sin^2\theta$) and the factor of $\ln M_W^2/m_{\mathcal{P}'}^2$. The term (7) contributes to the K_2-K_1 mass difference as follows:

$$\delta m = +\frac{1}{2m_K}\,(\langle K_2|\Delta|K_2\rangle - \langle K_1|\Delta|K_1\rangle). \qquad (8)$$

A naive estimate of (8) (obtained by inserting intermediate vacuum states in all possible ways[9]) yields an effect which is positive and more than an order of magnitude larger than the experimental number. This may not be a serious problem for several reasons.

1. Because of the tensor structure of (7), there is a partial cancellation among the various vacuum insertions. The contributions of the vacuum intermediate states would vanish in a theory with two colors (instead of the usual three). This partial cancellation may be affected by strong interactions which could increase or decrease the estimate of (8).

2. One-pion intermediate states give a negative contribution to (8). In general, CP-odd intermediate states give negative contributions while CP-even intermediate states give positive contributions. Though we know of no reason why the CP-even and odd contributions should cancel, the fact that the sign alternates casts considerable doubt on the usefulness of the vacuum approximation.

3. Except for the logarithmic enhancement factor, the term (7) is of the same order of magnitude as the second-order contribution of (5), the term responsible for $\Delta I = \frac{1}{2}$ nonleptonic strangeness-changing decays. In other words, the $\Delta S = 2$ contribution is about what we would expect on the basis of the observed enhancements of $\Delta I = \frac{1}{2}$ nonleptonic processes. The $\Delta S = 2$ prediction is smaller in the conventional theory simply because the conventional theory does not account for the $\Delta I = \frac{1}{2}$ rule, and in any theory which does give enough $\Delta I = \frac{1}{2}$ enhancement, the induced order G^2 $\Delta S = 2$ terms will be large. This apparent paradox, that either $\Delta I = \frac{1}{2}$, $\Delta S = 1$ is too small or $\Delta S = 2$ is too big, may be only a reflection of our ignorance of hadronic matrix elements, as discussed in 1 and 2.

4. Another possibility is that the naive estimate of (8) is more or less correct, but that there is some competing effect contributing to the K_2-K_1 mass difference with the opposite sign. In our theory there is a good candidate for such an ef-

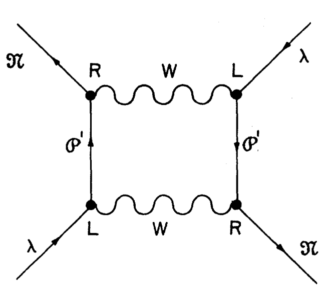

FIG. 2. A Feynman diagram which contributes to $\Delta S = 2$ processes in our model.

fect. The neutral member of the triplet of Higgs mesons whose vacuum expectation value contributes to the $\mathcal{P}'$ mass is coupled to a strangeness-changing scalar (and pseudoscalar) current. This meson must be very massive. If its mass is of the order of $G^{-1/2}$ it mediates $\Delta S=2$ effects with about the same strength as (7). Its contribution to the K_2-K_1 mass difference (if calculated naively as discussed above) is negative and could conceivably cancel (8).

The point of this long discussion of induced $\Delta S=2$ effects is that detailed calculations of order G^2 terms like (7) are premature in the sense that they cannot impose serious constraints on a theory of weak and electromagnetic interactions. It is enough for the theory to eliminate $\Delta S=2$ in order G and αG. We will state the argument once more. Even if we could reliably calculate hadronic matrix elements (which we cannot do at this time) the induced $\Delta S=2$ effects at the order G^2 level could always be canceled by the lowest-order effect of some new interaction which is too small to be observed in other processes. Indeed, in the theory we discuss in this paper, a new interaction with the required properties (but unknown strength) must exist if the quark masses are generated by Higgs meson couplings. The same logic can be applied to discussions of order G^2 contributions to the $K_L \to \mu^+\mu^-$ decay amplitude.

A naive calculation in the conventional theory gives the correct order of magnitude for a second-order effect of weak interactions (the K_1K_2 mass difference),[9] but does not adequately describe first-order phenomena (nonleptonic decays). The new theory gives a natural explanation of first-order processes, but fails (in an unjustified approximation) to adequately suppress second-order effects. Given the choice, we prefer the new theory.

III. EXTRA QUARKS: EXTRA LEPTONS

Just three kinds of quarks ($\mathcal{P}$, $\mathfrak{N}$, λ) suffice to construct the observed hadrons, excluding the newly found resonances ψ or J and ψ'. The necessity for a fourth charmed quark $\mathcal{P}'$ to make weak-interaction theory sensible was pointed out years ago.[5] The new resonances may confirm the existence of the new quark, for it offers a plausible explanation for what they are: states of charmonium.[10] Four quarks—each in three colors—and the observed leptons (30 Weyl spinors) are the basic building blocks of many a current theoretical development.

Beware that in no known way is nature limited to just these fundamental fermions. Other leptons or other quarks could exist. So also could more exotic things: fermions with their own strong interactions unshared by known hadrons, quarks without weak interactions leading to new and truly stable hadrons, quarks transforming as other than the fundamental three-dimensional representation of color SU(3), truly stable heavy leptons, etc. Unlike the case for the charmed quark, the logical necessity for the existence of such entities in conventional models is entirely absent.

On the other hand, it now seems that the cross section for e^+e^- annihilation into hadrons (or, whatever) may be too large to be explained (under the scaling hypothesis) in terms of the conventional four-quark types with color, and much less in terms of the original three. Instead of approaching the value 10/3 (the sum of the squares of the electric charges of the quarks) characteristic of four-quark types and no extra leptons, $R=[\sigma(\text{hadrons, etc.})/\sigma(\bar{\mu}\mu)]$ seems to remain near 5 at center-of-mass energies of 5–6 GeV.[11] It has been noted in the literature[12] that the *ad hoc* (i.e., motivated solely by this experiment and not at all by theory) introduction of extra quarks could explain the large cross section. So also could the existence of heavy charged leptons. Indeed, by positing the existence of not more than two new fermions chosen from among new quarks ($Q=\frac{2}{3}$ or $-\frac{1}{3}$) and new singly charged leptons, one obtains as a predicted asymptotic value for $3R$ any integer value between 10 and 18, inclusive (See Table I).

If our $V+A$ addition to the weak current is really present, a purely theoretical justification is found for the existence of extra fermions.

Our modified weak current is

TABLE I. Asymptotic value of R in several models with extra quarks or leptons. The model on top is the "old model." The model on bottom is our new "vector model."

Number of extra quarks with $Q=\frac{2}{3}$	Number of extra quarks with $Q=-\frac{1}{3}$	Number of extra charged leptons	Asymptotic value of R in e^+e^- annihilation
0	0	0	3.33
0	1	0	3.67
0	2	0	4.00
0	0	1	4.33
0	1	1	4.67
1	0	0	4.67
1	1	0	5.00
0	0	2	5.33
1	0	1	5.67
2	0	0	6.00
1	1	1	6.00

$$K_\alpha = J'_\alpha + J_\alpha + \bar{\nu}_e \gamma_\alpha (1+\gamma_5) e + \bar{\nu}_\mu \gamma_\alpha (1+\gamma_5)\mu\,, \quad (9)$$

where the first term is our addition. In order that the anomalies cancel [in SU(2)×U(1) models where all Weyl fermions are either weak doublets or singlets], it is necessary and sufficient for the sum of the electric charges of the left-handed fermions in weak doublets (Q_L) to equal the corresponding sum (Q_R) for right-handed fermions. For the conventional current $Q_L = Q_R = 0$; but for our new current $Q_L = 0$, $Q_R = 1$. (We use the fact that each type of quark is a color triplet.) There are no possible further additions to our weak current (9) involving only the four familiar quark types and the known leptons which satisfy the following three conditions:

(1) The weak current must reproduce the known phenomenology of leptonic and semileptonic decays [as is the case for (7)].

(2) The weak current must correspond to a generator of weak SU(2) and must be a color singlet.

(3) The assignments of fields to representations of weak SU(2) must satisfy $Q_L = Q_R$.

New fermions are needed. Observe that the conjectured new fermions are neither a replacement for nor an alternative to charm. They are merely an addition to the charm picture. The usual theoretical arguments resulting in the prediction of charm remain entirely unchanged. But both experiment (e^+e^- annihilation) and theory (the demand for renormalizability in a theory incorporating our new explanation for the nonleptonic $\Delta I = \frac{1}{2}$ rule) are telling us that there must exist more fermions.

A simple way to make our modified current anomaly-free involves the introduction of a new neutral heavy lepton coupled to a *right-handed* electron or muon:

$$I_\mu = K_\mu + [\bar{X}\gamma_\mu (1-\gamma_5) e \text{ or } \bar{Y}\gamma_\mu (1-\gamma_5)\mu]\,. \quad (10)$$

Either current is anomaly-free and leads to observable neutral-current phenomena violating electron-muon universality. But this possibility does not help us to understand the anomalously large value of R since the new fermion is electrically neutral.

Another possibility involves a new charged lepton L^- and its own neutrino ν'':

$$I_\mu = K_\mu + \bar{\nu}'' \gamma_\mu (1-\gamma_5) L^-\,.$$

This current yields a theory with an asymptotic value of R of $\frac{13}{3}$. To be relevant to the e^+e^- annihilation experiments and to be consistent with known limits, the mass of the new lepton must be near 2 GeV.[13] Note that L is coupled to a neutrino with the unorthodox helicity: The leptonic decays $L^- \to \bar{\mu} + \nu'' + \bar{\nu}'$ or $L^- \to e^- + \nu'' + \bar{\nu}$ will display a Michel parameter $\rho = 0$ and the mean energy of a decay electron (or muon) in the rest frame $\frac{3}{10} M_L$. On the basis of the nominal strengths of the currents, we estimate that the branching ratio into each leptonic mode is $\sim \frac{1}{5}$. Above L^+L^- threshold, e^+e^- annihilation should produce such a pair about 20% of the time. Experiment should quickly determine whether the new lepton exists.[14]

The anomaly may be removed in another way: with extra quarks rather than an extra lepton. Imagine a new pair of quark types (each a color triplet) $\mathcal{P}''$ with $Q = \frac{2}{3}$ and $\mathcal{N}''$ with $Q = -\frac{1}{3}$. The current

$$I_\mu = K_\mu + \bar{\mathcal{P}}'' \gamma_\mu (1+\gamma_5)\mathcal{N}'' \quad (12)$$

is anomaly-free with $Q_L = Q_R = 1$. This possibility gives $R = 5$, asymptotically. There are three varieties of charmonium ($\bar{\mathcal{P}}'\mathcal{P}'$, $\bar{\mathcal{P}}''\mathcal{P}''$, and $\bar{\mathcal{N}}''\mathcal{N}''$) in this model. Arguments based on asymptotic freedom would require that each charmonium state be made up of purely one kind of quark-antiquark, unless the masses of two or more of the new quarks were very nearby. Moreover, since we have not introduced (and may not, if we are to preserve universality) interactions permitting $\mathcal{N}''$ or $\mathcal{P}''$ quarks to change into other quark types, there is a new and rigorously conserved quantum number,

$$F = \int d^3x\,(\bar{\mathcal{P}}''\gamma_0\mathcal{P}'' + \bar{\mathcal{N}}''\gamma_0\mathcal{N}'')\,. \quad (13)$$

Among the hadrons containing just one $\bar{\mathcal{N}}''$ or $\mathcal{P}''$ quark, at least one (and probably two) will be *absolutely* stable.

The reader will have realized that there are many ways to choose a current that satisfies all our requirements. The particular choice which we shall describe in detail in the next section is evidently not unique, nor even the simplest. We choose it in part because it makes predictions about the neutral currents which differ radically from the conventional theory, and in part because it lends itself to the kind of unification which we believe necessary for the construction of a rational theory of elementary particle physics.

IV. OUR MODEL

We choose a model involving both extra quarks and extra leptons. The extra quarks are required in order that the charged weak current may be "vector-like,"[15] with all charged fermion fields transforming under weak SU(2) as doublets. There is a basis of Weyl spinors in terms of which the weak currents are purely vector currents are purely vector currents with no axial-vector admixture. Of course, this is not the basis in which the fermion mass matrix is diagonal. Extra neutral leptons are necessary for the leptonic weak current to be vector-like, and an extra charged lepton is required to preserve quark-lepton symmetry. Among theories incorporating the changed charmed current, ours has five evident virtues:

(1) Vector-like theories are always free of anomalies.[15] The anomaly is independent of the mass matrix, and clearly vanishes in the basis wherein the current is purely vector. For these theories, there is no seemingly accidental cancellation: The absence of anomalies is natural.[15]

(2) Vector-like theories may be approximated by lattice gauge theories, while many theories with intrinsic, irremovable γ_5's may not be.[16] Because many of our ideas about quark confinement emerge from the lattice picture,[17] it seems desirable—and may be essential—that the theory be put on a lattice.

(3) With a vector-like theory, it is possible that C and P are exact symmetries of the Lagrangian.[18] Observed violations of C, P, and CP may be due to spontaneous symmetry breaking. They too may be consequences of whatever mechanism it is that generates the observed hadron and lepton mass spectrum and the Cabibbo angle.

(4) Our model has a rich structure of unobserved massive leptons: three neutral ones and a charged one. We have noted elsewhere[19] that recent cosmic-ray evidence[20] suggests the existence of a long-lived neutral heavy lepton, and we are therefore inclined to favor a theory which incorporates such a particle.

(5) In a vector-like theory where all the fermions transform as weak doublets, it is necessarily true that the neutral current (except for terms involving electrically neutral fermions such as neutrinos) is purely a vector current. This is a virtue if only because it makes a number of very definite predictions:

(a) There should be no parity-violating interaction in order G between electrons (or muons) and nucleons.

(b) There should be no axial-vector contribution in the scattering of neutrinos or antineutrinos from hadrons via the neutral currents. In particular, the neutral-current cross section on any target must be the same for incident neutrino or antineutrino.

(c) In the scattering of muon neutrinos or antineutrinos from electrons, the effective coupling is simply proportional to $(\bar{e}\gamma_\mu e)[\bar{\nu}'\gamma_\mu(1+\gamma_5)\nu']$. That is to say, $g_A=0$.

(d) Nonleptonic $\Delta S=0$ parity violation arises solely from the charged-current weak interactions, and receives no contribution from the neutral current. But the usual argument[21] that the parity-violating one-pion exchange force should be $\sim\sin^2\theta$ fails. It receives a contribution from our new addition to the charged charmed current, not suppressed by $\sin\theta$.

In order to make the hadronic weak current vector-like, we need six kinds of quarks—two more than the usual four. Three kinds $(\mathcal{P}, \mathcal{P}', \mathcal{P}'')$ have charge $\frac{2}{3}$ and the other three kinds $(\mathcal{N}, \lambda, \mathcal{N}'')$ have charge $-\frac{1}{3}$. Each kind of quark is a color triplet. Six quark triplets are not too many from the point of view of asymptotic freedom. The asymptotic freedom of color SU(3) persists so long as the number of quark triplets is less than 16.

Under weak SU(2) the 18 quarks divide up into nine left-handed doublets and nine right-handed doublets. Assuming approximate CP conservation, we may specify the hadronic part of the charged weak current in terms of six angles: the left-handed partners of $\mathcal{P}$, $\mathcal{P}'$, and $\mathcal{P}''$ are determined by an arbitrary orthogonal transformation of the left-handed $\mathcal{N}$, λ, and $\mathcal{N}''$; the right-handed partners by an independent 3×3 orthogonal matrix. Both transformations commute with color SU(3). Two conditions must be met for the current to agree with observed phenomenology. The left-handed currents must reproduce universality: The sum of the squares of the coefficients of $\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{N}$ and $\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\lambda$ must be unity and the coefficients of $\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\mathcal{N}$ and $\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\lambda$ must vanish. We impose these conditions arbitrarily, hoping that a natural explanation for them (or for an approximate version of them) eventually will be found. We are led to the following three-parameter expression for the hadronic part of the charged weak current:

$$
\begin{aligned}
H_\mu = {} & \bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)[\mathcal{N}\cos\theta+\lambda\sin\theta]+\bar{\mathcal{P}}'\gamma_\mu(1+\gamma_5)[(\lambda\cos\theta-\mathcal{N}\sin\theta)\cos\varphi+\mathcal{N}''\sin\varphi] \\
& +\bar{\mathcal{P}}''\gamma_\mu(1+\gamma_5)[\mathcal{N}''\cos\varphi-(\lambda\cos\theta-\mathcal{N}\sin\theta)\sin\varphi]+\bar{\mathcal{P}}\gamma_\mu(1-\gamma_5)\mathcal{N}'' \\
& +\bar{\mathcal{P}}'\gamma_\mu(1-\gamma_5)[\mathcal{N}\cos\Psi+\lambda\sin\Psi]+\bar{\mathcal{P}}''\gamma_\mu(1-\gamma_5)[\lambda\cos\Psi-\mathcal{N}\sin\Psi]\ .
\end{aligned} \tag{14}
$$

The Cabibbo angle is θ. For the special choice $\varphi=\Psi=0$, our model reproduces the modified current of Ref. 1. More generally, the enhancement of $\Delta I=\frac{1}{2}$ decay rates (as well as the K_1K_2 mass difference) involves the factor

$$(\cos\Psi\cos\varphi\, m_{\mathcal{P}'}+\sin\Psi\sin\varphi\, m_{\mathcal{P}''})^2 .$$

We shall not concern ourselves with the values of these parameters.

For the leptonic part of the charged weak current to be vector-like, we must introduce two unobserved heavy neutral leptons, E^0 and M^0. Their left-handed parts participate in neither electromagnetism, weak interactions, nor strong interactions. Their right-handed parts transform as weak doublets together with the right-handed electron and muon. In addition, we assume the existence of a new family of leptons, L^- and L^0. This is necessary for lepton-quark symmetry, and allows us to embed our model in a superunified theory. Our theory involves three weak doublets of quarks of each color, and three weak doublets of leptons, for each handedness. We distinguish three simple possibilities for the structure of the weak leptonic charged current:

$$\begin{aligned}L_\lambda=&\bar{\nu}\gamma_\lambda(1+\gamma_5)e+\bar{E}^0\gamma_\lambda(1-\gamma_5)e\\&+\bar{\nu}'\gamma_\lambda(1+\gamma_5)\mu+\bar{M}^0\gamma_\lambda(1-\gamma_5)\mu\\&+\begin{cases}\bar{\nu}''\gamma_\lambda(1+\gamma_5)L^-+\bar{L}^0\gamma_\lambda(1-\gamma_5)L^-\;[\text{case(i)}]\,,\\ \bar{L}^0\gamma_\lambda(1+\gamma_5)L^-+\bar{\nu}''\gamma_\lambda(1-\gamma_5)L^-\;[\text{case(ii)}]\,,\\ \bar{L}^0\gamma_\lambda(1+\gamma_5)L^-+\bar{L}^0\gamma_\lambda(1-\gamma_5)L^-\;[\text{case(iii)}]\,.\end{cases}\end{aligned}\tag{15}$$

In cases (i) and (ii) the new family of leptons includes a third neutrino. The coupling of L^- to its neutrino is orthodox $(V-A)$ in case (i), and is $(V+A)$ in case (ii). In case (iii) we do without a third neutrino, so that the L^-L^0 coupling is purely vector. Suppose that L^- is heavier than L^0. In cases (i) and (ii) the L^0 is stable in order G, but decays according to the scheme

$$L^0\to\nu''+\gamma$$

in order eG. The effective coupling arises from one-loop diagrams in which L^0 virtually decomposes into L^-+W^+, which may interact electrically. In case (iii) L^0 is absolutely stable. (Of course, it might decay by mechanisms not yet explicitly included in our model, such as whatever it is that leads to CP violation.)

Our fundamental fermions fall into three sets of chiral states: ordinary quarks and electronic leptons, strange and charmed quarks and muonic leptons, and extra quarks and extra leptons. The theory fits into a superunified model based on O(9) or O(10) with the fermions transforming as three 16 dimensional spin representations, and two [case (iii)] or three [cases (i) and (ii)] singlets.

Both L_μ and H_μ are evidently anomaly-free. The corresponding neutral current, at least in its dependence on quark and charged lepton fields, is purely vector.

The asymptotic value of R in e^+e^- annihilation is 6, but it is unclear at presently studied energies just what thresholds have already been passed. It is possible that additional sharp $J^P=1^-$ resonances will be found at higher energies corresponding to $\bar{\mathcal{P}}''\mathcal{P}''$ or $\bar{\mathfrak{N}}''\mathfrak{N}''$ charmonium states. It is also possible that L^+L^- pairs are being copiously produced at presently studied energies. These will decay some of the time according to the schemes

$$L^-\to\begin{cases}\mu^-+\bar{\nu}'+\nu''\,,\\ \mu^-+\bar{\nu}'+L^0,\\ e^-+\bar{\nu}+\nu''\,,\\ e^-+\bar{\nu}+L^0,\end{cases}$$

depending on which of L^0 and L^- is heavier, and which of the three cases is chosen. These leptonic modes should have substantial branching ratios. Since the cross section for producing L^+L^- pairs in e^+e^- annihilation above threshold is known, it should be possible to determine whether such pairs are being produced. The production of L^+L^- pairs by cosmic rays, leading to the production of quasistable L^0's [case (iii)] might explain recent cosmic-ray data.[20]

The structure of the charged current requires that neutrino and antineutrino cross sections become equal at sufficiently high energies. The relevant ratio of charged-current cross sections, again called R, should become unity in the asymptotic region rather than the characteristic $\frac{1}{3}$ of more mundane energies. However, three (possibly quite different) new hadron threshold must be exceeded before this asymptotic result should be expected.

V. EXPERIMENTAL CONSEQUENCES

A. Charged-current phenomena

The predictions of the conventional theory and of our new theory for charged-current phenomena coincide until threshold for the production of new hadrons states (e.g., charmed particles) is reached. For neutrino energies below ~20 GeV, the ratio $R=\sigma(\bar{\nu})/\sigma(\nu)$ and the ratio of neutrino cross sections to electron cross sections are both consistent with the expectations of the naive quark-parton model, in which the target nucleon is approximated by just three valence quarks.[6]

In the conventional theory there is only one class of new hadrons involving $\mathcal{P}'$ quarks. Above charm threshold the contribution of the valence quarks to the production of final states containing one charmed particle vanishes for incident antineutrinos, and is suppressed by $\sin^2\theta$ for incident neutrinos. This is the only mechanism for quasielastic production of a single charmed particle.

Charm production may also occur in ν or $\bar{\nu}$ collisions by virtue of the "sea" of quark-antiquark pairs in the nucleon. In the conventional model it is the $\lambda\bar{\lambda}$ component of the sea which is most relevant. Recent experimental data suggest that charm production may be significant at high energies.[22] Dramatic effects with characteristic kinematic signatures have been reported as the energy of the incident neutrino exceeds an effective charm threshold of ~4 GeV. A recent analysis[23] of experimental data in the context of the conventional theory requires a suspiciously large admixture of $\lambda\bar{\lambda}$ quarks to obtain a good fit. This admixture is just compatible with upper bounds set by a comparison of electroproduction data with neutrino scattering data.[6] However, the amount of $\lambda\bar{\lambda}$ which is required far exceeds the upper limits that may be imposed on the $\bar{\mathcal{P}}\mathcal{P}$ and $\bar{\mathfrak{N}}\mathfrak{N}$ component. It seems unreasonable that the quark-antiquark sea should contain more $\lambda\bar{\lambda}$ quarks than light quark-antiquark pairs.

In our model there are several classes of new hadrons. We must suppose that the $\mathcal{P}'$ is considerably lighter than $\mathcal{P}''$ and $\mathfrak{N}''$, and we shall reserve the word charm for new hadrons containing $\mathcal{P}'$ quarks.

In our theory there is no $\sin^2\theta$ suppression of charm production by incident neutrinos off the valence quark constituents of the nucleon. Quasielastic production of single charmed hadrons by neutrinos could be quite copious. Moreover, both the $\bar{\mathfrak{N}}\mathfrak{N}$ and $\bar{\lambda}\lambda$ components of the sea are effective in charm production.

Barger, Weiler, and Phillips[7] have analyzed the data using our changed charmed current. They are able to explain the observed anomalous y distributions, both at small and large x. If the reported[22] dimuon events are assumed to have the same origin (production and leptonic decay of charmed particles), the angular and energy distributions of the fast muons are also consistent with theory.

Let us now recall that there must be more than one threshold for new hadron production. It seems that $W_1 \sim 4$ GeV is the effective threshold for charmed hadrons. At higher values of W the right-handed $\mathfrak{N}$ quark actively participates in deep-inelastic charged-current ν scattering. There will be a larger threshold, $W_2 > W_1$, for the production of new hadrons containing an $\mathfrak{N}''$ quark. Above this threshold, the right-handed valence $\mathcal{P}$ quark also participates, and contributes to inelastic $\bar{\nu}$ scattering. Thus, we distinguish three different energy regions:

I. below all new thresholds.
II. between W_1 and W_2.
III. above W_2.

Actually, these regions are slices in hadron mass, but they may be studied by varying E_ν and making appropriate kinematic cuts.

The following ratios of charged-current cross sections may be defined:

$$R = \frac{\sigma(\bar{\nu}p) + \sigma(\bar{\nu}n)}{\sigma(\nu p) + \sigma(\nu n)} ,\quad V = \frac{\sigma(\nu p)}{\sigma(\nu p) + \sigma(\nu n)} ,\quad U = \frac{\sigma(\bar{\nu}n)}{\sigma(\bar{\nu}p) + \sigma(\bar{\nu}n)} . \tag{16}$$

The quantities U and V should not depend on energy: They should each equal $\frac{1}{3}$ in any of the energy regions. However, R should be quite energy-dependent. We expect (and observe) $R \cong \frac{1}{3}$ in region I, and we predict $R \cong \frac{1}{4}$ in region II and $R \cong 1$ in region III. (Of course, smooth transitions are expected between the regions.)

If the Adler[24] and Gross-Llewellyn Smith[25] sum rules are sufficiently rapidly convergent, they also should display characteristic values in each of the three energy regions. With the notation

$$A = \tfrac{1}{2}\int_0^\infty \left[W_2^{\bar{\nu}}(\nu, q^2) - W_2^{\nu}(\nu, q^2)\right] \frac{d\nu}{m_p^2} ,\quad B = \tfrac{1}{2}\int_0^1 \left[F_3^{\bar{\nu}}(x) + F_3^{\nu}(x)\right] dx , \tag{17}$$

the expected results are shown in Table II. Methods of determining A and B from experimental data are discussed elsewhere.[26]

TABLE II. Values of sum rules in various energy regions. Region I is below charm threshold. Region II is above charm threshold and below threshold for production of hadrons containing $\mathfrak{N}''$ quarks. Region III is above both thresholds. $A(p)$ and $A(n)$ are values of the Adler sum rule off proton and neutron targets, as defined in text. $B(p)$ and $B(n)$ are values of Gross-Llewellyn Smith sum rules, also defined in text.

Energy region	$A(p)$	$A(n)$	$B(p)$	$B(n)$
I	1	−1	3	3
II	1	−3	2	1
III	0	0	0	0

B. Neutral-current phenomena

In this case the predictions of the conventional model and our new model are vastly different at all energies. We shall discuss the neutral-current predictions in terms of the naive quark-parton model. In this approximation, and for each mode, the description of neutral-current effects depends on two parameters: $\sin^2\theta$, the conventionally defined angle determining what combination of gauge fields is the photon, and M_Z, the mass of the neutral vector boson. It is only for the simplest Higgs meson structure—suggested by Weinberg and Salam—that there is a relation between these parameters:

$$M_Z = \tilde{M}_Z \equiv M_W \sec\theta = 76 \text{ GeV} \csc 2\theta \,. \tag{18}$$

More generally, the two parameters are logically independent and must be separately determined by experiment. It is useful to choose as parameters $\sin^2\theta$ and $\kappa = M_Z/\tilde{M}_Z$ (the ratio of the Z mass to its value as predicted by Weinberg and Salam). Evidently, κ^{-4} is a measure of the over-all strength of the neutral-current phenomena. In Sec. VI we argue that in the simplest unified models, we expect $\kappa \leq 1$ (i.e., *more* neutral currents than with the canonical value $M_Z = \tilde{M}_Z$).

In our comparisons of theory with experiment, we shall refer to the Weinberg-Salam model, as generalized to allow independent variation of M_Z and $\sin^2\theta$, as the "old model." Our new model is referred to as the "vector model."

1. Inclusive neutrino scattering

We concern ourselves with results off "matter" targets, containing equal numbers of protons and neutrons. We have noted elsewhere[6] that the study of inclusive neutrino scattering off separate proton and neutron targets is of little value in discriminating among different gauge theories.

The ratio $\tilde{R}$ of inclusive neutral-current cross sections of $\bar{\nu}$ to ν is independent of κ. Its measurement is a crucial test of the vector model. While in the old model $\tilde{R}$ is a simple function of $\sin^2\theta$

$$\tilde{R} = \frac{1 - 2\sin^2\theta + \frac{40}{9}\sin^4\theta}{3 - 6\sin^2\theta + \frac{40}{9}\sin^4\theta}\,, \tag{19}$$

in the vector model we unambiguously predict

$$\tilde{R} = 1\,. \tag{20}$$

Unfortunately, the current experimental status of $\tilde{R}$ is confused: Published results are in conflict[27]:

$$\tilde{R} = \begin{cases} \text{Harvard-Penn-Wisconsin} \\ \qquad \cong 1 \pm 0.2 \ (\langle E_\nu \rangle \sim 30 \text{ GeV}), \\ \text{Gargamelle} \cong 0.5 \pm 0.2 \ (\langle E_\nu \rangle \sim 2 \text{ GeV}). \end{cases}$$

At Gargamelle energies the recoil hadron energy distributions in charged-current events are known to disagree with the predictions of the naive quark picture. Thus it is perhaps unwise to compare the low-energy result for $\tilde{R}$ with naive theoretical expectations. On the other hand, high-energy data of the Caltech group[28] seem to favor the Gargamelle results (although the Caltech results are published with no error determinations).

Historically, the data have often been presented as a plot of the ratio of neutral-current to charged-current cross sections for ν and $\bar{\nu}$, with the theoretical prediction shown as a curve depending on $\sin^2\theta$ with $\kappa = 1$.

When both parameters are treated independently we have

$$\frac{\sigma(\bar{\nu})_{\rm NC}}{\sigma(\bar{\nu})_{\rm CC}} = \begin{cases} (\frac{1}{2} - \sin^2\theta + \frac{20}{9}\sin^4\theta)\kappa^{-4} \text{ (old)}, \\ (2 - 4\sin^2\theta + \frac{20}{9}\sin^4\theta)\kappa^{-4} \text{ (vector)}, \end{cases} \tag{21}$$

$$\frac{\sigma(\nu)_{\rm NC}}{\sigma(\nu)_{\rm CC}} = \begin{cases} (\frac{1}{2} - \sin^2\theta + \frac{20}{27}\sin^4\theta)\kappa^{-4} \text{ (old)}, \\ (\frac{2}{3} - \frac{4}{3}\sin^2\theta + \frac{20}{27}\sin^4\theta)\kappa^{-4} \text{ (vector)}. \end{cases} \tag{22}$$

Of course, charm production is not considered in these formulas: They are only valid predictions below charm threshold. In Fig. 3 we display the experimental data together with the theoretical predictions for each model, with $\kappa = 1$. Gargamelle data are compatible with $\sin^2\theta \sim 0.4$ in the old model, and disagree with the vector model whatever the value of κ. HPW data are compatible with the vector model with $\kappa \sim 1$. Again, we can come to no firm conclusion from such conflicting data.

Another crucial test of the vector model is in the hadron energy distribution of neutral-current events: The results for incident neutrino and antineutrino must be identical.

We return to the independent determination of κ and $\sin^2\theta$ as we describe what is known about neutrino scattering on electron targets.

2. Elastic neutrino-electron scattering

This is a very difficult process to measure, but one not obscured by strong-interaction effects. It offers itself to immediate and unambiguous theoretical analysis. The effective coupling is

$$2^{-1/2} G\{[\bar{\nu}_\mu \gamma^\alpha (1 + \gamma_5)\nu_\mu][\bar{e}\gamma_\alpha (g_V + g_A\gamma_5)e] + [\bar{\nu}_e \gamma^\alpha (1 + \gamma_5)\nu_e][\bar{e}\gamma_\alpha (G_V + G_A\gamma_5)e]\}\,, \tag{23}$$

and in both the old model and the vector model the following relations hold:

$$G_V = 1 + g_V\,,\qquad G_A = 1 + g_A\,. \tag{24}$$

The differential cross sections for incident muon neutrinos are

$$\frac{d\sigma}{dE_e}(\nu) = \frac{G^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{E_e}{E_\nu}\right)^2 - \frac{m_e E_e}{E_\nu^{\,2}}(g_V^{\,2} - g_A^{\,2})\right],$$

$$\frac{d\sigma}{dE_e}(\bar\nu) = \frac{G^2 m_e}{2\pi}\left[(g_V - g_A)^2 + (g_V + g_A)^2\left(1 - \frac{E_e}{E_\nu}\right)^2 - \frac{m_e E_e}{E_\nu^{\,2}}(g_V^{\,2} - g_A^{\,2})\right], \tag{25}$$

where E_e is the energy of the recoil electron. For electron neutrinos, substitute G_V and G_A for g_V and g_A. In the old model

$$g_V = \kappa^{-2}(-\tfrac{1}{2} + 2\sin^2\theta)\,,\qquad g_A = -\tfrac{1}{2}\kappa^{-2}\,, \tag{26}$$

while in the vector model

$$g_V = \kappa^{-2}(-1 + 2\sin^2\theta)\,,\qquad g_A = 0\,. \tag{27}$$

Measured upper limits[29] for $\nu_\mu e$ and $\bar\nu_e e$ cross sections can be converted into convex domains in the (g_V, g_A) plane within which, with 90% confidence, the actual values must lie. For $\bar\nu_\mu e$ scattering, there is a measured cross section based upon three observed events.[22] This constrains (g_V, g_A) to be within an annular domain excluding the origin. These constraints are displayed in Fig. 4.

Figure 5(a) gives a comparison of the old model with these experiments. Shown against the allowed (g_V, g_A) domain is the prediction of the old model with $\kappa = 1$, and the prediction of the old model as deduced from the inclusive scattering data with $\sin^2\theta$ and κ unconstrained. The consistency of this model with either set of hadronic data is marginal.

The vector model is compatible with only the HPW hadronic data. For each value of $\sin^2\theta$, there is a value of κ which fits the data, and a corresponding prediction of (g_V, g_A). These predictions are shown in Fig. 5(b) against the allowed leptonic domain. Values of $\sin^2\theta \leq 0.4$ ($\kappa \sim 1.4$) and $0.6 \leq \sin^2\theta \leq 0.7$ ($\kappa \sim 1$) are not in conflict with experiment.

It is revealing to combine the inclusive scatter-

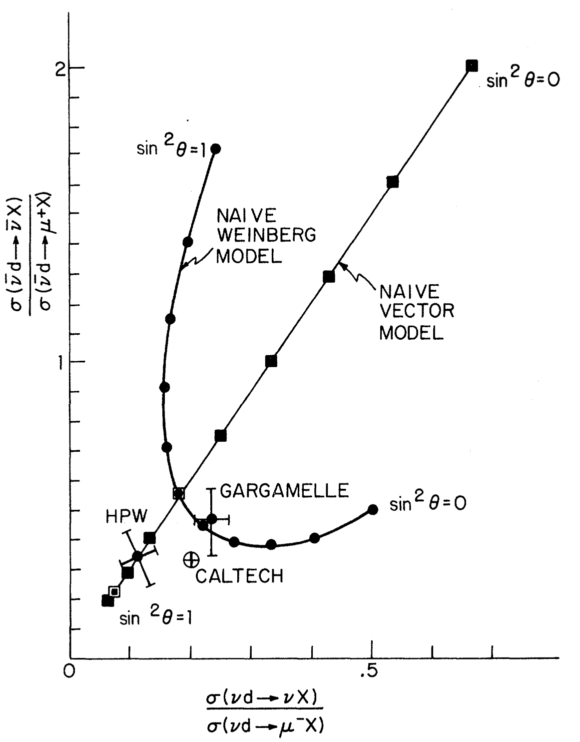

FIG. 3. Ratios of neutral-current to charged-current inclusive cross sections on matter. Predictions of the old model and of the vector model (both with $\kappa = 1$) are shown. Experimental determinations of these ratios are also shown.

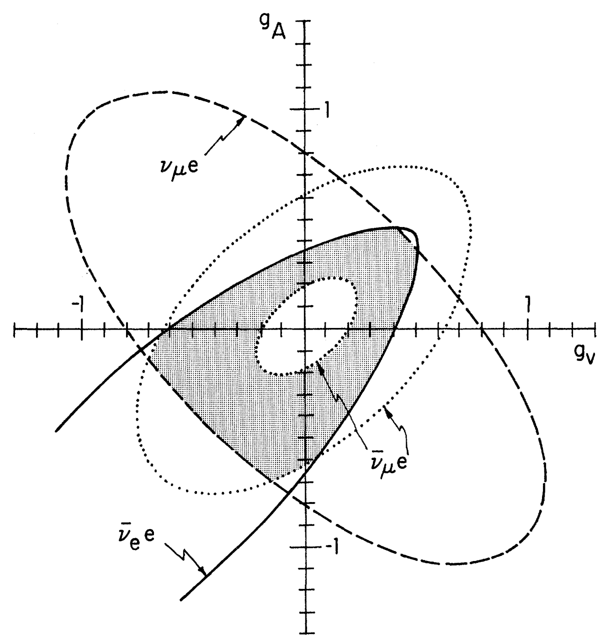

FIG. 4. Allowed domains (90% confidence level) for g_A and g_V from neutrino-electron scattering. This is an updated version of Fig. 9 in Ref. 6.

ing data and the leptonic neutral-current data into a figure displaying the allowed values of κ and $\sin^2\theta$. Figure 6(a) shows this analysis for the old model. The shaded region is allowed by observed νe scattering data. The two inclusive scattering data points are also shown. Figure 7 shows the allowed values of κ and $\sin^2\theta$ for the vector model. Only HPW data are shown since the Gargamelle data do not support this model. The disconnected shaded domain is consistent with the data. The region with $\kappa\sim 1$ and $\sin^2\theta\sim 0.68$ is favored (both in the value of κ and of $\sin^2\theta$) by arguments based on superunity presented in Sec. VI.

3. Single-pion production by neutral currents

We shall concern ourselves with pion production in the vicinity of the $\Delta(1236)$ resonance in the vector model. The hadronic neutral current is

$$J_Z^\alpha = 2(\bar{\mathcal{P}}\gamma^\alpha\mathcal{P} - \bar{\mathcal{N}}\gamma^\alpha\mathcal{N}) - 4\sin^2\theta J_{\rm em}^\alpha + 2(\bar{\mathcal{P}}'\gamma^\alpha\mathcal{P}' - \bar{\lambda}\gamma^\alpha\lambda + \bar{\mathcal{P}}''\gamma^\alpha\mathcal{P}'' - \bar{\mathcal{N}}''\gamma^\alpha\mathcal{N}''), \tag{28}$$

and it couples to the neutrino current $\bar{\nu}\gamma^\alpha(1+\gamma_5)\nu$ with an effective coupling strength $G/\sqrt{2}\,\kappa^2$. With the assumption that quarks other than $\mathcal{P}$ and $\mathcal{N}$ quarks do not contribute significantly to the relevant matrix elements of the neutral current, we can relate the neutral-current production of single pions to electroproduction data. Good electroproduction data exist for the reactions $ep\to ep\pi^0$ and $ep\to en\pi^+$ at low energies.[30] From the data we estimate that the $I=\frac{1}{2}$ background contributes 25% to the dominant $I=\frac{3}{2}$ resonant amplitude in the integrated cross section for $ep\to ep\pi^0+ep\pi^+$ at energies ~2 GeV. If it is assumed that the isovector part of the current dominates single-pion electroproduction, the neutrino-induced neutral-current cross sections are completely determined within our model.

Following Albright *et al.*,[31] we define

$$V_{\rm em}(N\pi) = \frac{G^2}{4\pi^2\alpha^2}\int Q^4 d\sigma(ep\to eN\pi)\,.$$

For electrons of ~2 GeV, $V_{\rm em}(p\pi^0)\sim 0.12\times 10^{-38}$ cm^2 and $V_{\rm em}(n\pi^+)\sim\frac{2}{3}V_{\rm em}(p\pi^0)$. In terms of these quantities we estimate the neutrino-induced cross sections at similar energies to be

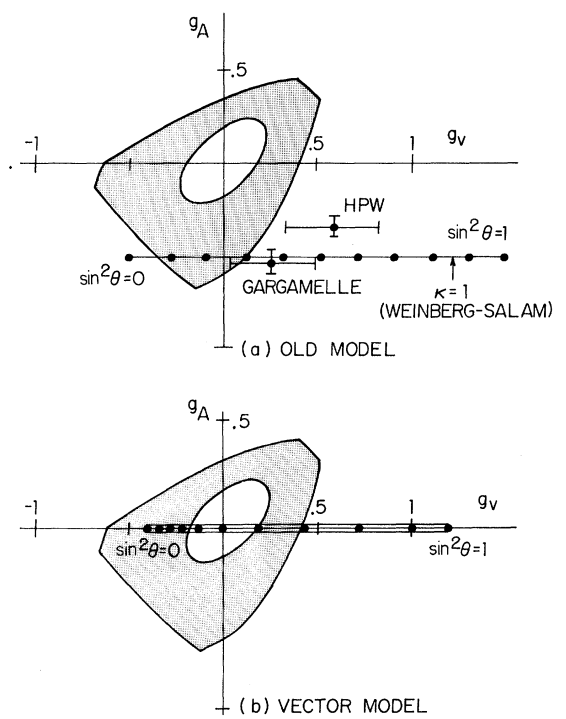

FIG. 5. Comparison of the experimentally allowed domain of g_A and g_V with theoretical models. In (a) we show the predictions of the old model with $\kappa=1$, as well as the predictions of the vector model with the relation between κ and $\sin^2\theta$ determined by the HPW inclusive scattering data.

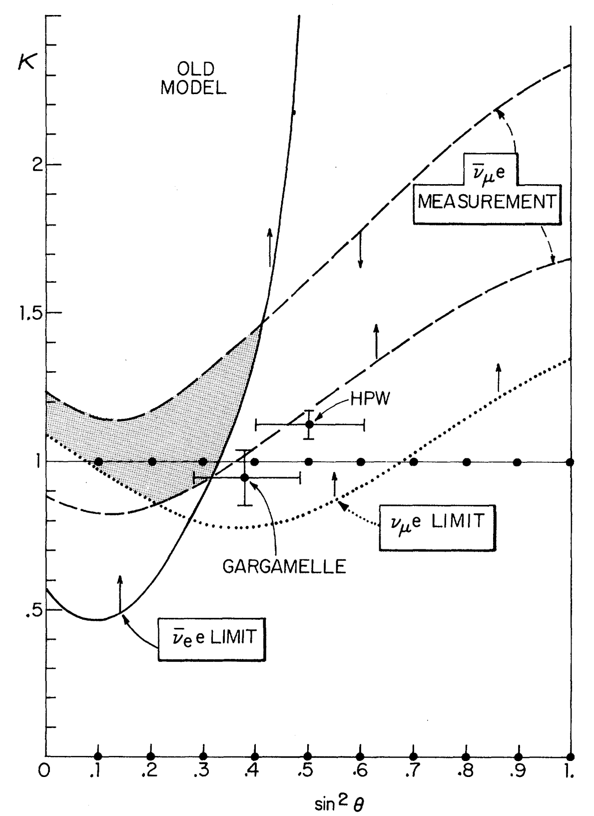

FIG. 6. Constraints on κ and $\sin^2\theta$ in the old model following from leptonic and inclusive data. The shaded region is allowed by all leptonic data. Gargamelle and HPW inclusive scattering results are also shown.

$$\begin{aligned}\sigma(\bar{\nu}n\to\bar{\nu}n\pi^0)&=\sigma(\nu n\to\nu n\pi^0)\\&=\tfrac{3}{2}\,\sigma(\bar{\nu}n\to\bar{\nu}p\pi^-)=\tfrac{3}{2}\,\sigma(\nu n\to\nu p\pi^-)\\&=\tfrac{3}{2}\,\sigma(\bar{\nu}p\to\bar{\nu}n\pi^+)=\tfrac{3}{2}\,\sigma(\nu p\to\nu n\pi^+)\\&=\sigma(\bar{\nu}p\to\bar{\nu}p\pi^0)=\sigma(\nu p\to\nu p\pi^0)\\&=\frac{4(1-2\sin^2\theta)^2}{\kappa^4}\,V_{\rm em}(p\pi^0)\\&\cong 1.04\,\frac{(1-2\sin^2\theta)^2}{\kappa^4}\,10^{-39}\ {\rm cm}^2. \qquad (29)\end{aligned}$$

Of more immediate experimental interest are ratios of neutral-current to charged-current single-pion production cross sections. These we expect to be energy-independent to a good approximation, for neutrino energies $\gtrsim 1$ GeV. The conventional definitions of these ratios as well as their predicted and observed values[32–36] are shown in Table III. For the charged-current cross sections, we use the values predicted by Adler[37] (choosing the characteristic mass of his axial form factor to be $M_A{}^2 = 0.88$ GeV^2): These are known to agree well with experiments.[22] Following

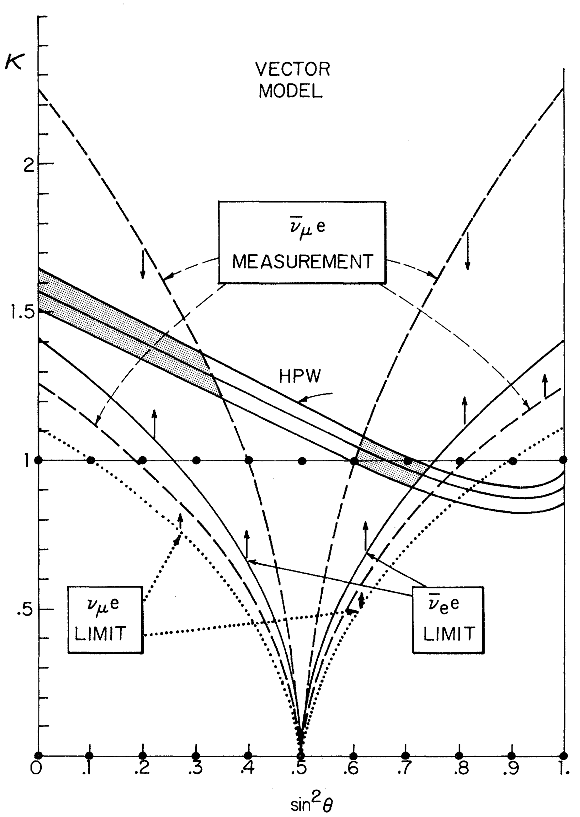

FIG. 7. Constraints on κ and $\sin^2\theta$ in the vector model following from leptonic and inclusive data. The shaded regions are allowed by all leptonic data in conjunction with the HPW measurements.

the above discussion of neutral-current deep-inelastic and leptonic phenomena, we set $\kappa = 1$ and $\sin^2\theta = 0.68 \pm 0.08$ in the vector model. Our results are compatible with experiment in cases where data exist. The large uncertainty in our theoretical predictions reflects the sensitivity of the results to the value of $\sin^2\theta$. In comparing R_N with the data (on a target consisting of 75% Al and 25% C), we have reduced the theoretical prediction by a factor of 2. This is because nuclear corrections (which have been worked out in the old model) are known to have approximately that effect.[38]

Perhaps more interesting is to use the HPW deep-inelastic data together with the observed single-pion production ratios to determine $\sin^2\theta$ and κ in our model. This analysis is shown in Fig. 8. The shaded regions in this figure are the allowed domains of $\sin^2\theta$ and κ allowing 1.2σ errors on the experimental data. The agreement of this figure with Fig. 7 is impressive. Our vector model is consistent with inclusive data, single-pion production, and νe scattering. Taking into account the superunification arguments in Sec. VI, we conclude that if our model is correct, $\sin^2\theta \sim 0.68$ and $\kappa \sim 1$. Corresponding to these values, we predict

$$\begin{aligned}M_W &= 2^{-1/4}\sqrt{\pi\alpha}\,G^{-1/2}\csc\theta = 45 \pm 1.5\ {\rm GeV},\\ M_Z &= \kappa\sec\theta M_W = 79 \pm 12\ {\rm GeV}.\end{aligned} \qquad (30)$$

4. Elastic scattering of neutrinos and antineutrinos by nucleons

Again, we need appropriate matrix elements of the weak current (28). The description of elastic neutrino-nucleon scattering involves the matrix elements of the isospin operator and the electromagnetic current—both of which are known from elastic electron scattering on nucleons. The remainder of the current measures the contribution of the strange, charmed, and extra quarks. Its matrix element at $Q^2 = 0$ must surely vanish. In our calculations we shall assume (plausibly, but without real justification) that this last term makes no contribution to elastic scattering. It follows that the differential cross sections for elastic neutrino-proton scattering are

$$\frac{d\sigma(\nu p\to\nu p)}{dQ^2}=\frac{d\sigma(\bar{\nu}p\to\bar{\nu}p)}{dQ^2}=\frac{G^2}{8\pi E_\nu{}^2\kappa^4}\,[A + C(4mE_\nu - Q^2)^2], \qquad (31)$$

where m is the proton mass, E_ν the neutrino energy, and the convention is such that $Q^2 > 0$. We use the following empirical scaling laws for nu-

TABLE III. Single-pion production by neutrinos or antineutrinos. Ratios of neutral-current to charged-current cross sections are defined. Their values as predicted by the vector model are given and compared with available experimental data. The predicted value of R_N has been reduced by a factor of 2 because these data were obtained from a complex nuclear target (see text).

Ratio	Definition	Prediction	Experiment	Reference
R_0	$\frac{\sigma(\nu p \to \nu p \pi^0)}{\sigma(\nu p \to \mu^- p \pi^+)}$	$0.13^{+0.14}_{-0.09}$	0.40 ± 0.22	32, 33, 36
R_+	$\frac{\sigma(\nu p \to \nu n \pi^+)}{\sigma(\nu p \to \mu^- p \pi^+)}$	$0.09^{+0.09}_{-0.06}$	0.13 ± 0.06	32, 33, 36
R_-	$\frac{\sigma(\nu n \to \nu p \pi^-)}{\sigma(\nu p \to \mu^- p \pi^+)}$	$0.09^{+0.09}_{-0.06}$	0.07 ± 0.03	32, 36
R_{0+}	$R_0 + R_+$	$0.22^{+0.23}_{-0.15}$	0.53 ± 0.23	32, 34, 36
R_N	$\frac{\sigma(\nu\text{“}p\text{”} \to \nu p \pi^0) + \sigma(\nu\text{“}n\text{”} \to \nu n \pi^0)}{2\sigma(\nu\text{“}n\text{”} \to \mu^- p \pi^0)}$	$\sim 0.16^{+0.16}_{-0.11}$	0.17 ± 0.06	34, 35, 36
$\bar{R}_0$	$\frac{\sigma(\bar\nu p \to \bar\nu p \pi^0)}{\sigma(\bar\nu p \to \mu^+ p \pi^-)}$	$0.85^{+0.91}_{-0.59}$		
$\bar{R}_+$	$\frac{\sigma(\bar\nu p \to \bar\nu n \pi^+)}{\sigma(\bar\nu p \to \mu^+ p n^-)}$	$0.59^{+0.61}_{-0.39}$		
$\bar{R}_-$	$\frac{\sigma(\bar\nu n \to \bar\nu p \pi^-)}{\sigma(\bar\nu p \to \mu^+ p \pi^-)}$	$0.59^{+0.61}_{-0.39}$		
$\bar{R}_{0+}$	$\bar{R}_0 + \bar{R}_+$	$1.44^{+1.52}_{-0.98}$		

cleon electromagnetic form factors[39]:

$$G_E^p(Q^2) \cong (1+\mu_p)^{-1} G_M^p(Q^2) \cong \mu_N^{-1} G_M^p(Q^2) \cong \left(1 + \frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2}, \quad G_E^n(Q^2) \cong 0\,. \tag{32}$$

The parameters in (31) are given by

$$A = \left(1 - \frac{Q^2}{4m^2}\right)\left(g_M{}^2 \frac{Q^2}{4m^2} - g_V{}^2 + \frac{Q^2}{m^2} g_V g_M\right) \times \frac{G_E^p(Q^2)}{(1+Q^2/4m^2)}, \quad C = [m^2 g_V{}^2 + \tfrac{1}{4} Q^2 g_M{}^2]\, \frac{G_E^p(Q^2)}{(4+Q^2/m^2)}, \tag{33}$$

where

$$g_V = 2\left[(1-2\sin^2\theta)(1+\mu_p)\left(1+\frac{Q^2}{4m^2}\right) - \frac{Q^2}{4m^2}\mu_n\right], \quad g_M = 2[(1-2\sin^2\theta)\mu_p - \mu_n], \tag{34}$$

and the μ_p and μ_n are the anomalous magnetic moments of the proton and neutron in nuclear magnetons.

The simplest prediction is for the forward cross section, which should be independent of energy,

$$\left.\frac{d\sigma}{dQ^2}\right|_{Q^2=0} = \frac{2G^2}{\pi}(1+\mu_p)^2 \frac{(1-2\sin^2\theta)^2}{\kappa^4}\,. \tag{35}$$

For $\sin^2\theta \cong 0.68$ and $\kappa \cong 1$ as estimated from inclusive and electron data, we find a forward cross section of $4.35 \times 10^{-39}\ \text{cm}^2\ \text{GeV}^{-2}$.

More easily measured are the ratios of cut cross sections:

$$\left[\frac{\sigma(\nu p \to \nu p)}{\sigma(\nu n \to \mu^- p)}\right]_{Q^2 > Q^2_{\min}} \quad \text{and} \quad \left[\frac{\sigma(\bar\nu p) \to \bar\nu p)}{\sigma(\bar\nu p \to \mu^+ n)}\right]_{Q^2 > Q^2_{\min}},$$

with Q^2 exceeding some minimum value. We present our results for these ratios in Figs. 9 and 10 for different values of the $Q^2_{\min}$ cut, as functions of neutrino energy. In computing charged-current cross sections we have neglected the muon mass and used the axial form factor $F_A = -1.23(1+Q^2/0.88\ \mathrm{GeV}^2)^{-2}$ which fits recent experimental data.[22]

In Fig. 11 we show the ratios of the corresponding differential cross sections

$$\frac{d\sigma(\nu p\to\nu p)/dQ^2}{d\sigma(\nu n\to\mu^- p)/dQ^2}\ \text{and}\ \frac{d\sigma(\bar\nu p\to\bar\nu p)/dQ^2}{d\sigma(\bar\nu p\to\mu^+ n)/dQ^2}$$

as functions of Q^2 for various values of the incident neutrino energy. In all our figures we have chosen $\sin^2\theta = 0.68$ and $\kappa = 1$. For antineutrino scattering the distributions show a rich and rapidly energy-dependent structure, providing an excellent test of the vector model.

VI. UNIFICATION

The 51 Weyl fields of our model fit naturally into three 16-dimensional spinor representations of the group O(9) and three singlets. The 36 generators of the 16 dimensional representation of O(9) are σ_i, τ_i, η_i, $\sigma_i\tau_j\rho_3$, $\tau_i\eta_j\rho_1$, and $\eta_i\sigma_j\rho_2$, where i and $j = 1, 2$, or 3 and σ_i, τ_i, η_i, and ρ_i are independent Pauli matrices. The 16 is a real representation, and the O(9) theory is vector-like.[15] The matrices η_i generate an SU(2) subgroup which we identify with the weak interactions SU(2). The matrices $\sigma_i(1-\tau_3\rho_3)$, $\tau_i(1-\sigma_3\rho_3)$, $(\sigma_1\tau_1+\sigma_2\tau_2)\rho_3$, and $(\sigma_1\tau_2-\sigma_2\tau_1)\rho_3$ generate the color SU(3). The U(1) generator is $(\sigma_3+\tau_3+\sigma_3\tau_3\rho_3)$. With these identifications, a possible assignment of the fermion fields into O(9) multiplets is shown in Table IV. It is easy to see that under the SU(2) $\times$SU(3) subgroup, the 16 transforms like (2, 3) + (2, $\bar{3}$) + (2, 1) + (2.1) so that all the fields except the E_{OR}, M_{OR}, and L_{OR} are members of SU(2) doublets.

The O(9) symmetry can be spontaneously broken down to SU(2)$\times$U(1)$\times$SU(3) by a Higgs meson multiplet transforming like the adjoint representation with a vacuum expectation value (VEV) in the "U(1) direction." Additional Higgs multiplets, or other mechanisms of symmetry breakdown, are required to give mass to the quarks and lep-

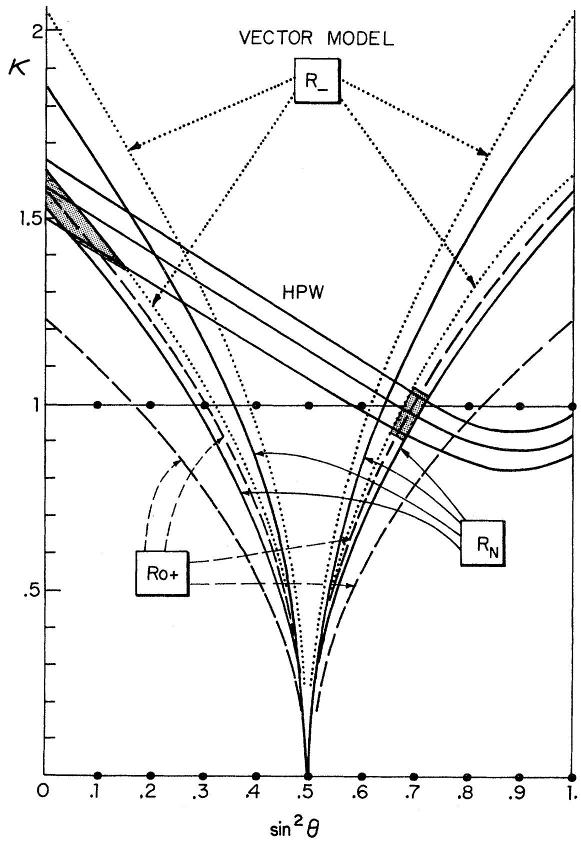

FIG. 8. Constraints on κ and $\sin^2\theta$ in the vector model following from exclusively hadronic data. The shaded regions are compatible both with HPW inclusive scattering data and with diverse experiments on single-pion production by neutrinos.

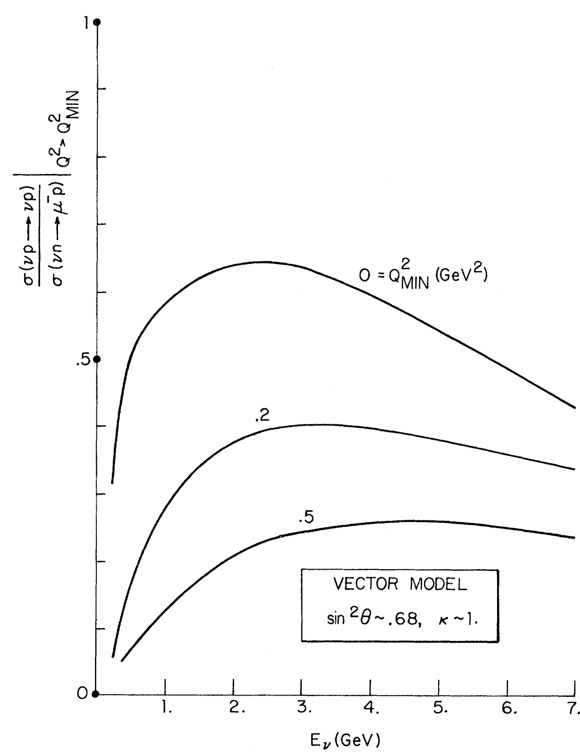

FIG. 9. Ratio of neutral-current to charged-current "elastic" cross sections for incident neutrinos as a function of neutrino energy, as predicted by the vector model. Results with different Q^2 cuts are shown.

tons. Two real 16's are required to give mass to the E^0, M^0, and L^0. All components of the 16's are members of SU(2) doublets, so their VEV's contribute to the W and Z masses just as do the Higgs mesons in the standard Weinberg model,[4] with $m_W/m_Z = \cos\theta$. To give the most general quark and lepton mass matrix, we also need additional Higgs mesons transforming like 9, 36, 84, and 126 dimensional representations of O(9). If some of these representations are absent, there will be zeroth-order relations[40] among the masses and angles. We have not found any of these to be particularly compelling and we will not discuss them further. The components of these "tensor" representations of O(9) transform like singlets and triplets under SU(2). Their VEV's can contribute to the W mass but not to the Z mass. Unless we include unnecessary Higgs mesons, *the mass will satisfy* $m_Z \leq m_W/\cos\theta$ *(i.e.,* $\kappa \leq 1$*)*. The inequality becomes an equality in the limit in which the VEV's of the 16's are much larger than the VEV's responsible for quark masses.

The VEV of the adjoint representation which breaks the O(9) symmetry down to SU(2)×U(1)

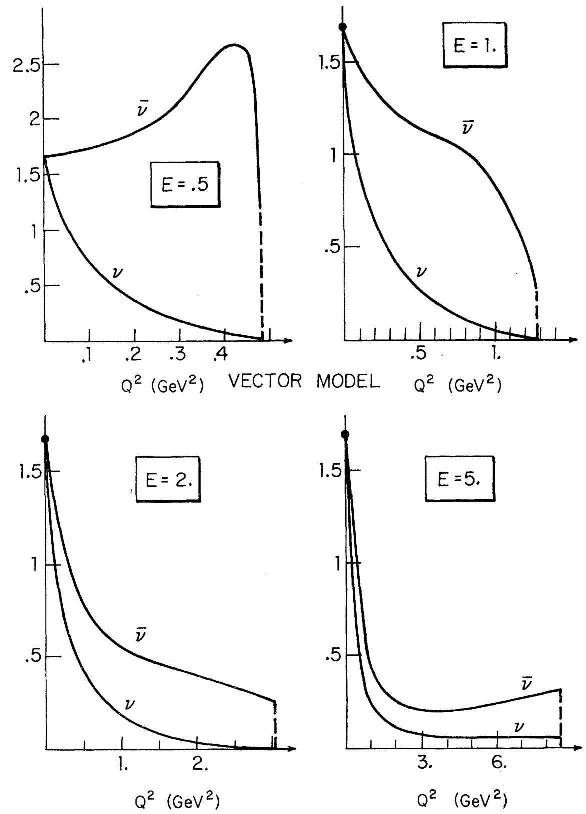

FIG. 11. Predictions of the vector model for the ratios of neutral-current to charged-current "elastic" differential cross sections as a function of Q^2 for different neutrino or antineutrino energies (in GeV). Results for neutrinos and antineutrinos are shown.

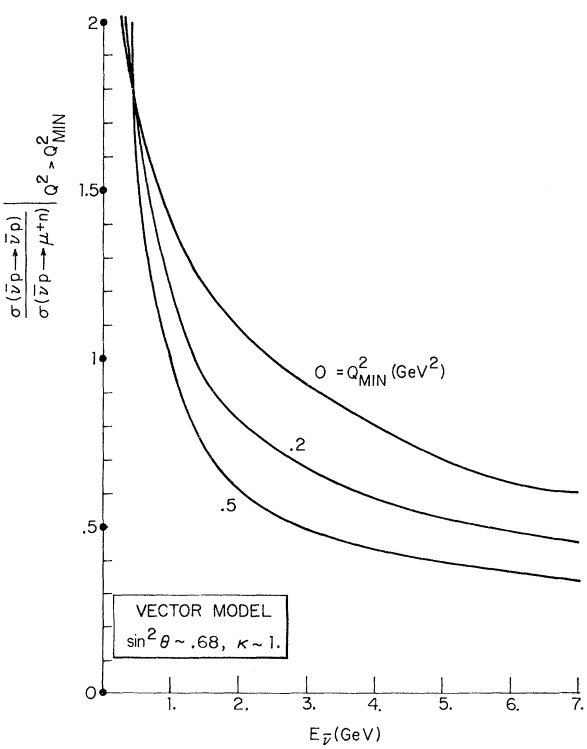

FIG. 10. Ratio of neutral-current to charged-current "elastic" cross sections for incident antineutrinos as a function of antineutrino energy, as predicted by the vector model. Results with different Q^2 cuts are shown.

TABLE IV. Assignment of fermions to O(9) 16's [case (i)]. $\bar{\Psi} = C\Psi^*$ where C is a charge-conjugation matrix.

σ_3	τ_3	η_3	ρ_3	M_1	M_2	M_3
1	1	1	1	ν	ν'	ν''
−1	1	1	1			
1	−1	1	1	$\mathcal{P}_\theta$	$\mathcal{P}'_\theta$	$\mathcal{P}''$
−1	−1	1	1			
1	1	−1	1	e^-	μ^-	L^-
−1	1	−1	1			
1	−1	−1	1	$\mathcal{N}$	λ	$\mathcal{N}''$
−1	−1	−1	1			
1	1	1	−1			
−1	1	1	−1	$\tilde{\lambda}$	$\tilde{\mathcal{N}}''$	$\tilde{\mathcal{N}}$
1	−1	1	−1			
−1	−1	1	−1	μ^+	L^+	e^+
1	1	−1	−1			
−1	1	−1	−1	$\tilde{\mathcal{P}}''$	$\tilde{\mathcal{P}}$	$\tilde{\mathcal{P}}'$
1	−1	−1	−1			
−1	−1	−1	−1	$\tilde{M}_0$	$\tilde{L}_0$	$\tilde{E}_0$

$\times$SU(3) must be very large for two reasons: to account for the observed difference in strength between the strong and electromagnetic interaction,[41] and to suppress the rate of proton decay by making the vector bosons which mediate this decay superheavy.[42] Ignoring the effect of Higgs meson couplings, we can estimate the mass of the superheavy vector bosons by the standard renormalization-group argument,[41] obtaining

$$\ln\left(\frac{M}{m}\right) \cong \frac{6\pi}{22}\left(\frac{1}{\alpha} - \frac{4}{3}\,\frac{4\pi}{g^2(m)}\right),$$

where M is the superheavy mass, m is an ordinary mass (a few GeV), and $g^2(m)$ is the effective strong coupling constant at m. A reasonable value of $g^2(m)$ gives $M \cong 10^{49}$ GeV. This number cannot be very reliable because it is very large compared to the Planck mass ($\sim 10^{19}$ GeV) so gravitational effects will presumably invalidate the naive calculation. But in any event the superheavy mass will probably be large enough to suppress proton decay to experimentally inaccessible levels.

The mixing angle is determined in principle by the unification.[43] In the absence of renormalization effects due to the large mass of the superheavy vector bosons, the mixing angle would be $\sin^2\theta = \frac{3}{4}$. Naive application of renormalization-group arguments changes this prediction to $\sin^2\theta \simeq \frac{1}{2}$. Again, this result may be meaningless since gravity has unjustifiably been ignored, but *we might expect* $\frac{3}{4} \geq \sin^2\theta \geq \frac{1}{2}$.

A similar unification can be achieved using the 16 dimensional representation of the group O(10).

VII. CONCLUSIONS

We have outlined an alternative to the conventional theory of weak and electromagnetic interactions. The impetus for the construction of the new model was our earlier suggestion that the addition of a $V+A$ term in the charm-changing charged weak current might explain the $\Delta I = \frac{1}{2}$. Our theory may seem radical—it involves six types of quarks and charged and neutral heavy leptons—but our approach is conservative in the sense that we have kept strictly within the framework of a specific theory of the strong interactions. Throughout this work we have assumed the following: that the strong interactions arise from a renormalizable asymptotically free[44] color-SU(3) gauge theory; that the color symmetry is exact,[45] the gauge gluons are massless and electrically neutral, and the quarks are fractionally charged; that color is completely screened because of the infrared structure of the theory so that only color-singlet states exist in the physical spectrum. These assumptions lead to plausible explanations of electroproduction scaling,[46] the new resonances,[47] and the mass spectrum of the observed low-mass hadrons,[48] among other things.

To illustrate the constraints imposed on us by our adherence to an unbroken-color-gauge model of the strong interactions, we will constrast our model with another six-quark model, proposed and discussed recently by Harari.[49] He suggests that in addition to the $\mathcal{P}$, $\mathfrak{N}$, and λ, there are three extra quark "flavors" with charges $\frac{2}{3}$, $\frac{2}{3}$, and $-\frac{1}{3}$, and that the strong interactions are such that the SU(3) which treats $\mathcal{P}\mathfrak{N}\lambda$ as a 3 and the extra quarks as a $\overline{3}$ is a useful approximate symmetry. He argues that the three new resonances J or $\psi(3.1)$, $\psi'(3.7)$ and the broader peak at 4.2 GeV are different linear combinations of heavy quark-antiquark pairs with definite SU(3) quantum numbers, like the π^0, η, η' system of pseudoscalar mesons in the light quark-antiquark sector.

In our theory the symmetry is SU(6)$\times$SU(6) broken only by quark mass terms. For example, if the $\mathcal{P}$, $\mathfrak{N}$, and λ quarks are approximately degenerate, while the $\mathcal{P}'$, $\mathcal{P}''$ and $\mathfrak{N}''$ quarks are heavier and not degenerate, the only useful (non-Abelian) symmetry group will be the ordinary SU(3) which acts only on the $\mathcal{P}\mathfrak{N}\lambda$ triplet and has nothing to do with the heavy quarks. If the $\mathcal{P}'$, $\mathcal{P}''$, and $\mathfrak{N}''$ are approximately degenerate, the symmetry is not SU(3), but SU(3)$\times$SU(3), one SU(3) acting on the light quark triplet and the other on the heavy quark triplet. We find our SU(3) [or SU(3)$\times$SU(3)] more plausible than Harari's SU(3) for the following reasons. To break the symmetry down to Harari's SU(3) in the context of renormalizable field theories would require (most simply) the introduction of SU(3) multiplets of fundamental bosons coupling strongly to both light and heavy quarks. These would tie together the transformation properties of both quark types, allowing only Harari's SU(3) as a conceivably useful symmetry group. But fundamental charged bosons would contribute to a longitudinal cross section in electroproduction, conflicting with experiment.

Moreover, the existence of such a second kind of fundamental strong interaction is difficult to reconcile with the notion of a renormalizable gauge theory of weak interactions. While the color SU(3) interactions are left invariant by weak gauge transformations, the interactions which couple together light and heavy quarks are not.

We also find the mixing scheme proposed by Harari unjustifiable. In the ρ, ω, φ system, the closest light-quark analogs of the J (Ψ), mixing between different quark-antiquark pairs is appreciable only for the ρ, ω pair and then only because the $\mathcal{P}$ and $\mathfrak{N}$ quarks are very nearly degenerate. The φ is to a good approximation pure $\lambda\bar{\lambda}$, and we expect that the J (ψ) is $\mathcal{P}'\overline{\mathcal{P}}'$ to an even better approximation.

The "success" of Harari's model is that it explains the large value of R above the resonance region, at the cost of giving up interpretation of the ψ's as a radial excitation of the J (ψ). But there are strong theoretical indications that the ψ' is a radial excitation.[50] Experimental indications (in the form of γ rays) may be difficult to observe.[13] In our theory we assume that the $\mathcal{P}''$ and $\mathfrak{N}''$ are heavier than the $\mathcal{P}'$. Narrow resonances associated with $\mathcal{P}''\overline{\mathcal{P}}''$ and $\mathfrak{N}''\overline{\mathfrak{N}}''$ should exist at large mass, but have not yet been seen. The fact that R is observed significantly to exceed $\frac{10}{3}$ may be due to the production of the heavy lepton $L^{\pm}$. Indeed, such a hypothesis yields a good fit to the data.[13] In view of the many successes of the color-gauge model, we feel that Harari's attempt to go outside this framework to explain the large value of R is misguided, especially since there is no satisfactory field theory realizing his model, and an adequate explanation exists involving the conventional model and one heavy lepton. Of course, we have no objection to the mere introduction of extra quarks such as has been proposed by Barnett and by Wilczek.[12]

Our vector model of weak and electromagnetic interactions is a viable alternative to the conventional theory. Both theoretical and experimental progress is necessary to determine which if either is correct. Experimental work is needed, particularly more neutrino scattering data, to check the detailed predictions of our model for charged- and neutral-current phenomena. Theoretical work is necessary to show whether our new explanation of the $\Delta I = \frac{1}{2}$ rule is consistent with nonleptonic decay data[51] and to complete our understanding of $\Delta S = 2$ effects.

A distrubing feature of the vector model is the apparent lack of naturalness.[40] This appears in several ways. In the vector model as in the old model, we have not been able to understand the hierarchy of quark masses in any simple way. Isospin symmetry, for example, is unnatural and must be put in by hand.

In the conventional model, and with the neglect of the Cabibbo angle, the $\mathcal{P}$ and $\mathfrak{N}$ quarks are symmetrically involved in the weak interactions. However, in the vector model, even this symmetry is lost by the couplings of the right-handed quarks. The quarks to which the right-handed $\mathcal{P}$ and $\mathfrak{N}$ are coupled (i.e., $\mathcal{P}'$ and $\mathfrak{N}''$) must be significantly different in mass. This makes the origin of isospin invariance even more mysterious. Moreover, the vector theory does not naturally guarantee Cabibbo universality, as the conventional model does. Further theoretical assaults on these questions are essential.

ACKNOWLEDGMENTS

In the course of completing this paper, we received a paper by Fritsch and Minkowski[52] which comes to similar conclusions. One of us (A.D.R.) wishes to thank E. Paschos for discussions.

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