Polarized electron-nucleus scattering and parity-violating neutral-current interactions*

G. Feinberg
Physics Department, Columbia University, New York, New York 10027
(Received 29 July 1975)

The polarization asymmetry in the scattering of polarized electrons from nuclei coming from the interference between electromagnetic interactions and parity-violating neutral-current interactions is discussed. It is shown how measurements of this asymmetry in various elastic and inelastic electron-nucleus scatterings can be used to determine specific spin and isospin terms in the hadronic neutral current. A specially interesting case would be a measurement in a $0^+ \rightarrow 0^-$ excitation, in which the asymmetry is 100 times larger than usual because the electromagnetic matrix element involves two-photon exchange.

I. INTRODUCTION

The recognition that neutral-current interactions between neutrinos and hadrons exist[1] has stimulated much thought about ways to detect such interactions in various contexts.[2,3] An important question is whether such nonelectromagnetic interactions also occur between charged leptons and hadrons, as predicted by some but not all unified gauge theories.[4] An aspect of the neutral-current interactions that has been used in various proposals to detect such interactions is the occurrence of parity-nonconserving terms, which can, through interference with the electromagnetic interaction, lead to the usual type of pseudoscalar asymmetries, such as dependence of cross sections on longitudinal polarizations, etc.

In particular, a number of theorists[5] have calculated the expected asymmetry between the deep-inelastic scattering of left-handed or right-handed leptons by nucleons. A proposal to measure such an asymmetry in electron scattering at the level expected in the Weinberg model has been made by a SLAC-Yale-Bielefeld group.[6] The asymmetry in the deep-inelastic region is expected[5] to be of order

$$R = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} \sim 10^{-5} \frac{q^2}{M_p^2}. \tag{1.1}$$

The measurement in the deep-inelastic region with a nucleon target is sensitive to a complicated combination of the various parity-violating terms in the neutral-current interaction, in addition to being relatively small. In this paper, we point out that measurements of polarized lepton scattering by a nucleus, both elastically and with the excitation of specific nuclear levels, can be used to disentangle the various possible parity-violating neutral-current interactions. In addition, the size of the asymmetry can in certain cases be one or two orders of magnitude greater than that given in (1.1). On the other hand, the nuclear experiments in some cases have much more severe background problems to contend with.

One advantage of nuclei as targets is the possibility of choosing the spin and isospin of the target at will. In addition, by observing the excitation of definite final states, one can isolate terms with specific isospin or parity properties in the weak and electromagnetic Hamiltonians. Furthermore, the presumed existence of an axial-vector neutral hadronic current, which has a different multipole expansion than the electromagnetic or weak vector currents, allows for enhancements in the relative strength of the weak matrix elements in certain transitions, compared to the electromagnetic matrix elements.

These advantages must be balanced against certain disadvantages of scattering from nuclei. One such disadvantage is the restriction to relatively small momentum transfers of $\lesssim 500$ MeV because of the rapid falloff of the nuclear form factors beyond such values. Since, in most cases, the relative strength of the weak and electromagnetic matrix elements increases as q^2, this falloff inhibits the observation of interference where it might be maximum. Another disadvantage in some cases is the small ratio of inelastic to elastic scattering, which exacerbates the problems of background to the processes of interest, and makes more difficult the measurement of the small polarization asymmetries. In spite of these difficulties, the information that might be obtained from electron-nucleus scattering is such as to make a serious effort to carry out such experiments worthwhile.

In this paper, I shall point out a few examples of possible experiments of this type that would furnish useful information about the parity-violating neutral-current interactions. I do not make a systematic analysis of electron-nucleus scattering, nor attempt precise calculations of the relevant matrix elements. Instead, my aim is to indicate a few places in which the special characteristics of nuclei allow for novel effects, and to estimate the expected size of these effects.

II. PARITY-VIOLATING LEPTON-HADRON INTERACTIONS

We first consider the effective Hamiltonian for the parity-violating interactions of charged leptons and hadrons. The notation will mostly follow that of an earlier paper on such interactions.[7] We shall not restrict ourselves to the Weinberg model, as one aim of the analysis is to find experiments that are sensitive to the specific assumptions of that model. However, we shall assume VA interactions, as only these are consistent with observed T invariance.[8] The formulas are written for electron-hadron interactions, but similar formulas, perhaps with different coupling constants, can be written for muon-hadron interactions. At the low-momentum transfers of interest, it is not, however, valid to neglect the muon mass, while I have neglected the electron mass everywhere:

$$H_{\text{p.v.}}=\frac{G_F}{2\pi\alpha\sqrt{2}}\sum_{i=1}^{2}(\epsilon_{ei}^{VA}\,\bar{\psi}_e\gamma^\rho\psi_e A_\rho^{(i)}+\epsilon_{ie}^{VA}\,\bar{\psi}_e\gamma^\rho\gamma_5\psi_e V_\rho^{(i)}). \tag{2.1}$$

In this equation, ϵ_{ei}^{VA} and ϵ_{ie}^{VA} are parameters measuring the strength of the interaction. Their magnitudes in the Weinberg model are given by

$$\epsilon_{1e}^{VA}=\pi\alpha(1-4\sin^2\theta_W),\quad \epsilon_{e2}^{VA}=\pi\alpha(1-4\sin^2\theta_W),$$
$$\epsilon_{1e}^{VA}=\pi\alpha(1-\tfrac{8}{3}\sin^2\theta_W),\quad \epsilon_{2e}^{VA}=-\pi\alpha(1-\tfrac{4}{3}\sin^2\theta_W). \tag{2.2}$$

The quantities $A_\rho^{(i)}$ and $V_\rho^{(i)}$ are hadronic axial-vector and vector currents. Their normalization and properties can most easily be explained by expressing them in terms of field operators for the nonstrange, noncharmed quarks that are presumably to be found in ordinary nuclei:

$$A_\rho^{(i)}=\bar{\psi}_i\gamma_\rho\gamma_5\psi_i\,,$$
$$V_\rho^{(i)}=\bar{\psi}_i\gamma_\rho\psi_i\,. \tag{2.3}$$

Here ψ_1 and ψ_2 are the two quark field operators corresponding to $T=\frac{1}{2}$ and $T=-\frac{1}{2}$. The somewhat unorthodox division of the coupling strengths used in Eq. (2.1) corresponds to having ϵ as the square of the dimensionless weak coupling constants. In writing (2.1), we are furthermore restricting q^2 to values much less than the mass of intermediate bosons, which is certainly justified for the scattering processes we shall consider. For reference, the one-photon-exchange electromagnetic interaction corresponds to an effective Hamiltonian

$$H_{1\gamma}=\frac{4\pi\alpha}{q^2}\bar{\psi}_e\gamma^\rho\psi_e J_\rho^{\text{em}}, \tag{2.4}$$

with

$$J_\rho^{\text{em}}=\tfrac{2}{3}\bar{\psi}_1\gamma_\rho\psi-\tfrac{1}{3}\bar{\psi}_2\gamma_\rho\psi_2$$
$$=\tfrac{2}{3}V_\rho^{(i)}-\tfrac{1}{3}V_\rho^{(2)}.$$

This assumption, that the electromagnetic and weak neutral hadron currents are linear combinations of the same operators, is not proved. Nevertheless, it is contained in most gauge models. The results given here are not qualitatively sensitive to this assumption. However, the ratios of form factors will not cancel as they do in certain places below if the assumption is false. This would show up as an additional dependence of $R(q^2)$ on q^2 beyond that given in (3.7), for example. This would itself be an interesting thing to discover.

Since we shall be interested in a pseudoscalar observable, and in low-momentum transfers, it is justified to neglect the parity-conserving neutral-current interactions, as the latter give no pseudoscalars through interference with the electromagnetic interaction, and, in the range of interest, are so much smaller than the electromagnetic interaction that we can neglect their interference with $H_{\text{p.v.}}$

The quantities of interest to us then will involve matrix elements of the currents $A_\rho^{(i)}$, $V_\rho^{(i)}$, and J_ρ^{em} evaluated between various nuclear states. A detailed analysis of such matrix elements in the context of electron scattering and neutrino scattering has been given.[9] That analysis uses a multipole expansion of the currents, together with nuclear model calculations of reduced matrix elements. For the purpose of this paper, such detailed results are not necessary, and only certain qualitative aspects of the matrix elements will be used.

The vector currents V_ρ and J_ρ^{em} have matrix elements between any two nuclear states, except for the case of a 0^+ to 0^- matrix element, which vanishes to all degrees of multipolarity. Therefore, all scatterings, elastic or inelastic, with the exception of 0^+ to 0^- transitions, can be induced by the vector currents, and interferences can be studied. In general, we should not expect any major fluctuations in the ratios of matrix elements of V_ρ and J_ρ^{em} from transition to transition, and it is probably sufficient to study a few convenient transitions to extract all the available information about those currents.

A very different situation exists for A_ρ, whose matrix elements all have an opposite parity to those of V_ρ. This implies first that A_ρ will have a matrix element linking a 0^+ to a 0^- state, but not a 0^+ to a 0^+ state. Therefore, in any 0^+ to 0^+ transition, only the terms involving V_ρ in $H_{\text{p.v.}}$ contribute, while in a 0^+ to 0^- transition, only those involving A_ρ will contribute. Therefore, by measurements of pseudoscalar corrections in 0^+ to 0^+ transitions, such as in elastic scattering on a spinless nucleus, and by choosing the isospin of the initial and final states suitably, it is possible to isolate the contributions of the two currents $V_\rho^{(1)}$, $V_\rho^{(2)}$, and to determine the two coupling constants multiplying them. Furthermore, we note that a 0^+ to 0^- transition, the matrix element $H_{1\gamma}$ vanishes also by parity conservation at least to the extent that the nuclear states have definite parity.[10] This means that in such a transition, the electromagnetic matrix element must involve two-or-more-photon exchange. This has the effect of decreasing the electromagnetic matrix element by a factor of 10^{-2} or more, and therefore increasing the relative size of the electromagnetic-weak interference by 100. It also has the effect of decreasing the over-all cross section for a 0^+ to 0^- transition by a factor of 10^4 or more, compared to elastic scattering, which makes it more difficult to observe any asymmetries. Some quantitative estimates of these effects will be given in Sec. IV. However, we may draw the qualitative conclusion that the measurement of a polarization asymmetry in a 0^+ to 0^- nuclear transition induced by electron scattering would isolate the contribution of the currents $A_\rho^{(1)}$ and $A_\rho^{(2)}$, and so determine the coupling constants multiplying them. Suitable 0^+ to 0^- transitions are difficult to come by, but there seem to be two candidates in O^{16}. A 0^- state with $T=0$ exists at 10.95 MeV, and a 0^- state with $T=1$ exists at 12.8 MeV,[11] while the ground state is of course 0^+. Similar situations may well exist in other nuclei.

Another interesting feature of the current A_ρ is that its "$M1$" matrix element is finite at zero momentum transfer. That is, the matrix element for a transition with $\Delta J=1$, and no parity change, is finite in the static limit, because the current $A_i \sim \sigma_i$ contributes to such transitions. This is in distinction to the case of V_ρ, where the $M1$ matrix element is proportional to q/M_p. Therefore, we might expect the weak axial-vector currents to get some relative enhancement at low momentum transfer in $M1$ transitions. However, we shall see below that this effect mostly disappears in the polarization asymmetry. In the following section, we consider more quantitative estimates of the relevant matrix elements for several elastic and inelastic scatterings.

III. ELASTIC LEPTON-NUCLEUS SCATTERING

The simplest case is the elastic scattering of leptons from a spinless nucleus. For this case, the matrix elements of $A_\rho^{(i)}$ must vanish by parity conservation provided that the nuclear ground state has a definite parity. The latter is not precisely true because of weak-interaction-induced parity mixing in nuclei. However, the mixing is not greater than 10^{-7}, except in very special cases of almost degenerate levels, and even then is still not more than 10^{-5} or so. Therefore, the matrix element of $A_\rho^{(i)}$ in the ground state of a spinless nucleus is much less then that of $V_\rho^{(i)}$, and we neglect it. We shall write the matrix elements in the style of particle physics, rather than that of nuclear physics. The matrix elements of $V_\rho^{(i)}$ are determined by Lorentz invariance, and time reversal invariance, to be

$$\langle p, 0^P, M_N | V_\rho^{(i)} | p', 0^P, M_N \rangle = \frac{(p+p')_\rho}{2M_N} F^{(i)}[(p-p')^2]. \tag{3.1}$$

Here 0^P is a state with $J=0$, and parity $P=\pm 1$. p and p' are the initial and final four-momentum of the nuclear center of mass. $F^{(i)}$ are form factors, which depend on the nucleus in question. Note that the internal nuclear initial and final states are identical here. M_n is the nuclear mass.

The corresponding lepton scattering matrix element from (2.1) and (2.4) is

$$M = \bar{U}\gamma^\rho U \frac{(p+p')_\rho}{2M_N} \frac{4\pi\alpha}{q^2}\left(\tfrac{2}{3}F^{(1)} - \tfrac{1}{3}F^{(2)}\right) + \bar{U}\gamma^\rho\gamma_5 U \frac{(p+p')_\rho}{2M_N} \frac{G_F}{2\pi\alpha\sqrt{2}}\left(\epsilon_{1e}^{VA}F^{(1)} + \epsilon_{2e}^{VA}F^{(2)}\right). \tag{3.2}$$

Here U is the lepton wave function. For a nucleus with ground-state isospin zero, the form factors are related by

$$F^{(1)} = F^{(2)} = 3ZF_{\text{ch}}, \tag{3.3}$$

where F_{ch} is the nuclear charge form factor normalized so $F(q^2=0)=1$. In this case, the matrix element reduces to

$$M = ZF_{\text{ch}}\frac{4\pi\alpha}{q^2}(p+p')_\rho\left[\bar{U}\gamma^\rho U + \frac{3G_F q^2}{8\pi\alpha^2\sqrt{2}}(\epsilon_{1e}^{VA} + \epsilon_{2e}^{VA})\bar{U}\gamma^\rho\gamma_5 U\right]. \tag{3.4}$$

The parity-violating term in the matrix element, and hence the pseudoscalar correlation, is sensitive to the isoscalar combination $\epsilon_{1e}^{VA} + \epsilon_{2e}^{VA}$ alone, and so isolates one combination of coupling constants.

The corresponding cross sections, for left-handed or right-handed electrons, neglecting terms quadratic in ϵ, and neglecting the electron mass, are

$$\frac{d\sigma_{L,R}}{d\Omega} \sim Z^2\sigma_M|F_{\text{ch}}|^2\left[1\mp\frac{3G_F q^2}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{1e}^{VA}+\epsilon_{2e}^{VA})\right]. \quad (3.5)$$

Here σ_M is the Mott cross section. The polarization asymmetry R is then given by

$$R(q^2) \equiv \frac{d\sigma_L(q^2)-d\sigma_R(q^2)}{d\sigma_L(q^2)+d\sigma_R(q^2)} = +\frac{3G_F|q^2|}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{1e}^{VA}+\epsilon_{2e}^{VA}) \quad (3.6)$$

$$= 10^{-2}\frac{|q^2|}{M_p^2}(\epsilon_{1e}^{VA}+\epsilon_{2e}^{VA}), \text{ where } M_p \text{ is the proton mass} \quad (3.7)$$

$$= 4\times10^{-4}\frac{|q^2|}{M_p^2}\sin^2\theta_W \text{ in the Weinberg model}$$

All references to nuclear properties other than that $T=0$, $J=0$ disappear from the expression for R. However, the nucleus is still relevant to determining how large a value of $|q^2|$ can be used, since the decrease of F_{ch} with q^2 depends on the nucleus. It might be feasible to obtain usable event rates at values of $|q^2|\simeq\frac{1}{4}M_p^2\sim10\ F^{-2}$, in light nuclei. This would correspond to an asymmetry,

$$R\simeq3\times10^{-5}, \quad (3.8)$$

which is comparable to that expected in the SLAC-Yale-Bielefeld proposal.[6] For an electron energy of 1 GeV and $q^2\sim10\ F^{-2}$ the cross section on a nucleus of $A=16$ would be approximately

$$\frac{d\sigma}{d\Omega}\sim4\times10^{-32}\ \text{cm}^2/sr, \quad (3.9)$$

or substantially higher than expected in the proposal of Ref. 6. It therefore appears feasible to measure asymmetries of the order of those given in (3.8).

When the target nucleus has $T\neq0$, then the form factors $F^{(1)}$ and $F^{(2)}$ are no longer equal, and the quantity $R(q^2)$ will in general depend on the ratios of these two form factors. The effect of this is to change Eqs. (3.6) to become

$$R(q^2)=\frac{3G_F|q^2|}{4\pi^2\alpha^2\sqrt{2}}\,\frac{\epsilon_{1e}^{VA}F^{(1)}+\epsilon_{2e}^{VA}F^{(2)}}{2F^{(1)}-F^{(2)}}. \quad (3.10)$$

The ratio of form factors may be rewritten as

$$\frac{(\epsilon_{1e}^{VA}+\epsilon_{2e}^{VA})F_S+(\epsilon_{1e}^{VA}-\epsilon_{2e}^{VA})F_V}{F_S+\frac{3}{2}F_V}$$

when F_S and F_V are isoscalar and isovector form factors of the nucleus. In principle, a measurement of $R(q^2)$, combined with other determinations of the ϵ_{ie}^{VA}, can then be used to determine these two form factors, whereas ordinary electron scattering fixes only one of them. The inverse procedure, of using $R(q^2)$ to determine ϵ_{1e}^{VA} and ϵ_{2e}^{VA} separately, appears less feasible, as it requires independent knowledge of $F^{(1)}$ and $F^{(2)}$. Perhaps it is possible to calculate $F^{(1)}$ from nuclear models with sufficient accuracy to do this. Alternative strategies for obtaining ϵ_{1e}^{VA} and ϵ_{2e}^{VA} will be discussed below.

Elastic scattering from nuclei with $J\neq0$ is more complicated to analyze because of the well known increase in the number of form factors. No discussions of this case will be attempted here, but the increased number of form factors, and the fact that both the terms with $V_\rho^{(i)}$ and $A_\rho^{(i)}$ of $H_{\text{p.v.}}$ can now contribute, make it more difficult to isolate contributions involving specific ϵ_{ie}.

IV. INELASTIC LEPTON-NUCLEUS SCATTERING

We consider here the excitation of specific nuclear states by scattering of polarized leptons. Again the simplest case to consider is when the initial and final states have $J=0$. But now there are two cases to consider, in which the parity of the final state is the same as or opposite to that of the initial state. For definiteness, and in conformity with the usual situation, we take the initial state to have $J^P=0^+$.

If the final state is also 0^+, then the situation is like that of elastic scattering, in that only $V_\rho^{(i)}$ will contribute to the weak or one-photon matrix elements. However, the inelastic scattering differs in that the $E0$ matrix element vanishes at $q^2=0$. This has the effect of decreasing both $H_{1\gamma}$ and $H_{\text{p.v.}}$ by the same factor, and does not affect $R(q^2)$, although it does decrease the cross section at low-momentum transfers. At the larger momentum transfers of interest to us, the inelastic excitation of a 0^+ level has a cross section close to that of elastic scattering, or other inelastic excitations, and the effect mentioned is less important.

The case of greatest interest here would be the excitation of a state with $J^P=0^+$ and isospin one, from a nucleus with ground state $J^P=0^+$ and isospin zero. For this case, we can write

$$\langle p,0^+,M_N|V_\rho^{(i)}|p',0^+,M_N'\rangle = \frac{(p+p')_\rho}{M_N+M_N'}G^{(i)}(q^2)+\frac{(p-p')_\rho}{M_N+M_N'}H^{(i)}(q^2), \quad (4.1)$$

where now we have by isospin invariance

$$G^{(1)} = -G^{(2)},$$
$$H^{(1)} = -H^{(2)}, \tag{4.2}$$

and if the vector current is conserved,

$$H^{(i)} = \frac{(M_N'^2 - M_N^2)}{q^2} G^{(i)}, \tag{4.3}$$

where $M_N^2 = p^2$, $M_N'^2 = p'^2$ are the masses of the ground state and excited state. Equation (4.3) implies that $G^{(1)}(q^2) \sim q^2$ for $q^2 \to 0$. One might hope that this property could be used as a test of the conservation of the weak neutral vector current. However, it turns out that almost any model of the vector current, conserved or not, gives this property.[12] Except at very small q^2, the term involving $H^{(i)}$ is negligible, as it eventually multiplies either zero or something involving the lepton mass. We therefore disregard it, and obtain for the over-all matrix element

$$M = \overline{U}\gamma^\rho U \frac{(p+p')_\rho}{M_N + M_N'} \frac{4\pi\alpha}{q^2} G^{(1)}$$
$$+ \overline{U}\gamma^\rho\gamma_5 U \frac{(p+p')_\rho}{M_N + M_N'} \frac{G_F}{2\pi\alpha\sqrt{2}} (\epsilon_{1e}^{VA} - \epsilon_{2e}^{VA}) G^{(1)} . \tag{4.4}$$

We note that the matrix now involves the difference $\epsilon_{1e}^{VA} - \epsilon_{2e}^{VA}$. Upon calculating $R(q^2)$, we find

$$R(q^2) = \frac{G_F |q^2|}{4\pi^2\alpha^2\sqrt{2}} (\epsilon_{1e}^{VA} - \epsilon_{2e}^{VA}) \tag{4.5}$$

$$= 3 \times 10^{-3} \frac{|q^2|}{M_p^2} (\epsilon_{1e}^{VA} - \epsilon_{2e}^{VA}) \tag{4.6}$$

$$= 1.2 \times 10^{-4} (1 - 2\sin^2\theta_W) \frac{|q^2|}{M_p^2} \text{ in the Weinberg model.}$$

The asymmetry here is comparable to that in elastic scattering, assuming similar values for the isoscalar and isovector combination of the ϵ^{VA}. The cross sections involved may be somewhat smaller than the estimates of (3.9) for elastic scattering.[13] Therefore, it may be more difficult, but perhaps not impossible, to measure $R(q^2)$ for such inelastic excitations, and so complete the determination of ϵ_{1e}^{VA} and ϵ_{2e}^{VA}.

We consider next the excitation of a 0^- state from a 0^+ ground state. If we neglect parity mixing in the nucleus due to weak interactions, the matrix elements of $V_\rho^{(i)}$ vanish by parity conservation. In this case, the matrix elements of $H_{1\gamma}$ and of the $V_\rho^{(i)}$ terms in $H_{\text{p.v.}}$ are zero. The effect of nuclear parity mixing will be discussed below.

The terms in $H_{\text{p.v.}}$ involving $A_\rho^{(i)}$ do not vanish for a 0^+ to 0^- transition. Instead, we have in analogy to (4.1)

$$\langle p, 0^+, M_N | A_\rho^{(i)} | p', 0^-, M_N' \rangle = \frac{(p+p')_\rho}{M_N + M_N'} A^{(i)}(q^2)$$
$$+ \frac{(p-p')_\rho}{M_N + M_N'} B^{(i)}(q^2), \tag{4.7}$$

where $A^{(i)}$ and $B^{(i)}$ are again form factors.

Since the axial-vector current is not expected to be conserved, $A^{(i)}$ and $B^{(i)}$ are not related by an equation similar to (4.3). If a PCAC (partially conserved axial-vector current) assumption is made for the isovector part of A_ρ, then some relation involving $A^{(i)}$, $B^{(i)}$, and other quantities might be found. However, in any case, the term with $B^{(i)}$ does not contribute, because in $H_{\text{p.v.}}$ it multiplies the vector lepton current $\overline{U}\gamma_\rho U$, whose divergence vanishes.

It is possible to relate $A^{(i)}$ and $B^{(i)}$ to nuclear matrix elements. From (4.7),

$$A^{(i)}(q^2) + \frac{q_0}{M_N + M_N'} B^{(i)}(q^2) = \langle p, 0^+, M_N | A_0^{(i)} | p, 0^+, M_N' \rangle ,$$

$$\frac{q_J}{M_N + M_N'} (A^{(i)} - B^{(i)}) = \langle p, 0^+, M_N | A_J^{(i)} | p', 0^-, M_N' \rangle , \tag{4.8}$$

$$A^{(i)} = \langle A_0^{(i)} \rangle + \frac{q_0}{q^2} \langle q_J A_J^{(i)} \rangle ,$$

where $q_0 = M_N - M_N' + q^2/2M_N$.

The two quantities $\langle A_0^{(i)} \rangle$ and $\langle q_J A_J^{(i)} \rangle$ are the matrix elements of a pseudoscalar and of a 3-divergence of an axial vector. They are essentially the matrix elements $\int \sigma \cdot p/M$ and $|q| \int \sigma_0$, discussed by Ericson *et al.*[14] in their treatment of muon capture from a 0^+ to a 0^- state. Indeed, for the case of electron scattering on O^{16} to excite the 0^- state at 12.8 MeV, which is an isospin partner of a state in N^{16}, the matrix elements presumably have values related by isospin rotation to those in the muon capture. The order of magnitude of these matrix elements is given by

$$\langle A_0^{(i)} \rangle \sim \frac{1}{M_p r_n} G_A(q^2),$$

$$\langle q_J A_J \rangle \sim q^2 r_n G_A(q^2),$$

where r_n is the nuclear radius, or about 2–3 F, and $G_A(q^2)$ is a form factor normalized by $G_A(0) = 1$. This implies that the two terms in (4.8) are comparable in magnitude, and perhaps 10% as big as the corresponding elastic matrix elements at small momentum transfer, with unknown form factor corrections at large momentum transfer.

Since the matrix elements of $V_0^{(i)}$ vanish, there is no contribution of $H_{1\gamma}$ to the excitation. However, there will be a contribution from 2γ exchange. At the least, we would expect this to be smaller than a one-photon exchange matrix element by a factor of α, so that the corresponding cross sections for the excitement of a 0^- state will be 10^4 times smaller than typical inelastic cross sections. On the other hand, we would correspondingly expect the interference between $H_{\text{p.v.}}$ and $H_{2\gamma}$ to be 100 times larger than that between $H_{\text{p.v.}}$ and $H_{1\gamma}$, thus increasing the size of $R(q^2)$ by such a factor. The parity-conserving neutral-current interaction is still negligible here and is expected to be 10^{-3} smaller than $H_{2\gamma}$.

In order to be slightly more quantitative, we may imagine the 2γ exchange to involve the virtual excitation of 1 or more states with $J^P=1^-$, i.e., to be a combination of an $E1$ with an $M1$ transition. The leading term in an expansion in power of q/M_p then has the form

$$M_{2\gamma} \simeq \alpha^2 \frac{(p+p')_\rho}{M_N+M_N'} \bar{U}\gamma_\rho\gamma_5 U \frac{r_n}{M_p} F_{2\gamma}(q^2, p_1\cdot p) \qquad (4.9)$$

Here $F_{2\gamma}$ is a form factor that may depend on energy as well as momentum transfer, as it arises from a box diagram. The occurrence of an axial-vector type lepton matrix element could have been predicted on the basis of charge conjugation invariance. This matrix element has a similar magnitude to that obtained for the two-photon dispersion corrections to elastic scattering by various authors.[15] Combining Eqs. (4.7)–(4.9), we obtain the over-all matrix element for the $0^+ \to 0^-$ excitation

$$M_{0^+\to 0^-} = \bar{U}\gamma_\rho\gamma_5 U \frac{(p+p')_\rho}{M_N+M_N'} \alpha^2 \left(\frac{r_N}{M_p}\right) F_{2\gamma}(q^2, p_1\cdot p) + \bar{U}\gamma_\rho U \frac{(p+p')_\rho}{M_N+M_N'} \frac{G_F}{2\pi\alpha\sqrt{2}} [\epsilon_{e1}^{VA} A^{(1)}(q^2) + \epsilon_{e2}^{VA} A^{(2)}(q^2)]. \qquad (4.10)$$

Again we may separate the isotopic dependence of $H_{\text{p.v.}}$ by considering final states with isospin zero or isospin one, excited from an isospin-zero ground state. In O^{16}, there is a 0^- state at 12.8 MeV, with isospin 1, and a 0^- state at 10.95 MeV with isospin zero.[11] The corresponding polarization asymmetries are

$$R(0^+ \to 0^-, T=0) = \frac{G_F}{\pi\alpha\sqrt{2}} \frac{(\epsilon_{e1}^{VA} + \epsilon_{e2}^{VA}) A^{(1)}(q^2)}{\alpha^2 (r_n/M_p) F_{2\gamma}(q^2, p_1\cdot p)}, \qquad (4.11)$$

$$R(0^+ \to 0^-, T=1) = \frac{G_F}{\pi\alpha\sqrt{2}} \frac{(\epsilon_{e1}^{VA} - \epsilon_{e2}^{VA}) A^{(1)}(q^2)}{\alpha^2 (r_n/M_p) F_{2\gamma}(q^2, p_1\cdot p)}, \qquad (4.12)$$

where the form factors $A^{(1)}(q^2)$ and $F_{2\gamma}(q^2, p_1\cdot p)$ depend on the states involved. If we use our previous estimates for $A^{(1)}$ and assume that the form factor ratios are slowly varying, and of order 1, then we obtain

$$R(0^+ \to 0^-) \simeq \frac{G_F}{\pi\alpha^3} \frac{(\epsilon_{e1}^{VA} \pm \epsilon_{e1}^{VA})}{\sqrt{2}\, r_n{}^2} \simeq 2\times 10^{-1} \frac{(\epsilon_{e1}^{VA} \pm \epsilon_{e2}^{VA})}{\pi}, \qquad (4.13)$$

which equals $3\times 10^{-3}(1-4\sin^2\theta_W)$ for a $T=0$ to $T=1$ transition in the Weinberg model, which equals 0 for a $T=0$ to $T=0$ transition in the Weinberg model.

As predicted, the asymmetry is much greater than in elastic scattering, or in the excitation of a 0^+ state. Furthermore, it is not proportional to q^2, so there is no necessity to go to large q^2. On the other hand, the cross sections are much smaller:

$$\frac{d\sigma}{d\Omega} \simeq \alpha^4 \left(\frac{r_n}{M_p}\right)^2 p^2 |F_{2\gamma}|^2 \simeq 10^{-34} |F_{2\gamma}|^2 \text{ cm}^2/\text{sr. for } p \sim 1 \text{ GeV}/c. \qquad (4.14)$$

This is several orders of magnitude smaller than the elastic cross section, with an unknown dependence of q^2 and also on energy.

We note that the extraction of the values of the ϵ's from the asymmetry R requires in this case a knowledge of the form factors $A^{(i)}$ and $F_{2\gamma}$. $F_{2\gamma}$ can be obtained, up to a sign from a measurement of the total cross section. The form factors $A^{(i)}$ would have to be either calculated from a nuclear model, or obtained from the corresponding muon capture or neutrino scattering excitation of the states.

The effect of parity mixing in the nucleus would be to generate a "pseudocharge" or "anapole" matrix element for the electromagnetic current between the "0^+" and "0^-" states. Qualitatively, we would expect the size of this matrix element to be of the order of magnitude

$$\langle \text{“}0^+\text{”} | J_\rho^{\text{em}} | \text{“}0^-\text{”} \rangle \simeq \frac{V_{\text{p.v.}}}{E_+ - E_-} \langle 0^+ | J_\rho^{\text{em}} | 0^+ \rangle,$$

where $V_{\text{p.v.}}$ is the parity-violating weak nuclear potential. Estimates of $V_{\text{p.v.}}$ give values of about 10^{-7} MeV[16] (see Ref. 16) so that we get

$$\langle \text{“}0^+\text{”} | J_\rho^{\text{em}} | \text{“}0^-\text{”} \rangle \simeq 10^{-8} G_{\text{ch}}(q^2) \frac{(p+p')_\rho}{2M_N}.$$

This matrix element, when introduced into (3.4), generates a parity-violating interaction that may be compared to the neutral-current-induced interaction given in (4.10). We find that

$$\frac{M^{\text{n.p.v.}}}{M^{\text{N.C.}}} \simeq \frac{10^{-8} G_{\text{ch}} 4\pi\alpha/q^2}{(G_F/2\pi\alpha\sqrt{2})(\epsilon_{11}^{VA}A^{(1)}+\epsilon_{12}^{VA}A^{(2)})}$$

$$\simeq 10^{-4}\frac{M_p^2}{q^2}\frac{G_{\text{ch}}}{A^{(1)}}$$

$$\simeq 10^{-4}\frac{M_p^3}{q^2} r_n \frac{G_{\text{ch}}}{G_A} \simeq 10^{-2},$$

where n.p.v. means nuclear parity-violating and N.C. means neutral-current–induced.

It would therefore appear that the effect of nuclear parity violation is a small fraction of the neutral-weak-interaction–induced matrix element, unless the estimates of nuclear parity violation are much too small. The reason for this suppression is essentially that $M^{\text{n.p.v.}}$ involves a photon exchange in addition to the weak interaction, and so is smaller by a factor of α.

What are the prospects for measuring the asymmetries in $0^+ \to 0^-$ transitions? It should first be noted that such transitions have not yet been observed at all in electron scattering. The reason is the small cross section given by (4.14), which is 10^5 times smaller than elastic or other inelastic cross sections at comparable energies and q^2. The problem does not seem to be the absolute event rate. For example, with a beam of 10^{13} electrons/sec, and a target containing 10^{24} atoms/cm^2, comparable to that in the proposal of Ref. 6, the event rate corresponding to (4.14), taking $F_{2\gamma} \simeq F_{\text{ch}}$, is approximately 3/sec, or some 10^7 events in 1000 hours of running. This would be enough to measure an asymmetry of the size estimated in (4.13). However, the problem appears to be distinguishing the events in which the 0^- level is excited, from the large radiative tail corresponding to the elastic or other inelastic scattering, which could be 10^3 larger than the 0^- excitation cross section. In order to distinguish the excitation, it would appear necessary to detect the excited nuclear state somehow, say through its subsequent decay by radiative or particle emission. Whether this is feasible can only be determined through a careful study by experimental physicists, and is beyond the scope of this paper. We believe that such a study is warranted, in view of the interesting possibilities for extracting the coupling constants of neutral axial-vector hadron currents with lepton currents.

Finally, we consider an alternative way to detect the axial-vector hadron currents by their contribution to an $M1$ nuclear excitation. In this case, both the currents $V_\rho^{(i)}$ and $A_\rho^{(i)}$ have nonvanishing matrix elements, but that of $A_\rho^{(i)}$ should be somewhat larger, especially at momentum transfers of 1 F^{-1} or so. For simplicity consider a $0^+ \to 1^+$ transition, in which the only electromagnetic matrix element is $M1$.

For such a transition we can write, again using the style of particle physics rather than of nuclear physics,

$$\langle p, 0^+, M_N | V_\rho^{(i)} | p, \phi, 1^+, M_N' \rangle = \frac{i\epsilon_{\rho\sigma\tau\lambda} p_\sigma q_\tau \phi_\lambda F_V^{(i)}(q^2)}{(M_N + M_N')M_p}, \qquad (4.15)$$

where ϕ_λ is the polarization axial 4-vector of the 1^+ state, the dimensional factors have been inserted as they might be expected to occur, and the factor i is required by T invariance.

Similarly, we have

$$\langle p, 0^+, M_N | A_\rho^{(i)} | p', \phi_\lambda, 1^+, M_N \rangle = \phi_\rho F_A^{(i)}(q^2). \qquad (4.16)$$

The relevant difference between (4.15) and (4.16) is the proportionality of (4.15) to q, a feature well known in the usual nonrelativistic description of $M1$ electromagnetic transitions. The corresponding electron scattering matrix elements are as follows:

(a) For a transition from an isospin-zero to isospin-zero state,

$$M_{0^+\to 1^+} = \frac{4\pi\alpha}{q^2}\overline{U}\gamma^\rho U \frac{i\epsilon_{\rho\sigma\tau\lambda} p_\sigma q_\tau \phi_\lambda}{M_p(M_N + M_N')}\frac{F_V^{(1)}}{3} + \overline{U}\gamma^\rho\gamma_5 U \frac{i\epsilon_{\rho\sigma\tau\lambda} p_\sigma q_\tau \phi_\lambda}{M_p(M_N+M_N')}\frac{G_F F_V^{(1)}}{2\pi\alpha\sqrt{2}}(\epsilon_{1e}^{VA} + \epsilon_{2e}^{VA})$$

$$+ \overline{U}\gamma^\rho U \phi_\rho \frac{G_F F_A^{(1)}}{2\pi\alpha\sqrt{2}}(\epsilon_{e1}^{VA} + \epsilon_{e2}^{VA}). \qquad (4.17)$$

The corresponding polarization asymmetry is

$$R(q^2) = -\frac{3G_F q^2}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{1e}^{VA} + \epsilon_{2e}^{VA}) \frac{+3\,G_F q^2}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{e1}^{VA} + \epsilon_{e2}^{VA})\frac{F_A^{(1)}}{F_V^{(1)}}\frac{M_p}{E}\left(1 + \frac{q^2}{4E^2}\right)^{-1}. \qquad (4.18)$$

(b) For a transition from an isospin-zero to an isospin-one state, the asymmetry is

$$R(q^2) = -\frac{G_F q^2}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{1e}^{VA}+\epsilon_{2e}^{VA}) + \frac{G_F q^2}{4\pi^2\alpha^2\sqrt{2}}(\epsilon_{e1}^{VA}+\epsilon_{e2}^{VA})\frac{F_A^{(1)}}{F_V^{(1)}}\frac{M_p}{E}\left(1+\frac{q^2}{4E^2}\right)^{-1}. \tag{4.19}$$

We note that both ϵ_{ie}^{VA} and ϵ_{ei}^{VA} contribute, and that the contributions do not involve different powers of q, although they depend differently on lepton energy. Note also that the two terms in the asymmetry end up proportional to equal powers of q, even though the matrix elements have different powers of q. Instead, there is the factor M_p/E which replaces the power of q^{-1}. In order to distinguish the contributions, it would be necessary to do experiments at equal q^2 but different lepton energy, which is similar to the situation for electron-nucleon scattering.[5] Alternatively, one could also do positron-nucleus scattering, in which the first term in (4.19) has opposite sign. It is possible to carry out similar analyses for the excitation of other states with various spins, starting with a nuclear ground state of arbitrary spin. In general the asymmetry in other cases will involve several independent form factors, and the extraction of the parity-violating coupling constants will not be simple. A case that may be worthy of mention is an excitation of the form $\frac{1}{2}^+ \to \frac{1}{2}^+$. In this case, there are contributions from the $E0$ and $M1$ terms in $V_\rho^{(i)}$, and from the $M1$ term in $A_\rho^{(i)}$. However, there is some suppression of the $E0$ term at low momentum transfer, for the reason indicated in Eq. (4.3) for $0^+ \to 0^+$ transitions. Therefore, there may be some enhancement of the polarization asymmetry for this case.

I conclude that the study of elastic and inelastic scattering of polarized electrons by nuclei, with a measurement of the difference in the cross sections for electrons of opposite helicity, is a promising method for detecting parity-violating weak neutral-current interactions, and for determining their magnitudes and detailed properties.

ACKNOWLEDGMENTS

I wish to thank Dr. M. Goldhaber for helpful suggestions on the possible use of $0^+ \to 0^-$ transitions for detecting neutral currents. I also wish to thank Professor V. Hughes for useful discussions on polarized electron scattering, and for making Ref. 6 available to me.

*This research was supported in part by the U. S. Energy Research and Development Administration.

[1]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973); B. Aubert *et al.*, Phys. Rev. Lett. 32, 1954 (1974); S. J. Barish *et al.*, *ibid.* 33, 448 (1974); B. C. Barish *et al.*, *ibid.* 34, 538 (1974); W. Y. Lee *et al.*, Proceedings of the Paris Neutrino Conference, 1975 (unpublished).

[2]See for example M. A. Bouchiat and C. C. Bouchiat, Phys. Lett. 48B, 111 (1974).

[3]A review of many such proposals is given by G. Feinberg, in *High-Energy Physics and Nuclear Structure —1975*, proceedings of the VI International Conference, Santa Fe and Los Alamos, edited by D. E. Nagle *et al.* (A.I.P., New York, 1975), p. 468.

[4]See the review article by M. A. B. Bég and A. Sirlin, Annu. Rev. Nucl. Sci. 24, 379 (1974).

[5]For example, S. M. Berman and J. R. Primack, Phys. Rev. D 9, 2171 (1974); 10, 3898 (1974); also E. Derman, *ibid.* 7, 2755 (1973).

[6]W. Ash *et al.*, SLAC proposal. I thank Professor V. Hughes for making this proposal available to me.

[7]M. A. B. Bég and G. Feinberg, Phys. Rev. Lett. 33, 606 (1974).

[8]T invariance of possible neutral current interactions, down to level of $10^{-3}G$, has been pointed out by P. Sandars [report (unpublished)], and by C. Bouchiat [Phys. Lett. 57B, 284 (1975)].

[9]T. W. Donnelly and J. D. Walecka, Annu. Rev. Nucl. Sci. 25, 329 (1975).

[10]The effect of relaxing this assumption is discussed below.

[11]See the summary by F. Ajzenberg-Selove, Nucl. Phys. 166, 1 (1971).

[12]T. de Forest, Jr. and J. D. Walecka, Adv. Phys. 15, 1 (1966).

[13]See for example the data given by J. H. Fregeau and R. Hofstadter, Phys. Rev. 99, 1503 (1955), in which the inelastic 0^+ excitation remains 10 times smaller than elastic scattering on C even at $q^2 \sim 4$ F^{-2}.

[14]T. E. O. Ericson *et al.*, Nuovo Cimento 34, 51 (1964).

[15]See the summary of dispersion corrections in Chap. 8 of Ref. 12.

[16]E. Henley, Annu. Rev. Nucl. Sci. 19, 367 (1969).