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Angular distributions in the decay $\psi' \to \psi\pi\pi$*

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A general analysis of the decay amplitudes for $\psi' \to \psi\pi\pi$ is presented. Angular distributions are calculated for $\psi' \to \psi\pi^+\pi^- \to \mu^+\mu^-\pi^+\pi^-$ in terms of partial-wave amplitudes diagonal in orbital and spin angular momentum. The determination of the partial-wave amplitudes for $\psi' \to \psi\pi\pi$ may yield the difference between s-wave and d-wave π-π phase shifts in the $I=0$ channel if there is sufficient π-π d-wave present to produce measurable interference. The angular distributions for decays of the form $\psi \to VPP$ follow, *mutatis mutandis*, although VP interactions may prevent the determination of the PP phase shifts. Determination of the partial-wave amplitudes will test the validity of chiral-symmetry models for the $\psi' \to \psi\pi\pi$ decay.

I. INTRODUCTION

The dominant decay of the $\psi' = \psi(3.7)$ is $\psi' \to \psi\pi\pi$.[1] The apparent quantum numbers of the ψ and ψ' are $J^P = 1^-$, $I^{GC} = 0^{--}$. The $\pi\pi$ system has $I^{GC} = 0^{++}$,[2] which requires that its angular momentum, in its rest frame, be even. The decay spectrum as a function of the $\pi\pi$ invariant mass, $m_{\pi\pi}$, does not conform to naive expectations[2] (phase space for the effective Lagrangian $g\psi'_\mu\psi^\mu\vec{\pi}\cdot\vec{\pi}$) even when final-state interactions are included. A reasonably satisfactory description is given by chiral symmetry[3] *if* the amplitudes which lead to anisotropic distributions are eliminated. Preliminary data indicate the anisotropies are small and are thus consistent with the chiral-symmetry picture. However, chiral symmetry offers no *a priori* reason for the absence of anisotropies: In general they are expected to be present.

Independent of chiral symmetry, the decay $\psi' \to \psi\pi\pi$ is a remarkable source of information both for the interactions of the new particles and of "old" particles—pions. Since the hadronic interactions of the ψ' and the ψ are feeble, we expect the decay amplitudes to be real except for rescattering corrections—a situation similar to that of K_{e4}. If the π-π d-wave contribution is strong enough, it will be possible to determine $\delta_s^{I=0} - \delta_d^{I=0}$ for $\pi^+\pi^-$ scattering in the region $m_{\pi\pi} \approx 500$ MeV.

From the decays in which the ψ decays leptonically ($\sim 7\%\,\mu^+\mu^-$, $\sim 7\%\,e^+e^-$) a good deal of polarization information is available. In addition, the ψ' produced by the e^+e^- annihilation is transversely polarized with respect to the beam. It is essential to exploit this polarization information to obtain the fullest understanding of the decay.

There are five invariant amplitudes for the decay $\psi' \to \psi\pi\pi$. To see this we set $q = \frac{1}{2}(q_1 - q_2)$, $Q = q_1 + q_2$ where the π^+ momentum is q_1 and the π^- momentum is q_2. Then denoting the ψ' and ψ polarizations by ϵ' and ϵ respectively we can form five invariants bilinear in $\epsilon'\epsilon^*$: $\epsilon^*\cdot\epsilon'$, $\epsilon^*\cdot q\,\epsilon'\cdot q$, $\epsilon^*\cdot Q\,\epsilon'\cdot Q$, $\epsilon^*\cdot q\,\epsilon'\cdot Q$, $\epsilon^*\cdot Q\,\epsilon'\cdot q$. Each of these can be multiplied by a function of the Lorentz invariants formed from the momenta—say Q^2 and $P'\cdot q$ where P' is the ψ' momentum.

An equally valid approach is to consider the crossed reaction $\pi\psi' \to \pi\psi$. The independent helicity amplitudes are $\langle 1|1\rangle$, $\langle 0|1\rangle$, $\langle -1|1\rangle$, $\langle 1|0\rangle$, and $\langle 0|0\rangle$; the others are related by parity. Again the helicity amplitudes are functions of two Lorentz invariants.

A more useful decomposition is in terms of partial waves. For fixed $m_{\pi\pi}^2$ we consider the $\pi\pi$ system as a superposition of eigenstates of angular momentum (in the $\pi\pi$ rest frame) $l = 0, 2, 4, \ldots$. The decay angle of the $\pi\pi$ system—the angle between one pion and some specified axis in the $\pi\pi$ rest frame—plays the role of the second variable. The partial-wave expansion can be truncated after a few terms substantially reducing the difficulty of the analysis. There are a variety of coupling schemes available for connecting the $\pi\pi$ system of "spin" l to the ψ of spin 1 to produce a total angular momentum 1 (= spin of ψ'). We may choose to diagonalize the helicities of the ψ and the π-π system in addition to J^2 and J_3. On the other hand, we may diagonalize L^2, the orbital angular momentum squared of the ψ $(\pi\pi)$ system and $\mathcal{S}^2$, the spin squared of the ψ and $(\pi\pi)$ systems in addition to J^2 and J_3.

If we diagonalize the helicities, λ_ψ and $\lambda_{\pi\pi}$, there are generally five amplitudes for each value of the π-π "spin", l. These correspond to $(\lambda_\psi, \lambda_{\pi\pi}) = (1,2)$, $(1,1)$, $(1,0)$, $(0,1)$, and $(0,0)$. All others are related by parity or disallowed by the requirement $|\lambda_\psi - \lambda_{\pi\pi}| \leq 1$ = spin of ψ'. For $l = 0$ (π-π s wave) only $(\lambda_\psi, \lambda_{\pi\pi}) = (1,0)$ and $(0,0)$ are permitted.

If, on the other hand, we diagonalize L^2 and $\mathcal{S}^2$ where $\vec{\mathcal{S}} = \vec{S}_\psi + \vec{l}$, for fixed l we of course have five amplitudes as well. Parity conservation requires $L+l=\text{even}$, while charge conjugation invariance requires $l=\text{even}$. Thus we have the allowed values $(L,\mathcal{S})=(l,l-1)$, (l,l), $(l,l+1)$, $(l+2,l+1)$, and $(l-2,l-1)$. There are only two $\pi\pi$ s-wave amplitudes $L=0$ ("relative s wave"), $\mathcal{S}=1$ and $L=2$ ("relative d wave"), $\mathcal{S}=1$.

The purpose of this paper is to relate these various amplitudes to the experimental observables. Although it is possible to deal directly with the three-body decay,[4] it is more useful here to consider sequential two-body decays $\psi'\to\psi\,(\pi\pi)$, $\psi\to\mu^+\mu^-$ and $(\pi\pi)\to\pi\pi$. Thus we shall always describe the $\mu^+\mu^-$ in the ψ rest frame, the π's in the π-π rest frame, and the ψ and the $(\pi\pi)$ in the ψ' rest frame.

The plan of this paper is as follows. In Sec. II decay amplitudes are calculated in terms of partial-wave amplitudes. In the following section, some of the angular distributions are presented. Implications for π-π phase shifts are discussed in Sec. IV. Section V is a summary.

The full angular distribution including the three lowest partial waves is presented in the Appendix.

II. DECAY AMPLITUDES

For the purposes at hand, it is simpler to deal with decay amplitudes rather than their squares. Our analysis is in terms of partial waves. We denote the π-π angular momentum in its rest frame by $\vec{l}$, the spin of the ψ by $\vec{s}$, that of the ψ' by $\vec{s}'$. The orbital angular momentum of the $\psi\,(\pi\pi)$ system is denoted by $\vec{L}$. Then if we define the channel spin, $\vec{\mathcal{S}}$, by

$$\vec{\mathcal{S}}=\vec{s}+\vec{l} \tag{1}$$

we have

$$\vec{s}'=\vec{\mathcal{S}}+\vec{L}. \tag{2}$$

As explained in the Introduction, both l and L are even.

An eigenstate of $J^2=s'^2$, L^2, $\mathcal{S}^2$, and J_z consisting of $\psi\pi^+\pi^-$ may be constructed by conventional techniques.[5]

The amplitude for a ψ' with $s_z(\psi')=s_z'$ (the z axis being defined by the incident e^+-e^- direction) to decay into a ψ with $s_z(\psi)=s_z$ and a pair of π's is

$$\langle\psi,s_z;\pi^+\pi^-(\Omega_\psi,\Omega_\pi)|\psi',s_z'\rangle=\sum_{\substack{l,L,\mathcal{S};\\ l_z,L_z,\mathcal{S}_z}} M_{l,L,\mathcal{S}}\left(\frac{2L+1}{4\pi}\right)^{1/2}D^L_{L_z0}(\Omega_\psi)^*\left(\frac{2l+1}{4\pi}\right)^{1/2}D^l_{l_z0}(\Omega_\pi)^*$$
$$\times\langle 1,s_z'|L,L_z;\mathcal{S},\mathcal{S}_z\rangle\langle\mathcal{S},\mathcal{S}_z|1,s_z;l,l_z\rangle, \tag{3}$$

where Ω_ψ describes the ψ direction in the ψ' rest frame and Ω_π describes the $\pi^\pm$ direction of the π-π rest frame. The coordinates with respect to which Ω_π is measured are obtained from the ψ' rest-frame coordinates by a pure boost in the direction of the π-π line of flight. This prescription makes Eq. (3) relativistically correct[5] in spite of its nonrelativistic appearance. We have abbreviated $D^j_{mm'}(\phi,\theta,0)=D^j_{mm'}(\Omega)$.

The subsequent decay of $\psi\to l^+l^-$ is most easily described by fixing the lepton helicities since the QED coupling requires $(\lambda^+,\lambda^-)=(\pm\frac{1}{2},\mp\frac{1}{2})$. Absorbing certain constants we have

$$\mathfrak{M}(\lambda,s_z')=\langle\mu^+\mu^-,\lambda;\pi^+\pi^-(\Omega_\psi,\Omega_\pi,\Omega_\mu)|\psi',s_z'\rangle$$
$$=\sum_{\substack{l,L,\mathcal{S};\\ l_z,L_z,s_z,\mathcal{S}_z}} M_{l,L,\mathcal{S}}\left(\frac{2L+1}{4\pi}\right)^{1/2}D^{L\,*}_{L_z0}(\Omega_\psi)\left(\frac{2l+1}{4\pi}\right)^{1/2}D^{l\,*}_{l_z0}(\Omega_\pi)\left(\frac{3}{4\pi}\right)^{1/2}D^{1\,*}_{s_z,\lambda}(\Omega_\mu)$$
$$\times\langle 1,s_z'|L,L_z;\mathcal{S}\mathcal{S}_z\rangle\langle\mathcal{S},\mathcal{S}_z|1,s_z;l,l_z\rangle. \tag{4}$$

Throughout, $\lambda=\lambda^+-\lambda^-$ is the difference of the μ^+ and μ^- helicities.

In e^+e^- annihilation, the ψ''s are produced with transverse polarization with respect to the beam and the outgoing lepton polarizations are not observed. Thus the angular distribution is

$$d\Gamma\propto[|\mathfrak{M}(1,1)|^2+|\mathfrak{M}(1,-1)|^2+|\mathfrak{M}(-1,1)|^2+|\mathfrak{M}(-1,-1)|^2]. \tag{5}$$

Since $|\mathfrak{M}(1,1)|=|\mathfrak{M}(-1,-1)|$ and $|\mathfrak{M}(1,-1)|=|\mathfrak{M}(-1,1)|$, we can write simply the first two terms on the right-hand side of Eq. (5). Including phase space we have

$$\frac{d\Gamma}{dm_{\pi\pi}\,d\Omega_\psi\,d\Omega_\pi\,d\Omega_\mu}\propto qp[|\mathfrak{M}(1,1)|^2+|\mathfrak{M}(1,-1)|^2], \tag{6}$$

where q is the $\pi^\pm$ momentum in the π-π rest frame and p is the ψ momentum in the ψ' rest frame.

III. ANGULAR DISTRIBUTIONS

The distribution as a function of Ω_ψ, Ω_π, or Ω_μ is obtained simply by integrating over the two other solid angles using orthogonality relations. In practice, the partial-wave series, Eq. (4), must be terminated after a few terms. We shall, for the purpose of demonstration, and with simplicity as a criterion, consider only $M_{I,L,S}=M_{001}$, M_{201}, and M_{021}. We find[6]

$$\frac{d\Gamma}{d\Omega_\psi} \propto [\,|M_{001}|^2 + |M_{201}|^2 + \tfrac{1}{4}|M_{201}|^2(5-3\cos^2\theta_\psi)]\,, \tag{7}$$

$$\frac{d\Gamma}{d\Omega_\pi} \propto [\,|M_{001}|^2 + \tfrac{1}{4}|M_{201}|^2(5-3\cos^2\theta_\pi) + |M_{021}|^2]\,, \tag{8}$$

$$\frac{d\Gamma}{d\Omega_\mu} \propto |M_{001}|^2(1+\cos^2\theta_\mu) + \tfrac{1}{10}(|M_{201}|^2 + |M_{021}|^2)(13+\cos^2\theta_\mu). \tag{9}$$

It is understood here that $M_{I,L,S}$ is a function of $m_{\pi\pi}$.

If the μ's are not observed, the distribution is

$$\begin{aligned}\frac{d\Gamma}{d\Omega_\pi d\Omega_\psi} \propto{}& |M_{001}|^2 + |M_{201}|^2(\tfrac{5}{4}-\tfrac{3}{4}\cos^2\theta_\pi) + |M_{021}|^2(\tfrac{5}{4}-\tfrac{3}{4}\cos^2\theta_\psi) + 2\,\mathrm{Re}M_{201}M_{001}^*\left[\frac{1}{\sqrt{2}}(\tfrac{3}{2}\cos^2\theta_\pi-\tfrac{1}{2})\right]\\ &+2\,\mathrm{Re}M_{021}M_{001}^*\left[\frac{1}{\sqrt{2}}(\tfrac{3}{2}\cos^2\theta_\psi-\tfrac{1}{2})\right]\\ &+2\,\mathrm{Re}M_{201}M_{021}^*[\tfrac{9}{8}\sin^2\theta_\pi\sin^2\theta_\psi\cos2(\phi_\mu-\phi_\psi)+\tfrac{9}{16}\sin2\theta_\pi\sin2\theta_\psi\cos(\phi_\pi-\phi_\psi)+\tfrac{1}{2}(\tfrac{3}{2}\cos^2\theta_\pi-\tfrac{1}{2})(\tfrac{3}{2}\cos^2\theta_\psi-\tfrac{1}{2})].\end{aligned} \tag{10}$$

The full decay distribution, $d\Gamma/d\Omega_\pi d\Omega_\psi d\Omega_\mu$, is presented in the Appendix.

The single-particle distributions, Eqs. (7)–(9), depend only on the magnitudes of M_{001}, M_{201}, and M_{021}. Joint distributions such as Eq. (10) depend on the relative phases of the amplitudes.

IV. π-π PHASE SHIFTS

The angular distributions in principle determine the phases of the partial-wave amplitudes—up to one over-all phase. If the ψ' and ψ are regarded as inert, then the usual final-state interaction argument requires $M_{I,L,S}=e^{i\delta_l^0(m_{\pi\pi})}|M_{I,L,S}|$, where δ_l^0 is the $I=0$, l wave $\pi\pi$ phase shift. Thus $\delta_0^0-\delta_2^0$ may be obtainable from $\psi'\to\psi\pi\pi$. This is in some ways similar to the Pais-Treiman[7] method for obtaining $\pi\pi$ phase shifts from K_{e4} decays. The viability of this technique in $\psi'\to\psi\pi\pi$ depends on a number of factors:

1. adequate data, especially for $\psi'\to\mu^+\mu^-\pi^+\pi^-$;
2. that some $l>0$ contributions be significant;
3. that a few terms in the partial-wave series suffice;
4. that the assumption of noninteraction ψ and ψ' be appropriate.

In principle similar techniques can be used for $\psi\to\omega\pi\pi$. The analysis is slightly different reflecting the replacement of $\psi\to\mu^+\mu^-$ by $\omega\to\pi^+\pi^-\pi^0$. However, item 4 above seems more dubious in this instance. The same is true for the SU(3) variants, e.g., $\psi\to K\bar{K}^*\pi$, $\psi\to\omega K\bar{K}$, etc.

V. SUMMARY

The results presented above and in the Appendix constitute a general treatment of the process $e^+e^-\to V'\to VP\bar{P}\to l^+l^-P\bar{P}$, although we have been primarily concerned with $\psi'\to\psi\pi\pi$. In advance of data analysis we cannot determine how many partial waves will be needed for an accurate description, but it is expected that $\psi'\to\psi\pi\pi$ will require fewer than would $\psi\to\omega\pi\pi$.

The determination of the partial-wave amplitudes for $\psi'\to\psi\pi\pi$ is essential for evaluating the treatment[3] by Brown and the present author of the spectrum $d\Gamma/dm_{\pi\pi}$, which speculated that only M_{001} is significant[8] and that its dependence on $m_{\pi\pi}$ is $\propto(m_{\pi\pi}^2-2m_\pi^2)$. The isolation of the partial-wave amplitudes would constitute some of the most refined information on the puzzling new particles.

The newly discovered particles may provide an opportunity to measure one of the "simplest" of hadronic quantities, the elastic π-π s-wave phase shift (for $I=0$) for $m_{\pi\pi}\sim 500$ MeV. If there is adequate π-π d wave to interfere with, this will be a valuable technique, perhaps supplanting K_{e4} decays as the best clean measurement of π-π phase shifts at low values of the π-π energy.

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APPENDIX

We display here the full spectrum obtained from Eqs. (4)–(6) which use only the first three partial waves $M_{l,L,S}$:

$$
\begin{aligned}
\frac{d\Gamma}{d\Omega_\pi d\Omega_\mu d\Omega_\psi} \propto\; & |M_{001}|^2[d^1_{11}(\theta_\mu)^2 + d^1_{-1,1}(\theta_\mu)^2] \\
& + |M_{201}|^2\{3d^2_{20}(\theta_\pi)^2[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2] + 3d^2_{10}(\theta_\pi)^2 d^1_{01}(\theta_\mu)^2 + \tfrac{1}{2}d^2_{00}(\theta_\pi)^2[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2] \\
& \quad + 2\sqrt{6}\, d^2_{20}(\theta_\pi)d^2_{00}(\theta_\pi)d^1_{1-1}(\theta_\mu)d^1_{11}(\theta_\mu)\cos 2(\phi_\pi - \phi_\mu) \\
& \quad + [-\sqrt{3}\, d^2_{00}(\theta_\pi) + 3\sqrt{2}\, d^2_{20}(\theta_\pi)]\, d^2_{10}(\theta_\pi)d^1_{01}(\theta_\mu)[d^1_{11}(\theta_\mu) - d^1_{-11}(\theta_\mu)]\cos(\phi_\pi - \phi_\mu)\} \\
& + |M_{021}|^2\{3d^2_{20}(\theta_\psi)^2[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2] + 3d^2_{10}(\theta_\psi)^2 d^1_{01}(\theta_\mu)^2 + \tfrac{1}{2}d^2_{00}(\theta_\psi)^2[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2] \\
& \quad + 2\sqrt{6}\, d^2_{20}(\theta_\psi)d^2_{00}(\theta_\psi)d^1_{1-1}(\theta_\mu)d^1_{11}(\theta_\mu)\cos 2(\phi_\psi - \phi_\mu) \\
& \quad + [-\sqrt{3}\, d^2_{00}(\theta_\psi) + 3\sqrt{2}\, d^2_{20}(\theta_\psi)]\, d^2_{10}(\theta_\psi)d^1_{01}(\theta_\mu)[d^1_{11}(\theta_\mu) - d^1_{-11}(\theta_\mu)]\cos(\phi_\psi - \phi_\mu)\} \\
& + 2\,\mathrm{Re} M_{201}M^*_{001}\{2\sqrt{3}\, d^2_{20}(\theta_\pi)d^1_{11}(\theta_\mu)d^1_{-11}(\theta_\mu)\cos 2(\phi_\pi - \phi_\mu) \\
& \qquad - (\tfrac{3}{2})^{1/2} d^2_{10}(\theta_\pi)d^1_{01}(\theta_\mu)[e^{i(\phi_\pi - \phi_\mu)}d^1_{11}(\theta_\mu) - e^{-i(\phi_\pi - \phi_\mu)}d^1_{-11}(\theta_\mu)] \\
& \qquad + (\tfrac{1}{2})^{1/2} d^2_{00}(\theta_\pi)[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2]\} \\
& + 2\,\mathrm{Re} M_{021}M^*_{001}\{2\sqrt{3}\, d^2_{20}(\theta_\psi)d^1_{11}(\theta_\mu)d^1_{-11}(\theta_\mu)\cos 2(\phi_\psi - \phi_\mu) \\
& \qquad - (\tfrac{3}{2})^{1/2} d^2_{10}(\theta_\psi)d^1_{01}(\theta_\mu)[e^{i(\phi_\psi - \phi_\mu)}d^1_{11}(\theta_\mu) - e^{-i(\phi_\psi - \phi_\mu)}d^1_{-11}(\theta_\mu)] \\
& \qquad + (\tfrac{1}{2})^{1/2} d^2_{00}(\theta_\psi)[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2]\} \\
& + 2\,\mathrm{Re} M_{201}M^*_{021}\{3d^2_{20}(\theta_\pi)d^2_{20}(\theta_\psi)[e^{-2i(\phi_\pi - \phi_\psi)}d^1_{11}(\theta_\mu)^2 + e^{2i(\phi_\pi - \phi_\psi)}d^1_{-11}(\theta_\mu)^2] \\
& \qquad + 3d^2_{10}(\theta_\pi)d^2_{10}(\theta_\psi)\cos(\phi_\pi - \phi_\psi)d^1_{01}(\theta_\mu)^2 + \tfrac{1}{2}d^2_{00}(\theta_\pi)d^2_{00}(\theta_\psi)[d^1_{11}(\theta_\mu)^2 + d^1_{-11}(\theta_\mu)^2] \\
& \qquad - \frac{3}{\sqrt{2}} d^2_{20}(\theta_\pi)d^2_{10}(\theta_\psi)d^1_{01}(\theta_\mu)[e^{i(2\phi_\pi - \phi_\psi - \phi_\mu)}d^1_{-11}(\theta_\mu) - e^{-i(2\phi_\pi - \phi_\psi - \phi_\pi)}d^1_{11}(\theta_\mu)] \\
& \qquad - \frac{3}{\sqrt{2}} d^2_{10}(\theta_\pi)d^2_{20}(\theta_\psi)d^1_{01}(\theta_\mu)[e^{i(\phi_\pi + \phi_\mu - 2\phi_\psi)}d^1_{-11}(\theta_\mu) - e^{-i(\phi_\pi + \phi_\mu - 2\phi_\psi)}d^1_{11}(\theta_\mu)] \\
& \qquad - \frac{\sqrt{3}}{2} d^2_{10}(\theta_\pi)d^2_{00}(\theta_\psi)d^1_{01}(\theta_\mu)[e^{i(\phi_\pi - \phi_\mu)}d^1_{11}(\theta_\mu) - e^{-i(\phi_\pi - \phi_\mu)}d^1_{-11}(\theta_\mu)] \\
& \qquad - \frac{\sqrt{3}}{2} d^2_{00}(\theta_\pi)d^2_{10}(\theta_\psi)d^1_{01}(\theta_\mu)[e^{i(\phi_\mu - \phi_\psi)}d^1_{11}(\theta_\mu) - e^{-i(\phi_\mu - \phi_\psi)}d^1_{-11}(\theta_\mu)] \\
& \qquad + \sqrt{6}\, d^2_{20}(\theta_\pi)d^2_{00}(\theta_\psi)d^1_{11}(\theta_\mu)d^1_{-11}(\theta_\mu)\cos 2(\phi_\pi - \phi_\mu) \\
& \qquad + \sqrt{6}\, d^2_{00}(\theta_\pi)d^2_{20}(\theta_\psi)d^1_{11}(\theta_\mu)d^1_{-11}(\theta_\mu)\cos 2(\phi_\psi - \phi_\mu)\}\,.
\end{aligned}
\tag{A1}
$$

The frames with respect to which angles are to be measured are discussed following Eq. (3).

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[6]Some of the single-particle distributions were communicated to me by F. J. Gilman.

[7]A. Pais and S. B. Treiman, Phys. Rev. 168, 1858 (1968).

[8]The amplitude retained in Ref. 3 is $\epsilon^* \cdot \epsilon' A(m_{\pi\pi}^2)$, which differs from M_{001} by terms of order $(P_\psi/M_\psi)^2 \lesssim 2\%$.