s-wave production of pions in $p + p \rightarrow \pi^+ + d$

J. Spuller and D. F. Measday
Department of Physics, University of British Columbia, Vancouver, B.C., Canada, V6T 1W5
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The data for the total cross section for $p + p \rightarrow \pi^+ + d$ are reanalyzed to obtain the component for s-wave pion production near threshold. This comprehensive analysis yields a value which is compatible with other low-energy pion-nucleon reactions.

INTRODUCTION

We suggest a possible solution to a long-standing puzzle in the low-energy scattering of pions off nucleons. It has been known for about twenty-five years that one can relate the cross sections for the following three processes near threshold: photomeson production on the proton ($\gamma + p \rightarrow \pi^+ + n$); the s-wave elastic scattering of pions off the proton ($\pi^\pm + p \rightarrow \pi^\pm + p$); and the s-wave production of pions in the reaction ($p + p \rightarrow \pi^+ + d$). The relations are given in Fig. 1, and the most recent complete discussion has been given by Rose.[1] The various reactions are related through a chain which connects s-wave cross sections (σ) or reaction rates for stopped negative pions (ω). Some of the relationships are based on procedures such as detailed balance, (D.B.), charge independence, (C.I.), or extrapolation to zero energy, (E.Z.E.), all of which are nowadays quite well understood. Other relationships are experimentally determined ratios of reaction rates (P and S) or of cross sections (R) for the pairs indicated in the figure. Lastly, there is one ratio of reaction rates (T) which cannot be measured directly, and we are forced to rely on a calculation. Although the two reactions ($\gamma p \rightarrow \pi^+ n$) and ($\pi^\pm p \rightarrow \pi^\pm p$) relate within the experimental errors, the most recent analysis by Richard-Serre *et al.*[2] of the third reaction ($p + p \rightarrow \pi^+ + d$) shows that it seems to have a cross section which is about 30% too low. This might be an accumulation of errors; (experimentally $S = 2.9 \pm 0.3$ and the calculation for T is 0.83 ± 0.08); but this seems to be an unlikely, although not impossible, cause of the puzzle.

We wish to note that a more likely reason for the discrepancy lies in the standard approach to the reaction $p + p \rightarrow \pi^+ + d$. It is normal to analyze this reaction, following in the footsteps of Gell-Mann and Watson,[3] who showed that, under certain assumptions, the total cross section for this reaction is given by

$$\sigma_T(pp \rightarrow \pi^+ d) = \alpha\eta + \beta\eta^3, \tag{1}$$

where η is the center-of-mass momentum of the pion in units of the pion mass. The first term comes from s-wave production and the second term from p-wave production, and in this phenomenological theory of Gell-Mann and Watson, α and β are constant. A more detailed calculation was made much later by Reitan[4] who showed that α could depend slightly on energy, but the effect was so small that it was not felt necessary to take it into account in the most recent experimental analysis by Richard-Serre *et al.*, in which it was found that $\alpha = 0.18 \pm 0.02$ mb and $\beta = 0.95 \pm 0.15$ mb. However, the relations between the low-energy pion reactions indicate a value for α of about 0.25 mb.

Now a recent calculation by Afnan and Thomas[5] has shown that α can vary quite significantly with energy, even near threshold. Although their results differ according to the nucleon-nucleon potential used to derive the deuteron wave function, nevertheless a common feature is that α falls monotonically and almost linearly with η for $0 < \eta < 0.6$. If one inspects the fits to the total cross sections obtained by Richard-Serre *et al.*, it is quite noticeable that the very-low-energy data lie above the fitted curve, while the medium energy data ($\eta \approx 0.4$) fall below. Afnan and Thomas themselves noted that their energy dependence of α could explain the discrepancy between the high value of α obtained from the low-energy data of Rose,[1] and the lower value of α obtained by Crawford and Stevenson[6] at a higher energy.

Rose found a value of α of 0.24 ± 0.02 mb when fitting to his own data only, i.e., $\eta < 0.5$, but he obtained a value for β of 0.52 ± 0.2 mb, which is not acceptable when higher-energy data are included. He then proceeded to fit higher-energy data ($\eta < 1.6$) by fixing α, but this is logically unsatisfactory within the framework of the model where α and β are constant as the fit can be significantly improved by letting α go free, and one then obtains $\alpha = 0.188 \pm 0.020$ mb (this is actually the value from CERN fit 1 which used data in the energy region $\eta < 1.0$). The value obtained by Rose is thus interesting but nevertheless it is based on an illogical approach. Crawford and Stevenson[6]

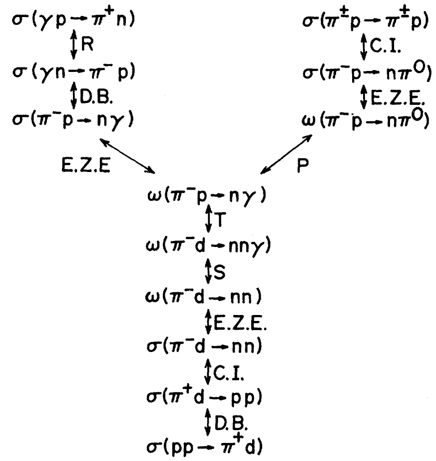

FIG. 1. Relations between low-energy pion reactions; C.I. = charge independence, D.B. = detailed balance, E.Z.E. = extrapolation to zero energy. The parameters R, P, T, and S are simply the ratios (experimental or calculated) for the processes as noted.

found $\alpha = 0.138 \pm 0.015$ mb in their analysis which covered a narrow energy band ($0.38 < \eta < 0.58$). Because the data at this energy are not very sensitive to the s-wave production, their result should again be taken as indicative but not conclusive. Thus within the framework of the Gell-Mann and Watson model the best value of the constant α is that obtained by Richard-Serre *et al.* who applied the model consistently with an extended data set. However, it is the basic model we are now questioning and so these other results have been discussed because they give premonitions of the breakdown of the model.

DISCUSSION OF EXISTING DATA

We have thus returned to this puzzle in order to ascertain whether it was possible to reconcile the low-energy pion cross sections. We started off using the data compilation of Richard-Serre *et al.*[2] as a basis. This compilation consisted of most of the data existing at the time except for some gentle pruning of very old data which were either incomplete in the angular distribution, or were completely inconsistent with the majority of the data. Note that the most vital set of data is the very-low-energy measurement of Rose; although we have criticized his analysis, nevertheless his data remain the keystone of our discussion.

The CERN fits used a data set in which the datum of Dolnick[7] at $\eta = 1.01$ was omitted. The reason was that it was available in a preliminary form at the time, but was not published; it was also somewhat higher than earlier data and so added significantly to the χ^2. However, we shall use it as there is indication from more recent work that the higher value is correct.

There are two experiments which at present are available only in preliminary form. From LAMPF (Los Alamos Meson Physics Facility) there are the results of Preedom *et al.*[8] on the reaction $\pi d \rightarrow pp$ at $E_\pi = 40$, 50, and 60 MeV. Their data at 50 MeV and 60 MeV are more in line with the measurement of Dolnick than with the earlier data. Unfortunately, their total cross section at 40 MeV is about 20% lower than typical fits, and these fits pass close to the nearby datum of Sachs[9] ($E_\pi = 37.6$ MeV), as well as to the data of Crawford and Stevenson[6] (equivalent $E_\pi = 23$ and 26 MeV) and of Durbin *et al.*,[10] ($E_\pi = 47$ and 63 MeV). Admittedly the lowest energy point of Durbin *et al.* at $E_\pi = 29$ MeV and also that of Cartwright *et al.*[11] (equivalent $E_\pi = 25$ MeV) have low cross sections as well, but they are in disagreement with the data of Crawford and Stevenson, the recent work of Rose, and the older measurement of Clark.[12] Many of these older measurements are omitted for various reasons from recent analyses, including this one. However, we resurrect them for completeness and to show that the weight of the evidence is clearly that the measurements of Cartwright *et al.* and the low-energy point of Durbin *et al.* are wrong. This opinion is uncompromisingly supported in Fig. 7 of Rose's paper. Because of this uncertainty casting a shadow over the LAMPF data we have omitted all of them from our analyses. We did, however, try fitting their data and the parameters we obtained were indistinguishable from the ones we shall present.

The second set of new data are from Geneva by Aebischer *et al.*[13] They have studied the reaction $pp \rightarrow \pi d$ at $E_p = 398$, 455, 497, 530, and 572 MeV at the CERN synchrocyclotron. The only analyzed data at present available are at 398 and 455 MeV (equivalent $E_\pi = 56$ and 84 MeV). These data are compatible with our fits, but no firm errors were given on the absolute cross section; the only comment available is that "the systematic error in the normalization is probably less than 10%." We tried fits using a ±10% error and again the parameters we obtained were indistinguishable from the one we shall present. We decided to omit these results because of the lack of information on the error.

Even though we omitted both the LAMPF and Geneva results, it was clear that they both strongly supported the measurement of Dolnick over those of Stadler[14] at $E_\pi = 91$ and 114 MeV. In fact, Stadler's results could be as much as 30% too low; furthermore, the angular distributions of Stadler's measurements give peculiar values of A, the isotropic component of the distribution. Thus we eagerly await the final results of the recent work, especially those from Geneva which could settle this question and bridge the gap in the data for $1.15 < \eta < 1.53$ which exists if one eliminates the results of Stadler. This gap ($88 < E_\pi < 142$ MeV) or ($460 < E_p < 570$ MeV) is quite large and comes in a very important region where the cross section reaches a maximum.

In conclusion, then, we omitted both of the most recent results from LAMPF and Geneva; our data set was thus identical to the CERN fits of Richard-Serre *et al.*,[2] except for the inclusion of the one measurement of Dolnick. We do not feel that any of our conclusions would be changed by making an alternative, yet reasonable, selection of the available data.

DATA FITTING

First we tried to reproduce the CERN fits of Richard-Serre *et al.*, to check our data set and the least-squares fitting routine. We thus fit the data to the form

$$\sigma_{\pi d \to pp} = \frac{2}{3} \frac{P_p^2}{\eta^2} (\alpha\eta + \beta_0\eta^3 + \beta_2\eta^5). \quad (2)$$

Table I gives the CERN fit 2 together with our fit for the same data set; the agreement is adequate and the small discrepancy is probably due to minor differences in the fitting techniques. One slight difference is that we have applied the Coulomb corrections of Reitan[4] to the data directly; this procedure is possible because the difference between the s-wave and p-wave corrections is negligible in the energy region where data exist.

The CERN fit 2, of course, omitted the datum of Dolnick and so therefore does our fit A; when it is included we obtain fit B, Table I, and note

TABLE I. Comparison of low-energy polynomial fits to Eq. (2) for $\eta \leq 1.55$. The first column is fit 2 of Richard-Serre *et al.*[2] The second column is our fit A using the same parameterization, while the third column is the same as fit A except the data set includes the total cross section of Dolnick.[7]

	CERN 2	Fit A	Fit B
α	0.188 ± 0.024	0.186 ± 0.010	0.180 ± 0.011
β_0	0.90 ± 0.16	0.93 ± 0.06	0.94 ± 0.07
β_2	-0.050 ± 0.045	-0.07 ± 0.03	-0.09 ± 0.03
χ^2/ν	1.05	1.1	1.4

that it makes little difference to the numerical value of the parameters, but it does add considerably to the χ^2 (which is why it was omitted from the CERN fits).

In order to test for the energy dependence of α, we now allow α to have an energy dependence of the form

$$\alpha = \alpha_0 + \alpha_1\eta. \quad (3)$$

To approximate the shape of the peak in the cross section [related to the $\Delta(1232)$] it was first decided to follow Richard-Serre *et al.*, using a polynomial expansion

$$\beta = \beta_0 + \beta_1\eta + \beta_2\eta^2. \quad (4)$$

However, it was found that this allowed too much freedom in the fitting function when all the coefficients were let free. It was then decided to investigate the prediction of Afnan and Thomas that α had a negative gradient. The data were fitted over $\eta \leq 1.55$ and included the datum of Dolnick. Holding α_1 fixed at the value of -0.2 we obtained the following result, fit C:

$$\sigma_{\pi d \to pp}\ (\text{mb}) = \frac{2}{3} \frac{P_p^2}{\eta^2} [(0.247 \pm 0.017)\eta - 0.2\eta^2 + (0.6 \pm 0.3)\eta^3 + (1.0 \pm 0.5)\eta^4 - (0.6 \pm 0.2)\eta^5], \quad (5)$$

where P_p is the proton c.m. momentum in units of $m_\pi c$. The χ^2/ν was 1.1 for 26 degrees of freedom.

Although this does not represent an improved (nor worse) fit than that of Richard-Serre *et al.*, as judged by a χ^2 criterion, it does indicate that the discrepancy among the processes $(\pi^+ + d \to p + p)$, $(\gamma + p \to \pi^+ + n)$, and $(\pi^\pm + p \to \pi^\pm + p)$, can be removed by allowing a significant energy dependence to the α parameter. Figure 2 shows a comparison between fit 2 of Richard-Serre *et al.*, and our fit C. We note that the difference is very small and we are forced to conclude that the value of α_0 is sensitive to the model used in the data analysis. It is also clear that if one wishes to distinguish between these solutions using total cross-section data only, then a factor of ten improvement in the experiments will be needed.

Since the use of nonorthogonal polynomials does not lead to easily interpretable results, we were not able to obtain consistent values of the coefficients by systematically increasing the domain of the fitting region, and correspondingly, the order of the fitting polynomial. (Note that an orthogonalization procedure does not help as it would still be necessary to extract the physical quantity of interest in a model-dependent fashion.) We were then led to try a different parameterization in an attempt to decouple the s- and p-wave pion interaction. We chose to add on a Breit-Wigner function to a low-order polynomial. The kinematics were handled in a relativistic manner, and the appropriate penetrability factors were included to ensure that the Breit-Wigner term would have the proper threshold behavior for p-wave pions (e.g., $\sigma \sim \eta$ for $\eta \rightarrow 0$). This was done by introducing a penetrability factor $(\eta R)^{2l}/D_l$ for the angular momentum barrier l, where R is a radius of interaction. The D_l were approximated by

$$D_0 = 1, \tag{6a}$$

$$D_1 = 1 + (\eta R)^2, \tag{6b}$$

$$D_2 = 9 + 3(\eta R)^2 + (\eta R)^4. \tag{6c}$$

The phase space introduces another factor proportional to η so that

$$\Gamma = \gamma[(\eta R)^{2l+1}/D_l], \tag{7}$$

where γ is assumed to be a constant. The fitting function for the cross section for the reaction $(\pi^+ + d \rightarrow p + p)$ was therefore

$$\sigma = \frac{2}{3}\frac{P_p^2}{\eta^2}(\alpha_0\eta + \alpha_1\eta^2 + \alpha_2\eta^3 + \alpha_3\eta^4) + \frac{G(s)\pi\lambda\!\!\!^{-}_\pi{}^2\Gamma_{el}\Gamma_r}{(E - E_R)^2 + \Gamma_T^2/4}, \tag{8}$$

where

$\Gamma_{el} = \gamma_{el}[(\eta R)^3/(1 + (\eta R)^2)]$,

$\Gamma_r = k\gamma_T(RP_p)$,

$\Gamma_T = \gamma_T[(\eta R)^3/(1 + (\eta R)^2) + k(RP_p)]$,

R = channel radius in units of $\hbar/m_\pi c$,

P_p = c.m. momentum of proton in units of $m_\pi c$,

η = c.m. momentum of pion in units of $m_\pi c$,

E_R = resonant energy,

$G(s) = \dfrac{2J+1}{(2S_\pi + 1)(2S_d + 1)} = \dfrac{5}{3}$,

k = fraction of total width from $\pi^+ + d \rightleftharpoons p + p$ channel.

We are assuming that the widths for most of the

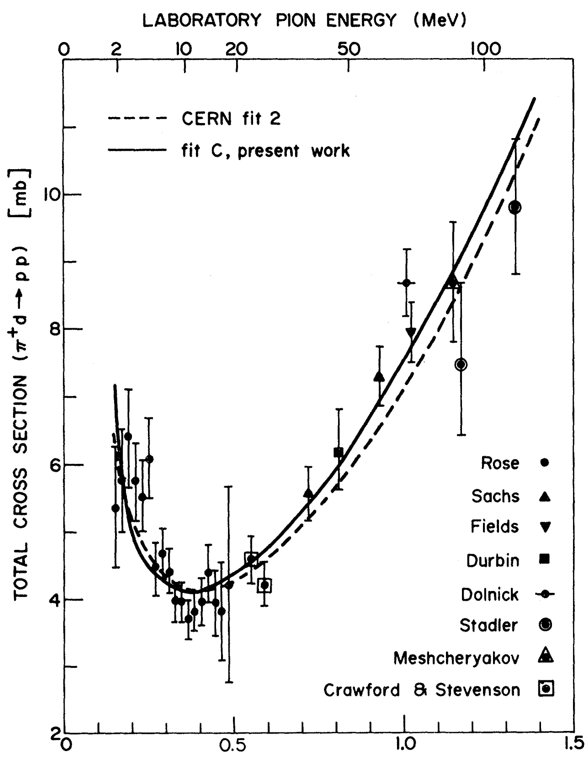

FIG. 2. Comparison of the CERN fit 2 of Richard-Serre *et al.* (Ref. 2) with our fit C. The experimental data points (see Ref. 2) have been corrected for Coulomb effects.

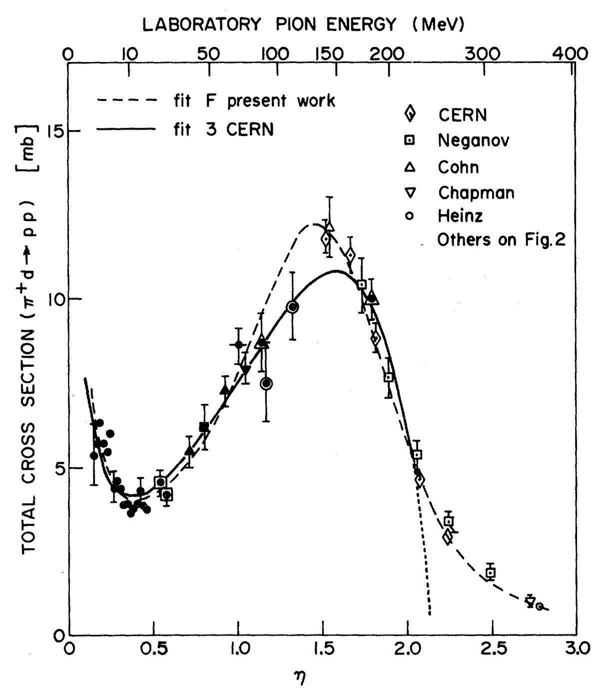

FIG. 3. A comparison of our fit F with the CERN fit 3 and with existing data (see Ref. 2) for the total cross section for $(\pi^+ + d \rightarrow p + p)$. The dotted portion of CERN fit 3 specifies a region of extrapolation. The experimental data points have been corrected for Coulomb effects.

TABLE II. Representative fits to the total cross section for the reaction ($\pi^+ + d \to p + p$) using a low-order polynomial with a Breit-Wigner function, i.e., Eq. (8). Quoted errors correspond to one standard deviation.

Fit	α_0 (mb)	α_1 (mb)	α_2 (mb)	α_3 (mb)	γ_{el} $(m_\pi c^2)$	γ_T $(m_\pi c^2)$	E_R (MeV)	R	k	χ^2/ν
D	0.29 ± 0.05	-0.8 ± 0.3	0.2 ± 0.1	···	0.5 ± 0.1	0.57 ± 0.04	2181 ± 7	1.4	0.05	1.2
E	0.30 ± 0.05	-0.8 ± 0.3	0.3 ± 0.1	-0.01 ± 0.04	0.5 ± 0.1	0.56 ± 0.05	2181 ± 7	1.4	0.05	1.2
F	0.27 ± 0.04	-0.5 ± 0.2	0.05 ± 0.1	0.03 ± 0.03	0.6 ± 0.15	0.71 ± 0.06	2183 ± 8	1.2	0.05	1.2
G	0.26 ± 0.04	-0.4 ± 0.2	0.08 ± 0.06	···	0.6 ± 0.15	0.92 ± 0.08	2179 ± 7	1.0	0.05	1.35

reactions (e.g., $\pi^+ + d \to \pi^+ + p + n$) have the same energy dependence as the elastic channel, but the reaction $\pi^+ + d \to p + p$ is treated differently because the phase space and penetrability of the $(p+p)$ channel will have little energy dependence. We emphasize that even though the Breit-Wigner function may adequately reproduce the shape of the bump in the cross section, it is very difficult to determine the actual values of the reduced widths, even when this formalism is supported on the basis of the reaction mechanism. Therefore, we ascribe no physical significance to the reduced widths. We feel this is not a serious fault as we merely wish to investigate the behavior of the α coefficient for low values of η.

The penetrability factor for Γ_r was set equal to 1 because $P_p = 2.63$ (≈ 370 MeV/c), even for the case of $\eta = 0$. It was not possible to allow the parameter k to be free in the fitting routine, and we set it to various values between $0.03 \leq k \leq 0.07$, these limits being educated guesses. It was found that the α coefficients were not sensitive to its value, although its inclusion in the expression for Γ_T was warranted on a χ^2 basis. Furthermore, the parameter R was treated in a similar fashion. With this parameterization it was possible to fit over the region $\eta \leq 2.76$ (41 data points), and obtain quite reasonable results ($\chi^2/\nu \approx 1.2$). Some representative fits are given in Table II and a plot of fit F is given in Fig. 3. The common feature of all the fits was the larger value of α_0 as compared to Richard-Serre *et al.* We note in passing that the new fits prefer the datum of Dolnick, and fall far from the earlier work of Stadler.[14] As we remarked earlier there is a dearth of data from $\eta = 1.15$ to 1.53 and it is particularly interesting to study this region as our new fits are substantially different here from the CERN fit 3 (see Fig. 3).

CONCLUSION

In conclusion it can be affirmed that the experimental data for the total cross section prefer a negative value of α_1 as predicted by Afnan and Thomas. According to the other assumptions made in the analysis one can obtain a value for α_0 between 0.2 mb and 0.3 mb and so it is probable, although not certain, that the relations between the low-energy pion reactions can now be satisfied. To fix the value of α_0 more firmly it is possible that careful measurements of the absolute differential cross section for the reaction ($p + p \to \pi^+ + d$) will help if accurate measurements can be made from the threshold at 287.5 MeV up to about 300 MeV (i.e., $\eta < 0.3$). However, it is likely that a definitive analysis will have to await experiments using polarized beams with polarized targets. Furthermore, to clarify the relations in Fig. 1 renewed investigations of the ratios S and T would do no harm.

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