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Application of current algebra to soft-pion production induced by the weak neutral current: Second-class *V*, *A* case*

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We apply current-algebra techniques to study neutrino-proton elastic scattering and neutral-current-induced soft-pion production in the case of second-class vector and axial-vector currents. (Such currents are *CP*-violating if the final and incident neutrinos are identical, but can be *CP*-conserving if the final and incident neutrinos are different particles.) The second-class currents are constructed phenomenologically from meson fields and the usual first-class *V*, *A* quark currents. The matrix elements of the second-class currents between one-nucleon states are estimated by inserting a complete set of intermediate states and saturating the sum with one-nucleon states. In this way the second-class matrix elements are expressed in terms of the experimentally measured renormalization constants of the first-class *V*, *A* currents and the pion-nucleon coupling constant. Using standard soft-pion techniques, we analyze recently reported Brookhaven National Laboratory results for neutral-current-induced soft-pion production under the simplifying assumption of a purely isoscalar second-class *V*, *A* neutral current. We find in this case that a second-class $V-A$ current is consistent with the reported results. Some qualitative features of second-class neutral currents are (i) equal cross sections for νp and $\bar{\nu} p$ elastic scattering, with only the axial-vector current contributing, and (ii) very small νp and $\bar{\nu} p$ elastic cross sections at energies of order 50 MeV, which are relevant for possible neutrino experiments of the Los Alamos Meson Physics Facility.

I. INTRODUCTION

Now that experiments have conclusively shown the existence of weak neutral currents,[1] neutral-current experiments are entering a new phase of sophistication which will eventually enable the determination of the phenomenology of the weak neutral interaction. This determination will largely follow from the analysis of the three most extensively studied semileptonic reactions: deep-inelastic neutrino-nucleon scattering, neutrino-proton elastic scattering, and neutral-current-induced weak-pion production.

This is the third in a series of three papers which develop the necessary apparatus to analyze and correlate data from these inclusive and exclusive neutral-current-induced semileptonic processes. In the first paper, these three processes are analyzed using standard current-algebra techniques in the framework of the conventionally assumed Weinberg-Salam model and more generally for neutral currents formed solely from members of the usual vector and axial-vector (V,A) nonets.[2] The remaining papers in this series explore the possibility of unconventional current structure. In the second paper, a similar apparatus is set up for correlating data from inclusive and exclusive semileptonic processes in the case of neutral currents with scalar, pseudoscalar, and tensor (S,P,T) Lorentz structure.[3] Since (V,A)-induced reactions leave the helicity of the neutrino unchanged while (S,P,T)-induced reactions flip neutrino helicity, the cases of V,A and S,P,T neutral currents do not interfere and may be studied independently.

In the present paper we calculate the cross sections for neutrino-proton elastic scattering and neutrino-induced pion production in the case of V,A neutral currents with abnormal G-parity. Since the existence of weak second-class *charged* currents is still an open question and the phenomenology of the weak neutral currents is as yet undetermined, we think it important to consider second-class V,A currents as possible candidates for the semileptonic currents participating in the weak neutral interaction.

The paper is organized as follows. In Sec. II we discuss our choice of phenomenological current and review other possible second-class currents discussed in the literature. We also set up the necessary apparatus for using current-algebra methods to obtain the cross sections. We derive the commutation relations, needed for the pion-production calculation, of our second-class current with the usual axial-vector current in the framework of the $SU(2)\times SU(2)$ σ model. We also estimate the necessary matrix elements of the second-class current between one-nucleon states. In Sec. III we calculate the elastic and pion-production cross sections. The quark-parton-model methods used in the previous papers of this series to calculate deep-inelastic cross sections cannot be applied here because of the phenomenological nature of the current.[2,3] In Sec. IV we analyze low invariant mass $[W=M(\pi N)\leq 1.4 \text{ GeV}]$ pion production for the Brookhaven National Laboratory (BNL) flux spectrum under the simplifying assumption of a pure isoscalar neutral current, describe some general features of second-class-current–induced reactions and compare results obtained here with those obtained from the usual V, A currents or from the S, T, P currents discussed in the previous two papers. The formulas for the pion-production amplitude are given in an appendix. Our metric and γ-matrix conventions follow those of Bjorken and Drell.

II. PHENOMENOLOGICAL SECOND-CLASS CURRENT

In this section, we discuss our choice of second-class current and lay some of the foundations for the cross-section calculations in Sec. III which employ current-algebra techniques. In Sec. II A we review some of the possible second-class-current structures suggested in the literature and discuss why we settle on a phenomenological choice. In Sec. II B we calculate the commutation relations of the second-class currents with the usual axial-vector charge, which we will need when we apply PCAC (partial conservation of axial-vector current) to calculate pion production. In Sec. II C we estimate the one-nucleon matrix elements of the second-class vector and axial-vector currents. These renormalization constants are necessary for both the elastic and pion-production cross-section calculations.

A. Choice of current

The classification of strangeness-conserving weak currents into two classes according to their G-parity was first introduced by Weinberg.[4] First-class vector currents are defined to have the same G-parity as the electromagnetic current $[G=(-)^{I+1}$, $I=$ isospin$]$ and first-class axial-vector currents are defined to have opposite G-parity $[G=(-)^{I}]$. These are in fact the G-parities of currents formed out of the usual quark fields, $\bar\psi\gamma_\mu \frac{1}{2}\lambda^i\psi$ and $\bar\psi\gamma_\mu\gamma_5\frac{1}{2}\lambda^i\psi$, where λ^i are the relevant SU(3) matrices and ψ are the quark fields. Second-class currents have opposite G-parity to first-class currents.

As it is not possible to build second-class currents in the usual way out of nonstrange quarks, we must try other constructions. Since we are interested in using these currents to calculate neutrino-proton scattering and pion production, the current we construct must have certain properties. It must have nonzero (and computable) matrix elements between one-nucleon states because these are ingredients in the elastic calculation and in the Born and PCAC-consistency-condition terms of the pion-production amplitude. Our experience in the previous two papers of this series indicates as well that the commutator of the current with the axial-vector charge should be nonzero, and should have a nonvanishing one-nucleon matrix element. This is because experimentally the pion-production cross section is large relative to the elastic cross section; in the isoscalar neutral-current theories we have previously considered,[5] it is difficult to achieve a sufficiently large pion-production cross section while at the same time satisfying the experimental upper bounds on the elastic and deep-inelastic cross section, without a sizable commutator-term contribution.

Although second-class currents cannot be built in the usual manner from nonstrange quarks, Okubo[6] suggested that a second-class axial-vector current could be constructed as the total derivative of the pseudotensor current built out of the usual quarks

$$\bar{A}^i_\lambda = \partial^\mu T^i_{\mu\lambda} = \partial^\mu(\bar\psi\sigma_{\mu\lambda}\gamma_5\lambda^i\psi)\,. \tag{1}$$

However, this choice is not appropriate for our purpose since it has zero commutator with the axial charge. Similarly, Maiani[7] suggested building second-class currents out of two types of quarks—the usual one plus their parity doublets. This model has the problem that the one-nucleon matrix elements of the current vanish in first approximation since the nucleon does not seem to contain these peculiar quarks.

Since no second-class current built out of quarks seems to work, we turn to the suggestion of Lipkin and others[8] that the second-class current be composed purely of meson fields. For example, the isovector second-class axial-vector current could be given by $\vec\pi\omega_\lambda$, the isoscalar axial-vector current given by $\vec\pi\cdot\vec\rho_\lambda$, etc. We will follow Lipkin's lead and construct our second-class currents (denoted by a bar) in a phenomenological manner as follows:

$$\begin{aligned}
\bar V^{I=0}_\lambda &= \frac{1}{2}\frac{1}{\langle\sigma\rangle}\left(\vec\pi\cdot\bar\psi\gamma_\lambda\gamma_5\frac{\vec\tau}{2}\psi+\bar\psi\gamma_\lambda\gamma_5\frac{\vec\tau}{2}\psi\cdot\vec\pi\right)\\
&=\frac{1}{2\langle\sigma\rangle}(\vec\pi\cdot\vec{\mathrm{A}}_\lambda+\vec{\mathrm{A}}_\lambda\cdot\vec\pi)\,,\\
\bar V^{I=1}_\lambda &= \frac{1}{2}\frac{1}{\langle\sigma\rangle}\left[\vec\pi\bar\psi\gamma_\lambda\gamma_5\frac{1}{2}\left(\frac{2}{3}\right)^{1/2}\psi+\bar\psi\gamma_\lambda\gamma_5\frac{1}{2}\left(\frac{2}{3}\right)^{1/2}\psi\vec\pi\right]\\
&=\frac{1}{2\langle\sigma\rangle}(\vec\pi A^{(0)}_\lambda+A^{(0)}_\lambda\vec\pi)\,,\\
\bar A^{I=0}_\lambda &= \frac{1}{2}\frac{1}{\langle\sigma\rangle}\left(\vec\pi\cdot\bar\psi\gamma_\lambda\frac{\vec\tau}{2}\psi+\bar\psi\gamma_\lambda\frac{\vec\tau}{2}\psi\cdot\vec\pi\right)\\
&=\frac{1}{2\langle\sigma\rangle}(\vec\pi\cdot\vec{\mathrm{V}}_\lambda+\vec{\mathrm{V}}_\lambda\cdot\vec\pi)\,,\\
\bar A^{I=1}_\lambda &= \frac{1}{2}\frac{1}{\langle\sigma\rangle}\left[\vec\pi\bar\psi\gamma_\lambda\frac{1}{2}\left(\frac{2}{3}\right)^{1/2}\psi+\bar\psi\gamma_\lambda\frac{1}{2}\left(\frac{2}{3}\right)^{1/2}\psi\vec\pi\right]\\
&=\frac{1}{2\langle\sigma\rangle}(\vec\pi V^{(0)}_\lambda+V^{(0)}_\lambda\vec\pi)\,.
\end{aligned} \tag{2}$$

Because of vector-meson dominance, our choice is essentially equivalent to Lipkin's, but is somewhat easier to use in calculating. The choice of $\langle\sigma\rangle$, the vacuum expectation value of the σ field in the SU(2) $\times$ SU(2) σ model,[9] is a convenient normalization of the currents, as will become clear in Sec. II B, where we will also eliminate it in terms of physically measurable quantities. In the remaining parts of this paper we restrict our attention to isoscalar currents only. The generalization to isovector currents is straightforward.

B. Commutation relations

To calculate the commutation relations of the second-class currents with the usual axial-vector current, we will work in the framework of the SU(2) $\times$ SU(2) σ model. We expect that our relations are more generally valid in any theory with the same underlying symmetry. We begin by writing down the σ-model Lagrangian[9]:

$$\mathcal{L} = -\bar{\psi}[-i\partial\!\!\!/ + g(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\psi + \tfrac{1}{2}(\partial\vec{\pi})^2 + \tfrac{1}{2}(\partial\sigma)^2 + \frac{\mu^2}{2}(\vec{\pi}^2 + \sigma^2) - \lambda(\vec{\pi}^2 + \sigma^2 - A)^2 - \epsilon\sigma . \tag{3}$$

The usual axial-vector current A_k^λ is generated by the field transformations

$$\delta\pi^i = -\delta^{ik}\sigma, \quad \delta\sigma = \pi^k, \quad \delta\psi = \frac{i}{2}\tau^k\gamma_5\psi . \tag{4}$$

Then the divergence of the axial-vector current can be computed and used to relate the parameter ϵ to the pion decay constant f_π via PCAC:

$$\begin{aligned}\partial_\lambda A_k^\lambda &= -\frac{\delta\mathcal{L}}{\delta\Lambda}\\ &= \epsilon\pi_k\\ &= f_\pi\pi_k\\ &= \frac{M_N M_\pi^2 g_A}{g_r}\pi_k .\end{aligned} \tag{5}$$

Similarly by requiring that the vacuum be a minimum of the potential, and choosing a vacuum for which $\langle\vec{\pi}\rangle = 0$, we can relate the vacuum expectation value of the σ field, $\langle\sigma\rangle$, to physically measurable quantities, as follows:

$$\begin{aligned}0 &= \left\langle\frac{\partial V}{\partial\sigma}\right\rangle\\ &= -\langle\sigma\rangle[\mu^2 - 4\lambda(\langle\sigma\rangle^2 - A)] + \epsilon\\ \Rightarrow \langle\sigma\rangle &= -\frac{M_N g_A}{g_r} .\end{aligned} \tag{6}$$

Using this relation, $\langle\sigma\rangle$ will be eliminated in the calculation of renormalization constants in Sec. II C.

Finally, using the fact that the axial-vector charge is the generator of the transformation of Eq. (4) and using the relation

$$[Q, \phi] = i\delta\phi , \tag{7}$$

for any field ϕ, we find

$$\begin{aligned}[Q^{5k}, \bar{A}_\lambda^{I=0}] &= -iV_\lambda^k + i\epsilon^{kjl}\frac{(\pi^j A_\lambda^l + A_\lambda^l\pi^j)}{2\langle\sigma\rangle} ,\\ [Q^{5k}, \bar{V}_\lambda^{I=0}] &= -iA_\lambda^k + i\epsilon^{kjl}\frac{(\pi^j V_\lambda^l + V_\lambda^l\pi^j)}{2\langle\sigma\rangle} .\end{aligned} \tag{8}$$

C. Estimation of second-class-current renormalization constants

To estimate the one-nucleon matrix elements of the second-class phenomenological current, we will write each form factor as a sum over intermediate states and truncate the sum at the one-nucleon intermediate state. (This method is analogous to that used to calculate nonleptonic hyperon decay induced by the weak current $\times$ current Lagrangian.[10] A sum over intermediate states is inserted between the two currents in the effective Lagrangian and the sum is truncated after the nucleon and decuplet intermediate states.)

The second-class form factors for the isoscalar currents of Eq. (2) will be defined as follows:

$$\begin{aligned}\langle N(p_2)|\bar{V}_\lambda^{I=0}|N(p_1)\rangle &= i\mathfrak{N}_N F_V^{(0)}(p^2)\bar{u}(p_2)\frac{p_\lambda}{2M_N} \times \tfrac{1}{2}(\tfrac{2}{3})^{1/2}u(p_1) ,\\ \langle N(p_2)|\bar{A}_\lambda^{I=0}|N(p_1)\rangle &= -\mathfrak{N}_N F_A^{(0)}(p^2)\bar{u}(p_2)\sigma_{\lambda\eta}\frac{p^\eta}{2M_N} \times \gamma_5\tfrac{1}{2}(\tfrac{2}{3})^{1/2}u(p_1) ,\end{aligned} \tag{9}$$

where $p = p_2 - p_1$ and

$$\mathfrak{N}_N = \left(\frac{M_N}{p_{10}}\frac{M_N}{p_{20}}\right)^{1/2} .$$

We proceed by writing each form factor as a sum over intermediate states and saturating the sum with one-nucleon states:

$$\langle N(p_2)|\overline{V}_\lambda^{I=0}(0)|N(p_1)\rangle=\frac{1}{2\langle\sigma\rangle}\langle N(p_2)|\vec{\pi}\cdot\vec{A}_\lambda(0)+\vec{A}_\lambda\cdot\vec{\pi}(0)|N(p_1)\rangle$$

$$=-\frac{g_r}{2M_Ng_A}\sum_{\text{states}}[\langle N(p_2)|\pi^i(0)|N(k)\rangle\langle N(k)|A_\lambda^i(0)|N(p_1)\rangle+\langle N(p_2)|A_\lambda^i(0)|N(k)\rangle\langle N(k)|\pi^i(0)||N(p_1)\rangle]\,, \tag{10}$$

$$\langle N(p_2)|\overline{A}_\lambda^{I=0}(0)|N(p_1)\rangle=\frac{1}{2\langle\sigma\rangle}\langle N(p_2)|\vec{\pi}\cdot\vec{V}_\lambda(0)+\vec{V}_\lambda\cdot\vec{\pi}(0)|N(p_1)\rangle$$

$$=-\frac{g_r}{2M_Ng_A}\sum_{\text{states}}[\langle N(p_2)|\pi^i(0)|N(k)\rangle\langle N(k)|V_\lambda^i(0)|N(p_1)\rangle+\langle N(p_2)|V_\lambda^i(0)|N(k)\rangle\langle N(k)|\pi^i(0)|N(p_1)\rangle]\,. \tag{11}$$

The quantities on the right-hand side of Eqs. (10) and (11) are experimentally known. The pion matrix element between nucleon states is given by

$$\langle N(p_2)|\pi^i(0)|N(k)\rangle=\left(\frac{M_N}{p_{20}}\frac{M_N}{k_0}\right)^{1/2}\frac{1}{m_\pi^2-(p_2-k)^2}g_r((p_2-k)^2)\overline{u}(p_2)i\gamma_5\tau^iu(k)\,, \tag{12}$$

where the momentum dependence of the form factor g_r is taken to be monopole (which together with the explicit pion pole gives a total dipole falloff):

$$g_r(q^2)=g_r/(1-q^2/M^2)\,. \tag{13}$$

We will leave the mass M as a parameter which we will vary to get an idea of the sensitivity of our calculation to this assumption. Quark-model considerations suggest the value $M\approx 0.9$ GeV.[11] The usual axial-vector current matrix element is

$$\langle N(k)|A_\lambda^i(0)|N(p_1)\rangle=\left(\frac{M_N}{k_0}\frac{M_N}{p_{10}}\right)^{1/2}\overline{u}(k)[g_A((p_1-k)^2)\gamma_\lambda\gamma_5+h_A((p_1-k)^2)(k-p_1)_\lambda\gamma_5]\frac{\tau^i}{2}u(p_1)\,. \tag{14}$$

We use PCAC to express h_A in terms of g_A;

$$h_A(q^2)=\frac{2M_Ng_A(q^2)}{m_\pi^2-q^2}\,, \tag{15}$$

and assume a dipole falloff, consistent with experiment,[12] for the momentum behavior of g_A:

$$g_A(q^2)=\frac{g_A}{(1-q^2/M^2)^2}\,. \tag{16}$$

The vector-current matrix element is well known experimentally:

$$\langle N(k)|V_\lambda^i(0)|N(p_1)\rangle=\left(\frac{M_N}{k_0}\frac{M_N}{p_{10}}\right)^{1/2}\overline{u}(k)\left[F_1((p_1-k)^2)\gamma_\lambda+iF_2((p_1-k)^2)\sigma_{\lambda\eta}\frac{(k-p_1)^\eta}{2M_N}\right]\frac{\tau_i}{2}u(p_1)\,. \tag{17}$$

The experimental behavior of the vector form factors may be parameterized as[13]

$$F_1(q^2)=\frac{F_1}{(1-q^2/M^2)^2}\,,\quad F_2(q^2)=\frac{F_2}{(1-q/4M_N^2)(1-q^2/M^2)^2}\,, \tag{18}$$

with F_1 and F_2 being constants.

To find the second-class matrix elements at zero momentum transfer, we will work in the brick-wall frame, $\vec{p}_1+\vec{p}_2=0$, and keep only leading powers of $|\vec{p}|=|\vec{p}_2-\vec{p}_1|$. Inserting Eqs. (12)–(18) into Eq. (11), we find for the zero component of the second-class axial-vector current matrix element

$$\langle N(p_2)|\overline{A}_0^{I=0}(0)|N(p_1)\rangle=\frac{-g_r}{M_Ng_A}\int\frac{d^3k}{(2\pi)^3}\frac{M_N}{k_0}g_r\frac{1}{m_\pi^2-(p_2-k)^2}\frac{1}{1-(p_2-k)^2/M^2}\frac{1}{[1-(p_1-k)^2/M^2]^2}$$

$$\times\sum_{\substack{\text{spin}\\\text{isospin}}}\overline{u}(p_2)i\gamma_5\tau_iu(k)\overline{u}(k)\left[F_1\gamma_0+\frac{iF_2}{1-(p_1-k)^2/4M_N^2}\sigma_{0j}\frac{(k-p_1)^j}{2M_N}\right]\frac{\tau_i}{2}u(p_1)\,. \tag{19}$$

In the brick-wall frame in the static limit, Eq. (9) for the isoscalar second-class axial-vector form factor becomes

$$\langle N(p_2)|\bar{A}_0^{I=0}|N(p_1)\rangle = -(\tfrac{2}{3})^{1/2}\,\bar{F}_A^{(0)}(0)\,\chi_2^\dagger\vec{\sigma}\cdot\vec{p}\,\chi_1 + O(|\vec{p}|^2)\text{ corrections}. \tag{20}$$

After some algebra and dropping nonleading terms in $|\vec{p}|$ from Eq. (19), we find

$$\bar{F}_A^{(0)}(0) = -\frac{2g_r}{g_A}\left(\frac{3}{2}\right)^{1/2}\frac{3}{128}\frac{g_r}{\pi^2}\left(\frac{M}{M_N}\right)^6 \times \int_0^1 \frac{dy(1-y^2)^{1/2}}{(1+fy)(1+gy)^3}\left\{F_1\left[1+\frac{1-y^2}{3}\left(\frac{1}{1+gy}-\frac{1}{1+fy}\right)\right]+F_2\frac{y(1-y)}{1+y}\left[1-\frac{(1-y^2)}{3y}\left(\frac{1}{1+y}+\frac{1}{1+gy}-\frac{1}{1+fy}\right)\right]\right\}, \tag{21}$$

where

$$f=\frac{m_\pi^2-2M_N^2}{2M_N^2},\quad g=\frac{M^2-2M_N^2}{2M_N^2}.$$

A similar calculation for the second-class vector current, performed for the z component, gives

$$\bar{F}_V^{(0)}(0) = -\frac{2g_r}{g_A}\left(\frac{3}{2}\right)^{1/2}\frac{3g_r g_A}{128\pi^2}\left(\frac{M}{M_N}\right)^6\int_0^1\frac{dy(1-y^2)^{1/2}}{(1+fy)(1+gy)^3}\left\{1+\frac{1-y^2}{3}\left(\frac{1}{1+gy}-\frac{1}{1+fy}\right)+\frac{1-y}{1+fy}\left[y-\frac{(1-y^2)}{3}\frac{1}{1+gy}\right]\right\}. \tag{22}$$

The integrals were done numerically for different values of the dipole mass M, using the experimental values $g_r=13.44$, $g_A=1.24$, $F_1=1$, $F_2=3.71$. The numerical results are given in Table I. There is approximately a 50% variation of the renormalization constants over the dipole-mass range of 0.8–1.0 GeV. A more serious uncertainty in our calculation arises from the neglect of higher-mass states in the sum over intermediate states. In attempting to include the decuplet we found that its contribution was highly sensitive to the momentum dependence of the form factors of the V,A currents sandwiched between nucleon and decuplet states. Different experimentally acceptable momentum falloffs[14] gave contributions which varied from finite to infinite. Therefore, we neglect the decuplet-and-higher-state contributions and expect that our calculation is good only up to order of magnitude, at best.[15]

TABLE I. Renormalization-constant values for different choices of the dipole mass M.

M (GeV)	$-\bar{F}_A^{(0)}(0)$	$-\bar{F}_V^{(0)}(0)$
0.8	5.1	8.2
0.9	6.4	10.0
M_N	7.0	10.7
1.0	8.0	11.9

III. CROSS-SECTION CALCULATIONS

In this section we set up the cross-section formulas needed for correlating neutrino-proton elastic scattering (Sec. III A) and neutral-current–induced pion production (Sec. III B) in the case of second-class V,A currents.

We specialize to the case of pure isoscalar currents. The salient new feature of the isovector case is the possibility of exciting the (3,3) resonance. The effects of (3,3) excitation can readily be calculated using the methods of Appendix B of Ref. 3. In the static limit, only the second-class axial-vector current excites the (3,3) resonance. If experimental results indicate the need for isovector neutral currents, our methods can easily be extended to include isovector second-class currents as well.

In principle, there may be interference with first-class V,A neutral currents if they are also present. Explicit calculation indicates that there is no interference for neutrino-proton elastic scattering, although there may be interference in the pion-production calculation between first- and second-class V,A currents. For the present we do not consider first- and second-class mixtures, although our calculation in the first-class V,A case in Ref. 2 can easily be combined with the results of Sec. III B to consider mixtures if more detailed experimental information than is now available warrants it. Of course, there is no interference with S, P, T currents in any exclusive (or inclusive) channels.

A. Elastic neutrino-nucleon scattering

We consider a general mixture of the isoscalar second-class V,A currents. We begin with the neutral-current effective Lagrangian

$$\mathcal{L}_{\rm eff}^{N}=\frac{2G}{\sqrt{2}}(h_0\bar{\nu}\gamma_\lambda\nu\bar{V}^{\lambda\, I=0}+g_0\bar{\nu}\gamma_\lambda\gamma_5\nu\bar{A}^{\lambda\, I=0}), \quad (23)$$

with $\bar{V}_\lambda$ and $\bar{A}_\lambda$ being the hadronic second-class vector and axial-vector currents of Eq. (2). The parameters g_0 and h_0 set the strength of the vector and axial-vector contributions to the neutral current. As written, the interaction of Eq. (23) is parity-conserving but CP-violating; Eq. (23) can be made CP-conserving if the outgoing "neutrino" is not the same species as the incident neutrino.[16] Since experimentally the incident neutrino is left-handed, the effective matrix element for accelerator neutrino reactions is obtained from Eq. (23) by substituting $\nu\to\frac{1}{2}(1-\gamma_5)\nu$ to give

$$\mathfrak{M}_{\rm eff}^{N}=\frac{G}{\sqrt{2}}\,\bar{\nu}\gamma^\lambda(1-\gamma_5)\nu(h_0\bar{V}_\lambda-g_0\bar{A}_\lambda)\,. \quad (24)$$

To calculate νN elastic scattering, we evaluate this amplitude between one-nucleon states using the current matrix elements given by Eq. (9). Squaring to get the differential cross section, we find

$$\frac{d\sigma}{dt}(\nu+N\to\nu+N)=\frac{-G^2}{8\pi E^2M_N^{\,2}}\frac{tg_0^{\,2}}{24M_N^{\,2}}|\bar{F}_A(t)|^2\times[-4M_N^{\,2}E^2+tM_N(2E+M_N)]\,, \quad (25)$$

where $t=-k^2$ and E is the incident neutrino energy. The second-class vector current does not contribute because the one-nucleon matrix element of the second-class vector current is proportional to $k_\lambda=k_{1\lambda}-k_{2\lambda}$, which is annihilated by the leptonic amplitude $\bar{\nu}(k_2)\gamma^\lambda(1-\gamma_5)\nu(k_1)$ for massless neutrinos. Since there is no V,A interference in the elastic channel, the neutrino and antineutrino elastic cross sections are identical.

B. Neutral-current pion production

We now turn to the main focus of this paper, the calculation of weak-pion production induced by second-class V,A currents. As in the two previous papers in this series, we use a pion-production model which includes without kinematic approximation the pseudoscalar coupling nucleon Born terms and pion-pole terms. To these are added the PCAC consistency-condition terms and current-algebra equal-time commutator terms so as to guarantee that the pion-production amplitude has the correct soft-pion limit. This model when applied to the vector current in pion photoproduction and electroproduction and to pion production induced by the charged weak $V-A$ current[17] is in good agreement with experiment in the low-invariant-mass ($W\leq 1.4$ GeV) region.

We begin with the standard soft-pion formula for the process $\mathcal{J}+N\to\pi^j+N$, with $\mathcal{J}$ being a general external current and N being a nucleon:

$$\begin{aligned}\mathfrak{M}_\pi&=\langle N(p_2)\pi^j(q)|\mathcal{J}(0)|N(p_1)\rangle\\&=-\mathfrak{N}_{N\pi}\bar{u}(p_2)\left[\frac{g_r}{M_Ng_A}J_j'(k-q)+\frac{g_r}{2M_N}\{\gamma_5\tau_j,J(k)\}_++\frac{g_r}{2M_N}\gamma_5\tau_j\frac{\not p_2+\not q+M_N}{\nu-\nu_B}J(k)-J(k)\frac{\not p_1-\not q+M_N}{\nu+\nu_B}\frac{g_r}{2M_N}\gamma_5\tau_j\right.\\&\qquad\left.+\text{possible additional pion-pole "seagull" contributions}\right]u(p_1)\psi_j^*+O(q)\,,\end{aligned} \quad (26)$$

with

$$\begin{aligned}&\nu=(p_1+p_2)\cdot k/(2M_N)\,,\quad \nu_B=-q\cdot k/(2M_N)\,,\quad \mathfrak{N}_{N\pi}=\mathfrak{N}_N(2q_0)^{-1/2}\,,\\&\langle N(p_2)|\mathcal{J}(0)|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)J(p_2-p_1)u(p_1)\,,\quad \langle N(p_2)|[F_j^5,\mathcal{J}(0)]|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)J_j'(p_2-p_1)u(p_1)\,.\end{aligned} \quad (27)$$

In these equations, $k=p_2=q-p_1$ denotes the four-momentum carried by the external current, $g_r\approx 13.5$ is the pion-nucleon coupling constant, and ψ_j is the isospin wave function of the emitted pion. On the right-hand side of Eq. (26) the first term is the current-algebra equal-time commutator, the second term is the PCAC consistency-condition contribution, and the the third and fourth terms are the usual pseudoscalar coupling nucleon Born terms. The additional pion-pole terms are needed only when the first four terms do not contain all the possible pion-pole contributions expected for the reaction. In the case at hand, parity considerations rule out pion-pole contributions from the axial-vector current. Bose symmetry for the pion together with the masslessness of the neutrino rule out a pion-pole contribution for the isoscalar vector current.

In our case the external current is given by

$$\mathcal{J}^{\text{s.c.}}(0)=\frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\lambda}(1-\gamma_5)\nu(h_0\bar{V}_{\lambda}^{I=0}-g_0\bar{A}_{\lambda}^{I=0})\,. \tag{28}$$

Using the σ-model result of Eq. (8), the relevant commutator is

$$\langle N(p_2)|[F_j^5,\mathcal{J}(0)^{\text{s.c.}}]|N(p_1)\rangle$$
$$=\frac{G}{\sqrt{2}}\bar{\nu}\gamma_{\lambda}(1-\gamma_5)\nu\left\langle N(p_2)\left|h_0\left(-iA_j^{\lambda}+i\epsilon_{jkl}\frac{(\pi_kV_l^{\lambda}+V_l^{\lambda}\pi_k)}{2\langle\sigma\rangle}\right)-g_0\left(-iV_j^{\lambda}+i\epsilon_{jkl}\frac{(\pi_kA_l^{\lambda}+A_l^{\lambda}\pi_k)}{2\langle\sigma\rangle}\right)\right|N(p_1)\right\rangle. \tag{29}$$

The one-nucleon matrix elements of the usual vector and axial-vector currents V_j^{λ} and A_j^{λ} are defined in Eqs. (14)–(18). The matrix elements of $\epsilon_{jkl}\pi_kV_l^{\lambda}$ and $\epsilon_{jkl}\pi_kA_l^{\lambda}$ are related by simple isospin considerations to the second-class matrix elements computed in Sec. II C,

$$\epsilon^{jkl}\left\langle N(p_2)\left|\frac{\pi^kV_{\lambda}^l+V_{\lambda}^l\pi^k}{2\langle\sigma\rangle}\right|N(p_1)\right\rangle=-\mathfrak{N}_N\tfrac{1}{3}(\tfrac{2}{3})^{1/2}\bar{F}_A((p_2-p_1)^2)\bar{u}(p_2)i\frac{\sigma_{\lambda\eta}p^{\eta}}{2M_N}\gamma_5\tau^ju(p_1)\,,$$
$$\epsilon_{jkl}\left\langle N(p_2)\left|\frac{\pi^kA_{\lambda}^l+A_{\lambda}^l\pi_k}{2\langle\sigma\rangle}\right|N(p_1)\right\rangle=-\mathfrak{N}_N\tfrac{1}{3}(\tfrac{2}{3})^{1/2}\bar{F}_V((p_2-p_1)^2)\bar{u}(p_2)\frac{p_{\lambda}}{2M_N}\tau^ju(p_1)\,. \tag{30}$$

Substituting Eqs. (28)–(30) into Eq. (26) and neglecting $O(q)$ pion recoil corrections, we arrive at the following expression for the pion-production amplitude:

$$\mathfrak{M}_{\pi}=-\mathfrak{N}_{N\pi}\frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\lambda}(1-\gamma_5)\nu\bar{u}(p_2)(\mathfrak{M}_{\lambda}^{\text{COM}}+\mathfrak{M}_{\lambda}^{\text{PCAC}}+\mathfrak{M}_{\lambda}^{\text{BORN}})\frac{\tau_j}{2}u(p_1)\psi_j^*\,, \tag{31}$$

where $\mathfrak{M}_{\lambda}^{\text{COM}}$, $\mathfrak{M}_{\lambda}^{\text{PCAC}}$, and $\mathfrak{M}_{\lambda}^{\text{BORN}}$, the commutator, consistency-condition, and nucleon Born terms, respectively, are given by

$$\mathfrak{M}_{\lambda}^{\text{COM}}=\frac{-ig_r}{M_Ng_A}\left[h_0g_A(k^2)\left(\gamma_{\lambda}\gamma_5-\frac{2M_Nq_{\lambda}\gamma_5}{m_{\pi}^2-(q-k)^2}\right)+h_0(\tfrac{2}{3})^{3/2}\,i\bar{F}_A(k^2)\sigma_{\lambda\eta}\frac{(k-q)^{\eta}}{2M_N}\gamma_5\right.$$
$$\left.-g_0\left(F_1(k^2)\gamma_{\lambda}+iF_2(k^2)\frac{\sigma_{\lambda\eta}}{2M_N}(k-q)^{\eta}\right)+g_0(\tfrac{2}{3})^{3/2}\bar{F}_V(k^2)\frac{q_{\lambda}}{2M_N}\right], \tag{32}$$

$$\mathfrak{M}_{\lambda}^{\text{PCAC}}=\frac{g_r}{M_N}(\tfrac{2}{3})^{1/2}g_0\bar{F}_A(k^2)\frac{\sigma_{\lambda\eta}k^{\eta}}{2M_N}, \tag{33}$$

$$\mathfrak{M}_{\lambda}^{\text{BORN}}=\frac{-g_r}{(2M_N)^2}(\tfrac{2}{3})^{1/2}g_0\bar{F}_A(k^2)\left(\frac{\not{q}}{\nu-\nu_B}\sigma_{\lambda\eta}k^{\eta}+\sigma_{\lambda\eta}k^{\eta}\frac{\not{q}}{\nu+\nu_B}\right). \tag{34}$$

Note that the vector current contributes only through the commutator term for massless neutrinos.

To calculate the pion-production cross section we express the matrix element in terms of a convenient set of nine invariants defined by

$$\mathfrak{M}_{\pi}=-\mathfrak{N}_{N\pi}\frac{G}{\sqrt{2}}\bar{\nu}\gamma^{\lambda}(1-\gamma_5)\nu\bar{u}(p_2)(A\gamma_{\lambda}-2BP_{\lambda}+Fp_{\lambda}+C\gamma_{\lambda}\gamma_5+Dp_{\lambda}\gamma_5+2EP_{\lambda}\gamma_5+G\sigma_{\lambda\eta}q^{\eta}+Hq_{\lambda}\not{q}+2JP_{\lambda}\not{q})u(p_1)\,, \tag{35}$$

where $P_{\lambda}=\frac{1}{2}(p_{1\lambda}+p_{2\lambda})$. Expressions for $A,B,\ldots,J$ are given in the Appendix. The pion-production differential cross section is then computed to give

$$\frac{d\sigma}{dt\,dW}=\frac{1}{16\pi^3}\frac{|\vec{q}|}{E^2}\frac{G^2}{8M_N^2}\frac{1}{4\pi}\int d\Omega_{\pi}\Sigma_{\pi}\,, \tag{36}$$

with Σ_{π} given by

$$\Sigma_{\pi}=\frac{8M_N^2m_{\nu}^2}{G^2}\sum_{\substack{\text{spin}\\\text{average}}}\langle\mathfrak{M}_{\pi}\rangle^2\,. \tag{37}$$

Σ_{π} was calculated from $\mathfrak{M}_{\pi}$ using standard trace techniques. The result, which is complicated, was checked by independently calculating the amplitude acting on nucleon Pauli spinors, assembling it using the complex arithmetic feature of Fortran 4, and then checking this program against the program for the covariant cross-section evaluation. In Eq. (36), W is the invariant mass of the outgoing pion-nucleon isobar, and $|\vec{q}|$ and $d\Omega_{\pi}=\sin\phi\,d\phi\,d\delta$ are the pion momentum and solid angle in the isobaric rest frame.

Finally, we note that all formulas in this section are for incident neutrinos. For incident antineutrinos, all axial-vector amplitudes are reversed in sign.

IV. NUMERICAL RESULTS

In this section we give sample numerical results obtained from the formulas developed in Sec. III, as applied to the analysis of low-invariant-mass ($W \leq 1.4$ GeV) pion production in the BNL neutrino flux. In particular, we study three choices of second-class isoscalar currents: pure V, pure A, and the combination $V-A$.

Recently, the Columbia-Illinois-Rockefeller collaboration at BNL reported a measurement of the ratio

$$R_0' = \frac{\sigma(\nu + T \to \nu + \pi^0 + \cdots)}{2\sigma(\nu + T \to \mu^- + \pi^0 + \cdots)}, \quad (38)$$

$$T = \tfrac{1}{4}[{}_6\mathrm{C}^{12}] + \tfrac{3}{4}[{}_{13}\mathrm{Al}^{27}]$$

with the preliminary result[18]

$$R_0' = 0.17 \pm 0.06 . \quad (39)$$

This measured value is consistent with the value expected[19] in the Weinberg-Salam model when $\sin^2\theta_W$ is in the currently favored range of 0.3–0.4. Hence if (3,3)-resonance excitation is observed, which is expected in the Weinberg-Salam model, then this measurement would support the standard gauge-theory interpretation of neutral currents. However, preliminary BNL invariant-mass spectra for π^0 production show a clear (3,3) excitation peak for charged-current–induced pion production, but no comparable excitation in the neutral-current case. Our discussion focuses on issues which will be raised if this preliminary result is confirmed by a more detailed analysis of the BNL data.

The simplest interpretation of nonexcitation of the (3,3) resonance by the neutral current is that the neutral current is pure isoscalar. However, as mentioned earlier, the presence of a sizable second-class isovector vector-current piece need not lead to a sizable (3,3) peak in the neutral-current sector, since the second-class vector current does not excite the (3,3) resonance in the static limit. Nonetheless, all of our numerical work is done with pure isoscalar currents.

Applying nuclear final-state corrections as described in Appendix C of Ref. 2, we find that the nuclear-target ratio given in Eq. (39) implies a nucleon-target ratio

$$2R_0 \equiv \frac{\sigma^{\mathrm{BNL}}(\nu + n \to \nu + n + \pi^0) + \sigma^{\mathrm{BNL}}(\nu + p \to \nu + p + \pi^0)}{\sigma^{\mathrm{BNL}}(\nu + n \to \mu^- + p + \pi^0)}$$
$$= 2R_0' \times 1.4$$
$$= 0.48 \pm 0.17 , \quad (40)$$

with the superscript BNL denoting averaging over the BNL neutrino flux. Using the theoretical estimate for the usual $V-A$ charged-current–induced reaction[20]

$$\sigma^{\mathrm{BNL}}(\nu + n \to \mu^- + p + \pi^0,\ W \leq 1.4\ \mathrm{GeV}) \approx 0.14 \times 10^{-38}\ \mathrm{cm}^2 , \quad (41)$$

we get from Eq. (40) the cross section for neutral-current π^0 production

$$\sigma^{\mathrm{BNL}}(\nu + n \to \nu + n + \pi^0,\ W \leq 1.4\ \mathrm{GeV}) + \sigma^{\mathrm{BNL}}(\nu + p \to \nu + p + \pi^0,\ W \leq 1.4\ \mathrm{GeV}) = (68 \pm 24) \times 10^{-41}\ \mathrm{cm}^2 . \quad (42)$$

This is one of the basic numbers which must be approximated by an acceptable model.

We will simultaneously take into account the available data on neutrino-proton elastic scattering[21]:

$$\sigma^{\mathrm{CERN}}(\nu + p \to \nu + p,\ \text{Cundy cuts}) \leq 0.24\sigma^{\mathrm{CERN}}(\nu + n \to \mu^- + p,\ \text{Cundy cuts})$$

at 95% confidence level; Cundy cuts are

$$1 \leq E \leq 4\ \mathrm{GeV},\quad 0.3 \leq |k^2| \leq 1\ (\mathrm{GeV}/c)^2 . \quad (43)$$

Neglecting possible distorting effects of the cuts and using the fact that the CERN and BNL neutrino flux shapes are similar,[22] Eq. (43) gives an approximate upper bound on the BNL neutrino-proton elastic cross section

$$\sigma^{\mathrm{BNL}}(\nu + p \to \nu + p) \lesssim 0.24\sigma^{\mathrm{BNL}}(\nu + n \to \mu^- + p) = 0.21 \times 10^{-38}\ \mathrm{cm}^2 \quad (44)$$

at 95% confidence level.

In our numerical work, all form factors are chosen to be dipoles:

$$F(k^2) = F(0)(1 - k^2/M^2)^{-2} , \quad (45)$$

with dipole mass $M = 0.9$ GeV, a value suggested by experimental and quark-model considerations.[11] The second-class form factors, $\overline{F}_A(0)$ and $\overline{F}_V(0)$, are given the values calculated in Sec. II C for the same value of M. Although we expect that our calculation is only good up to order of magnitude, varying these form factors is not likely to change dramatically the shape of the experimental distributions. The over-all strength of the current

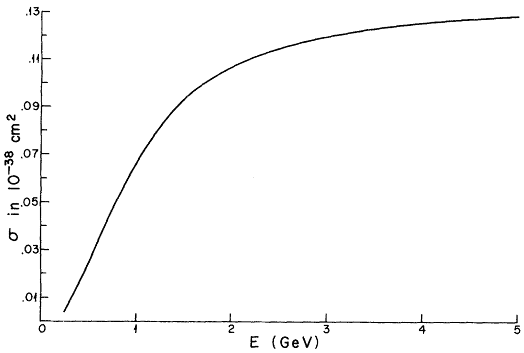

FIG. 1. Cross section $\sigma(\nu+p\to\nu+p)$ versus incident neutrino energy E for an A or $V-A$ current. The cross section vanishes for V current. The same graph applies for antineutrino-proton elastic scattering.

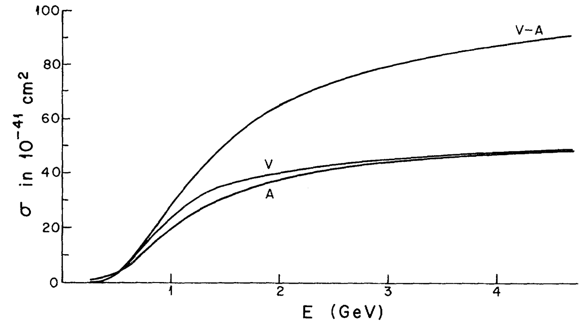

FIG. 3. Cross section $\sigma(\bar\nu+n\to\bar\nu+n+\pi^0)$ $+\sigma(\bar\nu+p\to\bar\nu+p+\pi^0)$ versus incident neutrino energy E for three current choices.

determined by the parameters g_0 and h_0 was not adjusted in our sample fits but rather was taken to be either 0 or 1 depending on the choice of current as follows:

$$\begin{array}{lll} V & g_0=0, & h_0=1, \\ A & g_0=1, & h_0=0, \\ V-A & g_0=1, & h_0=1. \end{array} \qquad (46)$$

The features of these three choices of neutral current are given in Figs. 1–5 and Table II. The general shapes of the distributions for the three choices are not that dissimilar, except that there is no elastic scattering for the pure vector second-class current case. In Fig. 1 we give the elastic neutrino-proton scattering cross section, which is the same as the elastic antineutrino-proton cross section. In Figs. 2 and 3 the pion-production cross sections induced by neutrinos and by antineutrinos are drawn. For pure vector or pure axial-vector currents the neutrino and antineutrino cross sections are equal, while in the $V-A$ case the interference is constructive for neutrinos and destructive for antineutrinos, giving $\sigma(\nu n\pi^0+\nu p\pi^0)\geq\sigma(\bar\nu n\pi^0+\bar\nu p\pi^0)$. In Figs. 4 and 5 the pion-production distributions in invariant mass for the Brookhaven neutrino and antineutrino fluxes are given. These curves resemble those calculated for isoscalar first-class currents, particularly the one obtained in Ref. 2 for an isoscalar vector current dominated by the anomalous magnetic-moment form-factor term. In Table II we list some of the theoretical values for experimentally measurable quantities. We note that the $V-A$ example is consistent as it stands with the preliminary BNL data, while pure V or A examples can be made to agree with the BNL neutrino pion-production data by respectively scaling up h_0 or g_0 from the values given in Eq. (46), without violating the elastic bounds in either case. In all three cases the elastic cross sections at LAMPF energies

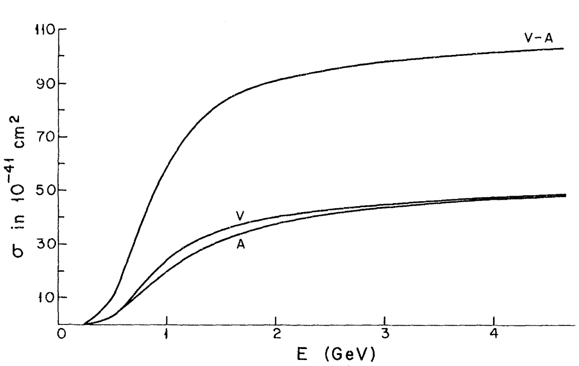

FIG. 2. Cross section $\sigma(\nu+n\to\nu+n+\pi^0)$ $+\sigma(\nu+p\to\nu+p+\pi^0)$ versus incident neutrino energy E for three current choices.

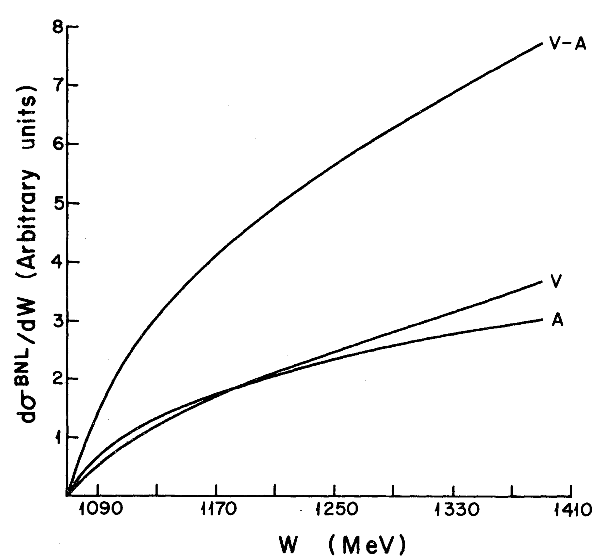

FIG. 4. Differential cross section $d\sigma/dW$ for the reaction $\nu+p\to\nu+p+\pi^0$, averaged over the BNL neutrino flux for three current choices.

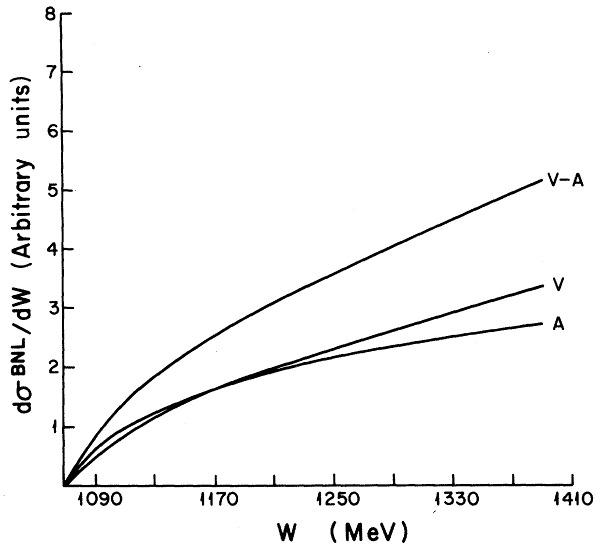

FIG. 5. Differential cross section $d\sigma/dW$ for the reaction $\bar{\nu}+p\rightarrow\bar{\nu}+p+\pi^0$, averaged over the BNL neutrino flux for three current choices.

($E\sim 50$ MeV) are very small, being roughly two to three orders of magnitude smaller than the corresponding cross sections expected for first class V,A or S,P,T neutral couplings.[2,3]

As in the case of the first-class V,A and S,P,T phenomenologies discussed in the earlier two papers of this series, we have written general computer programs embodying the calculations of the present paper. These programs will be applied to try to make more definite statements about the structure of the weak neutral interaction when improved experimental data become available.

APPENDIX

In this appendix we give the expressions for the amplitudes $A,\ldots,J$ which parameterize the pion-production amplitude [Eq. (35)].

The amplitudes are as follow:

$$A=ig_0a_E^{(0)}\frac{g_r}{M_Ng_A}[F_1(k^2)+F_2(k^2)]\,,$$

$$B=ig_0a_E^{(0)}\frac{g_r}{2M_N^2}\left[\frac{1}{g_A}F_2(k^2)-(\tfrac{2}{3})^{1/2}F_A(k^2)\right],$$

$$C=-ih_0a_E^{(0)}\frac{g_r}{M_Ng_A}g_A(k^2)\,,$$

$$D=-ih_0a_E^{(0)}\frac{2g_r}{g_A}\frac{g_A(k^2)}{m_\pi^2-(q-k)^2}\,,$$

$$E=ih_0a_E^{(0)}(\tfrac{2}{3})^{3/2}\frac{g_r}{2M_N^2g_A}F_A(k^2)\,, \qquad \text{(A1)}$$

$$F=ig_0a_E^{(0)}(\tfrac{2}{3})^{3/2}\frac{g_r}{2M_N^2g_A}F_V(k^2)\,,$$

$$G=g_0a_E^{(0)}(\tfrac{2}{3})^{1/2}\frac{g_r}{2M_N^2}F_A(k^2)\,,$$

$$H=-ig_0a_E^{(0)}(\tfrac{2}{3})^{1/2}\frac{g_r}{4M_N^2}F_A(k^2)\left(\frac{1}{\nu-\nu_B}-\frac{1}{\nu+\nu_B}\right),$$

$$J=-ig_0a_E^{(0)}(\tfrac{2}{3})^{1/2}\frac{g_r}{4M_N^2}F_A(k^2)\left(\frac{1}{\nu-\nu_B}+\frac{1}{\nu+\nu_B}\right),$$

TABLE II. Values of experimentally measurable quantities for three choices of neutral current.

		Choice of current[a]		
	Expt.	V	A	$V-A$
$\sigma^{\rm BNL}(\nu p)$ in 10^{-38} cm^2	≤ 0.21	0.0	0.099	0.099
$\sigma^{\rm BNL}(\bar{\nu}p)$ in 10^{-38} cm^2		0.0	0.092	0.092
$\sigma^{\rm BNL}(\nu n\pi^0+\nu p\pi^0$, $W\leq 1.4$ GeV) in 10^{-41} cm^2	68 ± 24 (preliminary)	34.2[b]	36.8[b]	83.3
$\sigma^{\rm BNL}(\bar{\nu}n\pi^0+\bar{\nu}p\pi^0$, $W\leq 1.4$ GeV) in 10^{-41} cm^2		31.5[b]	34.2[b]	53.1
$\sigma(\nu p)$ at $E=50$ MeV in 10^{-43} cm^2		0.0	2.2	2.2

[a] The parameter values g_0, h_0 used in these examples are given in Eq. (46) of the text.

[b] The small differences between the flux-averaged ν- and $\bar{\nu}$-induced cross sections calculated in the pure-V and pure-A cases arise from differences in the shapes of the BNL ν- and $\bar{\nu}$-flux distributions.

where $a_E^{(0)}$ is the isospin wave function

$$a_E^{(0)} = \chi_2^* \psi_j^* \frac{\tau_j}{2} \chi_1 , \qquad \text{(A2)}$$

with $\chi_{1,2}$ being the initial and final nucleon isospinors and ψ_j being the final pion isospin wave function.

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$$F_1 = \frac{G_E - (q^2/4M_N^2)G_M}{1 - q^2/4M_N^2} .$$

This reduces to Eq. (18) in the text only for $G_M(0)/G_E(0) = 1$, which does not hold experimentally. Presumably the small difference between Eq. (18) and the exact q^2 behavior is absorbed into the variation of M, the dipole mass.

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