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Absorptive corrections to the pion-exchange Deck amplitude*

E. L. Berger and A. C. Irving†
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
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Consequences of absorbing the pion-exchange amplitude in Deck models are investigated. We make quantitative estimates of the largest corrections to be reasonably expected from an approach to absorption which has been successful in two-body processes. We conclude that the major features of the low-mass enhancement (e.g., spin-parity content, mass, and angular distributions) obtained from the model are not altered by absorption.

I. INTRODUCTION

The pion-exchange Deck amplitude has been studied extensively in connection with the description of enhancements observed at low subenergy in two- to three-particle reactions.[1] Double-Regge-exchange diagrams such as drawn in Fig. 1(a), with trajectories $\alpha_2=\pi$ and $\alpha_1=$ Pomeron, provide significant "diffractive" enhancements near threshold in the variable s_2. Properties of these enhancements, including both production and decay angular distributions, are in at least crude agreement with data on $\pi^{\pm}p\rightarrow(\rho\pi)^{\pm}p$, $K^{\pm}p\rightarrow(K^*\pi)^{\pm}p$, and $pp\rightarrow(N\pi)^+p$ over a range of energies.

While far from providing a full description of the data, the pion-exchange graph is expected to continue to play a major role in attempts to understand low-mass enhancements in terms of "t (or u) channel dynamics."[2]

The behavior of the Deck amplitude as a function of t_2 [cf. Fig. 1(a)] is crucial in the model since it influences both the *shape* of the mass distribution $d\sigma/ds_2$ and the *spin-parity content* (i.e., decay angular distributions) of the low-mass enhancement.[1] Because of this, one is led to question the validity of the simple pion- (Regge-) pole-exchange approximation used in most work to date. Indeed, a pure π pole, vanishing at $t=0$, is well known to be a poor description of some π-exchange processes, such as $\gamma p\rightarrow\pi^+n$ and $np\rightarrow pn$, which show sharp peaks at very small t in their differential cross sections. More generally, the presence of some nonevasive (Regge cut) correction is required in the phenomenological description[3-5] of all known helicity-nonflip evasive π-exchange amplitudes.[6]

In Deck amplitudes in which evasive π-exchange amplitudes are important, the presence of a contribution with the t_2 dependence

$$A_\pi^{\rm abs}\sim\frac{\rm const}{\mu^2-t_2} \tag{1}$$

rather than the pure pole form

$$A_\pi\sim\frac{-t_2}{\mu^2-t_2} \tag{2}$$

could, *a priori*, change significantly the predicted properties of the low-mass enhancement. Turning to specific cases, we remark that in the processes $\pi^{\pm}p\rightarrow(\rho\pi^{\pm})p$ and $K^{\pm}p\rightarrow(K^*\pi^{\pm})p$, the vector mesons are produced dominantly in helicity-zero states, by a π-exchange amplitude which is finite as $t_2\rightarrow0$. In these cases, evasive pion exchange is unimportant, and the net cut correction is effectively only a mild "form factor" modification of the t_2 dependence

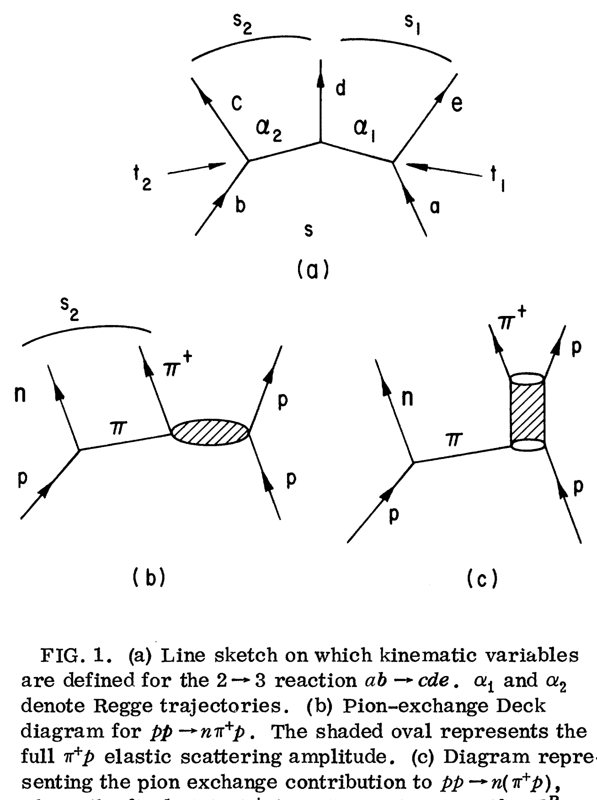

FIG. 1. (a) Line sketch on which kinematic variables are defined for the $2\rightarrow3$ reaction $ab\rightarrow cde$. α_1 and α_2 denote Regge trajectories. (b) Pion-exchange Deck diagram for $pp\rightarrow n\pi^+p$. The shaded oval represents the full π^+p elastic scattering amplitude. (c) Diagram representing the pion exchange contribution to $pp\rightarrow n(\pi^+p)$, where the final state (π^+p) system is in a specific J^P state.

dence.

In $pp \to (n\pi^+)p$, sketched in Fig. 1(b), the left vertex $pn\pi^+$ has unit helicity flip in the s channel, expressed as a factor of $(-t_2')^{1/2}$ in the amplitude. If the right vertex $p(\pi^+p)\pi^+$ is pure s-channel nonflip, the net cut correction will again be unimportant. However, helicity-flip couplings at the right vertex are not forbidden. The extent to which a piece with unit s-channel flip is present [providing a second factor of $(-t_2')^{1/2}$] governs the importance of evasive π exchange [cf. Eq. (2)] in the over-all amplitude.

In this paper we treat the process $pp \to (n\pi^+)p$ in some detail. First, we extract an expression for the portion of the right-hand vertex in Fig. 1(b) which is proportional to $(-t_2')^{1/2}$. This is described in Sec. II. Absorption of the amplitude is treated in Sec. III. We employ the Williams absorption model[7,8] for pion-exchange amplitudes. It should be noted that our approach to the absorption of diffractive amplitudes thus differs markedly from other attempts.[9,10] Our approach is a straightforward if technically cumbersome generalization of what has been successful in $2\to 2$ reactions. Conclusions are discussed in Sec. IV.

II. EVASIVE AND NONEVASIVE π EXCHANGE IN $pp\to n\pi^+p$

The Williams[7,8] model gives a good quantitative account of absorptive corrections to evasive π-exchange amplitudes occurring in various processes.[4,8,11] Its only major phenomenological shortcoming is its inability to reproduce the decrease in absorption observed with increasing mass of the produced object.[5,12] In this sense, the model provides an estimate of the *maximum* absorption expected at a given mass.

To apply the model to $pp \to n\pi^+p$ we require the s-channel π-exchange couplings (evaluated at $t_2 = \mu^2$)[13] $\pi^s_{\lambda\mu}(J^P, m)$ for each π^+p state J^P of mass m. Since the Reggeized t-channel π-exchange couplings are

$$\pi^t_{\lambda'\mu'}(J^P, m) \propto [\alpha_\pi(t_2)]^{|\lambda'-\mu'|}, \tag{3}$$

the only contributions surviving at $t_2 = \mu^2$ arise from the t-channel nonflip couplings:

$$\pi^s_{\lambda\mu}(J^P, m) = [d^J_{1/2\lambda}(X_{\pi p})d^{1/2}_{1/2\mu}(X_p) - \tau d^J_{-1/2\lambda}(X_{\pi p})d^{1/2}_{-1/2\mu}(X_p)]\pi^t_{1/2,1/2}(J^P, m), \tag{4}$$

where τ [$=P(-)^{J-1/2}$] is the naturality of the particular π^+p partial wave, and $X_{\pi p}$ and X_p are crossing angles evaluated at $t_2 = \mu^2$.

A complete description $pp \to n\pi^+p$ is in principle afforded by the diagram of Fig. 1(c), given a knowledge of each partial wave at all values of m ($\le\sqrt{s} - m_n$). In practice the dominant Deck contribution for low $M_{n\pi^+}$ is associated with m values near $\frac{1}{2}\sqrt{s}$. For typical s, m is above the value at which phase-shift analyses are available. The standard Deck procedure[1] is to employ the diagram of Fig. 1(b) in which the entire πp elastic scattering amplitude is input, usually in the form of a fit to data.

In the present work we use only a rough knowledge of the πp partial-wave structure to estimate, via Eq. (4), the *average* s-channel couplings of the π to the $\bar{p}(\pi p)$ system. This is effected by ignoring all but the nonflip πp amplitude.[14] We calculate the impact parameter, and hence partial-wave, distribution

$$\pi^t_{1/2,1/2}(b^{\tau=+}; m) = \pi^t_{1/2,1/2}(b^{\tau=-}; m) \propto e^{-b^2/2B}, \tag{5}$$

where

$$\frac{d\sigma}{dt}(\pi^+p \to \pi^+p) \propto e^{Bt} \tag{6}$$

and

$$l = [q_{\pi p}b], \quad J = l \mp \tfrac{1}{2}, \quad \tau = \pm. \tag{7}$$

As an example, we consider $pp \to n\pi^+p$ at 15 GeV/c. The πp mass distribution contributing to the low-s_2 enhancement peaks near m ($\equiv\sqrt{s_1}$) $= 2.6$ GeV,[1] where π^+p elastic scattering data are described by $B = 7.0$ GeV^{-2}. Since, away from the edges of phase space, the crossing relation [Eq. (4)] depends less on the mass m than on the angular momentum J, we achieve a reasonable estimate of the coupling structure by using a single (average) m value (2.6 GeV) to evaluate the effect of crossing the t-channel couplings [Eq. (5)].

After a numerical summation, we find the following values for the s-channel vertices of $\pi^+\bar{p}$ (π^+p):

$$\begin{aligned} V_1 &\equiv \pi^s_{1/2,1/2}\big|_{t_2=\mu^2} = 1 \text{ (normalization)},\\ V_2 &\equiv [\pi^s_{-1/2,1/2}/(-t_2')^{1/2}]\big|_{t_2=\mu^2} = 1.40,\\ V_3 &\equiv [\pi^s_{3/2,1/2}/(-t_2')^{1/2}]\big|_{t_2=\mu^2} = -1.22. \end{aligned} \tag{8}$$

Those for the π^+p^-n vertex are

$$\begin{aligned} U_1 &\equiv \pi_{1/2,1/2} = (-t_{\min})^{1/2},\\ U_2 &\equiv [\pi_{-1/2,1/2}/(-t_2')^{1/2}]\big|_{t_2=\mu^2} = 1 \text{ (normalization)}. \end{aligned} \tag{9}$$

The nonflip amplitude combinations $U_2V_2(-t_2')$ and $U_2V_3(-t_2')$ are the evasive amplitudes of interest.

III. ABSORPTION

In the Williams model one replaces t by μ^2 in all *evasive* amplitudes. Thus a Deck pion pole squared amplitude, behaving as $F_\pi = [\sqrt{-t}/(\mu^2 - t)]^2$, becomes after absorption

$$F_\pi^{abs} = \{V_1^2(-t) + (V_2^2 + V_3^2)[\tfrac{1}{4}(-t - \mu^2 + 2t_{min})^2 + t' t_{min} + \tfrac{1}{4}(t - \mu)^2]\}/[\lambda(t - \mu^2)^2], \quad (10)$$

where

$$\lambda = [V_1^2 + (-t')(V_2^2 + V_3^2)].$$

Here, as elsewhere in this article, $(-t_{min})$ is the minimum momentum transfer carried by the pion-exchange link.

In Fig. 2, we compare numerical values of F_π and F_π^{abs} at 15 GeV/c, at two typical values of m. The difference between F_π and F_π^{abs} at very small t is an increasing function of $(-t_{min})$ and hence of m. However, inside the physical region $|t| \geq |t_{min}|$ there is no significant difference between the shapes and magnitudes of F_π and F_π^{abs}. The relatively large values of $|t_{min}|$ associated with the inelastic kinematics removes any dramatic effect associated with the π-exchange cut outside the physical region. The value of t_{min} is relatively large because when $M_{n\pi^+}$ is small (<2 GeV), the relevant values of m are a substantial fraction of $\sqrt{s}$. Since for fixed $M_{n\pi^+}$ the average value of m grows with $\sqrt{s}$ (roughly as $\frac{1}{2}\sqrt{s}$), these conclusions are also essentially energy independent.

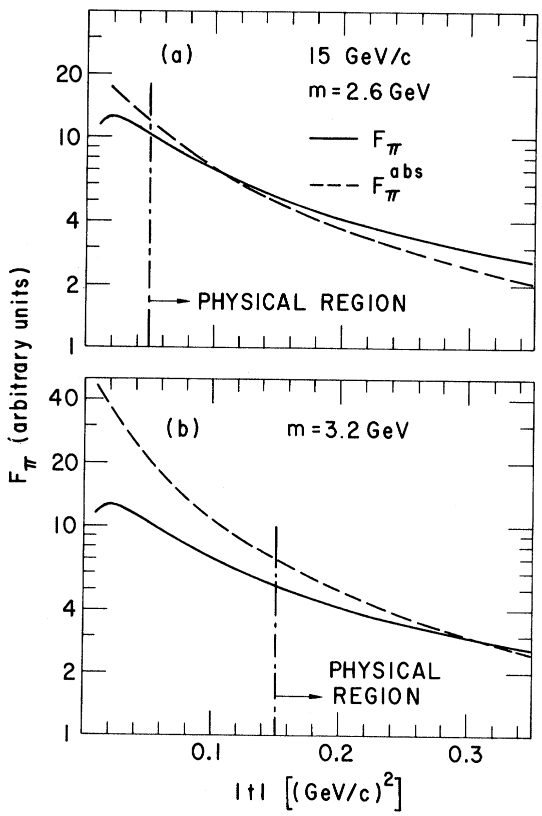

FIG. 2. Comparison of F_π (solid line) and F_π^{abs} (dashed line) as a function of t for two values of the mass of π^+p: (a) m=2.6 GeV and (b) m=3.2 GeV. The beginning of the physical region is marked by a vertical line.

IV. CONCLUSIONS

The character of pion exchange in Deck amplitudes[1] may be one of two types, which are exemplified by

$$\pi^\pm p \to (\rho\pi)^\pm p \text{ and } K^\pm p \to (K^*\pi)^\pm p. \quad (11)$$

In these cases, evasive pion exchange is negligible. Absorptive effects are not an important consideration, except for normalization:

$$pp \to (n\pi^+)p. \quad (12)$$

In principle, evasive amplitudes may be important. However, we have shown here that there is little quantitative difference between F_π and F_π^{abs} in the physical region. We conclude that pion absorption produces insignificant changes in predicted distributions which depend sensitively on the t_2 structure of the π-exchange Deck amplitude. In particular, the shape of the production momentum transfer distribution $d\sigma/dt_1$, that of the mass distribution $d\sigma/dM_{n\pi^+}$, and the spin-parity content of the low-mass enhancement will show negligible modifications. This conclusion may be generalized readily to inclusive reactions, such as $pp \to nX$ and $\pi p \to \rho X$.

We emphasize that we have considered a particular type of absorption which is connected directly to the pion-exchange line in the 2- to 3-particle amplitude. Our aim was to ascertain whether such pion absorption effects, well established in 2-particle reactions, are also relevant in 2- to 3-body processes. In this study, we employ the fairly successful Williams absorption model,[7,8] extended, with consequent uncertainties to be sure, into the as-yet-untried domain of very large recoil mass ($\sim\frac{1}{2}\sqrt{s}$) and significant t_{min}. We find that these absorptive effects appear not to play an important role. Other absorptive effects, such as final-state rescattering of the np system in Fig. 2(b), do seem to be essential. As shown elsewhere,[15] they affect the structure of the distribution $d\sigma/dt_1$ significantly but, again, leave the average t_2 distribution, the $M_{n\pi^+}$ distribution, and the spin-parity content of the low-mass enhancement basically unaltered.

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[1]For a review and list of references to earlier work, consult E. L. Berger, in Argonne Report No. ANL/HEP 7506, Daresbury Study Weekend Series No. 8, edited by J. B. Dainton and A. J. G. Hey (unpublished).

[2]We do not question whether an "s-channel" (e.g., resonant) interpretation of the low-mass enhancement may make more sense physically, nor do we take up the issue of whether other exchanges in addition to π are important in a t-channel model. See Ref. 1 for a discussion of these matters. Our aim is to investigate corrections to the pion amplitude, assuming that it is relevant.

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[13]The Williams model requires that all t-dependent factors, except that from the pion propagator and angular momentum conservation, be evaluated at $t=\mu^2$.

[14]This crude approximation is justified in obtaining a rough estimate of the size of possible effects.

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