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Comment on "Group theory of the spontaneously broken gauge symmetries"

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Several additions are made to Li's analysis of the patterns of symmetry breaking in SU(n).

In a recent paper[1] Li examined systematically the pattern of symmetry breaking in the general rotation group O(n) and unitary group SU(n). His comprehensive analysis forms a very useful guide for anyone interested in studying symmetry breaking in large groups. In the course of our work we have discovered that in a couple of respects his discussion of SU(n) is incomplete.

When SU(n) is broken by a scalar field ϕ^{ij} that transforms as a second-rank antisymmetric tensor, Li shows that the symmetry SU(n) → SU($n-2$) when $\lambda_2 < 0$. [λ_2 is one of the parameters in the potential $V(\phi)$.] The correct result is SU(n) → SU($n-2$) × SU(2). The extra SU(2) appears because, as shown by Li, the vacuum expectation value (VEV) of ϕ^{ij} can be chosen to be $\phi^{12} = -\phi^{21} = \sigma$. Since ϕ^{ij} is antisymmetric, it is a singlet under SU(2) transformations of the 1, 2 indices and remains invariant under SU($n-2$) transformations of the remaining $n-2$ indices.

Li's results are also incomplete when SU(n) is broken by a scalar field ϕ_i^j that transforms as a tensor belonging to the adjoint representation. The correct patterns of symmetry breaking are

$$\mathrm{SU}(n) \to \mathrm{SU}(l) \times \mathrm{SU}(n-l) \times \mathrm{U}(1)$$

or

$$\mathrm{SU}(n) \to \mathrm{SU}(n-1) \times \mathrm{U}(1),$$

where $l = n/2$ (n even), $(n+1)/2$ (n odd). Which case is realized depends on the parameters of the potential $V(\phi)$. The extra U(1) in either case results from the fact that the VEV of ϕ_j^i can be chosen proportional to a diagonal generator of SU(n). This diagonal generator commutes with the VEV of ϕ_j^i and generates a U(1) symmetry which is unbroken.

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[1] Ling-Fong Li, Phys. Rev. D 9, 1723 (1974).