PHYSICAL REVIEW D VOLUME 12, NUMBER 10 15 NOVEMBER 1975

Direct-channel tests of the strong-coupling Pomeron*

J. L. Cardy
Department of Physics, University of California, Santa Barbara, California 93106
(Received 19 May 1975)

We discuss how the strong-coupling renormalization-group solution of Reggeon field theory satisfies some of the simplest constraints of direct-channel unitarity, including those which lead to the decoupling theorems for the simple Pomeron pole.

The construction of a theory of high-energy diffraction scattering which is consistent with the constraints of unitarity in both the direct and crossed channels (hereafter referred to as the s and t channel, respectively) is an old problem. In s-channel models, it is difficult to see how to implement t-channel unitarity. In the Reggeon calculus,[1] which is based on the constraints of t-channel unitarity, it is not a question of enforcing s-channel unitarity, but testing to see whether it can be satisfied. While the solution of a Pomeron pole with intercept below unity appears to present a consistent picture, the weak-coupling solution of a pole exactly at unity, with asymptotically negligible cuts, runs into difficulties, chiefly with the decoupling theorems.[2]

However, it has recently become clear that this solution is not, in general, correct.[3,4] Instead, the partial-wave amplitudes, which correspond to the Green's functions of Reggeon field theory, asymptotically obey scaling laws, which, in particular, predict that the total and elastic cross sections should behave as powers of $\ln s$. Since Reggeon field theory can be derived without reference to the s channel, and claims to give a full description of the scattering amplitude in certain kinematic regions, the constraints of s-channel unitarity form an effectively independent test of the theory. Indeed, since it is likely that the predictions of the theory in its simplest form are applicable only to superhigh energies, it may be that s-channel unitarity is the only laboratory where it will ever be tested.

The renormalization-group solution of the Reggeon calculus in its purest form makes statements about only exclusive processes in which the transverse momenta are near zero, and the rapidity gaps between the particles (or, more generally, clusters) are large and of the same order of magnitude. It can also be applied to inclusive processes in certain kinematic limits, for example the triple-Regge region.[5,6] At this level, we can ask only a limited number of s-channel questions of the theory. However, since the answers to these questions are essentially independent of the details of the theory, a violation of one of the constraints would amount to a disproof of the validity of the theory in its present form. By contrast, more detailed s-channel questions would involve more model-dependent answers in the Reggeon calculus, and such constraints would serve merely to distinguish between different detailed possibilities, without modifying the whole theory. In this sense, the more restricted questions are the more important ones.

The constraints we shall consider here are the following.

1. Positivity of inclusive cross sections, including the total cross section σ_{tot}.
2. $\sigma_{el} < \sigma_{tot}$: the elastic is less than the total cross section.
3. $\tilde{\sigma}_n < \sigma_{tot}$: $\tilde{\sigma}_n$ is the cross section for the production of n particles (in general, n low-mass clusters) with large rapidity gaps of the order of the total rapidity. The theory predicts that $\tilde{\sigma}_n/\sigma_{el} \propto (\ln s)^{-n\beta}$, so it is sufficient to demand that $\beta > 0$.
4. The Froissart bound: in D dimensions of transverse momentum this is $\eta \leq D$, where $\sigma_{tot} \propto (\ln s)^{\eta}$.
5. $\sigma_{TR} < \sigma_{tot}$: σ_{TR} is the cross section obtained from integrating over the triple-Regge region (large mass diffraction).
6. $\sigma_{DTR} < \sigma_{tot}$: σ_{DTR} is the cross section obtained from integrating over the di-triple-Regge region (central production of a large-mass cluster).
7. $\tilde{\sigma}_N < \sigma_{tot}$: $\tilde{\sigma}_N$ is the cross section for the production of N large-mass clusters with large rapidity gaps between them.
8. Schwarz-inequality arguments: e.g., the 2 $\to$ 3 production amplitude is bounded by the one-particle inclusive amplitude in the triple-Regge region.

Several of these constraints have been discussed elsewhere and we shall mention them only briefly. Constraints 5 to 8 are the usual alternative starting points on the road to the decoupling theorems for the simple Pomeron pole.[2] From them one deduces the vanishing of various Pomeron couplings at zero momentum, proceeds (by more questionable arguments[7]) to the vanishing of Pomeron-Reggeon-particle vertices, and finally obtains the decoupling of the Pomeron pole from total cross sections. In this paper we shall go no further than the first step, since, in the theory under consideration, Reggeons have a sufficiently complicated structure to defy the arguments of the further stages, which assume a simple pole behavior. We note that in this theory the full Pomeron-Reggeon-particle vertex *does* vanish, but this comes about because of screening of the bare vertex by the cuts, not in spite of the cuts. As a result one cannot continue this result onto the particle pole on the Reggeon trajectory, since the cuts decouple there, leaving the bare vertex uncanceled.[8]

The derivation of the constraints 1–8 is in general quite difficult, since Reggeon field theory is not Hermitian and one cannot use positivity. The arguments are usually based on the universality hypothesis (which can be verified in the ϵ expansion) that higher-order Pomeron couplings and the couplings of more than one Pomeron to the external particles are asymptotically negligible.

With these assumptions, the elastic amplitude evaluated at rapidity Y and impact parameter $\vec{b}$ is proportional to the two-point function of the Reggeon field theory, which has the scaling form as $Y, \vec{b} \to \infty$

$$\langle 0 | \psi(Y, \vec{b}) \psi^{+}(0,0) | 0 \rangle = Y^{\eta - D\nu/2} f(\vec{b}^2/Y^{\nu}). \qquad (1)$$

The total and elastic cross sections are then proportional to Y^{η} and $Y^{2\eta - D\nu/2}$, respectively. (In this paper we use the exponent notation of Ref. 3.) Universality tells us that the asymptotic sign of the total cross section is the same as would be obtained in a theory with just a constant triple-Pomeron coupling. In such a theory we can use the renormalization group[9] to relate $i\Gamma^{(1,1)}(E,0)$, the Fourier transform of the inverse two-point function evaluated at zero momentum, to its ultraviolet behavior as $E \to -\infty$ (although this has no physical meaning)

$$i\Gamma^{(1,1)}(E,0,g,\alpha') = i\Gamma^{(1,1)}(\lambda E, 0, \tilde{g}(\lambda)) \times \exp\left\{-\int_{g}^{\tilde{g}} \frac{dg'}{\beta(g')}[1+\eta(g')]\right\}. \qquad (2)$$

So long as $\beta(g)$ and $\eta(g)$ have no singularities between the origin and the infrared-stable zero, we see that $i\Gamma^{(1,1)}(E,0)$ has the same sign for all negative values of E. Since the theory with only a constant triple-Pomeron coupling is ultraviolet free, i.e., $\tilde{g}(\lambda) \to 0$ as $\lambda \to \infty$, we have $i\Gamma^{(1,1)} \sim E$ as $|E| \to \infty$. Since this corresponds to the exchange of a simple Pomeron pole, the coefficient of the power behavior of $i\Gamma^{(1,1)}$ as $|E| \to 0$ is of the correct sign to give a positive contribution to σ_{tot}. [We must also demand that $\eta > -1$, since a factor $\Gamma(1+\eta)$ appears in taking the inverse Mellin transform to obtain the cross section. When $\eta \leq -1$ it can be shown that $\Gamma^{(1,1)}$ no longer dominates the cross section. However, all approximation schemes indicate that $\eta > 0$.] One can use similar arguments to show that other inclusive cross sections to which the calculus can be directly applied (e.g., in the triple-Regge region) are positive. In particular, the Pomeron-Pomeron total cross section is positive. This was a delicate point in the weak-coupling theory.[10]

The constraint $\sigma_{el} < \sigma_{tot}$ is satisfied because the left-hand side of Eq. (1) is bounded. In a Hermitian theory such a bound would follow immediately, since, by the Schwarz inequality, the two-point function is bounded by $\langle |\psi|^2 \rangle$, which is finite (with a suitable ultraviolet cutoff on the theory). In the non-Hermitian case, we use the functional integral representation of the two-point function to show that in the strong-coupling limit this is proportional to the spin-spin correlation function of a system of Ising spins on a lattice, which is of course bounded by unity.[11] Taking the limit $Y \to \infty$ with $\vec{b}^2/Y^{\nu}$ fixed on the right-hand side of Eq. (1), we obtain

$$\eta - \tfrac{1}{2} D\nu \leq 0 \ . \qquad (3)$$

If the inequality is strict, then $\sigma_{el}/\sigma_{tot} \to 0$. If equality is obtained, we can always choose the couplings g to the external particles small enough so that $\sigma_{el} < \sigma_{tot}$. In Ref. 11 it is also shown that the lattice model predicts $\nu \leq 2$, which, together with Eq. (3), implies the Froissart bound $\eta \leq D$.

Leaving aside constraint 3 for the moment, we turn to the decoupling arguments. The constraint $\sigma_{TR} < \sigma_{tot}$ is pictorially represented in Fig. 1. Rules for the construction of the inclusive cross section in the triple-Regge region have been given in Refs. 5 and 6. The important point is that Reggeon energy is not conserved at the central vertex. This is essentially because the rapidity of the central vertex is fixed in terms of the external invariants. The dashed line in Fig. 1 represents a discontinuity in the missing mass. This

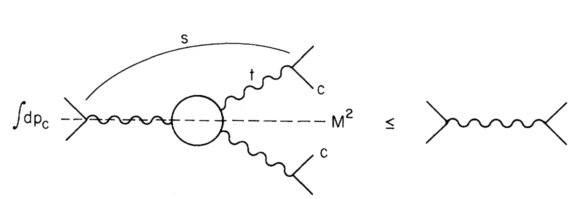

FIG. 1. Representation of the constraint $\sigma_{TR} < \sigma_{tot}$.

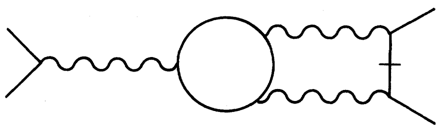

FIG. 2. Diagram giving an upper bound on σ_{TR}.

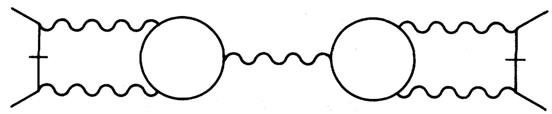

FIG. 3. Diagram giving an upper bound on σ_{DTR}.

can be taken in a completely general way using signature factors, and does not involve assuming model-dependent cutting rules.

The integration over particle c may be represented by Fig. 2. In asserting this we must show that the leading singularities in Fig. 2 are really generated in the region of the phase space implied by Fig. 1. In general, we would expect the scaling law for the triple-Regge cross section to be valid only in the region where t is small [less than $(\ln s)^{-\nu}$] and $\ln M^2$ is of the same order as $\ln s$. However, since the contributions to Fig. 2 from all parts of phase space are positive, it is clear that if we can show that the cross section represented by Fig. 2 is less than the total cross section, this will also be true for the small part of phase space in which we have a reliable formula for the inclusive cross section. If, on the other hand, we found that Fig. 2 gave a contribution which exceeded σ_{tot}, we would first have to carefully examine the phase-space integral before concluding that unitarity is violated. In fact, this will turn out not to be the case. This is a reversal of the usual logic: we are testing to see whether the theory satisfies the unitarity constraints, rather than using unitarity to constrain the theory.

In Fig. 2, the rapidity of the central vertex is integrated over, and so Reggeon energy is once again conserved, and the vertex becomes the usual full vertex which appears in the Reggeon calculus for the elastic amplitude. The dependence on Reggeon energy E of the diagram is simply determined, the anomalous dimension γ of the vertex dependence $(-E)^{\gamma}$ being given by the scaling relation[3,4]

$$\gamma = 1 + \tfrac{3}{2}\eta - \tfrac{1}{4}D\nu. \tag{4}$$

We then have for the behavior of Fig. 2

$$\sigma_{TR} \leqslant \sigma^{(2)} \propto Y^{3\eta/2 - D\nu/4}, \tag{5}$$

which is less than σ_{tot} if

$$\tfrac{1}{2}\eta - \tfrac{1}{4}D\nu \leqslant 0 \tag{6}$$

with the condition that the coupling to the external particles be small enough, if equality is obtained. We recognize this as the condition that $\sigma_{el} < \sigma_{tot}$.

This constraint has also been checked[3,6] by assuming the form for the inclusive cross section given by the ϵ expansion, but performing the phase-space integration directly in $D=2$. In this approach, the sum rule is satisfied only because of powers of $\ln\ln s$ which appear in restricting the phase-space integral. In our method, Eq. (6) is seen to be trivially satisfied within the ϵ expansion, so that the sum rule is satisfied by powers of $\ln s$. However, our argument, together with $\sigma_{el} < \sigma_{tot}$, shows that the result holds independent of the ϵ expansion.

The constraint (6) on the two-particle inclusive cross section can be tested by calculating Fig. 3. We find that it has the same power behavior as σ_{el}, so this constraint is also satisfied if $\sigma_{el} < \sigma_{tot}$. It is amusing to note that the decoupling constraints 5 and 6 in the case of a simple Pomeron pole are only effective for $D \leqslant 2$. If physics were in $2+\epsilon$ dimensions, such arguments for the triple-Pomeron zero would not have appeared. This is not true of the next constraint.

We consider[12] the contribution to the total cross section from the production of two large-mass clusters such that the rapidity gap between the clusters is of the same order as the total rapidity. This involves the absorptive part of the Pomeron-particle elastic amplitude, which at large rapidities is presumably dominated by the exchange of a single Pomeron. This argument is illustrated in Fig. 4. It needs three important qualifications:

(i) To avoid double counting, it is necessary to localize the centers of the clusters at fixed points on the rapidity axis, and also to limit their size. However, we can appeal to a positivity argument similar to the above to ignore this restriction unless a violation is obtained.

(ii) It is not proven that the exclusive production of large-mass clusters is mediated by single-Pomeron exchange. In the case of low-mass clusters one can argue this on the basis of universality. However, it can be seen that any more complicated contributions would be reflected in higher-order contributions to the diagram on the right, which are either of the same order of magnitude by scal-

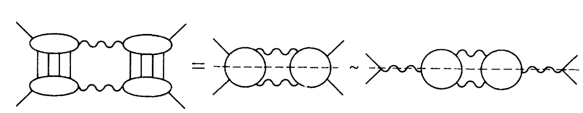

FIG. 4. Contribution of two high-mass clusters to σ_{tot}.

ing, or negligible by universality.

(iii) The discontinuity indicated by the dashed line in the diagram on the right cannot be taken by simply using the signature factors, since the diagram has other discontinuities in the total energy. Instead, it must be calculated by assuming some kind of cutting rules for the bare Pomeron, for example those suggested by Abramovskii, Gribov, and Kancheli. We assume that, whatever convention is used, the discontinuity indicated by the dashed line is proportional to the total discontinuity.

With these provisos, we see that the constraint is saturated up to a constant, since the scaling relation (4) implies that the diagram on the right of Fig. 4 has the same power behavior as the total cross section. Without them, it is difficult to see how the argument places any constraint on the theory. These arguments can be extended without any further difficulty to the multiple production of widely spaced high-mass clusters considered in Ref. 12. Despite the fact that this constraint is saturated, the result does not give any information as to how σ_{tot} is built up by production processes, because the internal structure of the high-mass clusters is undetermined. Since the Pomeron-Pomeron and Pomeron-particle cross sections are presumably built up in the same way as the particle-particle cross section, any statement based on the above result becomes a tautology.

Finally, we consider the Schwarz-inequality constraints. These are illustrated in their simplest form in Fig. 5. In these diagrams all momentum transfers are zero, and dashed lines represent discontinuities in the relevant channels. In the case of a simple Pomeron pole, this inequality can be shown to place no constraint on the full Pomeron-Pomeron-particle vertex.[7] In the strong-coupling case the vertex structure is more complicated and this argument probably does not work. The constraint can be simply expressed in terms of the anomalous dimensions γ and γ' of the triple-Pomeron vertex and the Pomeron-Pomeron-particle vertex, respectively: namely $2\gamma' \geq \gamma$. This inequality is saturated[3] to $O(\epsilon)$, but strictly satisfied[13] to $O(\epsilon^2)$. The reasons for this are discussed in Ref. 13. It appears difficult to establish

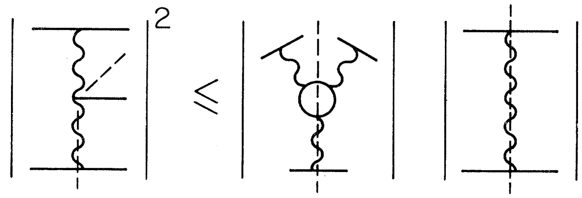

FIG. 5. Schwarz inequality.

this result in general, and this has not yet been achieved.

We note that if the two-loop calculations[14] give an accurate estimate of γ' and γ in $D=2$, then the inequality will apply by the arguments of Ref. 13.

It is interesting to rewrite the above inequality in another way. γ' appears in the cross section $\tilde{\sigma}_1$ for producing a particle in the central region, and can be shown by straightforward scaling arguments to be related to the exponent β by

$$\beta = 2\gamma' - 2\eta + \tfrac{1}{2}D\nu - 1. \qquad (7)$$

Using Eq. (4), the Schwarz inequality is equivalent to

$$\beta \geq -\tfrac{1}{2}\eta + \tfrac{1}{4}D\nu. \qquad (8)$$

The right-hand side is positive if $\sigma_{el} < \sigma_{tot}$, implying that $\beta > 0$. Therefore constraints 2 and 8 together imply 3. The Schwarz inequality is also satisfied by the results of the high-temperature expansion,[15] since the left- and right-hand sides of Eq. (8) are approximately 0.94 and 0.42 respectively. The inequality is strongly violated if the mass term is taken to be zero. The value of β is then far too small. The $O(\epsilon^2)$ value of β at $D=2$ is 0.74.

We have discussed how the strong-coupling solution of Reggeon field theory satisfies some of the simplest constraints of s-channel unitarity, which can be answered in a largely model-independent fashion. While some of these points have been previously considered, it is interesting to bring them together and show how they logically relate to each other. The result $\sigma_{el} < \sigma_{tot}$, which follows from the field-theoretic formulation of the Reggeon calculus, appears to play a central role. These studies show that the strong-coupling Pomeron is probably the leading candidate for a consistent theory of high-energy diffraction scattering, and that it is sufficient to take a quite general Reggeon field theory, without assuming any decouplings at the level of the bare theory.[16] The outstanding problem is to show how the rising total cross section is built up out of the partial cross sections, which means obtaining a reliable expression for σ_n when n is near the mean multiplicity. Such an expression will no doubt depend on more model-dependent details, but may also give information on the predictions of the consistent theory in the nonasymptotic regime.

The author would like to thank the Theoretical Physics Department of the Fermi National Accelerator Laboratory, where this work was carried out, for its hospitality, and also acknowledges stimulating conversations with J. Bartels and E. Rabinovici.

*Work supported by the National Science Foundation.

[1]For a review of the Reggeon calculus, as well as references to earlier work, see H. D. I. Abarbanel, J. B. Bronzan, R. L. Sugar, and A. R. White, Phys. Rep. 21C, 119 (1975).

[2]For a review of the decoupling arguments, see R. C. Brower and J. H. Weis, Rev. Mod. Phys. 47, 605 (1975).

[3]A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan, Phys. Lett. 48B, 239 (1974); Zh. Eksp. Teor. Fiz. 67, 848 (1974) [Sov. Phys.—JETP 40, 420 (1974)].

[4]H. D. I. Abarbanel and J. B. Bronzan, Phys. Lett. 48B, 345 (1974); Phys. Rev. D 9, 2397 (1974).

[5]J. L. Cardy, R. L. Sugar, and A. R. White, Phys. Lett. 55B, 384 (1975).

[6]H. D. I. Abarbanel, J. Bartels, J. B. Bronzan, and D. Sidhu, Phys. Rev. D 12, 2798 (1975).

[7]J. L. Cardy and A. R. White, Nucl. Phys. B80, 12 (1974).

[8]J. Bartels and E. Rabinovici, Phys. Lett. 58B, 171 (1975).

[9]R. L. Sugar and A. R. White, Phys. Rev. D 10, 4074 (1974).

[10]C. T. Sachrajda, Nucl. Phys. B83, 321 (1974).

[11]J. L. Cardy and R. L. Sugar, Phys. Rev. D 12, 2514 (1975).

[12]R. C. Brower, A. H. Mueller, A. Sen, and J. H. Weis, Phys. Lett. 46B, 105 (1973).

[13]M. Baker, Phys. Lett. 51B, 158 (1974); Nucl. Phys. B80, 61 (1974).

[14]J. W. Dash and S. J. Harrington, Report No. OITS-75-2; Phys. Lett. 57B, 78 (1975).

[15]J. Ellis and R. Savit, Nucl. Phys. B94, 477 (1975).

[16]R. C. Brower and J. Ellis, Phys. Lett. 51B, 242 (1974).