SU(4)-symmetric strong-coupling theory

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SU(4)-symmetric strong-coupling theory is investigated using the method of induced representations. Isobar content is discussed for various choices of the little group. For a particularly interesting case, the unitarity condition is solved.

I. INTRODUCTION

It appears that SU(4) symmetry may have application in classifying hadronic states. Since SU(3)-symmetric strong-coupling theory had some success in classifying isobar states,[1] in relating meson-baryon coupling constants, and in deducing properties of the electromagnetic current,[2] it seems reasonable to consider the SU(4)-symmetric model.

Cook, Goebel, and Sakita[1] formulated the static strong-coupling model in algebraic language, which permits the techniques of representation theory to be used to obtain solutions. This algebraic formulation of the strong-coupling model leads to the following equations:

$$[A_\beta, A_\alpha] = 0\,, \tag{1}$$

$$\Lambda_{\beta\alpha} = \Lambda_{\beta\gamma}\Lambda_{\gamma\alpha}\,, \tag{2}$$

where

$$\Lambda_{\beta\alpha} = [A_\beta[\mathfrak{M}, A_\alpha]] \tag{3}$$

and $\mathfrak{M}$ is the mass operator. These operators are to be considered as operators acting in isobar space. The A_β are proportional to the meson currents, and thus their matrix elements in isobar space are proportional to meson-isobar-isobar coupling constants. Equation (1) is referred to as the strong-coupling condition, and Eq. (2) is the unitarity condition. The imposition of an internal-symmetry group comes from two requirements; namely, A_α transform in a particular way with respect to the group, and $\mathfrak{M}$ be an invariant. That is, for internal-symmetry generators F_α

$$[F_\alpha, A_\beta] = i f_{\alpha\beta\gamma} A_\gamma \tag{4}$$

and

$$[F_\alpha, \mathfrak{M}] = 0\,. \tag{5}$$

Since in this paper we are concerned with p-wave mesons forming a 15-dimensional representation of SU(4) (the adjoint representation), we attach a vector index on A_β and have

$$[J_i, A_{\beta j}] = i\epsilon_{ijk} A_{\beta k}\,, \tag{4$'$}$$

$$[J_i, \mathfrak{M}] = 0\,. \tag{5$'$}$$

Thus the SU(4) strong-coupling theory has a group structure, G, defined by the semidirect product of $K = \mathrm{SU}(4)\times \mathrm{SU}_J(2)$ and a 45-dimensional Abelian group T_{45}, that is,

$$G = K \times T\,. \tag{6}$$

The Lie algebra structure is given by Eqs. (1), (4), (4′), and

$$[F_\alpha, F_\beta] = i f_{\alpha\beta\gamma} F_\gamma \tag{7}$$

$$[J_i, J_j] = i\epsilon_{ijk} J_k \tag{8}$$

$$[F_\alpha, J_i] = 0\,. \tag{9}$$

The F_α are chosen so that $f_{\alpha\beta\gamma}$ are odd under permutation of any two indices (see Appendix). We consider mass operators of the form

$$\mathfrak{M} = aF^2 + bJ^2\,, \tag{10}$$

where $F^2 = \sum_\alpha F_\alpha F_\alpha$, the quadratic Casimir operator of SU(4), and $J^2 = \sum_i J_i J_i$.

We will investigate various unitary irreducible representations (UIR's) of G by the induced representation technique.[3] With a mass operator of the form given by G, if the unitarity condition, Eq. (2), is satisfied at one point of the orbit it will be satisfied identically.

Because the various SU(2) subgroups of SU(4), modulo conjugation, are of particular interest in this problem, the nature of these SU(2) subgroups is discussed in Sec. II. In Sec. III we investigate the various possibilities for little groups and their general implications for isobar content. In Sec. IV we consider in more detail the isobar content when the little group contains an SU(2) group. The mass operators that satisfy Eq. (2) are given for the most physically interesting case. Some concluding remarks are given in Sec. V.

II. SU(2) SUBGROUPS OF SU(4)

In this section we study two questions concerning the SU(2) subgroups of SU(4), namely, what are the different SU(2) subgroups and what are their representation contents in the 15-dimensional representation of SU(4)?

To determine the classes of conjugate SU(2) subgroups of SU(4) we merely note that there are four unitary, nontrivial, nonequivalent 4-dimensional representations of SU(2): (a) $\underline{4}$, (b) $\underline{3} \oplus \underline{1}$, (c) $\underline{2} \oplus \underline{2}$, and (d) $\underline{2} \oplus \underline{1} \oplus \underline{1}$. The third component, L_3, of the generator of the SU(2) subgroup can always be chosen from the maximal Abelian subalgebra of the SU(4) algebra; that is, L_3 can be chosen to be a linear combination of F_3, F_8, and F_{15}. (See Appendix.) A knowledge of the eigenvalues of L_3 for any representation of SU(4) permits one to deduce the SU(2) representation content. In the following Y is the usual hypercharge, a generator of SU(3), and $Z=(\frac{3}{2})^{1/2}F_{15}$ agrees with the definition in Ref. 4. F_{15} (and thus Z) commutes with all SU(3) generators.

Case (a): The generators of the SU(2) subgroup can be chosen to be

$$\begin{aligned} L_3 &= F_3 + \sqrt{3}\,F_8 + (\tfrac{2}{3})^{1/2} 3F_{15} \\ &= F_3 + \tfrac{3}{2}Y + 2Z\,, \\ L_2 &= \sqrt{3}\,F_2 + 2F_7 + \sqrt{3}\,F_{14}\,, \\ L_1 &= \sqrt{3}\,F_1 + 2F_6 + \sqrt{3}\,F_{13}\,. \end{aligned} \tag{11}$$

Consideration of the eigenvalues of L_3 in the 15-dimensional UIR of SU(4) shows that this SU(2)'s representation content is

$$\underline{15} = \underline{7} \oplus \underline{5} \oplus \underline{3}\,. \tag{12}$$

Case (b): The SU(2) generators are

$$\begin{aligned} L_3 &= F_3 + \sqrt{3}F_8 = F_3 + \tfrac{3}{2}Y\,, \\ L_2 &= \sqrt{2}F_2 + \sqrt{2}F_7\,, \\ L_1 &= \sqrt{2}F_1 + \sqrt{2}F_6\,, \end{aligned} \tag{13}$$

and the reduction of the $\underline{15}$ UIR with respect to this SU(2) is

$$\underline{15} = \underline{5} \oplus \underline{3} \oplus \underline{3} \oplus \underline{3} \oplus \underline{1}\,. \tag{14}$$

Case (c): The SU(2) generators are

$$\begin{aligned} L_3 &= F_3 + (\tfrac{2}{3})^{1/2}F_{15} - (\tfrac{1}{3})^{1/2}F_8 = F_3 + \tfrac{2}{3}Z - \tfrac{1}{2}Y\,, \\ L_2 &= F_2 + F_{14}\,, \\ L_1 &= F_1 + F_{13}\,, \end{aligned} \tag{15}$$

and the reduction of the $\underline{15}$ UIR with respect to this SU(2) is

$$\underline{15} = \underline{3} \oplus \underline{3} \oplus \underline{3} \oplus \underline{3} \oplus \underline{1} \oplus \underline{1} \oplus \underline{1}\,. \tag{16}$$

Case (d): The SU(2) generators are

$$\begin{aligned} L_3 &= F_3\,, \\ L_2 &= F_2\,, \\ L_1 &= F_1\,. \end{aligned} \tag{17}$$

The SU(2) subgroup is isospin. The reduction of the $\underline{15}$ UIR with respect to this SU(2) is

$$\underline{15} = \underline{3} \oplus \underline{2} \oplus \underline{2} \oplus \underline{2} \oplus \underline{2} \oplus \underline{1} \oplus \underline{1} \oplus \underline{1} \oplus \underline{1}\,. \tag{18}$$

III. REPRESENTATIONS AND THEIR ISOBAR CONTENT

We have seen in Sec. I that we must consider UIR's of $G = K \times T$. Cook and Sakita[3] have investigated the representation of such groups by the method of induced representations. We will indicate their results.

A representation g of G is determined by specifying an orbit, and thus a little group $K_0 \subset K$ for one point on the orbit, and a representation k_0 of K_0. The induced UIR, g, of G, when reduced with respect to K, will contain any UIR of K, k, that has in it a representation k_0 of K_0. The UIR of K, k, will occur as many times as k_0 is in k. Thus, to investigate the possible isobar content of the UIR's of G we merely have to investigate the possible little groups and their representations.

Let us first consider the implications of the content of K_0 with respect to $\mathrm{SU}_J(2)$. Suppose $K_0 \subset \mathrm{SU}(4)$; then in the UIR, g, of G, there is no connection between the isobar's spin and its SU(4) content. Thus, all spins have the same SU(4) content. We will not consider this case.

Recall that the $A_{\alpha i}$ transform like a vector with respect to $\mathrm{SU}_J(2)$. The little group $K_0 \subset K$ is that group which at a particular point on the orbit of characters of T leaves the point unchanged. For instance, if the only nonzero characters at a point in the orbit are those of $A_{\alpha 3}$, then $K_0 \supset \mathrm{U}(1)$ with U(1) generated by J_3. Then again all spins that occur in the induced UIR of G have the same SU(4) content. Another possibility would be that the $\mathrm{U}(1) \subset K_0$ may have as a generator a linear combination of J_3 with a generator of SU(4). In this case if a given spin, J_0, has a given SU(4) multiplet, then all higher spins have at least the same SU(4) multiplet present. We will consider these cases no further.

The cases to be considered have been reduced to include only those such that K_0 contains an $\mathrm{SU}_{I_0}(2)$ generated by $I_{0i} = L_i + J_i$, where L_i are generators of a $\mathrm{SU}(2) \subset \mathrm{SU}(4)$. As shown in Sec. II there are four distinct conjugate SU(2) subgroups. Let $(b_0)_{\beta i} F^\beta$ be the generator L_i for a particular point on the orbit with character $(a_0)_{\alpha i}$; then

$$(b_0)_{\alpha i} f_{\alpha\rho\Sigma}(a_0)_{\Sigma j} + \epsilon_{ijk}(a_0)_{\rho k} = 0 \tag{19}$$

is the condition that the point transforms like a scalar under $SU_I(2)$. One possibility for $(a_0)_{\rho k}$ is

$$(a_0)_{\rho k} = C(b_0)_{\rho k} \tag{20}$$

with C some arbitrary constant.

For the cases (a) and (d) of Sec. II, since the reduction of the $\underline{15}$ UIR of SU(4) with respect to SU(2) contains only one triplet, condition (20) is the only possibility. We will consider only cases for which Eq. (20) is satisfied. Furthermore, Eq. (20) fixes a point in the orbit and thus determines the complete little group. The isobar content of UIR's of G for this little group is discussed in the next section.

IV. UIR FOR WHICH $K_0 \supset SU_{I_0}(2)$

From the discussion of the previous section it is clear that the interesting little groups to consider are those containing an $SU_{I_0}(2)$. Additional generators of the little group must transform like scalars with respect to the SU(2) subgroup of SU(4). Thus, from the knowledge of the SU(2) content of the 15-dimensional UIR of SU(4), Eqs. (12), (14), (16), and (18), for case (a) $K_0 = SU_{I_0}(2)$; for case (b) $K_0 = SU_{I_0}(2) \times U(1)_Z$ where $U(1)_Z$ is generated by Z; for case (c) $K_0 = SU_{I_0}(2) \times SU'(2)$ where $SU'(2)$ has for generators

$$\frac{A_1^3 + A_3^1}{2} + \frac{A_2^4 + A_4^2}{2},$$

$$i\left(\frac{A_1^3 - A_3^1}{2} + \frac{A_2^4 - A_4^2}{2}\right),$$

and

$$-(\tfrac{2}{3}Z + Y);$$

and for case (d) $K_0 = SU_{I_0}(2) \times U(1)_{Y+2Z/3} \times SU'(2)$ where $SU'(2)$ is generated by

$$\frac{A_3^4 + A_4^3}{2},$$

$$i\frac{A_4^3 - A_3^4}{2},$$

and

$$-\tfrac{1}{2}Y + \tfrac{2}{3}Z.$$

We will now note the low-dimensional representation content of K contained in the UIR of G for particular choices of K_0 and representations of K_0. Generally, we consider the smallest-dimensional representation of the little group, consistent with half-integer-spin multiplets, since this implies the smallest multiplicity of isobar representation. We list in Table I the isobar content for such cases. The notation designating the SU(4) representation is that of Ref. 4.

TABLE I. Low-dimensional representation content of K contained in the UIR of G for particular choices of K_0 and representations of K_0. (a) $K_0 = SU_{I_0}(2)$. (b) $K_0 = SU_{I_0}(2) \times U(1)_Z$. For this case a UIR of SU(4) contains only integer representations of SU(2). We must then consider half-integer representations of $SU_{I_0}(2)$. (c) $K_0 = SU_{I_0}(2) \times SU'(2)$. (d) $K_0 = SU_{I_0}(2) \times SU'(2) \times U(1)_{Y+2Z/3}$.

Rep. of K_0	SU(4) rep.	Spin	Multiplicity
	(a)		
$I_0 = 0$	$\underline{4}$	$\frac{3}{2}$	1
	$\underline{20}$	$\frac{3}{2}$	1
	$\underline{20}$	$\frac{5}{2}$	1
	$\underline{20}$	$\frac{7}{2}$	1
	$\underline{20'}$	$\frac{3}{2}$	1
	$\underline{20'}$	$\frac{5}{2}$	1
	$\underline{20'}$	$\frac{7}{2}$	1
	(b)		
$I_0 = \frac{1}{2}$	$\underline{20}$	$\frac{1}{2}$	1
$Z = \frac{3}{4}$	$\underline{20}$	$\frac{3}{2}$	1
	$\underline{20}$	$\frac{5}{2}$	1
	$\underline{20'}$	$\frac{1}{2}$	1
	$\underline{20'}$	$\frac{3}{2}$	1
	...	...	...
$I_0 = \frac{1}{2}$	$\underline{20}$	$\frac{3}{2}$	1
$Z = \frac{1}{4}$	$\underline{20}$	$\frac{5}{2}$	1
	$\underline{20'}$	$\frac{1}{2}$	1
	$\underline{20'}$	$\frac{3}{2}$	2
	...	...	...
	(c)		
$I_0 = 0$	$\underline{20}$	$\frac{3}{2}$	1
	$\underline{20'}$	$\frac{1}{2}$	1
$L' = \frac{3}{2}$	$\overline{\underline{20}}$	$\frac{3}{2}$	1
	$\overline{\underline{20'}}$	$\frac{1}{2}$	1
	...	...	...
	(d)		
$I_0 = 0$	$\underline{20}$	$\frac{3}{2}$	1
$L' = 0$	$\underline{20'}$	$\frac{1}{2}$	1
	$\overline{\underline{36}}$	$\frac{1}{2}$	1
$Y + \frac{2}{3}Z = \frac{3}{2}$	$\overline{\underline{56}}$	$\frac{3}{2}$	1
	...	...	...

Cases (a)–(d) refer to the classification of the SU(2) subgroups of Sec. II.

V. MASS OPERATORS FOR CASE (d)

It is clear from the previous section that the most interesting representation occurs for case (d), for which the little group is $SU_{I_0}(2)\times U(1)\times SU'(2)$. For case (c) the spin-$\frac{1}{2}$, $\underline{20}'$, and $\overline{\underline{20}'}$ representations are degenerate since the quadratic Casimir operator has the same value for conjugate representations. For the mass operator given by Eq. (10) in general

$$\Lambda_{\beta j,\alpha i}=af_{\beta\sigma\tau}f_{\alpha\rho\tau}A_{\sigma j}A_{\rho i}+b\epsilon_{jnt}\epsilon_{imt}A_{\beta n}A_{\alpha m}\,. \tag{21}$$

Defining

$$\alpha^2=\tfrac{1}{3}A_{i\alpha}A_{i\alpha}$$

for one point on the orbit, for case (d)

$$A_{i\alpha}=\alpha\delta_{i\alpha}$$

and thus

$$\Lambda_{\beta j,\alpha i}=\alpha^2 af_{\beta j\tau}f_{\alpha i\tau}+\alpha^2 b\epsilon_{j\beta t}\epsilon_{i\alpha t}\,. \tag{22}$$

The unitarity condition, Eq. (2), then determines the allowed values of a and b. A particularly simple way of solving this condition is to consider the $f_{\beta\tau j}$, for $j=1,2,3$, as a set of SU(2) operators in a 15-dimensional space. Let $(J_i)_{\beta\tau}=f_{\beta\tau i}$. J has a reduction as noted by Eq. (18). Also let $(j_i)_{\beta\tau}=\epsilon_{\beta\tau i}$, i.e., they are SU(2) generators in this 15-dimensional space which has a reduction to one spin 1 and 12 scalars. Now the unitarity condition is

$$\sum_i(\alpha^2 aJ_jJ_i+\alpha^2 bj_jj_i)(\alpha^2 aJ_iJ_k+\alpha^2 bj_ij_k)=\alpha^2 aJ_jJ_k+\alpha^2 bj_jj_k\,.$$

Thus

$$\alpha^4a^2J_jJ^2J_k+2\alpha^4abj_jj\cdot JJ_k+\alpha^4b^2j_jj^2j_k=\alpha^2aJ_jJ_k+\alpha^2bj_ij_k\,.$$

Since the j_i acting on any part of the space except the spin-part-1 give zero we must have

$$\alpha^4a^2JjJ^2J_k=\alpha^2aJjJ_k$$

for $J^2=0$ and $\frac{1}{2}(\frac{1}{2}+1)=\frac{3}{4}$ and thus

$$\alpha^2a=\tfrac{3}{4}\alpha^4a^2\,. \tag{23}$$

For the spin-1 part of the space $J^2=j\cdot J=j^2=2$ we must have

$$2\alpha^4a^2+4\alpha^4ab+2\alpha^4b^2=\alpha^2a+\alpha^2b\,. \tag{24}$$

Equations (23) and (24) have four solutions[5]:

	$\alpha^2 a$	$\alpha^2 b$	Λ
(I)	0	0	0
(II)	0	$\frac{1}{2}$	Λ^{II}
(III)	$\frac{4}{3}$	$-\frac{4}{3}$	Λ^{III}
(IV)	$\frac{4}{3}$	$-\frac{5}{6}$	Λ^{IV}

It should be noted that for any SU(N) strong-coupling model using as a little group $K_0=SU_{I_0}(2)\times U(1)\times SU(N-2)$ the above arguments apply and the same four solutions for the mass matrix result.

Goebel,[6] using a Hamiltonian formulation of the static strong-coupling model, argues that the physical solutions are such that the projection operator Λ (note $\Lambda^2=\Lambda$) should project at each point on the orbit onto a space whose dimension should be the dimension of K minus the dimension of K_0, the little group. For case (d) this is $18-7=11$.

From Eq. (22)

$$\begin{aligned}\mathrm{Tr}\Lambda&=\alpha^2af_{\beta i\tau}f_{\beta i\tau}+\alpha^2b\epsilon_{i\beta t}\epsilon_{i\beta t}\\&=\alpha^2a\,\mathrm{Tr}J^2+\alpha^2b\,\mathrm{Tr}j^2\\&=\alpha^2a[6+(4)(2)(\tfrac{1}{2})(\tfrac{1}{2}+1)]+\alpha^2b[6]\\&=12\alpha^2a+6\alpha^2b\,.\end{aligned}$$

$\mathrm{Tr}\Lambda^{\mathrm{II}}=3$, $\mathrm{Tr}\Lambda^{\mathrm{III}}=8$, and $\mathrm{Tr}\Lambda^{\mathrm{IV}}=11$, implying that IV is the solution that would result from a Hamiltonian formulation. Again it is easy to see that this trace condition holds for Λ^{IV} for the SU(N) strong-coupling model.

For SU(4) F^2 has value $\frac{63}{8}$ for the $\underline{20}$ UIR of SU(4) and $\frac{39}{8}$ for $\overline{\underline{20}'}$, the smallest UIR present in case (d). Thus, for solutions II, III, and IV the $(\underline{20}, J=\frac{3}{2})$ states have larger mass than the $(\overline{\underline{20}'}, J=\frac{1}{2})$ states.

VI. DISCUSSION

We have seen that in the SU(4) strong-coupling theory the requirements that the low-lying spin-$\frac{1}{2}$ states contains only one octet and no decuplet and that the low-lying spin-$\frac{3}{2}$ states contain only one decuplet and no octet leads to a unique little group, case (d), and a unique representation of this little group. It appears to be the least degenerate representation of the strong-coupling group possible— the little group is the largest. The unitarity condition was solved for this case, with the result that the $(\underline{20}', \frac{3}{2})$ has a higher mass than the $(\underline{20}', \frac{1}{2})$. It is premature to investigate the mass relationships of solution IV further than this. It was noted that the resulting mass operator in fact solves any SU(N) strong-coupling model.

It is also interesting to note that the electromagnetic current is uniquely defined in the sense of Ref. 2 if one assumes that the current transforms like the 15-dimensional UIR of SU(4).

Also for this case the charge operator, Q, must be of the form

$$Q = I_3 + \tfrac{1}{2}Y + \beta(\tfrac{1}{3}Z - \tfrac{1}{4}B)$$

if the decuplet and octet are to have the correct charge assignments. This charge operator is that defined by Ref. 7 for $\beta = 1$, and is that of Ref. 8 for $\beta = -2$.

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APPENDIX

The 16 generators of U(4), A^i_k ($i, k = 1, 2, 3$, or 4), satisfy the commutation relation

$$[A^i_k, A^j_l] = \delta^i_l A^j_k - \delta^j_k A^i_l \ .$$

The generators, F_α ($\alpha = 1, 2, \ldots, 15$), of SU(4) are defined by

$$F_1 = \tfrac{1}{2}(A^1_2 + A^2_1), \quad F_2 = \tfrac{1}{2}i\,(A^1_2 - A^2_1),$$
$$F_3 = \tfrac{1}{2}(A^1_1 - A^2_2), \quad F_4 = \tfrac{1}{2}(A^3_1 + A^1_3),$$
$$F_5 = \tfrac{1}{2}i(A^1_3 - A^3_1), \quad F_6 = \tfrac{1}{2}(A^2_3 + A^3_2),$$
$$F_7 = \tfrac{1}{2}i(A^2_3 - A^3_2), \quad F_8 = (1/\sqrt{3})(\tfrac{1}{2}A^1_1 + \tfrac{1}{2}A^2_2 - A^3_3),$$
$$F_9 = \tfrac{1}{2}(A^4_1 + A^1_4), \quad F_{10} = \tfrac{1}{2}i(A^1_4 - A^4_1),$$
$$F_{11} = \tfrac{1}{2}(A^4_2 + A^2_4), \quad F_{12} = \tfrac{1}{2}i(A^2_4 - A^4_2),$$
$$F_{13} = \tfrac{1}{2}(A^4_3 + A^3_4), \quad F_{14} = \tfrac{1}{2}i(A^3_4 - A^4_3),$$
$$F_{15} = (\tfrac{3}{2})^{1/2}(\tfrac{1}{6}A^1_1 + \tfrac{1}{6}A^2_2 + \tfrac{1}{6}A^3_3 - \tfrac{1}{2}A^4_4)\,.$$

F_α, ($\alpha = 1, \ldots, 8$) are the generators of SU(3). $F_8 = \tfrac{1}{2}\sqrt{3}\,Y$, where Y is the usual hypercharge, and $Z = (\tfrac{3}{2})^{1/2}F_{15}$. F_α's satisfy commutation relations

$$[F_\alpha, F_\beta] = i f_{\alpha\beta\gamma} F_\gamma\,,$$

where the structure constants, $f_{\alpha\beta\gamma}$, are antisymmetric under the interchange of any two indices.

The quadratic Casimir operator of SU(4), F^2, is given by

$$F^2 = \sum_\alpha F_\alpha F_\alpha$$

$$F^2 = \tfrac{1}{2}\sum_{\mu,\nu} A^\mu_\nu A^\nu_\mu$$

for all U(4) representations for which $A^\mu_\mu = 0$.

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