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Scalar bound states and dynamical symmetry breaking*

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The possibility of a model with a massive fermion, axial-vector and scalar mesons, and a massless vector meson originating as a dynamical consequence of a chiral U(1) × U(1) gauge-invariant model with a massless fermion and vector and axial-vector mesons is explored. A recent approach due to Cornwall which makes use of an effective Lagrangian combined with the Callan-Symanzik equations, and its extension, are used with an assumption similar to that of the Baker-Johnson finite quantum electrodynamics. The physical quartic scalar coupling constant and the ratios of physical masses of axial-vector and scalar mesons with respect to that of fermions are expressed in terms of the vector and axial-vector gauge coupling constants.

I. INTRODUCTION

At the present time there are two ways of inducing spontaneous symmetry breakdown in gauge theories. One way is to introduce elementary Higgs scalar fields[1] with negative mass squared which lead to nonzero vacuum expectation values for scalar fields and generation of mass for vector mesons; such theories have been shown to be renormalizable and unitary.[2] We shall term this mechanism Higgs symmetry breaking (HSB) for brevity.[1] The other way is dynamical symmetry breaking[3] (DSB) which envisages the possibility of existence of nonperturbative solutions (with massive vector mesons) to the field equations of a symmetric theory without elementary scalar fields. DSB is attractive because it does not necessitate the existence of elementary scalar particles which do not seem to have been observed experimentally and which, in general, spoil the asymptotic freedom of non-Abelian gauge theories. The main hurdle one faces in any investigation of DSB is that it involves solutions to homogeneous integral equations for which conventional perturbation theory fails. Recently Cornwall[4] (JMC) has elucidated a novel method to deal with DSB without any scalar bound states. He starts out with an O(2)×O(2) gauge-invariant model consisting of a doublet of fermions with the same mass and two massless vector mesons. Each of the vector mesons corresponds to a separate O(2) symmetry. One of the symmetries is spontaneously broken giving a mass to the associated vector meson and mass difference between fermions. In order to show how the latter originates, JMC writes an effective Lagrangian which has the required mass spectrum and which is gauge-invariant (in a restricted sense) and renormalizable but nonlocal and nonpolynomial. Using the Callan-Symanzik (CS) equations with β coefficients for gauge couplings equal to zero, he shows that the bare mass terms in the effective Lagrangian vanish in the limit of infinite cutoff even though the corresponding renormalized masses are finite, and thus he demonstrates that the effective Lagrangian is a phenomenological representation of the nonperturbative solution with relevant finite physical masses of the original symmetric theory. Note that having zero β coefficients in the CS equations is analogous to the Baker-Johnson[5,6] approach to finite quantum electrodynamics.

In this paper we investigate the possibility of having a scalar meson as a bound state in the framework of a dynamical symmetry broken model using the approach of JMC outlined above and its extension. We start out with a chiral U(1)×U(1) gauge-invariant model with a fermion and vector and axial-vector mesons all of which are massless. (The triangle anomaly in this model can be removed by increasing the number of fermions.) The desired physical mass spectrum after DSB consists of a massless vector meson in addition to a fermion and scalar and axial-vector mesons all of which are massive. We assume that β coefficients for gauge couplings and quartic scalar coupling in the CS equations are zero. Using methods analogous to that of JMC, we demonstrate that, in a Lagrangian which has the desired mass spectrum, under certain conditions various bare masses and other symmetry-breaking terms vanish in the limit of infinite cutoff, even though the corresponding physical masses and coupling constants are finite. We use the vanishing of the wave-function renormalization constant ($Z_{2\sigma}$), the vertex renormalization constant ($Z_{1\lambda}$) of quartic coupling of the scalar meson, and the ratio $Z_{1\lambda}/Z_{2\sigma}$ as criteria for compositeness.[7,8] Our analyses lead to expressions for the fermion–axial-vector-meson mass ratio, the fermion–scalar-meson mass ratio, and the value of the renormalized quartic scalar coupling constant in terms of vector and axial-vector coupling constants.

In the next section we describe symmetric models with and without scalar fields and give an expression for the symmetry-broken Lagrangian in the unitary gauge due to the Higgs mechanism. Guided by this, in Sec. III we construct the effective Lagrangian and give a discussion of masses and renormalization constants. The CS equations for the effective Lagrangian are written in Sec. IV. After calculating the coefficients in the CS equations, asymptotic expressions for various quantities and constraints for DSB are obtained. The effective Lagrangian is compared with the symmetric Lagrangian without scalars in Sec. V. Section VI contains remarks on the relevance of symmetry-breaking terms in establishing the compositeness criteria for the scalar meson and their relevance in non-Abelian theories.

II. SYMMETRIC MODELS AND HSB

We begin by considering chiral U(1)×U(1) gauge-invariant models with and without scalar mesons.[9] The corresponding Lagrangian densities are

$$\mathcal{L}_S = i\bar{\Psi}\not{\partial}\Psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} - g_{V0}\bar{\Psi}\gamma_\mu\Psi V^\mu - g_{A0}\bar{\Psi}\gamma_\mu\gamma_5\Psi A^\mu \tag{1}$$

and

$$\mathcal{L}_H = \mathcal{L}_S + \tfrac{1}{2}\partial_\mu\Pi\partial^\mu\Pi + \tfrac{1}{2}\partial_\mu\Sigma\partial^\mu\Sigma - \tfrac{1}{2}\tilde{\mu}_0^2(\Sigma^2+\Pi^2) - \frac{\tilde{\Lambda}_0}{4}(\Sigma^2+\Pi^2)^2 - 2g_{A0}(\Sigma\partial^\mu\Pi - \Pi\partial^\mu\Sigma)A_\mu + 2g_{A0}^2(\Sigma^2+\Pi^2)A^2 - G_0\bar{\Psi}(\Sigma + i\gamma_5\Pi)\Psi\,, \tag{2}$$

where Ψ, V_μ, and A_μ are massless spinor, vector, and axial-vector fields, and Σ and Π are massive scalar and pseudoscalar fields, $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We assume $\tilde{\mu}_0^2 < 0$ and $\tilde{\Lambda}_0 > 0$. Equations (1) and (2) are invariant under infinitesimal gauge transformations:

$$\text{Vector: } \delta\Psi = i\alpha\Psi\,,\quad \delta V_\mu = -\frac{1}{g_{V0}}\partial_\mu\alpha\,,\quad \delta\bar{\Psi} = -i\bar{\Psi}\alpha\,,\quad \delta A_\mu = 0\,,\quad \delta\begin{pmatrix}\Sigma\\ \Pi\end{pmatrix} = 0\,; \tag{3a}$$

$$\text{Axial vector: } \delta\Psi = i\beta\gamma_5\Psi\,,\quad \delta V_\mu = 0\,,\quad \delta\bar{\Psi} = i\beta\bar{\Psi}\gamma_5\,,\quad \delta A_\mu = -\frac{1}{g_{A0}}\partial_\mu\beta\,,\quad \delta\begin{pmatrix}\Sigma\\ \Pi\end{pmatrix} = 2\beta\begin{pmatrix}\Pi\\ -\Sigma\end{pmatrix}. \tag{3b}$$

Note that if β is finite, the transformation on Σ and Π can be expressed as

$$\begin{pmatrix}\Sigma\\ \Pi\end{pmatrix} \to \exp(2i\sigma_2\beta)\begin{pmatrix}\Sigma\\ \Pi\end{pmatrix}, \tag{4}$$

where σ_2 is a Pauli matrix. We take the spontaneous symmetry-breaking solution of (2) to correspond to

$$\langle 0|\Sigma|0\rangle = \Sigma_0 \neq 0\,. \tag{5}$$

We make the following change of variables:

$$\begin{pmatrix}\Sigma\\ \Pi\end{pmatrix} = \exp\left[i\sigma_2\frac{\tilde{\Pi}(x)}{\Sigma_0}\right]\left[\begin{pmatrix}\Sigma_0\\ 0\end{pmatrix} + \begin{pmatrix}\tilde{\Sigma}(x)\\ 0\end{pmatrix}\right]. \tag{6}$$

Note that under a finite gauge transformation

$$\tilde{\Pi}(x) \to \tilde{\Pi}(x) + 2\Sigma_0\beta \tag{7}$$

and $\tilde{\Sigma}(x)$ does not change. We can choose the gauge parameter β to be $-\tilde{\Pi}(x)/2\Sigma_0$ for which $\tilde{\Pi}(x) \to 0$. Then, indicating the transformed Ψ and A_μ by the same symbols, the Lagrangian becomes

$$\mathcal{L}'_H = \bar{\Psi}(\not{p} - G_0\Sigma_0)\Psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} + \tfrac{1}{2}(4g_{A0}^2\Sigma_0^2)A^2 + \tfrac{1}{2}\partial_\mu\tilde{\Sigma}\partial^\mu\tilde{\Sigma} - \tfrac{1}{2}(2\tilde{\Lambda}_0\Sigma_0^2)\tilde{\Sigma}^2 - \tilde{\Lambda}_0\Sigma_0\tilde{\Sigma}^3 - \frac{\tilde{\Lambda}_0}{4}\tilde{\Sigma}^4 - g_{V0}\bar{\Psi}\gamma_\mu\Psi V^\mu - g_{A0}\bar{\Psi}\gamma_\mu\gamma_5\Psi A^\mu - G_0\bar{\Psi}\tilde{\Sigma}\Psi + 2g_{A0}^2(\tilde{\Sigma}^2 + 2\Sigma_0\tilde{\Sigma})A^2\,. \tag{8}$$

This symmetry-broken Lagrangian density describes massive Fermi, scalar, and axial-vector fields in addition to a massless vector field. The massless pseudoscalar Goldstone meson which results due to (5) has completely decoupled from the system. Note that (1) is renormalizable. Because the symmetric Lagrangian (2) is renormalizable, the theory described by (8) is also renormalizable.[2]

III. EFFECTIVE LAGRANGIAN

Now we specify our massive model by an effective Lagrangian which has massive Fermi, axial-vector, and scalar fields and which is U(1)×U(1) gauge-invariant and renormalizable. Such a Lagrangian is not unique and could contain many parameters (masses and coupling constants). We construct a Lagrangian[10] which is similar to (8) but U(1)×U(1) gauge-invariant, and for which the gauge-independent β coefficients for vector and axial-vector couplings are the same as for symmetric theory given by (1). (In this latter aspect we are guided by the desire to have asymptotic freedom in the presence of scalars in non-Abelian cases.) Such a Lagrangian is analogous to that of JMC but more complicated:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_S - \bar{\Psi} m_0(1 + 2g_{A0}\sigma/\mu_0)\exp(2i\gamma_5 g_{A0}\phi)\Psi + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - (\lambda_0\mu_0^2/4g_{A0}^2)\sigma^2 - (\lambda_0\mu_0/2g_{A0})\sigma^3 - \tfrac{1}{4}\lambda_0\sigma^4 + (\tfrac{1}{2}\mu_0^2 + 2g_{A0}\mu_0\sigma + 2g_{A0}^2\sigma^2)(A^\mu - \partial^\mu\phi)^2\Big|_{\phi=\Box^{-1}\partial\cdot A}\,. \tag{9}$$

The infinitesimal gauge transformations on Ψ, V_μ, and A_μ are the same as in (3) but for the scalar we have now $\delta\sigma = 0$ for both the vector and axial-vector cases. Note that (9) is invariant under this gauge transformation in a restricted sense, i.e., as long as $\Box\beta \neq 0$. The implications of such a restriction are discussed in JMC. Note that if we choose the gauge in which $\partial_\mu A^\mu = 0$, (9) formally seems to become the same as (8) on identifying σ with $\tilde{\Sigma}$ and having

$$\tilde{\Lambda}_0 = \lambda_0\,,\quad G_0 = (2g_{A0}m_0/\mu_0)\,,\quad \Sigma_0 = (\mu_0/2g_{A0})\,. \tag{10}$$

But we would like to emphasize that the theories described by (8) and (9) are physically different as would be apparent from the Feynman rules for the axial-vector propagator (see below). In our work we are concerned only with (9). We shall call the terms in $(\mathcal{L}_{\text{eff}} - \mathcal{L}_S)$ of (9) "symmetry-breaking terms." The nonlocality of Lagrangian (9) can be removed by adding to (9) a Lagrange multiplier term

$$-\chi(\Box\phi - \partial\cdot A)\,.$$

In order to arrive at Feynman rules we must add gauge-breaking terms to (9). The final effective Lagrangian is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_S - \bar{\Psi} m_0(1 + 2g_{A0}\sigma/\mu_0)\exp(2i\gamma_5 g_{A0}\phi)\Psi + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - (\lambda_0\mu_0^2/4g_{A0})\sigma^2 - (\lambda_0\mu_0/2g_{A0})\sigma^3 - \tfrac{1}{4}\lambda_0\sigma^4 + (\tfrac{1}{2}\mu_0^2 + 2g_{A0}\mu_0\sigma + 2g_{A0}^2\sigma^2)(A^\mu - \partial^\mu\phi)^2 - \chi(\Box\phi - \partial\cdot A) - \frac{1}{2\eta}[(\partial\cdot V)^2 + (\partial\cdot A)^2]\,, \tag{11}$$

where η is the gauge parameter. We take the gauge-invariant field σ to be normal-ordered. The free propagators are

$$\begin{aligned} iG &= i(\not{p} - m_0)^{-1}\,,\quad D^\sigma = i(k^2 - M_{\sigma0}^2)^{-1}\,,\\ &\qquad M_{\sigma0}^2 = \lambda_0\mu_0^2/2g_{A0}^2\,,\\ iD^V_{\mu\nu} &= -i[g_{\mu\nu} - (1-\eta)k_\mu k_\nu k^{-2}]k^{-2}\,,\\ iD^A_{\mu\nu} &= -i[(g_{\mu\nu} - k_\mu k_\nu k^{-2})(k^2 - \mu_0^2)^{-1} + \eta k_\mu k_\nu k^{-4}]\,,\\ iD^\phi &= -i\eta k^{-4}. \end{aligned} \tag{12}$$

We note that the source of the axial-vector field in (9) is conserved. Also we can formally derive the Ward-Takahashi (WT) identities for proper axial-vector vertices. The proper vertex for axial-vector–fermion coupling, $g_A\Gamma^A_\mu$, obeys the same WT identity as given by (1):

$$q^\mu\Gamma^A_\mu(p, p+q) = \gamma_5 S^{-1}(p+q) + S^{-1}(p)\gamma_5\,, \tag{13}$$

where $S^{-1} = G^{-1} + i\Sigma(p)$. To the first order in g_{A0},

$$\Gamma^A_\mu = i\gamma_\mu\gamma_5 + 2i\gamma_5 m_0 q_\mu q^{-2}\,, \tag{14}$$

which satisfies (13). One can verify that it is satisfied in higher orders also. We define the necessary renormalization constants:

$$\begin{aligned} &\Psi = Z_{2F}^{1/2}\Psi_R\,,\quad V_\mu = Z_{2V}^{1/2}V_{R\mu}\,,\quad A_\mu = Z_{2A}^{1/2}A_{R\mu}\,,\\ &\sigma = Z_{2\sigma}^{1/2}\sigma_R\,,\quad g_{V0} = Z_{1V}Z_{2F}^{-1}Z_{2V}^{-1/2}g_V\,,\\ &g_{A0} = Z_{1A}Z_{2F}^{-1}Z_{2A}^{-1/2}g_A\,,\quad \lambda_0 = Z_{1\lambda}Z_{2\sigma}^{-2}\lambda\,. \end{aligned} \tag{15}$$

Because of the WT identities Z_{2F}, Z_{1V}, and Z_{1A} have the same cutoff dependence. There are many more contributions to the axial-vector-meson self-energy than to that of the vector meson. But the divergences that come in are logarithmic and can be removed by mass renormalization, and Z_{2V} and Z_{2A} differ by finite factors from the corresponding ones for the symmetric theory. This is not true of Z_{2F}, Z_{1V}, and Z_{1A} and we shall comment on this aspect later. Only $Z_{2\sigma}$ and $Z_{1\lambda}$ have no analogs in the symmetric theory given by (1).

Our object is to show that (9) is a representation of spontaneous-symmetry-broken solutions of (1). For this purpose the necessary conditions seem to be (in the limit of infinite cutoff) as follows: (a) m_0, μ_0, and $M_{\sigma0}$ are zero even though the renormalized counterparts m, μ, and M_σ are nonzero, and (b) $Z_{2\sigma}$ and $Z_{1\lambda}$ are zero. Note that the latter condition is understood to be the criterion for compositeness in field theory.[7] We could require in addition that $Z_{1\lambda}Z_{2\sigma}^{-1}$ vanish.[8]

In order to see whether these conditions are satisfied we use the CS equations.

IV. CALLAN-SYMANZIK EQUATIONS AND ASYMPTOTIC SOLUTIONS

We have three coupling constants and two masses in (9). The CS equation[11] for a renormalized one-particle irreducible, amputated vertex Γ is

$$\left(D+\beta_V\frac{\partial}{\partial g_V}+\beta_A\frac{\partial}{\partial g_A}+\beta_\lambda\frac{\partial}{\partial\lambda}+\beta_\eta\frac{\partial}{\partial\eta}-\sum N_i\gamma_i\right)\Gamma=\Delta\Gamma, \tag{16}$$

where Γ describes a process with N_i fields of type i each having anomalous dimension γ_i $(i=F,V,A,\sigma)$. β_V, β_A, β_η, and γ_i are dimensionless functions of renormalized coupling constants and mass ratios: $D=m(\partial/\partial m)+\mu(\partial/\partial\mu)$ (with renormalized coupling constants fixed). The right-hand side involves the operation of $\bar{D}=m(\partial/\partial m)+\mu(\partial/\partial\mu)$ (with unrenormalized coupling constants fixed) acting on the unrenormalized vertex Γ_μ expressed in terms of bare masses and coupling constants. We refer the reader to JMC for details. In our case the right-hand side of the CS equation is given in terms of $\bar{D}\mathfrak{M}$ with

$$\mathfrak{M}=\mathfrak{M}_1+\mathfrak{M}_2+\mathfrak{M}_3+\mathfrak{M}_4+\mathfrak{M}_5+\mathfrak{M}_6, \tag{17}$$

where

$$\begin{aligned}
\mathfrak{M}_1&=\bar{\Psi}m_0\exp(2i\gamma_5 g_{A0}\phi)\Psi,\\
\mathfrak{M}_2&=-\tfrac{1}{2}\mu_0^2(A_\mu-\partial_\mu\phi)^2,\\
\mathfrak{M}_3&=(2g_{A0}m_0/\mu_0)\bar{\Psi}\sigma\exp(2i\gamma_5 g_{A0}\phi)\Psi,\\
\mathfrak{M}_4&=-2g_{A0}\mu_0\sigma(A_\mu-\partial_\mu\phi)^2,\\
\mathfrak{M}_5&=(\lambda_0\mu_0^2/4g_{A0}^2)\sigma^2,\\
\mathfrak{M}_6&=(\lambda_0\mu_0/2g_{A0})\sigma^3,
\end{aligned} \tag{18}$$

where $\phi=\Box^{-1}\partial\cdot A$. $\bar{D}\mathfrak{M}$ can be expressed in terms of two functions δ_1 and δ_2 defined by

$$\begin{aligned}
\bar{D}\ln m_0&=1+\delta_1,\\
\bar{D}\ln\mu_0&=1+\delta_2.
\end{aligned} \tag{19}$$

Then we get

$$\begin{aligned}
&\bar{D}\mathfrak{M}_1=(1+\delta_1)\mathfrak{M}_1, \quad \bar{D}\mathfrak{M}_2=2(1+\delta_2)\mathfrak{M}_2,\\
&\bar{D}\mathfrak{M}_3=(\delta_1-\delta_2)\mathfrak{M}_3, \quad \bar{D}\mathfrak{M}_4=(1+\delta_2)\mathfrak{M}_4,\\
&\bar{D}\mathfrak{M}_5=2(1+\delta_2)\mathfrak{M}_5, \quad \bar{D}\mathfrak{M}_6=(1+\delta_2)\mathfrak{M}_6,
\end{aligned} \tag{20}$$

the sum of which is $\bar{D}\mathfrak{M}$. Also we have

$$\begin{aligned}
\gamma_i&=\bar{D}\ln Z_{2i}^{1/2} \quad (i=F,V,A,\sigma),\\
\beta_V&=g_V\bar{D}\ln(Z_{2F}Z_{2V}^{1/2}Z_{1V}^{-1}),\\
\beta_A&=g_A\bar{D}\ln(Z_{2F}Z_{2A}^{1/2}Z_{1A}^{-1}),\\
\beta_\lambda&=\lambda\bar{D}\ln(Z_{2\sigma}^{2}Z_{1\lambda}^{-1}).
\end{aligned} \tag{21}$$

Because of the comments after (15), β_V, β_A, γ_V, and γ_A are the same as in the symmetric theory. γ_F is not the same, but as we shall see, in the lowest-order perturbation calculation, it is gauge-dependent as is its counterpart in symmetric theory. This does not matter for us because we are interested in comparing symmetry-violating terms with symmetric terms and this does not involve γ_F.

In our model we assume

$$\beta_i=0 \quad (i=V,A,\lambda) \tag{22}$$

to mimic a situation in which there is an ultraviolet-stable fixed point. This assumption leads to

$$\gamma_A=0=\gamma_V, \quad \beta_\eta=0, \tag{23}$$

the latter following from $\beta_\eta=-\eta\gamma_{A,V}$. Hence our model takes after the Baker-Johnson finite quantum electrodynamics.[5,6] Also, assumption (22) brings us closer to what happens in non-Abelian gauge theories. We do know that $\beta_V=0=\beta_A$ corresponds to the approximation of neglecting self-energy corrections to internal vector and axial-vector lines.[5,6] It would be interesting to investigate whether our assumption of $\beta_\lambda=0$ could follow as a consequence of this approximation when one treats the bound-state problem using the Bethe-Salpeter equation.

A. Expressions for masses in the asymptotic limit

Making use of the CS equation for the fermion proper self-energy $\Sigma(p)$ which receives contributions of $O(\mathfrak{M}_1)$ and $O(\mathfrak{M}_1\mathfrak{M}_3^2)$, we can arrive at its asymptotic form. Let

$$\Sigma(p)=\Sigma_1(p)+\Sigma_2(p), \tag{24a}$$

where

$$\begin{aligned}
\Sigma_1(p)&\sim O(\mathfrak{M}_1),\\
\Sigma_2(p)&\sim O(\mathfrak{M}_1\mathfrak{M}_3^2).
\end{aligned} \tag{24b}$$

The CS equation for $\Sigma(p)$ reads

$$(D-2\gamma_F)\Sigma(p)=(1+\delta_1)\Sigma_1(p)+(1+3\delta_1-2\delta_2)\Sigma_2(p). \tag{25}$$

Because there is no physical significance to separating $\Sigma(p)$ into two parts and because we have one equation involving $\Sigma(p)$ only, we can arrive at an expression for it if

$$(1+\delta_1)=(1+3\delta_1-2\delta_2), \tag{26a}$$

which implies

$$\delta\equiv\delta_1=\delta_2. \tag{26b}$$

Then we have

$$(D-2\gamma_F)\Sigma(p)=(1+\delta)\Sigma(p), \tag{27}$$

which gives the asymptotic form for large spacelike p

$$\Sigma(p) \sim m B(-p^2/M^2)^{-\gamma_F - \delta/2}, \tag{28}$$

where B is a dimensionless function of mass ratios and coupling constants (DFMC). M^2 is any combination with dimension 2 of masses in the theory. The anomalous dimension of the fermion field, γ_F, can be calculated in a general gauge using (21) in the lowest order:

$$\gamma_F = \frac{1}{4\pi}(\alpha_A + \alpha_V)\eta + \frac{1}{2\pi}\alpha_A(m^2/\mu^2), \tag{29}$$

where $\alpha_i \equiv g_i^2/4\pi$ $(i=A, V)$. It is to be noted that $(-p^2/M^2)^{-\gamma_F}$ factors out of the full inverse propagator. Hence the ratio of the symmetry-violating term with the symmetric term does not involve γ_F and is gauge-invariant. If $\delta > 0$, $\Sigma(p)$ vanishes with respect to the $\not p$ part.

Now we turn our attention to the symmetry-violating part of the axial-vector polarization $\Pi^A_{SB\mu\nu}$, which is that part of polarization which vanishes when μ_0 and m_0 are zero. This receives contributions of $O(\mathfrak{M}_1^2)$, $O(\mathfrak{M}_2)$, and $O(\mathfrak{M}_4^2)$. Defining Π^A_{SB} as

$$\Pi^A_{SB\mu\nu}(k) = (k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi^A_{SB} \tag{30}$$

and using (26), we get the CS equation for Π^A_{SB},

$$(D - 2\gamma_A)\Pi^A_{SB} = 2(1+\delta)\Pi^A_{SB}. \tag{31}$$

Hence the asymptotic solution is

$$\Pi_{SB}(k) \sim \mu^2 C(-p^2/M^2)^{-\gamma_A - \delta}, \tag{32}$$

where C is a DFMC. In our model $\gamma_A = 0$. Π_{SB} vanishes in the limit of large spacelike p for $\delta > 0$.

We calculate δ_1 and δ_2 in the lowest-order perturbation theory. This involves the evaluation of self-energy diagrams for the fermion and the axial-vector meson. We emphasize that for fermion self-energy, the contact term from $-2g_{A0}^2 m_0 \bar\Psi\Psi(\Box^{-1}\partial\cdot A)^2$ is essential for a gauge-invariant answer. (Note that a similar situation arises in the scalar quantum electrodynamics.) In a general gauge there are eight fermion self-energy diagrams. Ignoring finite terms we find

$$m = m_0\left[1 + \left(\frac{3\alpha_{V0}}{4\pi} - \frac{3\alpha_{A0}}{4\pi} - \frac{3}{2\pi}\alpha_{A0}\frac{m_0^2}{\mu_0^2}\right)L\right], \tag{33}$$

where $L \equiv \ln(\Lambda^2/M^2)$. Using (19) we find

$$\delta_1 = \frac{3}{2\pi}(\alpha_V - \alpha_A) - \frac{3}{\pi}\alpha_A\frac{m^2}{\mu^2}. \tag{34}$$

We have computed the axial-vector polarization in a general gauge. It involves 21 diagrams, and it is gauge-invariant. The computation in the Landau gauge ($\eta = 0$) is much simpler and obtains the same answer. The result is

$$\Pi^A_{\mu\nu}(k) = \frac{1}{\pi}(2\alpha_{A0}m_0^2 - 3\alpha_{A0}\mu_0^2 - \tfrac{1}{3}\alpha_{A0}k^2) \times \alpha_A L(g_{\mu\nu} - k_\mu k_\nu k^{-2}). \tag{35}$$

This gives

$$\mu^2 = \mu_0^2\left[1 + \left(\frac{2\alpha_{A0}}{\pi}\frac{m_0^2}{\mu_0^2} - \frac{3\alpha_{A0}}{\pi}\right)L\right]. \tag{36}$$

Use of (19) gives

$$\delta_2 = \frac{2\alpha_A}{\pi}\frac{m^2}{\mu^2} - \frac{3\alpha_A}{\pi}. \tag{37}$$

Combining (26), (34), and (37) we get

$$\frac{m^2}{\mu^2} = \frac{3}{10}\left(\frac{\alpha_V}{\alpha_A} + 1\right). \tag{38}$$

We have already seen that in order to satisfy the conditions for DSB, we must have $\delta > 0$. With (26), (34), and (37) this implies

$$\left(\frac{\alpha_V}{\alpha_A} - 1\right) > 2\frac{m^2}{\mu^2} > 3, \tag{39}$$

which is consistent with (38).[12] Integration of (19) gives

$$m_0 = mD(\Lambda^2/M^2)^{-\delta/2}, \quad \mu_0 = \mu E(\Lambda^2/M^2)^{-\delta/2}, \tag{40}$$

where D and E are DMFÇ. Hence m_0 and μ_0 vanish in the limit of infinite cutoff even though their ratio is finite.

B. Asymptotic expressions for $Z_{1\lambda}$ and $Z_{2\sigma}$

The renormalization-group equation for $Z_{1\lambda}$ can be easily obtained by operating D on both sides of $Z_{1\lambda}^{-1}Z_{1\lambda} = 1$ and using the chain rule. As β's are zero we get

$$(D - 2\gamma_{1\lambda})Z_{1\lambda} = 0, \tag{41}$$

where $\gamma_{1\lambda} = \bar D \ln Z_{1\lambda}^{1/2}$. The lowest-order diagrams which contribute to $Z_{1\lambda}$ are shown in Fig. 1. We have ignored the diagrams of $O(\lambda_0^3)$ because the bare σ mass $M_{\sigma 0}$ of (12) should not become infinite in the limit $g_{A0} \to 0$, and this requires $\lambda_0 \sim O(g_{A0}^2)$. They give

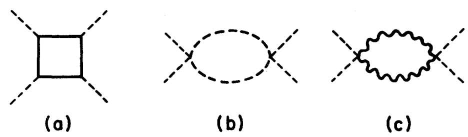
$$Z_{1\lambda} = 1 + \frac{L}{\pi^2}[\tfrac{29}{16}\lambda + g_A^4\lambda^{-1}(3 - 4m^4\mu^{-4})], \tag{42}$$

which gives

$$\gamma_{1\lambda} = -\frac{1}{4\pi^2}[\tfrac{9}{4}\lambda + 4g_A^4\lambda^{-1}(3 - 4m^4\mu^{-4})]. \tag{43}$$

Now (41) yields

$$Z_{1\lambda} \underset{\Lambda^2\to\infty}{\sim} H(\Lambda^2/M^2)^{-\gamma_{1\lambda}}, \tag{44}$$



FERMION:——, SCALAR:---, AXIAL-VECTOR: ∿∿

FIG. 1. One diagram from each of the classes of diagrams which contribute to vertex renormalization constant $Z_{1\lambda}$ in the lowest order. There are five more diagrams in class (a), and two more in each of the rest.

where H is a DFMC. In order to satisfy the DSB condition $Z_{1\lambda}\to 0$ and hence we require $\gamma_{1\lambda}>0$. Combining this with (43), we get

$$\lambda<\lambda_1\,, \tag{45}$$

where

$$\lambda_1\equiv\tfrac{4}{3}g_A{}^2\frac{m^2}{\mu^2}\left(4-3\frac{\mu^4}{m^4}\right)^{1/2}. \tag{46}$$

This tells us that λ can at most be of $O(g_A{}^2)$ which is consistent with neglecting the diagrams of $O(\lambda_0{}^3)$ mentioned above.

The renormalization-group equation for $Z_{2\sigma}$ is

$$(D-2\gamma_\sigma)Z_{2\sigma}=0\,. \tag{47}$$

In the computation of γ_σ we ignore the self-energy diagram of $O(\lambda^2)$ as it would be of $O(g_A{}^4)$ by the above consideration. In a general gauge there are 12 self-energy diagrams of $O(g_A{}^2)$ but only the fermion loop contributes to $Z_{2\sigma}$:

$$Z_{2\sigma}=1-\frac{2\alpha_A}{\pi}\frac{m^2}{\mu^2}L\,. \tag{48}$$

This gives

$$\gamma_\sigma=\frac{2\alpha_A}{\pi}\frac{m^2}{\mu^2}\,. \tag{49}$$

Thus the asymptotic solution for $Z_{2\sigma}$ is

$$Z_{2\sigma}\underset{\Lambda^2\to\infty}{\sim}J(\Lambda^2/M^2)^{-\gamma_\sigma}\,, \tag{50}$$

where J is a DFMC. As $\gamma_\sigma>0$, $Z_\sigma\to 0$ as $\Lambda^2\to\infty$ thus satisfying one of the compositeness conditions for σ trivially.

Because $\beta_\lambda=0$ in our model, we infer from (21) that the ratio $(Z_{2\sigma}{}^2/Z_{1\lambda})$ must be cutoff-independent. Hence we must have

$$2\gamma_\sigma=\gamma_{1\lambda}\,. \tag{51}$$

This condition yields

$$\lambda=\lambda_2\,, \tag{52}$$

where

$$\lambda_2\equiv\tfrac{8}{9}g_A{}^2\left[-\frac{m^2}{\mu^2}+\left(10\frac{m^4}{\mu^4}-\tfrac{27}{4}\right)^{1/2}\right], \tag{53}$$

which can be expressed in terms of only α_A and α_V using (38). It is easy to see that

$$\lambda_2<\lambda_1\,. \tag{54}$$

Expressing (53) in terms of α_A and α_V we have

$$\lambda=\frac{32\pi}{9}\alpha_A\left\{-\frac{3}{10}\left(1+\frac{\alpha_V}{\alpha_A}\right)+\left[\frac{9}{10}\left(1+\frac{\alpha_V}{\alpha_A}\right)^2-\frac{27}{4}\right]^{1/2}\right\}. \tag{55}$$

Because of (54), (45) is automatically satisfied by (55). For the value of λ given in (55)

$$\frac{Z_{1\lambda}}{Z_{2\sigma}{}^2}\sim K\,, \tag{56}$$

where K is a finite DFMC. From (44) and (50) it follows that

$$\frac{Z_{1\lambda}}{Z_{2\sigma}}\to 0 \tag{57}$$

in the limit of infinite cutoff. Thus all of the compositeness criteria are satisfied by $Z_{2\sigma}$ and $Z_{1\lambda}$.

The asymptotic expression for Z_{2F} could be obtained by solving the corresponding renormalization-group equation. We get

$$Z_{2F}\underset{\Lambda^2\to\infty}{\sim}N(\Lambda^2/M^2)^{-\gamma_F}\,, \tag{58}$$

where N is a DFMC. The renormalization constants, Z_{2V} and Z_{2A}, are independent of cutoff as $\gamma_V=0=\gamma_A$.

V. RELATIONSHIP BETWEEN $\mathcal{L}_S$ AND $\mathcal{L}_{\rm eff}$

At the end of Sec. III we gave the necessary conditions for spontaneous dynamical symmetry breaking. The only condition we have not dealt with so far is the vanishing of $M_{\sigma 0}$ given in (12). In terms of renormalized quantities we have

$$M_{\sigma 0}{}^2=M_\sigma{}^2E\frac{Z_{1\lambda}}{Z_{2\sigma}{}^2}\left(\frac{\Lambda^2}{M^2}\right)^{-\delta}, \tag{59}$$

where

$$M_\sigma{}^2=\lambda\mu^2/2g_A{}^2\,. \tag{60}$$

Using (56), we see that $M_{\sigma 0}$ vanishes in the limit of infinite cutoff. Combining (38) and (60) we get

$$\frac{m^2}{M_\sigma{}^2}=\frac{12\pi}{5}\left(\frac{\alpha_V+\alpha_A}{\lambda}\right), \tag{61}$$

where λ is given by (55).

Now let us look at each term in the effective Lagrangian (9). We compare the symmetry-violating terms having $\bar\Psi\Psi$ with the corresponding symmetric term. Because $(\gamma_\sigma-\delta)>0$, $m_0(1+2g_A\sigma/\mu_0)\to m_0\to 0$ asymptotically. Thus these symmetry-violating terms vanish with respect to the $\not{p}$ part of the symmetric term. It is trivial to see that all other symmetry-breaking terms in (9) vanish. For example, consider the σ^3 term. The renormalized coupling is $(\lambda\mu/2g_A)$ which is finite, and the cutoff dependence of the corresponding vertex renormalization constant is given by $Z_{1\lambda}Z_{2\sigma}^{-1/2}Z_{2A}^{1/2}(\Lambda/M)^{-\delta}$ which vanishes because $(\gamma_{1\lambda}-\frac{1}{2}\gamma_\sigma+\delta)>0$ as $\delta>0$ and $\gamma_{1\lambda}>\frac{1}{2}\gamma_\sigma$ from (51).

VI. CONCLUSION

We have deduced a set of conditions (38), (39), and (55) at the fixed point in order to have dynamical symmetry breaking with a scalar meson as a bound state. This necessary set does not form a sufficient one any more than the necessary condition that a bound-state Schrödinger wave function behave like $\exp(-|K|r)$ at infinite r ensures the existence of the bound state. The scalar meson is a bound state in the sense that $Z_{1\lambda}$, $Z_{2\sigma}$, and $Z_{1\lambda}/Z_{2\sigma}$ vanish asymptotically.[13] This is due to the symmetry-breaking Yukawa term corresponding to $\mathfrak{M}_3$ of (18) which is proportional to the gauge coupling constant g_{A0}. Thus this symmetry-breaking term of the Yukawa type seems essential for the composite character of the scalar meson.

We can generalize our treatment to non-Abelian gauge theories. The method consists of (a) constructing an effective gauge-invariant Lagrangian (in a restricted sense) and (b) showing that the symmetry-breaking terms vanish asymptotically and scalars satisfy the compositeness criteria. Note that in this case there is no need to assume β coefficients vanish because of the possibility of the existence of the ultraviolet-stable fixed point at the origin. In our preliminary investigation we find that the symmetry-breaking terms similar to the Yukawa term mentioned above play an essential role in satisfying the compositeness conditions mainly because they are proportional to the gauge coupling constant, and hence their contributions survive in the asymptotic limit.

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[9]We could equally well consider the $O(2)\times O(2)$-invariant model of Ref. 4. In the corresponding $\mathfrak{L}_H$, we have $\bar\Psi(\Sigma+i\gamma_5\Pi)\Psi$ of (2) replaced by $\bar\Psi(\Sigma\tau_3+\Pi\tau_1)\Psi$.

[10]This effective Lagrangian is the same as one constructed in J. Lemmon and K. T. Mahanthappa (unpublished) if one identifies π of this reference as $\mu_0\phi$.

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[12]It is to be noted that for the case of Refs. 3 and 4, which corresponds to setting $\sigma=0$ (i.e., having only a composite Goldstone boson), we obtain their condition, $\alpha_V>\alpha_A$, for the existence of DSB, whereas with the scalar bound state present the condition becomes $\alpha_V>4\alpha_A$ as in (39). The question whether the latter can be obtained as a condition for the existence of the bound state with DSB directly from the Bethe-Salpeter equation needs investigation.

[13]These criteria for the bound state have been shown to be satisfied for $L_{\rm eff}$ of (9) in which the various coupling constants are expressed only in terms of g_{A0}, λ_0, m_0, and μ_0. If one asks whether the implied constraints between the coupling constants are derivable and/or necessary, one is forced to deal with the difficult problem of homogeneous integral equations which we have tried to avoid in this work.