Neutrino pair bremsstrahlung by nucleons in neutron-star matter

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We calculate neutrino pair bremsstrahlung from nucleons in neutron-star matter via weak neutral currents. We find bremsstrahlung in neutron-neutron collisions to be unimportant, but find that the bremsstrahlung emissivity in neutron-proton collisions is of the same order as that for the modified URCA process.

I. INTRODUCTION

Several experiments[1-3] strongly suggest the existence of neutral hadronic currents coupled weakly to neutral leptonic currents with a strength comparable to that of the ordinary weak interaction. If such an interaction exists, it can have important consequences for neutrino opacity in dense stellar matter[4] and admits a new type of neutrino pair emissivity due to the decay of excited nuclear levels.[5,6] It may also significantly modify[7] the neutrino pair emissivities for the conventional processes—e^--e^+ annihilation, photoneutrino process, plasmaneutrino process—originally considered in terms of the weak interaction of purely leptonic charged currents. In this paper we consider a different neutrino pair radiation process, namely that due to bremsstrahlung of a $\nu\bar{\nu}$ pair in nucleon-nucleon collisions:

$$n+n\to n+n+\nu+\bar{\nu}, \tag{1a}$$

$$n+p\to n+p+\nu+\bar{\nu}. \tag{1b}$$

This is to be compared with the production of ν and $\bar{\nu}$ through the modified URCA process[8]:

$$\begin{aligned} &n+n\to n+p+e^-+\bar{\nu},\\ &n+p+e^-\to n+n+\nu. \end{aligned} \tag{2}$$

This latter process can play a significant role in the cooling of young neutron stars,[9] although the transition of the nucleons to a likely superfluid state greatly suppresses the rate of emission.

By adopting a standard $V-A$ model for the neutral nucleon current (the parameters of which may be simply expressed in terms of those entering into the gauge theories[10] of the weak and electromagnetic interactions) we calculate the emissivities associated with (1a) and (1b). We find that for degenerate neutron-star matter both processes have emissivities proportional to T^8. This is the same temperature dependence as that found for the modified URCA process. For (1a) the coefficient of T^8 (determined by densities of states and an averaged neutron-neutron cross section) is much smaller than that for (2), but for (1b) the coefficient of T^8 is comparable to that for (2). Our results differ from those reported by Bond[11]: The differences are in the direction of making the bremsstrahlung process a less significant one than implied by his results.

II. CALCULATION OF $\nu\bar{\nu}$ BREMSSTRAHLUNG

We adopt the following form for the interaction between a neutral nucleon current and the neutrino current:

$$\mathcal{H}_{\rm int}=\frac{G}{\sqrt{2}}(\bar{\psi}_i\gamma^\mu(g_i-g_i'\gamma_5)\psi_i)\times[(\bar{\psi}_{\nu_e}\gamma_\mu(1-\gamma_5)\psi_{\nu_e})+(\bar{\psi}_{\nu_\mu}\gamma_\mu(1-\gamma_5)\psi_{\nu_\mu})], \tag{3}$$

where G is the Fermi coupling constant ($G\simeq 10^{-5}/m_p^2$) and g_i, g_i' (i = neutron, proton) are reduced coupling constants for the vector and axial-vector currents of the nucleons. In terms of a conventional gauge theory[10] we have

$$g_n=-\tfrac{1}{2},\quad g_n'=-1.23/2,$$

$$g_p=\tfrac{1}{2}-\sin^2\theta_W,\ \text{and}\ g_p'=1.23/2.$$

We evaluate the nucleon current in the nonrelativistic limit in which

$$\bar{\psi}\gamma^\mu(g-g'\gamma_5)\psi\to\delta_0^\mu g\psi^\dagger\psi-\delta_k^\mu g'\psi^\dagger\sigma^k\psi. \tag{4}$$

The nucleons in neutron-star matter are very degenerate and therefore the $\nu\bar{\nu}$ pairs that are radiated have very low energies $\sim kT$ (much less than the nucleon Fermi energies). Thus we have *soft* $\nu\bar{\nu}$ bremsstrahlung, and the amplitude for this may be expressed as a product of three factors: the T-matrix element for the scattering of nucleons on the mass shell, a nucleon propagator, and a radiation matrix element. (In fact four amplitudes must be added, corresponding to the emission of the $\nu\bar{\nu}$ pair from each of the four external legs of the T matrix.) The emissivity (power radiated per unit volume) for nonrelativistic nucleons is given by

$$\dot{\mathcal{E}}^{V,A}(1,2\to 1',2')=2\left(\frac{G}{\sqrt{2}}\right)^2\sum_{\text{spins}}\int\frac{d^3p_1d^3p_2d^3p_1'd^3p_2'}{(2\pi)^{18}}f_1f_2(1-f_{1'})(1-f_{2'})$$
$$\times\int d^3q\int dq_0q_0(2\pi)^4\delta^4(p_1'+p_2'+q-p_1-p_2)\int\frac{d^3q_1}{2q_{10}}\frac{d^3q_2}{2q_{20}}\delta^4(q_1+q_2-q)|\mathfrak{M}^{V,A}|^2. \tag{5a}$$

The matrix element for emission via the vector current is

$$\mathfrak{M}^V=\langle\vec{q}_1,\vec{q}_2|\bar{\psi}_\nu\gamma^0(1-\gamma_5)\psi_\nu|0\rangle\langle\vec{p}_1',\vec{p}_2'|T|\vec{p}_1,\vec{p}_2\rangle[g_1(G(1)+G(1'))+g_2(G(2)+G(2'))] \tag{5b}$$

and that for emission via the axial-vector current is

$$\mathfrak{M}^A=\langle\vec{q}_1,\vec{q}_2|\bar{\psi}_\nu\gamma^k(1-\gamma_5)\psi_\nu|0\rangle\langle\vec{p}_1',\vec{p}_2'|[(g_1'G(1')\sigma_1^k+g_2'G(2')\sigma_2^k)T+T(g_1'G(1)\sigma_1^k+g_2'G(2)\sigma_2^k)]|\vec{p}_1,\vec{p}_2\rangle. \tag{5c}$$

The emissivities of (5a) are to be reduced by $\frac{1}{2}$ if the nucleons are identical. The factor of 2 in (5a) allows for the emission of both electron and muon neutrinos. Extra integration variables d^4q are introduced along with an additional δ function to facilitate the integration over neutrino momenta[12]:

$$\int\frac{d^3q_1}{2q_{10}}\frac{d^3q_2}{2q_{20}}q_1^\mu q_2^\nu\delta^4(q_1+q_2-q)=\frac{\pi}{24}(2g^{\mu\nu}q^2+q^\mu q^\nu). \tag{6}$$

In (5a) the f's are Fermi-Dirac functions

$$f(\epsilon)=[\exp((\epsilon-\mu)/kT)+1]^{-1}, \tag{7}$$

where μ is the chemical potential of the neutron or proton quasiparticles. In the low-temperature limit the only important nucleon states are those near the Fermi surfaces, and in this limit the nonrelativistic quasiparticle propagators take a simple form, for example

$$G(1)=-(q_0-\vec{p}_1\cdot\vec{q}/m_1+\vec{q}^2/2m_1)^{-1}, \tag{8}$$

where $\vec{p}_1$ is the incoming momentum of the first nucleon of effective mass m_1, before the emission of a $\nu\bar{\nu}$ pair of total momentum $\vec{q}$ and total energy q_0.

The total neutrino momentum $|\vec{q}|$ is of order kT, and we may safely neglect it in the momentum-conserving δ function and put the remaining nucleon momenta on their respective Fermi surfaces. Then the description of the nucleon phase space and scattering is standard[13]:

$$\int d^3p_1d^3p_2d^3p_{1'}d^3p_{2'}\delta^3(\vec{p}_1'+\vec{p}_2'-\vec{p}_1-\vec{p}_2)=4\pi m_1^2m_2^2p_{f1}p_{f2}[p_{f1}^2+p_{f2}^2+2p_{f1}p_{f2}\cos\theta]^{-1/2}\sin\theta\,d\theta\,d\phi\,d\phi_2d\epsilon_1d\epsilon_2d\epsilon_{1'}d\epsilon_{2'}. \tag{9}$$

The T matrix depends only upon the two angles θ and ϕ; θ is the angle between the incoming momenta, ϕ is the angle between the planes defined by the incoming and outgoing nucleon momenta, and ϕ_2 is the azimuthal angle of $\vec{p}_2$ with respect to $\vec{p}_1$. The remaining phase-space integral is[14]

$$\int d\epsilon_1d\epsilon_2d\epsilon_{1'}d\epsilon_{2'}f_1f_2(1-f_{1'})(1-f_{2'})\delta(\epsilon_{1'}+\epsilon_{2'}+q_0-\epsilon_1-\epsilon_2)=q_0\frac{(kT)^2}{6}[(2\pi)^2+(q_0/kT)^2](\exp(q_0/kT)-1)^{-1}. \tag{10}$$

If the spin dependence of the T matrix is parameterized by

$$T=\left(\frac{3T_1+T_0}{4}\right)+\left(\frac{T_1-T_0}{4}\right)\vec{\sigma}_1\cdot\vec{\sigma}_2, \tag{11}$$

the nucleon spin sums are readily performed. For neutron-neutron scattering the amplitudes T_1 and T_0 are just the isospin-1 amplitudes T_1^1 and T_0^1 for scattering in spin-triplet and spin-singlet states, respectively. For neutron-proton scattering the amplitudes are the sum of isospin-1 and isospin-0 amplitudes: $T_1 = (T_1^1 + T_1^0)/2$ and $T_0 = (T_0^1 + T_0^0)/2$.

We now turn to the results of these calculations. The vector current contribution to $\nu\bar{\nu}$ bremsstrahlung in neutron-neutron collisions is suppressed because the leading terms from the propagators cancel. The lowest-order contribution is

$$\dot{\mathcal{E}}_{nn}^{V} = \frac{31\pi}{53\,460}\left(\frac{G}{\sqrt{2}}\right)^2 g_n^{\,2} m_n^{\,2} p_{fn} (kT)^{10} \left\langle \frac{3\,|T_1^1(\theta,\phi)|^2 + |T_0^1(\theta,\phi)|^2}{\cos\frac{1}{2}\theta} \right\rangle , \tag{12}$$

where the brackets denote an average of the T-matrix elements over the angles θ, ϕ. There is an additional $(kT)^{10}$ contribution in this case arising from the emission of the neutrino pair from internal lines of the T matrix; this contribution can be expressed in terms of derivatives of the T matrix.[15] In this paper we neglect this correction and (12) itself because the axial-vector current contribution is more important. The emissivity due to the axial-vector current is

$$\dot{\mathcal{E}}_{nn}^{A} = \frac{451}{1\,190\,700\pi}\left(\frac{G}{\sqrt{2}}\right)^2 p_{fn}^{\,3} m_n^{\,2} (kT)^8 g_n'^{\,2} \left\langle \left(|T_1^1|^2 + |T_0^1|^2 - 2\,\mathrm{Re}(T_0^1 T_1^1)\cos\phi \right) \frac{1-\cos\theta}{\cos\frac{1}{2}\theta} \right\rangle . \tag{13}$$

The reason for this essential difference between the vector current and the axial-vector current neutrino radiation is that vector current radiation (as in the case of photons) from identical "charge-to-mass" particles must proceed through quadrupole terms as the dipole moment is conserved; the axial-vector current radiation is not inhibited because there is no corresponding constant dipole moment.

A complete expression for the emissivity from neutron-proton scattering is complicated largely by phase-space factors. We present results correct only to lowest order in $(v/c)^2 \sim \frac{1}{10}$ and in $(p_{fp}/p_{fn})^2 \sim \frac{1}{10}$. We find

$$\dot{\mathcal{E}}_{np}^{V} = \frac{41}{476\,280\pi}\left(\frac{G}{\sqrt{2}}\right)^2 m_n^{\,2} m_p^{\,2} p_{fp}^{\,3} (kT)^8 \left(\frac{g_n}{m_n} - \frac{g_p}{m_p}\right)^2 \left\langle \left(3\,|T_1^1 + T_1^0|^2 + |T_0^1 + T_0^0|^2\right)\sin^2\theta(1-\cos\phi) \right\rangle \tag{14}$$

and

$$\dot{\mathcal{E}}_{np}^{A} = \frac{41}{37\,800\pi}\left(\frac{G}{\sqrt{2}}\right)^2 m_n^{\,2} m_p^{\,2} p_{fp} (kT)^8 (g_n' - g_p')^2 \left\langle |(T_1^1 + T_1^0) - (T_0^1 + T_0^0)|^2 \right\rangle . \tag{15}$$

The neutron-neutron vector current emissivity is clearly negligible for temperatures $\ll 10^{13}$ K. The neutron-neutron axial-vector current and the neutron-proton vector current emissivities are proportional to $(v/c)^2$ for the nucleons and thus are small in comparison with the neutron-proton axial-vector current contribution. To give numerical estimates for this latter process one requires detailed information on the strength and angular dependence of the T matrix for neutron-star matter. This information is not available, although Fermi-liquid parameters (essentially the forward-scattering T-matrix elements) have been calculated for pure neutron matter.[16] We have estimated $\dot{\mathcal{E}}_{nn}^{A}$ from these Fermi-liquid parameters and find, at nuclear matter density,

$$\dot{\mathcal{E}}_{nn}^{A} \sim 2\times 10^{18} T_9^{\,8}\ \mathrm{erg\,cm^{-3}\,sec^{-1}},$$

with T_9 the temperature in units of 10^9 K. To estimate the more interesting $\dot{\mathcal{E}}_{np}^{A}$ we use the same parameters and find

$$\dot{\mathcal{E}}_{np}^{A} \sim 10^{20} T_9^{\,8}\ \mathrm{erg\,cm^{-3}\,sec^{-1}}.$$

We have also confirmed this crude estimate by using laboratory neutron-proton cross sections. More reliable estimates await detailed calculations of Fermi-liquid parameters for neutron-star matter. We note that the neutron-proton axial-vector current emissivity is comparable to that obtained by Bahcall and Wolf for the modified URCA process. Thus to the extent that the modified URCA process plays an important role in the cooling of a young neutron star, so too should the bremsstrahlung of neutrino pairs in neutron-proton collisions be important. A more detailed comparison of these two processes will be reported elsewhere.

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[1]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).
[2]A. Benvenuti *et al.*, Phys. Rev. Lett. 32, 800 (1974).
[3]S. J. Barish *et al.*, Phys. Rev. Lett. 33, 448 (1974).
[4]D. Z. Freedman, Phys. Rev. D 9, 1389 (1974).
[5]B. Pontecorvo, Usp. Fiz. Nauk 79, 3 (1963) [Sov. Phys.—Usp. 6, 1 (1963)].
[6]J. N. Bahcall, S. B. Treiman, and A. Zee, Phys. Lett. 52B, 275 (1974).
[7]D. Dicus, Phys. Rev. D 6, 941 (1972).

[8]J. N. Bahcall and R. A. Wolf, Phys. Rev. 140, B1452 (1965).
[9]S. Tsuruta, V. Canuto, J. Lodenquai, and M. Ruderman, Astrophys. J. 176, 739 (1972).
[10]S. Weinberg, Phys. Rev. D 5, 1412 (1972).
[11]J. R. Bond, Bull. Am. Phys. Soc. 20, 18 (1975).
[12]A. Lenard, Phys. Rev. 90, 968 (1953).
[13]D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Benjamin, New York, 1966); cf. p. 57.
[14]P. Morel and P. Nozieres, Phys. Rev. 126, 1909 (1962).
[15]F. E. Low, Phys. Rev. 110, 974 (1958).
[16]S.-O. Backman, C.-G. Kallman, and O. Sjoberg, Phys. Lett. 43B, 263 (1973).