Short-distance symmetries, the axial anomaly, and the conformal group*

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We discuss symmetry at small distances, focusing in particular on the implementation of the conformal group. In the framework of a theory which generates strong interactions from a non-Abelian gauge group, we show that the existence of the axial anomaly—which determines the amplitude for $\pi^0 \to 2\gamma$—as well as the nonrenormalization theorem for this anomaly may be regarded as consequences of conformal symmetry.

I. INTRODUCTION

It has often been suggested that the symmetries which govern elementary particle interactions are far more extensive than anything visible to the naked eye.[1] However, few attempts have been made to substantiate such suggestions. The readily apparent symmetry signals are phenomena in the low-frequency domain. We have in mind orderings in particle spectra, small renormalization of comparative half-lives in superallowed β transitions,[2] existence of low-mass zero-spin mesons (near-Goldstone modes), the remarkable accuracy of some Goldberger-Treiman formulas, etc. Of these, the first two signals are such as to indicate an internal symmetry of the conventional or Wigner-Weyl type, and the last two signals would be indicative of a spontaneously broken or Nambu-Goldstone symmetry.[3]

Less readily apparent, but still clearly discernible, would be the spoor of a spontaneously broken local gauge symmetry or a Higgs-Kibble[4] realization.

The major problem associated with symmetries that are known to have a low-frequency signature is the following: Why is the symmetry not masked completely by radiative corrections[2] involving the symmetry-breaking interaction? Is there a higher symmetry that leads to a diminution, if not cancellation, of radiative effects? While systematic treatments are lacking, answers to such problems have been given in a few specific cases. (See, for example, the discussion in Refs. 2, 3, and 4.)

Very little is known about the physical relevance of symmetries which, by their very nature, can only make themselves manifest at high frequencies. We have in mind (a) dilatational[1] symmetry, which may or may not have anything to do with the phenomenon of Bjorken scaling[5] *at present energies*, and (b) full conformal[1] symmetry which—despite heroic efforts by a number of workers[6]—has hitherto remained an enigma.

Our purpose in this paper is to examine some aspects of scale and conformal symmetry in the context of a hadrodynamics in which all interactions are generated from a semisimple gauge group. The genesis of the Bell-Jackiw-Adler anomaly[7] as well as the nonrenormalization theorem[8] for the anomaly are tied to conformal symmetry. (Our considerations constitute a proof of the nonrenormalization theorem which happens to be manifestly free of infrared problems.)

This paper is organized as follows: In Sec. II we formulate the notion of short-distance symmetry in the context of the Wilson expansion,[9] and sketch a brief argument to indicate that the nature of symmetry realization (Wigner-Weyl vs Nambu-Goldstone) is irrelevant in discussing this type of symmetry. In Sec. III we start with the conventional wisdom on scale and conformal symmetry and proceed to formulate, what appear to us to be, the relevant problems in the study of conformal symmetry. The availability of solutions in specific situations is discussed. In Sec. IV we study the genesis of the Bell-Jackiw-Adler anomaly, and the nonrenormalization theorem for the anomaly, within the framework of conformal symmetry. Section V is devoted to a summary of the conclusions and an appraisal of the outlook.

II. SHORT-DISTANCE SYMMETRY

For our purposes, the most convenient formulation of short-distance symmetry (SDS) is provided by the Wilson expansion near the tip of the light cone (in x space). For a pair of local operators A and B we have

$$A(x)B(0) = \sum_n a_n(x, g)O_n(0) , \tag{2.1}$$

where the set of local operators $O_n(0)$ is presumed to be complete and closed, in the sense that the short-distance behavior of the product of any two operators is representable in the form (2.1). It is assumed that the operators O_n can be ordered by the degree of singularity of the c-number functions which multiply them. The leading operators on the right-hand side of (2.1) are those which multiply the most singular functions; we shall say that O_m leads over O_n if a_m is more singular than a_n. It is evident, of course, that if naive dimensional analysis were to hold, ordering as introduced above would coincide with the ordering of operators by their physical dimension. Note that operators which are expected to be dimensionally nondegenerate can easily become degenerate in this naive limit.

We develop our formulation of SDS by requiring, first of all, the following: If the symmetry group is G, and if A, B, and each of the O_n are well-defined tensor operators under G, then the leading O_n on the right-hand side of (2.1) are the ones which occur in the Clebsch-Gordan decomposition of $A \otimes B$ under G. In other words, operators which do not occur in the Clebsch-Gordan series, and which are therefore generated by the action of symmetry-breaking spurions are presumed to be unimportant in the short-distance limit. For our purposes, this last statement is the operational content of the statement that all symmetry-breaking is stemming from (generalized) mass terms.[9]

Next, we observe that information about the *nature* of the underlying symmetry is coded in the c-number functions a_n. Starting with this observation we can examine the validity of a conjecture[10] that was inspired by the work of Weinberg on spectral-function sum rules[11]: *At small distances the Nambu-Goldstone way merges with the Wigner-Weyl way; one can talk of symmetry without specifying the nature of the realization.* To see this, let us start with a Nambu-Goldstone realization, with the spontaneous-breakage mechanism triggered in the usual way via explicit introduction of σ-like fields. All Green's functions of the theory are then festooned with tadpoles, and the renormalization-group equations for the a_i may be written in the form[12]

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_{O_n}(g) + \gamma_A(g) + \gamma_B(g) - \tilde{\gamma}_\sigma(g) v \frac{\partial}{\partial v}\right] a_n = 0. \quad (2.2)$$

Here, we have simplified matters by assuming that the theory contains just one coupling constant and a single σ-like field. The notation is as follows: μ is the mass scale introduced to define the Green's functions of the theory, v is the vacuum expectation value of the σ field, and

$$\beta(g) = \frac{dg}{d\ln \mu}, \quad (2.3)$$

$$\gamma_i(g) = \frac{d\ln Z_i}{d\ln \mu} \qquad (i \equiv O_n,\ A,\ \text{or } B), \quad (2.4)$$

$$\tilde{\gamma}_\sigma(g) = \frac{d\ln(Z\sigma)^{1/2}}{d\ln \mu}. \quad (2.5)$$

Here Z_i is the constant which renormalizes the operator labeled by the index i and Z_σ is the wave-function renormalization constant for the q-number part of the σ field.

Equation (2.2) can be readily solved:

$$a_n = (\mu^2 x^2)^{\gamma(g_\infty)/2} \exp\left\{-\int_0^t [\gamma(\bar{g}) - \gamma(g_\infty)]dt\right\} \Phi_n(\bar{g}, \bar{\rho}), \quad (2.6)$$

where $\gamma \equiv \gamma_{O_n} - \gamma_A - \gamma_B$,

$$t \equiv -\tfrac{1}{2}\ln(x^2\mu^2) = \int_g^{\bar{g}} \frac{dg'}{\beta(g')}, \quad (2.7)$$

$$\bar{\rho} = \frac{\bar{v}}{\mu} = \frac{v}{\mu} \exp\left\{-\int_0^t [1 + \tilde{\gamma}_\sigma(\bar{g})]dt\right\}, \quad (2.8)$$

and Φ_n is an arbitrary function. We have assumed that there exists an ultraviolet-stable zero of β at $g = g_\infty$ so that $\bar{g} \to g_\infty$ as $x^2 \to 0$.

Observe that the positivity condition

$$1 + \tilde{\gamma}_\sigma(g_\infty) > 0 \quad (2.9)$$

guarantees the vanishing of $\bar{\rho}$, and hence freedom from tadpoles, in the short-distance limit. In other words, Eq. (2.9) ensures the merger of Nambu-Goldstone and Wigner-Weyl realizations at small distances. For theories quantized in a Hilbert space with a positive metric Eq. (2.9) is always valid and this merger always takes place. For other theories, and this includes all gauge theories, Eq. (2.9) can be checked only if some reliable procedure for calculation of γ is available. If the interactions are generated via a semisimple gauge group the theory may be[13] asymptotically free and one might be able to use perturbative arguments to verify Eq. (2.9).

To summarize: Within the framework provided by the Wilson expansion, a sufficient condition, for the short-distance merger of Wigner-Weyl and Nambu-Goldstone realizations of symmetry, is that the (q-number part of the) triggering field of the latter realization admit of a Källén-Lehmann representation.

III. DILATIONS AND CONFORMAL TRANSFORMATIONS

In canonical field theory, with manifestly renormalizable couplings, the generator of dilatations is associated with a current

$$D_\lambda = x^\mu \theta_{\mu\lambda}, \quad (3.1)$$

where $\theta_{\mu\lambda}$ is the symmetric Gürsey-Huggins form[14] of the energy-momentum tensor. (In terms of D_λ, the dilatation-generator $D \equiv \int D_0 d^3x$.) Also, the generators of special conformal transformations are associated with the currents

$$K_{\alpha\lambda}=(x^2 g_{\alpha\mu}-2x_\alpha x_\mu)\theta^\mu_\lambda . \qquad (3.2)$$

Combining Eqs. (3.2) and (3.1) we obtain

$$\partial^\lambda K_{\alpha\lambda}=-2x_\alpha \partial^\lambda D_\lambda , \qquad (3.3)$$

a result which embodies the *Mack-Salam theorem*[6] In canonical, renormalizable field theories the extension of the Poincaré group with the dilatation group induces invariance under the full conformal group SO(4, 2) ~SU(2, 2).

This section is devoted to the following question: To what extent can one salvage the Mack-Salam theorem in a real anomaly-infested world?

Consider first the group of scale transformations. The anomalies which preclude a straightforward implementation of the dilatation group, in *quantum* field theory, are made manifest by the Callan-Symanzik equation[15]

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-n y(g)\right]\Gamma^{(n)}=\Gamma^{(n)}_\Delta . \qquad (3.4)$$

Here $\Gamma^{(n)}$ is an amputated n-point Green's function, $\Gamma^{(n)}_\Delta$ an "$n+1$ point" Green's function obtained from $\Gamma^{(n)}$ by including a term

$$\Delta=\int d^4x : \theta_{\mu\nu}(x)g^{\mu\nu} : \qquad (3.5)$$

in the defining chronological product. (Note that the double dots indicate the *soft part* of the normal product. See, for example, the discussion by Schroer.[16]) In writing (3.4) we have assumed, in the interest of simplicity, that there is just one field, one mass scale, and one coupling constant.

Now, in a canonical scale-invariant theory Δ, β, and γ all vanish. In the real world we can justify the neglect of Δ at high frequencies, at least for nonexceptional momenta, via the Weinberg theorem[17]; however, nothing so innocuous can be used for discarding the anomaly functions β and γ.

The solution of Eq. (3.4) can be written as

$$\Gamma^{(n)}(\xi x_1,\xi x_2,\ldots,\xi x_n;g)=\xi^{-D+n\gamma(g_\infty)}\exp\left[-n\int_0^t(\gamma(\bar g)-\gamma(g_\infty))\,dt\right]\times\Gamma^{(n)}(x_1,x_2,\ldots,x_n;\bar g(t))\ \ (\text{in the limit of } \xi\to 0), \qquad (3.6)$$

where D is the canonical dimension of $\Gamma^{(n)}$ and $\bar g(t)$ is defined by

$$t=-\ln|\xi|=\int_g^{\bar g(t)}\frac{dg'}{\beta(g')} . \qquad (3.7)$$

Three cases need to be distinguished:

(a) The theory is at a Gell-Mann-Low[18] fixed point, $g=g_c$, i.e., $\beta(g_c)=0$.

(b) The theory is in the domain of attraction of a nontrival ultraviolet-stable fixed point.

(c) The origin is ultraviolet-stable and the theory is in the domain of attraction of the origin.

In each of these cases the scaling limit is attained at small distances but not with normal physical dimensions; it is important to bear in mind that even in case (c) the dimensional anomalies leave a nonvanishing trace and the naive scaling which would be possible if the theory were free—instead of just asymptotically free—is never attained.

We have, in essence, reviewed the conventional wisdom for the scale group in order to ask the right questions for the full conformal group. Two of these warrant explicit statement:

1. Is there a "sensible" differential equation, analogous to Eq. (3.4), for special conformal transformations?[16] (By sensible we mean an equation that will permit us to study the existence, and the possible onset, of the conformal limit at small distances.)

2. Is the conformal limit attained in cases (a), (b), and (c) discussed above?

A detailed discussion of these questions is reserved for a future publication; in the present paper we confine ourselves to the following remarks.

For case (a) the conformal limit is attained to the same extent that the scaling limit is attained, i.e., with anomalous dimensions. In a sense, therefore, the Mack-Salam theorem goes through.

For case (c) a weak form of the Mack-Salam theorem is immediately obvious: The theorem is operative for those Green's functions that do not involve dimensional anomalies. This means, in particular, that all Green's functions involving weak and electromagnetic currents attain the conformal limit if the strong interactions are generated in an asymptotically free way.

It is worth noting that the conformal symmetry alone can only determine the n-point Green's functions ($n\geq 4$) up to arbitrary functions of harmonic ratios. However, in case (c) the Callan-Symanzik equation determines these functions uniquely.

IV. THE AXIAL ANOMALY

We restrict our discussion to a specific dynamical situation. We assume that weak and electromagnetic interactions have a common origin in a gauge group realized in the manner of Higgs and Kibble. Furthermore, we assume that strong interactions are generated via a semisimple gauge group which commutes with the weak-electromagnetic gauge group and remains unbroken. Even in this model our results are complete only in the situation in which the strong coupling is such that the theory is in the domain of attraction of the origin. [This permits us to expand the relevant Green's function in powers of $\bar{g}$, the invariant coupling constant defined in Eq. (3.7).] Several authors have speculated that the situation envisaged here may, in fact, correspond to physical reality.[4]

The internal hadronic group relevant to our considerations is

$$G = SU(3)_L \otimes SU(3)_R \otimes SU(3)_C , \tag{4.1}$$

the first two factors corresponding to the usual chiral group and the last factor is the so-called "color" group which is gauged to generate strong interactions. (Our discussion is without prejudice to the possibility—which seems well nigh to be a certainty—that the actual internal hadronic group is larger than G.) All vector and axial-vector currents, in the following discussion, are presumed to transform as $(8, 1, 1,) \oplus (1, 8, 1)$ under G.

Consider now the three-point function

$$\Gamma^{\alpha\beta\gamma}_{\mu\nu\lambda}(x, y, z) = \langle 0 | T\{V^{\alpha}_{\mu}(x) V^{\beta}_{\nu}(y) A^{\gamma}_{\lambda}(z)\} | 0 \rangle , \tag{4.2}$$

where μ, ν, λ are Lorentz indices and α, β, γ are SU(3) indices (1–8).

The Callan-Symanzik equation tells us how $T(x, y, z)$ behaves when $x-y, x-z \to 0$ at the same rate. The case corresponding to $x-y \to 0$ and $x-z \to 0$ at different rates can be analyzed by the operator-product expansion (OPE) of Wilson. We find that $T(x, y, z)$ is conformal-symmetric[19] in the short-distance limit and may be written as follows[20,21]:

$$T^{\alpha\beta\gamma}_{\mu\nu\lambda}(x, y, z) = \frac{N d^{\alpha\beta\gamma}}{16\pi^6} \left\{ \mathrm{Tr}(\gamma_5 \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma \gamma_\lambda \gamma_\tau) \frac{(x-y)^\rho (y-z)^\sigma (z-x)^\tau}{[(x-y)^2 - i\epsilon]^2 [(y-z)^2 - i\epsilon]^2 [(z-x)^2 - i\epsilon]^2} + c\epsilon_{\mu\nu\lambda\rho} [\partial^{\rho}_{(x)} \delta^4(x-z)\delta^4(y-z) - \partial^{\rho}_{(y)} \delta^4(y-z)\delta^4(x-z)] \right\}. \tag{4.3}$$

The first term in the curly brackets in Eq. (4.3) is manifestly identical to the expression for the triangle graph in massless free spinor theory, the second term—enclosed in square brackets—is a contact term permitted by conformal symmetry and parity. Note that the normalization constants N and c are not specified by the conformal group per se. (However, read on.) Note also that the contact terms play the role of counterterms that are needed to give meaning to the triangle graph. (The $i\epsilon$ prescription in the two-point function is not sufficient to specify this graph; the ambiguities can be seen most clearly in momentum space, where they stem from the fact that the graph has a superficial linear divergence and different routing of the internal momenta will lead to different results—results which differ by polynomials in the external momenta.)

Next, we note that it is impossible to choose c so that the vector and axial-vector currents are simultaneously conserved—unless we are willing to set $N=0$. In other words, the constraint of conformal symmetry forces the chiral group to yield—if the theory is to be nontrivial.

If $N \neq 0$ and we insist that conservation of the vector current be sacrosanct, the numerical value of c is specified and is such that conservation of the axial-vector current breaks down. The continuity equation relevant to $\pi^0 \to 2\gamma$ takes the form

$$\partial^\lambda A^{(3)}_\lambda = N\left(\frac{\alpha}{8\pi}\right) F_{\mu\nu} \tilde{F}^{\mu\nu} , \tag{4.4}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}_{\mu\nu}$ is its dual.

Finally we discuss the magnitude of the constant N. Since the conformal-symmetric solution for $T^{\alpha\beta\gamma}_{\mu\nu\lambda}(x, y, z)$, displayed in Eq. (4.3), must satisfy the renormalization-group equation, we have

$$\left[\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g}\right] N = 0 . \tag{4.5}$$

Observe now that N is dimensionless and, therefore, cannot depend on μ; if we make the innocuous assumption that it is differentiable at $g=0$, Eq. (4.5) implies that

$$N(g) = N(0) = 1 . \tag{4.6}$$

Equation (4.6) establishes the nonrenormalization theorem for the axial anomaly. This theorem is here seen to emerge as a consequence of conformal symmetry implemented within the framework of the renormalization group.

Note added in proof. It should be stressed that conformal symmetry is here a property of the "summed-up" theory; it does *not* hold order by order in the ordinary coupling constant. The situation is analogous to that for the anomalous algebras discussed in Ref. 24, except that here the symmetry-violating graphs make their appearance in *fourth* order. That the parameter N should equal unity, in an asymptotically free theory, is pretty much self-evident; the "formal argument" leading to Eq. (4.6) is included only for the sake of completeness.

V. COMMUTATOR ANOMALIES

Following Ref. 21 we may parametrize the anomalies, associated with the names of Schwinger[22] and of Johnson and Low,[23] through the relevant terms in the Wilson expansion:

$$V_\mu^\alpha(x)V_\nu^\beta(0)=S_{VV}\delta^{\alpha\beta}\frac{(g_{\mu\nu}x^2-2x_\mu x_\nu)}{2\pi^4(-x^2+i\epsilon x_0)^4} - K_{VV}d^{\alpha\beta\gamma}\frac{g^{\rho\tau}\epsilon_{\mu\nu\rho\sigma}A_\tau^\gamma(0)x^\sigma}{2\pi^2(-x^2+i\epsilon x_0)^2} + \cdots . \tag{5.1}$$

Similarly one can define S_{AA}, K_{AA}, and K_{VA}. ($S_{VA}=0$ by parity.) The terms indicated by dots in Eq. (5.1) are such that their singularity structure or their transformation property makes them irrelevant for our purpose. For a more complete display of terms in the Wilson expansion, see for example, Ref. 24.

Assuming that the "triple-point" limit in Eq. (4.3) can be taken sequentially (e.g., $y\to z$ followed by $x\to y$), Eqs. (4.2), (4.3), (5.1), etc., lead to the Crewther[25] formula

$$N=S_{VV}K_{VA}. \tag{5.2}$$

Equation (4.6) above implies, however, that the conformal group actually leads to the stronger formula

$$S_{VV}K_{VA}=1. \tag{5.3}$$

Note that in an asymptotically free theory[24] $K_{VA}=1$ so that S_{VV} also equals unity; this, of course, is just the content of the so-called parton model.

VI. CONCLUDING REMARKS

We have presented a formulation of the notion of short-distance symmetry within the framework of the Wilson expansion and have shown that the nature of symmetry realization, Wigner-Weyl vs Nambu-Goldstone, is irrelevant for this type of symmetry. This permits us to discuss scale and conformal symmetry without getting involved with spectral problems and entities such as "dilatons." Furthermore, since we insist only on symmetry at small distances, our discussion bypasses completely the usual conceptual difficulties that have afflicted many of the proposals, which have been made in the past, to investigate the physical relevance of the conformal group. For example, one need not worry about any possible clash with the principle of causality.

We have also discussed the question: When does the underlying dynamics permit the theory to achieve conformal invariance at small distances? For asymptotically free theories a sufficiently definitive answer is available; for situations in which the theory is in the domain of attraction of a nontrivial fixed point, the question is under study and will be discussed elsewhere.

If the theory is such that the conformal limit is indeed attained at small distances, the symmetry picture which emerges is quite striking. The conformal group clashes with the chiral group, the latter yields, and an anomaly is generated. The proper orientation of the anomaly is fixed by the electromagnetic gauge group. Furthermore, not only does the conformal group engender the anomalous continuity equations, it is also responsible for the nonrenormalization theorem which prevents the metamorphosis of these equations into relationships rendered useless by radiative corrections. (Without freedom from incalculable radiative corrections, one could never use the anomaly prediction for $\pi^0\to 2\gamma$ to rule out certain gauge theories and models of hadronic structure.)

We note, also, that the conformal group relates the lack of closure of the $U(6)\otimes U(6)$ algebra[24] of Feynman, Gell-Mann, and Zweig to the deviations of the Schwinger anomaly from the parton model.

It would be surprising indeed if a structure as rich as that furnished by the conformal group did not have implications, of physical relevance, other than those outlined in this paper.

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