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Problems with the Achiman model in the deep-inelastic region*

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It is pointed out that the prediction of a large charge-symmetry violation by the Achiman gauge-symmetry model also leads to a violation of the positivity condition on the structure function $F_2^{\nu N}$. A search for the origins of this inconsistency is made.

In a recent paper by Aubrecht and this author[1] (hereafter referred to as AR) a study of the Achiman model[2] in the deep-inelastic region was made using the techniques of light-cone algebra.[3] Recall that the Achiman model employs Han-Nambu quarks[4] to construct an extension of the Weinberg-Salam model[5] to the hadronic sector. The gauge group is SU(2)⊗U(1), and the gauge symmetry is broken in a simple way by a doublet of Higgs fields.[6] Using the CERN data[7] on the weak charged and neutral current processes

$$\nu_\mu(\overline{\nu}_\mu)+N\to\mu^-(\mu^+)+X,\quad \nu_\mu(\overline{\nu}_\mu)+N\to\nu_\mu(\overline{\nu}_\mu)+X,$$

and the SLAC data[8] on $e+N\to e+X$, we pointed out in AR that the Achiman model predicts a large charge-symmetry violation $\gtrsim 30\%$ even below the threshold of producing SU(3)″ nonsinglet states. There is nothing in the experimental data available at present to suggest such a big violation of charge symmetry.

In this note I shall show that this prediction of a large violation of charge symmetry by the Achiman model also leads to a violation of the positivity condition on the structure function F_2. Since the positivity conditions on the structure functions are model independent statements, we may conclude that the Achiman model is ruled out experimentally. I shall also discuss in some detail the possible origin of this conflict between experiment and the model under consideration.

For notation and details, the reader is referred to AR. The structure functions below and above the threshold of producing SU(3)″ nonsinglet states are given as follows.

Structure functions below the SU(3)″ *threshold.*

$$F_2^{eN}(\xi)=-\xi[(\tfrac{2}{3}A^{00}+\tfrac{1}{6}\sqrt{2}A^{80})],\tag{1}$$

$$F_2^{\nu N}(\xi)=-\xi[2A^{00}+\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta_C)A^{80}-\tfrac{3}{2}\sqrt{2}\,\sin^2\theta_C S^{80}],\tag{2}$$

$$F_3^{\nu N}(\xi)=[2S^{00}+\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta_C)S^{80}-\tfrac{3}{2}\sqrt{2}\,\sin^2\theta_C A^{80}],\tag{3}$$

$$F_2^{\overline{\nu}N}(\xi)=-\xi[2A^{00}+\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta_C)A^{80}+\tfrac{3}{2}\sqrt{2}\,\sin^2\theta_C S^{80}],\tag{4}$$

$$F_3^{\overline{\nu}N}(\xi)=[2S^{00}+\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta_C)S^{80}+\tfrac{3}{2}\sqrt{2}\,\sin^2\theta_C A^{80}],\tag{5}$$

$$G_2^{\nu N}(\xi)=-\tfrac{1}{3}\xi(1-4z+8z^2)(A^{00}+\tfrac{1}{4}\sqrt{2}\,A^{80}),\tag{6}$$

$$G_3^{\nu N}(\xi)=\tfrac{1}{3}(1-4z)(S^{00}+\tfrac{1}{4}\sqrt{2}\,S^{80}),\tag{7}$$

where $z=\sin^2\theta_W$ and $\xi=q^2/2M\nu$.

Structure functions above the SU(3)″ *threshold.*

$$F_2^{eN}(\xi)=-\tfrac{1}{3}\xi(4A^{00}+\tfrac{1}{2}\sqrt{2}\,A^{80}),\tag{8}$$

$$F_2^{\nu N}(\xi)=-3\xi(4A^{00}+\tfrac{1}{2}\sqrt{2}\,A^{80}-\tfrac{3}{2}\sqrt{2}\,S^{80}),\tag{9}$$

$$F_3^{\nu N}(\xi)=3(4S^{00}+\tfrac{1}{2}\sqrt{2}\,S^{80}-\tfrac{3}{2}\sqrt{2}\,A^{80}),\tag{10}$$

$$F_2^{\overline{\nu}N}(\xi)=-3\xi(4A^{00}+\tfrac{1}{2}\sqrt{2}\,A^{80}+\tfrac{3}{2}\sqrt{2}\,S^{80}),\tag{11}$$

$$F_3^{\overline{\nu}N}(\xi)=3(4S^{00}+\tfrac{1}{2}\sqrt{2}\,S^{80}+\tfrac{3}{2}\sqrt{2}\,A^{80}),\tag{12}$$

$$G_2^{\nu N}(\xi)=-\tfrac{1}{3}\xi[2(1-2z+8z^2)A^{00}+\tfrac{1}{4}\sqrt{2}\,(1-8z+8z^2)A^{80}],\tag{13}$$

$$G_3^{\nu N}(\xi)=\tfrac{2}{3}(1-2z)S^{00}+\tfrac{1}{12}\sqrt{2}\,(1-8z)S^{80}.\tag{14}$$

Let us assume that the CERN and SLAC data relate to the scaling region below the SU(3)″ threshold. Using Eqs. (1) through (7) we can now evaluate z and the integrals of the various structure functions, or equivalently the quantities

$$I_{A,S}^{i0}\equiv\int_0^1 d\xi\,\xi(A^{i0},S^{i0}),\quad i=0,8.$$

We find that

$$z=0.68\pm0.07,\tag{15}$$

$$I_A^{00}+\tfrac{1}{4}\sqrt{2}\,I_A^{80}=-0.217\pm0.015,\tag{16}$$

$$I_A^{80}=-0.10\pm0.063,\tag{17}$$

$$I_S^{00}+\tfrac{1}{4}\sqrt{2}\,I_S^{80}=0.063\pm0.056,\tag{18}$$

$$I_S^{80}=-1.727\pm0.341.\tag{19}$$

Note the large value for the quantity I_S^{80}. It is this object which is responsible for the charge-symmetry violation in the structure function F_2, both below and above the SU(3)″ threshold. For instance, from Eqs. (2) and (4) we see that

$$\int_0^1 d\xi\,(F_2^{\nu N}-F_2^{\overline{\nu}N})=3\sqrt{2}\,\sin^2\theta_C I_S^{80}=-0.41\pm0.09\tag{20}$$

below the SU(3)″ threshold.

It is important to realize that although the values of a given structure function above and below the SU(3)″ threshold are in general different [above the SU(3)″ threshold the structure functions will receive additional contributions from SU(3)″ nonsinglet states], the values of the quantities z, $I^{00}_{A,S}$, $I^{80}_{A,S}$ are the same in the two scaling regions. This allows us to use the values of Eqs. (15) through (19) in Eqs. (8) through (14) and obtain quantitative predictions on the high-energy neutrino and antineutrino production cross sections of SU(3)″ nonsinglet states. Accordingly, from Eq. (9) we get

$$\int_0^1 d\xi F_2^{\nu N}(\xi) = -12[(I_A^{00}+\tfrac{1}{4}\sqrt{2}\,I_A^{80}) - \tfrac{1}{8}\sqrt{2}\,I_A^{80} - \tfrac{3}{8}\sqrt{2}\,I_S^{80}]$$

$$= -12[-(0.217\pm 0.015)+(0.018\pm 0.011) + (0.916\pm 0.181)]$$

$$= -8.59\pm 2.18\,, \qquad (21)$$

which is negative. On the right-hand side of Eq. (21), we have exhibited the various numerical contributions separately. The largest contribution (and the source of trouble) comes from the piece containing I_S^{80}, which is also responsible for the large charge-symmetry violation [see Eq. (20)]. To guarantee the positivity of $F_2^{\nu N}(\xi)$, the value of I_S^{80} would have to be smaller at least by a factor of 5. Of course, the smaller value of I_S^{80} would also mean smaller charge-symmetry violation.[9]

Since the Achiman model is in direct clash with the positivity requirement on the structure function $F_2^{\nu N}$, let us search for the origin of this disagreement.

(1) The first possibility we may consider is that the CERN data do not scale (after all, they have neutrino and antineutrino energies in the range 1–10 GeV only). If this were the case, we could not analyze the data with the help of Eqs. (1) through (7) since they are derived from the leading light-cone singularities of the current commutators and that means scaling. In support of this possibility, we might be tempted to point out the discrepancy between the published CERN and Fermilab data.[10] The latter involves neutrinos and antineutrinos of much higher energies and gives values for the slopes α_ν and $\alpha_{\bar{\nu}}$ two standard deviations below the corresponding values in the CERN data. Moreover, the ratios of the neutral to charged current cross sections for neutrino as well as antineutrino in the Fermilab data are smaller than those in the CERN data. The situation has recently changed, however. With the latest Fermilab data[11] reported at the London conference, this discrepancy has virtually disappeared (albeit with large error bars), and so is not a problem. We also mention here the fact that recent experiments show some breakdown of scaling in the electron data.[12] Again, it is too small to be a source of trouble. To summarize, as far as the scaling aspect is concerned, we see no serious problem in analyzing the available data with Eqs. (1) through (7). We must therefore, turn our attention to other things.

(2) As a second possibility, let us consider the (rather unlikely) explanation that the data under consideration do not relate to the scaling region below the SU(3)″ threshold, that the SU(3)″ threshold has been crossed, and channels for the production of SU(3)″ nonsinglet states have opened up. If this were the case, Eqs. (1) through (7) would be of no use to us in analyzing the data. If we are just above the SU(3)″ threshold, scaling has broken down and even Eqs. (8) through (14) are of no use. However, well above the SU(3)″ threshold scaling sets in again and we can work with Eqs. (8) through (14). This would circumvent the problem of a clash between the Achiman model and the positivity requirement on $F_2^{\nu N}$. Even so, the problem of a large charge-symmetry violation is still there. In AR we showed that

$$\gamma_2 = \frac{\int_0^1 d\xi (F_2^{\nu N} - F_2^{\bar{\nu} N})}{\int_0^1 d\xi (F_2^{\nu N} + F_2^{\bar{\nu} N})} = 0.27\pm 0.07 \qquad (22)$$

and

$$\gamma_3 = \frac{\int_0^1 d\xi\, \xi (F_3^{\nu N} - F_3^{\bar{\nu} N})}{\int_0^1 d\xi\, \xi (F_3^{\nu N} + F_3^{\bar{\nu} N})} = 6.52\pm 4.30 \qquad (23)$$

above the SU(3)″ threshold.

Experimentally, there is no evidence for such a large charge-symmetry violation. Note that the expression of charge-symmetry violation in terms of the ratios γ_2 and γ_3 is but one of the various ways of parameterizing this effect. In the same context quantities such as

$$\langle \xi y\rangle,\ \langle E'/E\rangle\,,\ \langle (E'/E)^2\rangle\,,\dots\,,$$

which are directly measurable, are of great interest.[13] Let us compute the Achiman model value of $\langle \xi y\rangle$ and compare it with experiment. Now,

$$\langle \xi y\rangle_\nu + R\langle \xi y\rangle_{\bar{\nu}} = \frac{G^2 ME}{24\pi\sigma^{\nu N}}\left[7\int_0^1 d\xi\, \xi\left(F_2^{\nu N}+F_2^{\bar{\nu} N}\right) - 5\int_0^1 d\xi\, \xi^2\left(F_3^{\nu N}-F_3^{\bar{\nu} N}\right)\right], \qquad (24)$$

where $R=\sigma^{\bar{\nu}N}/\sigma^{\nu N}$. From Eqs. (8), (9), and (11) we have the sum rule

$$F_2^{\nu N}+F_2^{\bar{\nu} N} = 18F_2^{eN}\,, \qquad (25)$$

which gives

$$\int_0^1 d\xi\, \xi(F_2^{\nu N}+F_2^{\bar{\nu} N}) = 18\int_0^1 d\xi\, \xi F_2^{eN} = 0.648\pm 0.046\,, \tag{26}$$

where we have used the SLAC data to evaluate the right-hand side in (26). Unfortunately, we cannot evaluate the second integral on the right-hand side of (24) equally cleanly. We make the additional assumption that

$$\frac{\int_0^1 d\xi\, \xi^2(F_3^{\nu N}-F_3^{\bar{\nu} N})}{\int_0^1 d\xi\, \xi(F_3^{\nu N}-F_3^{\bar{\nu} N})} = \frac{\int_0^1 d\xi\, \xi F_2^{eN}}{\int_0^1 d\xi\, F_2^{eN}}\,. \tag{27}$$

Actually this boils down to assuming the same functional dependence for $A^{00}(\xi)$ and $A^{80}(\xi)$, apart from an overall multiplicative factor. Of course, this is a reasonable assumption since the strange quark contribution in the nucleon matrix elements is generally negligible in comparison to the nonstrange quarks. We thus find that

$$\int_0^1 d\xi\, \xi^2(F_3^{\nu N}-F_3^{\bar{\nu} N}) = 0.124\pm 0.082\,. \tag{28}$$

From (24), (26), and (28) we finally get

$$\langle \xi y\rangle_\nu + R\langle \xi y\rangle_{\bar{\nu}} = 0.343\pm 0.147\,, \tag{29}$$

which is to be compared with the CERN result

$$\langle \xi y\rangle_\nu + R\langle \xi y\rangle_{\bar{\nu}} = 0.146\pm 0.01\,. \tag{30}$$

The Achiman model prediction is a minimum of 35% off the experimental result.[14]

(3) The most likely explanation is that there is something wrong with the Achiman model at a probably fundamental level. It will be recalled that below the SU(3)″ threshold the weak hadronic neutral current in the Achiman model is given by

$$J_\lambda^Z = \tfrac{1}{2}(V_\lambda{}^3 + A_\lambda{}^3) - 2zJ_\lambda^{\text{em}}\,, \tag{31}$$

whereas the canonical form of the neutral current in the original Weinberg-Salam theory is

$$J_\lambda^Z = (V_\lambda{}^3 + A_\lambda{}^3) - 2zJ_\lambda^{\text{em}}\,. \tag{32}$$

Here, as before, $z=\sin^2\theta_W$. This relative suppression of the neutral current in the Achiman model was originally a welcome feature since it served to lower the theoretical prediction on the ratio

$$R = \frac{\sigma(\nu p\to \nu\pi^0 p)+\sigma(\nu n\to\nu\pi^0 n)}{2\sigma(\nu n\to \mu^-\pi^0 p)}\,,$$

within the experimental upper bound. However, the cost is too high since it leads to much more serious difficulties elsewhere. Moreover, the new refined calculations of Adler *et al.*[15] give a more acceptable number for R even with the form (32). Using this form and analyzing neutrino scattering from an aluminum target, they find that

$$R \simeq 0.18 \text{ for } z=0.4\,, \tag{33}$$

whereas the experimental upper limit of W. Lee[16] is

$$R<0.14 \quad (90\% \text{ confidence level})\,. \tag{34}$$

The discrepancy between (33) and (34) is hardly significant since it may be attributed to the approximations made in the work of Adler *et al.*

As far as the question of charge-symmetry violations is concerned, the form (32) also does much better than the Achiman form (31). The structure functions corresponding to (32) are given by[17]

$$G_2^{\nu N}(\xi) = -\xi[(1-2z+\tfrac{8}{3}z^2)A^{00} + \tfrac{1}{2}\sqrt{2}\,(1-2z+\tfrac{4}{3}z^2)A^{80}]\,, \tag{35}$$

$$G_3^{\nu N}(\xi) = (1-2z)(S^{00}+\tfrac{1}{2}\sqrt{2}\,S^{80})\,. \tag{36}$$

When we analyze the data with the help of Eqs. (1) through (5) and (35) and (36), we find that the charge-symmetry violation is about 5% and $z\sim 0.4$. Of course, some charge-symmetry violation must be there, induced as it is by the strangeness-changing part of the Cabibbo current.

To overcome this drawback of the Achiman model, perhaps one needs more than just a doublet of Higgs fields to break the gauge symmetry realistically. The point is that with the simple symmetry-breaking model employed by Achiman, one gets the relation

$$m_W{}^2/m_Z{}^2 = g^2/(g^2+g'^2) = \cos^2\theta_W\,,$$

and so the interaction Lagrangian for the process $\nu+N\to\nu+X$ is given by

$$\mathcal{L}_I = \frac{G}{\sqrt{2}}\, i\bar{\nu}\gamma_\lambda(1+\gamma_5)\nu J_\lambda^Z\,. \tag{37}$$

In a more general scheme of symmetry breaking, however, we need not have a relation between the W and Z vector boson masses. We could then write

$$m_W{}^2/m_Z{}^2 = ag^2/(g^2+g'^2)\,,$$

where a is arbitrary, and the factor a would appear multiplied on the right-hand side of the interaction Lagrangian (37). This may provide the necessary enhancement factor to compensate for the suppression of the weak hadronic neutral current in the Achiman model.

In summary, the Achiman model in its present form cannot be reconciled with the experimental data available at present. It probably needs a more elaborate scheme of gauge-symmetry breaking to be salvaged.

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[9] Actually, the large value of I_S^{80} may also imply a clash with the normalization condition

$$\int_0^1 d\xi\, S^{80}(\xi) = -1/\sqrt{2}\,,$$

and, therefore, with the Adler sum rule

$$\int_0^1 \frac{d\xi}{\xi}\,(F_2^{\bar{\nu}N} - F_2^{\nu N}) = 3\sin^2\theta_C\,,$$

and the Gross–Llewellyn-Smith sum rule

$$\int_0^1 d\xi\,(F_3^{\nu N} + F_3^{\bar{\nu}N}) = -6 + 3\sin^2\theta_C.$$

This will happen if the function $S^{80}(\xi)$ does not change sign in the interval $0 \le \xi \le 1$ because in that case

$$|I_S^{80}| = \left|\int_0^1 d\xi\,\xi\,S^{80}(\xi)\right| \le \left|\int_0^1 d\xi\,S^{80}(\xi)\right|.$$

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[14] One can also calculate the values of quantities such as

$$\langle E'/E\rangle,\ \langle (E'/E)^2\rangle, \ldots,$$

etc. in a straightforward manner. We find, for example, that

$$\langle E'/E\rangle_\nu + R\,\langle E'/E\rangle_{\bar{\nu}} = 0.765 \pm 0.167,$$

$$\langle E'/E\rangle_\nu - R\,\langle E'/E\rangle_{\bar{\nu}} = 0.133 \pm 0.134,$$

$$\langle (E'/E)^2\rangle_\nu + R\,\langle (E'/E)^2\rangle_{\bar{\nu}} = 0.546 \pm 0.117,$$

$$\langle (E'/E)^2\rangle_\nu - R\,\langle (E'/E)^2\rangle_{\bar{\nu}} = 0.014 \pm 0.076,$$

below the SU(3)″ threshold. Unfortunately, no experimental data on the measurement of these quantities have been reported so far.

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