# Relativistic superdense matter in cold systems: Theory*

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The extension of relativistic quantum fields to dense systems of fermions strongly interacting via meson exchange is discussed. The direct application of quantum field theory methods in this context fails. The failure of this approach leads to an effective Lagrangian based on an approximation scheme naturally suited to high-density matter. The resulting model can be solved exactly for cold systems, and small temperature corrections may be added. The applications of this model to neutron stars and problems in elementary particle physics will be made in the following paper.

## I. INTRODUCTION

The observation of exotic astrophysical phenomena during the last decade has stimulated an interest in the investigation of matter under extreme conditions. In this paper we study what is commonly called superdense matter.[1] These are the states of matter which occur near the limit of gravitational collapse. We are particularly interested in superdense matter in neutron stars because they play a central role in current models of pulsars and compact x-ray sources in binary systems. In addition, we mention briefly the application of this kind of matter to models of elementary particles.

A general feature shared by all previous models of neutron stars is that the more massive have average core densities greater than $10^{15}$ g/cm$^3$. With few exceptions these models all predict stars with masses less than $1.4M_\odot$ to $1.76M_\odot$. These models are developed from nonrelativistic theories and are based on our understanding of the properties of nuclear matter.[1,2] A disturbing feature of this situation is that the high-mass models involve densities which are above nuclear density where relativistic effects are significant and cannot be ignored. It is therefore necessary to approach the problem from a fully relativistic interacting many-body theory which incorporates approximations suited to the supernuclear density regime.

In this and the following paper we report the results of a study of superdense matter based on a fully relativistic phenomenological description of the strong interactions. The strengths of these interactions are determined by a fit to a simple description of nuclear matter, and to one-boson-exchange potentials which fit high-energy nucleon-nucleon scattering. These are then employed within the framework of a many-body theory, which is also relativistic, to calculate baryon composition and the pressure-energy density equation of state for stellar matter. The relativistic stellar structure equations at $T=0$ are then integrated using this equation of state; the results are presented in the following paper.

The predictions of this model differ significantly from previously described results,[2] and stand in more satisfactory agreement with current evolutionary predictions and observational bounds. The stability limit against gravitational collapse is $2.39M_\odot$. This higher mass limit is particularly important since it implies that black-hole formation may be less frequent than current theories of superdense matter suggest. The moments of inertia for intermediate- and high-mass stars ($\gtrsim 10^{15}$ g/cm$^3$) bring models of pulsars as rapidly rotating neutron stars well within the limits set by observations.[3,4] Finally, we note that our model leads to a picture of compact object formation which is in general accord with the results of evolutionary studies.[5]

The results of our investigations will be discussed in the following order. In Sec. II we use a phenomenological Lagrangian to explore the properties of strongly interacting superdense matter in the Born approximation. This approach, which is found to be inadequate, leads to the consideration of an expansion in density rather than in the coupling constants. This expansion in density then motivates construction of an effective Lagrangian which retains the major features of the strong interactions for densities relevant to neutron stars. This construction is given in Sec. III. From this Lagrangian we obtain relativistic finite-density Green's functions. These are used to obtain expressions for the baryon effective masses and to obtain self-consistency conditions on the baryon number and proper number densities. In Sec. IV we derive expressions for the chemical potentials, pressure, energy density, and speed of sound.

In the following paper[6] the results of this model are used to construct neutron-star models and possible application to models of elementary particles. We then discuss the sensitivity of our results in terms of the available methods for determining the coupling constants. We consider corrections to the chemical potentials and effective masses due to finite-temperature effects in the high-density, low-temperature limit in Sec. V.

## II. FUNDAMENTAL INTERACTIONS

In a field-theoretic approach to many-body theory the boundary conditions which reflect the finite density and temperature of the system are most naturally incorporated through the use of Green's functions. The relativistic formulation of this problem has already been discussed.[7,8]

It is natural, in formulating a model of superdense matter, to try to include a large fraction of the known particles and resonances as possible constituents. A reasonable trial group might include the first SU(3)-symmetric octet of baryons, the electron and muon, and low-lying mesons. Adopting these as fundamental fields, the system can be described by the Lagrangian density

$$\mathcal{L}(x)=\sum_B \bar{\psi}_B(x)(i\partial\!\!\!/-m_B)\psi_B(x)+\sum_L \bar{\psi}_L(x)(i\partial\!\!\!/-m_L)\psi_L(x)+\sum_P \mathcal{L}_P+\mathcal{L}_I+\mathcal{L}_C. \quad (2.1)$$

Baryons are denoted by $B$, leptons by $L$, and mesons by $P$. $\mathcal{L}_C$ contains all mass counterterms.

The term $\mathcal{L}_I$ represents the meson exchanges among the hadrons. In the density range of interest, $\epsilon\sim10^{14}-10^{16}$ g/cm$^3$, the important exchanged mesons are the $\pi$, $\eta$, $\rho$, $\omega$, $\delta$, and $\phi$. Because of charge constraints the presence of the leptons has an important effect on the relative concentrations of the other species, but their interactions with the baryons are negligible. The interaction term can then be written as

$$\mathcal{L}_I=\sum_{B,P}\bar{\psi}_B\Gamma_{BB'P}\psi_{B'}\phi_P g_{BB'P}. \quad (2.2)$$

The $\Gamma_{BB'P}$ include pseudoscalar and vector couplings, and the strengths $g_{BB'P}$ may be fixed by requiring that (2.2) reproduce the nucleon-nucleon scattering data in the Born approximation.

The two-point functions are defined in terms of the elementary fields by

$$G_F^{(B)}(x-x')=-i\langle T\psi_B(x)\bar{\psi}_B(x')\rangle \quad (2.3)$$

and

$$D_F^{(P)}(x-x')=-i\langle T\phi_P(x)\phi_P(x')\rangle, \quad (2.4)$$

where the angular brackets denote a suitably defined relativistic thermodynamic average.[7] At zero temperature the latter is over the baryon $N$-body ground state, and $T$ is the time-ordering operator. The equations of motion for $D_F^{(P)}$ and $G_F^{(B)}$ which couple to higher-order $N$-point functions are generated from (2.1) in the usual way. Approximations must be made if we are to solve these equations. One approach is to solve the Green's-function Dyson equations truncated to second order in perturbation theory.

The appearance of real mesons in the system would introduce complications. Being bosons, they may form condensates, which would require the incorporation of non-normal ground states. These effects may be included in our approach. However, their presence is not expected to alter the structure of neutron stars in the density range of interest. We therefore set $\mathcal{L}_P=0$ in (2.1) and consider only virtual mesons, described by the usual vacuum propagators, in this calculation.

The noninteracting spin-$\frac{1}{2}$ baryon propagator corresponding to a finite-density $T=0$ system is

$$S_F(p,q_F)=\frac{p\!\!\!/+m}{2E_p}\left[\frac{1-n_F(\vec{p})}{p^0-E_{\vec{p}}+i\epsilon}+\frac{n_F(\vec{p})}{p^0-E_{\vec{p}}-i\epsilon}-\frac{1}{p^0+E_{\vec{p}}-i\epsilon}\right], \quad (2.5)$$

where $n_F(\vec{p})$ is the $T=0$ Fermi distribution step function, and all other variables have their usual meaning.[7] The number of baryons $N$ in a system of volume $V$ is related to the Fermi wave vector by

$$3\pi^2N/V=q_F{}^3. \quad (2.6)$$

Standard perturbation theory applied to a model of this kind leads to baryon self-energy diagrams such as those shown in Fig. 1. Tadpole diagrams [Fig. 1(a)] vanish identically for the exchanged mesons discussed above, since a non-vacuum-valued quantum number is carried by each. To lowest order, the self-energy contains only bubble diagrams and $G_F^{(B)}(p,q_F)$ will be given by[9]

$$\{[p\!\!\!/-\Sigma\!\!\!/^{(B)}(p,q_F)]-[m_B+\Sigma^{(B)}(p,q_F)]\}\times G_F^{(B)}(p,q_F)=1, \quad (2.7)$$

where $\Sigma\!\!\!/^{(B)}$ and $\Sigma^{(B)}$ contain a contribution from each of the exchanged mesons. In each of these self-energies we can uniquely separate out a finite-density-dependent part:

$$\Sigma\!\!\!/^{(B)}(p,q_F)=p\!\!\!/S_2^{(B)}(p^2)+\Sigma\!\!\!/_f^{(B)}(p,q_F), \quad (2.8)$$

$$\Sigma^{(B)}(p,q_F)=m_BS_1^{(B)}(p^2)+\Sigma_f^{(B)}(p,q_F). \quad (2.9)$$

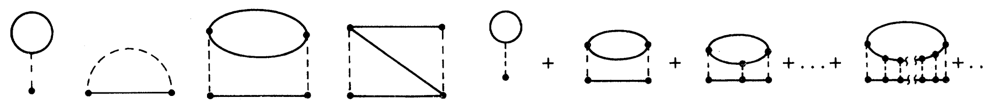


FIG. 1. Typical low-order irreducible self-energy diagrams resulting from (2.1) for exchange of vector, scalar, or pseudoscalar particles [diagrams (a)–(d)]. Leading-order contributions to the expansion in density are given in (e).

The terms $S_1$ and $S_2$ represent the elementary-particle self-energy which survives at zero baryon density. These terms are divergent, and are renormalized by subtraction to the baryon physical mass. Although $S_1$ and $S_2$ are not explicitly density-dependent, there is an implicit dependence, since they are sensitive to the value of $p^2$ which is driven off the mass shell by the density effects.

The excitation energies and chemical potentials of the baryons may be obtained from the poles of $G_F^{(B)}(p, q_F)$. In the interest of simplicity let us discuss the exchange of only one meson. We find that

$$G_{F\alpha\beta}^{(B)}(p, q_F) = \frac{N(p, q_F)_{\alpha\beta}}{D(p, q_F)}, \tag{2.10}$$

where the denominator is given by

$$\begin{aligned} D(p, q_F) = \{p^0[1 - S_2(p^2)] - \Sigma_f^0(p, q_F)\}^2 \\ - \{|\vec{p}|[1 - S_2(p^2)] - \Sigma_f^v(p, q_F)\}^2 \\ - \{m_B[1 + S_1(p^2)] + \Sigma_f(p, q_F)\}^2 . \end{aligned} \tag{2.11}$$

The term $\Sigma_f^v$ represents the three-vector magnitude of $\Sigma_f^\mu$ ($\mu = 1, 2, 3$). The excitation energies $p^0 = p^0(q_F, p)$ are obtained as solutions of

$$D(p^0, \vec{p}, q_F) = 0, \tag{2.12}$$

and the chemical potential (equal to the Fermi energy at $T = 0$) is given by

$$\mu(q_F) = p^0(|\vec{p}| = q_F, q_F). \tag{2.13}$$

The solutions of (2.12) for pseudoscalar, scalar, and vector mesons have been investigated numerically for densities $\lesssim (q_F/m_B \sim 1)$. Low-density expansions have also been made for the region $q_F/m_B \ll 1$. We find generally that (a) the terms $S_i(p^2)$ are sensitive to deviations away from the mass shell; and (b) $\Sigma_f^0$, $\Sigma_f^v$, and $\Sigma_f$, though strongly density-dependent, beat against one another in such a way that their leading-order contributions cancel.

For the dominant exchanges of $\pi$ and $\rho$ we found several additional features. For some range of the renormalized coupling constant $g_{\pi NN}$ a ghost develops and solutions to (2.12) are not found for real values of $p^0$. For sufficiently weak coupling, solutions are found in the density range considered ($0.05 < q_F/m_N < 1.0$). The leading-order terms in the $\pi$-exchange model yield attractive nucleon-nucleon interactions. However, these terms cancel in (2.12), and the next-order terms correspond to an effective increase in the value of $p^0$. In fact, the chemical potential $\mu$ is greater than that of a free gas at the same density. Analysis of $\rho$ exchange leads to much the same conclusion: Leading-order effects tend to cancel, and higher-order terms yield a chemical potential for vector exchange which lies below that of a free gas, which is the opposite of the expected result of repulsion.

A model based on exchange of observed mesons in the Born approximation is clearly inadequate.[10] At the very least we should retain terms representing fourth-order processes [Figs. 1(c) and 1(d)] in a calculation of the Green's functions. Analysis of these terms for baryons shows that they have leading-order density dependence proportional to baryon number density. Furthermore, in the density range of interest for massive neutron stars the dominant term in each is, to within a density-independent factor, the same as the tadpole term [Fig. 1(a)]. In fact the finite $N$th-order contribution should also show a density dependence at most proportional to $q_F{}^3$ in this range.

These observations suggest that the model obtained by expanding in powers of the renormalized coupling constants of observed mesons should be replaced by one which leads to an expansion in density, as illustrated schematically in Fig. 1(e). The first term in this series is formally equivalent to the tadpole graph which was discarded previously. Moreover, this simple term is proportional to the leading-order corrections from the higher-order expressions in Fig. 1(e). Thus we approximate the interactions due to meson exchange in superdense matter by expressions proportional to the primitive tadpole diagram represented in Fig. 1(a). The constants of proportionality may be determined phenomenologically, as will be discussed in Sec. VI. Since we treat only spin-$\frac{1}{2}$ baryons the only phenomenological fields of interest are the scalar and vector fields.

When the self-energy corresponding to Fig. 1(a) is examined for these cases it is found to be repulsive for vector exchange. Scalar exchange produces attraction in the lower-density region, but becomes repulsive at larger densities. The system treated in this fashion thus enjoys all the expected behavior for the exchanges studied, and does not suffer from the difficulties of nearby ghost states with their associated anomalous properties.

## III. PHENOMENOLOGICAL MODEL

The analysis of the previous section suggests that we consider as a model of the strong interactions in superdense matter the baryons coupled via a vector and a scalar field which have vacuum quantum numbers. A universal SU(3)-symmetric coupling will be assumed. The effective Lagrangian density then has the form

$$\mathcal{L}(x)=\sum_B \bar\psi_B(x)(i\partial\!\!\!/-m_B)\psi_B(x)-\sum_B\left[g_S\bar\psi_B(x)\psi_B(x)\phi_S(x)+g_V\bar\psi_B(x)\gamma_\mu\psi_B(x)\phi_V^\mu(x)\right]$$
$$+\tfrac{1}{2}[\partial_\mu\phi_S(x)\partial^\mu\phi_S(x)-\mu_S^2\phi_S(x)^2]-\tfrac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)+\tfrac{1}{2}\mu_V^2\phi_V^\nu(x)\phi_V(x)_\nu\,, \tag{3.1}$$

where

$$F^{\mu\nu}(x)=\partial^\mu\phi_V^\nu(x)-\partial^\nu\phi_V^\mu(x)\,. \tag{3.2}$$

The counterterms have been dropped since all expressions to be considered below are finite. Accordingly we understand $m_B$ to represent the baryon physical masses.

A more appropriate effective Lagrangian, which is consistent with our earlier statements about the nonobservability of the meson degrees of freedom, should contain only baryon fields. A nonlocal Lagrangian, equivalent to the above, is

$$\mathcal{L}(x)=\sum_B \bar\psi_B(x)(i\partial\!\!\!/-m_B)\psi_B(x)$$
$$-\sum_{BB'}\left[\int d\xi\, g_S^2\bar\psi_B(x)\psi_B(x)\Delta(x-\xi)\bar\psi_{B'}(\xi)\psi_{B'}(\xi)+\int d\xi\, g_V^2\bar\psi_B(x)\gamma_\mu\psi_B(x)\Delta^{\mu\nu}(x-\xi)\bar\psi_{B'}(\xi)\gamma_\nu\psi_{B'}(\xi)\right]. \tag{3.3}$$

The $\Delta$'s appearing in (3.3) are the time-symmetric vacuum two-point functions for phenomenological scalar and vector fields. We thus retain the role of the mesons as mediating the interactions without allowing their presence as physical particles.[11] The Lagrangian (3.3) thus emphasizes the fact that we no longer consider the interactions as resulting from the exchange of observed particles. Therefore analogies between our scalar or vector fields and observed mesons are at best a convenient device for fixing the scale of these couplings. A functional variation of the thermodynamic average of this Lagrangian yields the equations of motion for the baryon Green's functions. These are

$$(i\partial\!\!\!/-m_B)G_F^{(B)}(x-x')$$
$$=\delta(x-x')$$
$$+\sum_{B'}g_S^2\int d^4\xi\,\Delta(x-\xi)G_F^{(BB')}(x\xi,x'\xi^-)$$
$$+\sum_{B'}g_V^2\int d^4\xi\,\Delta_{\mu\nu}(x-\xi)\gamma^\mu G_F^{(BB')}(x\xi,x'\xi^-)\gamma^\nu\,, \tag{3.4}$$

where we have defined the four-point function by

$$G_F^{(BB')}(xy,x'y')\equiv(1/i)^2\langle T\psi_B(x)\psi_{B'}(y)\bar\psi_{B'}(y')\bar\psi_B(x')\rangle\,, \tag{3.5}$$

and the notation $\xi^-$ signifies that the time component is infinitesimally earlier than $\xi$.

The Hartree-Fock approximation to (3.4) results if we set

$$G_F^{(BB')}(xy,x'y')=G_F^{(B)}(x-x')G_F^{(B')}(y-y')-G_F^{(B)}(x-y)G_F^{(B')}(y'-x')\delta_{BB'}\,. \tag{3.6}$$

The notation in the last term reflects the fact that the two-point functions are diagonal in the baryon index. This term leads to self-energy contributions like the bubble diagram shown in Fig. 1(b), while the first term leads to tadpole diagrams [Fig. 1(a)]. Motivated by the discussion of the previous section, we retain the first, or Hartree, term in (3.6), since it represents the leading-order correction in powers of the density. The resulting equations of motion become

$$(i\not{\partial} - m_B)G_F^{(B)}(x-x') = \delta(x-x') + \sum_{B'}\left[g_S{}^2\int d^4\xi\,\Delta(x-\xi)G_F^{(B')}(\xi-\xi^-) + g_V{}^2\int d^4\xi\,\Delta_{\mu\nu}(x-\xi)\gamma^\mu G_F^{(B')}(\xi-\xi^-)\gamma^\nu\right]G_F^{(B)}(x-x'). \tag{3.7}$$

A substantial simplification results if we consider (3.7) in momentum space. Using

$$G_F^{(B)}(x-x') = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-x')}G_F^{(B)}(p) \tag{3.8}$$

and

$$\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-x')}\Delta(k) \tag{3.9}$$

for the scalar and vector propagators, it immediately follows that

$$(\not{p} - m_B)G_F^{(B)}(p, q_{FB}, q_{F\alpha}) = 1 + \lim_{\eta\to 0^+}\sum_{B'}\left\{i\frac{g_S{}^2}{\mu_S{}^2}\int\frac{d^4q}{(2\pi)^4}\,e^{iq^0\eta}\,\mathrm{tr}[G_F^{(B')}(q, q_{FB'}, q_{F\alpha})] - i\gamma^0\frac{g_V{}^2}{\mu_V{}^2}\int\frac{d^4q}{(2\pi)^4}\,e^{iq^0\eta}\,\mathrm{tr}[\gamma^0 G_F^{(B')}(q, q_{FB'}, q_{F\alpha})]\right\}G_F^{(B)}(p, q_{FB}, q_{F\alpha}). \tag{3.10}$$

The factors $-\mu_S{}^{-2}$ and $-\mu_V{}^{-2}$ are the $k^2=0$ scalar and vector propagators, and the traces over $G_F^{(B')}$ and $\gamma^0 G_F^{(B')}$ have been shown explicitly. The factors $e^{iq^0\eta}$ specify the integration contour in the $q^0$ plane, and result from the temporal constraint on $\xi$ in (3.7). The density dependence of $G_F^{(B)}(p, q_{FB}, q_{F\alpha})$ has been explicitly exhibited. It should be emphasized that each baryon Green's function depends on its density $q_{FB}$ and, through the interactions, on the density of all other baryons actually present in the system. The latter are collectively denoted by $q_{F\alpha}$. The terms in curly brackets are the vector and scalar self-energies, which we denote by

$$-i\Sigma_S^{(BB')} = \frac{g_S{}^2}{\mu_S{}^2}\lim_{\eta\to 0^+}\int\frac{d^4q}{(2\pi)^4}\,e^{iq^0\eta}\,\mathrm{tr}[G_F^{(B')}(q)], \tag{3.11}$$

and

$$i\Sigma_V^{(BB')} = \frac{g_V{}^2}{\mu_V{}^2}\lim_{\eta\to 0^+}\int\frac{d^4q}{(2\pi)^4}\,e^{iq^0\eta}\,\mathrm{tr}[\gamma^0 G_F^{(B')}(q)]. \tag{3.12}$$

In terms of $\Sigma_V^{(BB')}$ and $\Sigma_S^{(BB')}$ the Green's functions are given by

$$\left[\not{p} - m_B - \sum_{B'}(\Sigma_S^{(BB')} + \gamma^0\Sigma_V^{(BB')})\right]G_F^{(B)}(p) = 1. \tag{3.13}$$

The last equation may be inverted directly to obtain

$$G_F^{(B)}(p) = \frac{\gamma^0(p^0-\Sigma_V^{(B)}) - \vec{\gamma}\cdot\vec{p} + m_B + \Sigma_S^{(B)}}{(p^0-\Sigma_V^{(B)})^2 - p^2 - (m_B+\Sigma_S^{(B)})^2}, \tag{3.14}$$

where the obvious notation $\Sigma^{(B)} = \sum_{B'}\Sigma^{(BB')}$ has been used for scalar and vector self-energies of the baryon $B$, and explicit dependence on all baryon densities has been dropped for notational simplicity. The finite-density boundary conditions[7] are introduced through (2.5) and Dyson's equation

$$G_F^{(B)}(p) = S_F^{(B)}(p) + S_F^{(B)}(p)\Sigma^{(B)}G_F^{(B)}(p). \tag{3.15}$$

Denoting the excitation energy and effective mass for each baryon by

$$\mathcal{E}_B(\vec{p}) \equiv (p^2 + m_{e,B}{}^2)^{1/2}, \tag{3.16}$$

$$m_{e,B} \equiv m_B + \Sigma_S^{(B)}, \tag{3.17}$$

the finite-density baryon Green's functions are

$$G_F^{(B)}(p) = \frac{\gamma^0(p_0-\Sigma_V^{(B)}) - \vec{\gamma}\cdot\vec{p} + m_{e,B}}{2\mathcal{E}_B(\vec{p})}\left[\frac{1-n_F^{(B)}(\vec{p})}{p^0-\Sigma_V^{(B)}-\mathcal{E}_B(\vec{p})+i\epsilon} + \frac{n_F^{(B)}(\vec{p})}{p^0-\Sigma_V^{(B)}-\mathcal{E}_B(\vec{p})-i\epsilon} - \frac{1}{p^0-\Sigma_V^{(B)}+\mathcal{E}_B(\vec{p})-i\epsilon}\right]. \tag{3.18}$$

The zero-temperature distribution functions and the Fermi wave vector of the baryon $B$ are defined according to

$$n_F^{(B)}(p) \equiv \theta(q_{F,B} - p)\,, \tag{3.19}$$

$$q_{F,B}{}^3 \equiv 3\pi^2 N_B/V\,. \tag{3.20}$$

The total number of baryons in the system is given by

$$N = \sum_B N_B\,. \tag{3.21}$$

We observe that the three terms in the propagator (3.18) correspond to (1) positive-energy excitations above the filled Fermi sea, (2) positive-energy excitations below the Fermi sea (holes), (3) and negative-energy excitations. The ground state is devoid of physical antiparticles, although they may contribute through virtual states.

The baryon number and proper number densities $n^{(B)}$ and $\bar{n}^{(B)}$, defined by

$$n^{(B)} = \langle \psi_B^\dagger(x)\psi_B(x)\rangle = -i \lim_{x' \to x+0} \mathrm{tr}[\gamma^0 G_F^{(B)}(x - x')] \tag{3.22}$$

and

$$\bar{n}^{(B)} = \langle \bar{\psi}_B(x)\psi_B(x)\rangle = -i \lim_{x' \to x+0} \mathrm{tr}[G_F^{(B)}(x - x')] = -i \lim_{\eta \to 0} \int \frac{d^4p}{(2\pi)^4}\, e^{ip^0\eta}\, \mathrm{tr}[G_F^{(B)}(p)]\,, \tag{3.23}$$

are related to $\Sigma_V^{(B)}$ and $\Sigma_S^{(B)}$ through the $G_F^{(B)}$ as in (3.11) and (3.12):

$$\Sigma_V^{(BB')} = \frac{g_V{}^2}{\mu_V{}^2}\, n^{(B')} \tag{3.24}$$

and

$$\Sigma_S^{(BB')} = -\frac{g_S{}^2}{\mu_S{}^2}\, \bar{n}^{(B')}\,. \tag{3.25}$$

Through $\mathcal{E}_B(p)$ and $m_{e,B}$, as given in (3.16) and (3.17), the $\bar{n}$ and $n$ are themselves functions of the $\Sigma$'s.

In evaluating (3.11) and (3.12) we have dropped infinite terms which arise from the number density of filled negative-energy states. The appearance of these terms is a direct consequence of the form of our Lagrangian, and may be eliminated at the outset in either of two ways: (1) by normal ordering the Lagrangian, or (2) by defining the physical number densities as the difference between $n$ or $\bar{n}$ and the corresponding quantity at zero density. In either case we find that the appropriate prescription[7] to take the place of (3.11) or (3.12) is to replace the factor $e^{ip^0\eta}$ by $\frac{1}{2}(e^{ip^0\eta} + e^{-ip^0\eta})$.

The solutions for the Green's functions will be complete if they are required to self-consistently reproduce the $n^{(B)}$ and $\bar{n}^{(B)}$. Using (3.18) in (3.22), it immediately follows that

$$n^{(B)} = q_{F,B}{}^3/3\pi^2\,. \tag{3.26}$$

This is identical to (3.20) and shows that the number density of each species is automatically self-consistent. Thus (3.22) is a trivial constraint and simply serves to define $q_{F,B}$. The proper number densities (3.23) lead to the consistency condition

$$\bar{n}^{(B)} = \int_0^{q_{F,B}} \frac{p^2\,dp}{\pi^2}\, \frac{m_{e,B}}{\mathcal{E}_B(p)} = \frac{m_{e,B}}{2\pi^2}\left(q_{F,B}\mathcal{E}_{F,B} - m_{e,B}{}^2 \ln\left|\frac{q_{F,B} + \mathcal{E}_{F,B}}{m_{e,B}}\right|\right). \tag{3.27}$$

To complete the solution we require that $\bar{n}^{(B)}$ [or equivalently $m_{e,B} = m_B - (g_S{}^2/\mu_S{}^2)\bar{n}^{(B)}$] be given by this transcendental algebraic equation. It is easily shown that in the low-density limit ($q_F/m_B \to 0$) $m_{e,B}$ approaches $m_B$ and $\bar{n}^{(B)}$ approaches $n^{(B)}$.

The consequences of (3.27) are significant: In the high-density limit ($q_{F,B}/m_B \to \infty$) the number density $n^{(B)} \to \infty$. However, the only consistent solution of (3.27) in this limit is $m_{e,B} \to 0$. It will be observed that this does not occur in the absence of the self-consistency requirement,[12] since then $\bar{n}^{(B)}$ goes asymptotically as $q_F{}^2$. Because $\bar{n}^{(B)}$ is proportional to $\Sigma_S$ and enters only as a correction to the physical mass, we may interpret (3.27) as a self-consistency condition on the effective masses $m_{e,B}$.

For a system comprised of $N$ different types of baryons there will be $N$ conditions of the form (3.27). Each is an explicit function of one baryon mass $m_{e,B}$, and an implicit function of all others. Equations (3.27) therefore represent a set of coupled transcendental equations for the $m_{e,B}$. The relative simplicity of (3.27) is dependent on the fact that in this model both self-energies $\Sigma_S^B$ and $\Sigma_V^B$ are independent of external momentum. Generally speaking, they could contain explicit dependence on the baryon four-momentum. The self-consistency conditions would then be replaced by a coupled set of transcendental integral equations whose solution would be extremely difficult, even by numerical means.

We conclude this section by emphasizing that the results above are fully relativistic, as regards both finite density and strong-interaction effects, and treat the baryon physical degrees of freedom consistently throughout.

## IV. EQUATIONS OF STATE

The formal Green's-function approach developed in the previous section will be used below to study the bulk properties of a system of superdense matter. To this end we review the physical information contained in the baryon Green's functions. We then fit the parameters of our model to nuclear matter.

### A. Physical information contained in $G_F^{(B)}(p)$

Inspection of (3.18) shows that the excitation energies of the system as given by the poles of $G_F^{(B)}(p)$ are

$$\mathcal{E}_B(\vec{p}) = \Sigma_V^{(B)} + (p^2 + m_{e,B}{}^2)^{1/2} . \tag{4.1}$$

Since the self-energies are real, the excitations have an infinite lifetime, and represent eigenstates of the effective Hamiltonian obtained from (3.3) treated in the Hartree approximation. In accordance with (2.13) the chemical potentials of the baryons are given by

$$\mu^{(B)} = \Sigma_V^{(B)} + (q_{F,B}{}^2 + m_{e,B}{}^2)^{1/2} . \tag{4.2}$$

The chemical potentials determine the equilibrium concentrations for the baryons in the system.

### B. Bulk properties determined by $G_F^{(B)}(p)$

We have already discussed the number densities $n^{(B)}$ and $\bar{n}^{(B)}$ as determined by (3.22) and (3.23). The chemical potential (4.2) and $n^{(B)}$ constitute $N-2$ of the $N$ equations of state needed for a complete thermodynamic description of the system. Of the remaining two, the temperature is trivial: $T=0$. The final equation of state is given by the pressure,

$$\begin{aligned} P &= \sum_B \int_0^{q_{F,B}} n^{(B)} \frac{d\mu^{(B)}}{dq_{F,B}} dq_{F,B} \\ &= -i \lim_{\eta\to 0^+} \sum_B \int d\mu^{(B)} \int \frac{d^4p}{(2\pi)^4} e^{ip^0\eta} \operatorname{tr}[\gamma^0 G_F^{(B)}(p)] . \end{aligned} \tag{4.3a}$$

The summation includes only those baryons actually present in the system at a given density. Equation (4.3a) involves an integration over densities, through $\mu^{(B)}$, which is not always convenient. An alternate expression for $P$ which involves integration over momenta follows from the stress-energy tensor. As shown in the Appendix, the total baryon pressure is

$$\begin{aligned} P &= \tfrac{1}{3}\langle \Phi_0 | \mathcal{T}^{ii} | \Phi_0 \rangle \\ &= \sum_B P^{(B)} \\ &= -i \lim_{\eta\to 0^+} \int \frac{d^4p}{(2\pi)^4} e^{ip^0\eta} \\ &\quad \times \sum_B \operatorname{tr}[\tfrac{1}{3}\vec{\gamma}\cdot\vec{p} + \Sigma_S^{(B)} + \gamma^0\Sigma_V^{(B)}]\, G_F^{(B)}(p) . \end{aligned} \tag{4.3b}$$

When using (4.3b) the effective mass entering through $G_F^{(B)}$ and $\Sigma_S^{(B)}$ is constant so that the momentum integrations may be carried out analytically. Equation (4.3a) requires that $m_{e,B}$ be known for each value of $\mu^{(B)}$. The Green's function is given by (3.18), so that (4.3b) yields

$$\begin{aligned} P^{(B)} &= 2\int \frac{d^3p}{(2\pi)^3} \frac{p^2}{\mathcal{E}_B(p)} n_F^{(B)}(p) + \frac{g_V^2}{\mu_V^2} n^{(B)} \int \frac{d^3p}{(2\pi)^3} n_F^{(B)}(p) - \frac{g_S^2}{\mu_S^2} n^{(B)} \int \frac{d^3p}{(2\pi)^3} \frac{m_{e,B}}{\mathcal{E}_B(p)} n_F^{(B)}(p) \\ &= \frac{g_V^2}{2\mu_V^2} n^{(B)2} - \frac{g_S^2}{2\mu_S^2} \bar{n}^{(B)2} + \frac{1}{3\pi^2} \int_0^{q_F} \frac{p^4\, dp}{(p^2 + m_{e,B}{}^2)^{1/2}} \\ &= \frac{1}{3}[\epsilon^{(B)} - m_{e,B}\bar{n}^{(B)}] + \frac{g_V^2}{3\mu_V^2} n^{(B)2} - \frac{2}{3}\frac{g_S^2}{\mu_S^2} \bar{n}^{(B)2} , \end{aligned} \tag{4.4}$$

where $\epsilon^{(B)}$ is defined below. Equation (3.27) has been used to introduce the term $m_{e,B}\bar{n}^{(B)}/3$. A term corresponding to the infinite pressure of the filled negative-energy states is included in (4.3b), but is easily removed by the method described in Sec. III. The result, Eq. (4.4), is finite.

All of the equations of state are now known, at least in principle, and the thermodynamic potentials could be found. However, it is more direct to proceed with the Green's functions. The thermodynamic potential which we need to complete our analysis is the ground-state energy density, which is just the expectation value of the Hamiltonian. We show in the Appendix that the ground-state energy is given by

$$\begin{aligned} \epsilon &= \langle \Phi_0 | \mathcal{H} | \Phi_0 \rangle \\ &= \sum_B \epsilon^{(B)} , \end{aligned} \tag{4.5a}$$

$$\begin{aligned} \epsilon^{(B)} &= -\frac{i}{2} \lim_{\eta\to 0^+} \int \frac{d^4p}{(2\pi)^4} e^{ip^0\eta} \\ &\quad \times \operatorname{tr}[(\gamma^0 p_0 + \vec{\gamma}\cdot\vec{p} + m_B) G_F^{(B)}(p)] . \end{aligned} \tag{4.5b}$$

In addition to the physically occupied positive-energy states, the equations above contain contributions from all filled negative-energy states, and these are removed as before. Notice that the physical mass $m_B$ enters in the trace in (4.5b).

The last equation may be applied directly to $G_F^{(B)}(p)$, and leads to

$$\begin{aligned}\epsilon^{(B)} &= \int \frac{d^3p}{(2\pi)^3}\frac{n_F^{(B)}(p)}{\mathcal{E}_B(\vec{p})}\left[p^0(p^0-\Sigma_V^{(B)})+p^2+m_B m_{e,B}\right]\\ &= \frac{q_{F,B}^3}{6\pi^2}\Sigma_V^{(B)} - \frac{m_{e,B}}{4\pi^2}\left(q_{F,B}\mathcal{E}_{F,B} - m_{e,B}^2\ln\left|\frac{q_{F,B}+\mathcal{E}_{F,B}}{m_{e,B}}\right|\right)\\ &\quad + \frac{1}{4\pi^2}\left(q_{F,B}\mathcal{E}_{F,B}^3 - \frac{1}{2}m_{e,B}^2 q_{F,B}\mathcal{E}_{F,B} - \frac{m_{e,B}^4}{2}\ln\left|\frac{q_{F,B}+\mathcal{E}_{F,B}}{m_{e,B}}\right|\right). \end{aligned} \tag{4.6}$$

An alternate expression for the pressure, which follows from the first law of thermodynamics, uses the ground-state energy density and is

$$P = n^2\frac{\partial(\epsilon/n)}{\partial n}. \tag{4.7}$$

Finally consider the speed of sound $v_s$ defined thermodynamically by the derivative at constant entropy,

$$v_s^2 \equiv (\partial P/\partial\epsilon)_s, \tag{4.8}$$

for a system consisting of one baryonic species. Then $P$ and $\epsilon$ are functions of $q_F$, and

$$\begin{aligned} v_s^2 &= \frac{(\partial P/\partial q_F)_s}{(\partial\epsilon/\partial q_F)_s}\\ &= \frac{n\,d\mu/dq_F}{\mu\,dn/dq_F}, \end{aligned} \tag{4.9}$$

as follows from (4.3a) and the definition of the chemical potential $\mu \equiv (\partial\epsilon/\partial n)_s$. Equations (4.2) and (2.6), the self-consistency relation (3.27) for $m_e$, and straightforward algebra lead to the following expression for the adiabatic speed of sound:

$$v_s^2 = \frac{1}{3}\,\frac{q_F^2 + (g_V^2/\mu_V^2)(q_F^3\mathcal{E}_F/\pi^2) + (g_S^2/\mu_S^2)(m_e^2 q_F^3/\pi^2\mathcal{E}_F)(A/B)}{q_F^2 + m_e^2 + (g_V^2/\mu_V^2)(q_F^3\mathcal{E}_F/3\pi^2)}, \tag{4.10}$$

$$A = 1 - \frac{m_e\mathcal{E}_F}{q_F^2}\ln\left|\frac{q_F+\mathcal{E}_F}{m_e}\right|, \tag{4.11}$$

$$B = 1 + \frac{g_S^2}{\mu_S^2}\frac{m_e^2 q_F}{\pi^2 m_B\mathcal{E}_F}. \tag{4.12}$$

In the high-density limit we find that $A$ and $B\to 1$, and consequently

$$\lim_{q_F/m_B\to\infty} v_s^2 = 1. \tag{4.13}$$

Inspection of (4.13) shows that this is in fact just the asymptotic sound speed for a pure vector coupling $g_S=0$. The scalar coupling ($g_V=0$) would yield an asymptotic sound speed $v_s^2\to\frac{1}{3}$ equivalent to that of a relativistic free gas. It is notable that without the self-consistency imposed on $m_e$ by (3.27), the scalar and the vector coupling would separately have the same limit (4.13). The limit (4.13) is significant since it shows that a consistent treatment based on Lorentz-invariant nonlocal interactions at the Lagrangian level automatically yields results consistent with macroscopic causality.

## V. FINITE-TEMPERATURE CORRECTIONS

Applications of superdense matter may be made to systems with some finite temperature. For instance, matter inside neutron stars will not be in its ground state during the initial formation process. Current evolutionary models suggest that the dense core will cool rapidly. Except for a short period following collapse, the matter will be at nearly zero temperature.[13] It is therefore reasonable to assume that such systems will have temperatures satisfying $kT \ll \mu$ (where $\mu$ is the baryon chemical potential) when densities $\gtrsim \epsilon_N$. We present below the lowest-order temperature corrections to our model, restricting attention to a single-component system. The inclusion of additional components offers no formal obstacles and may be achieved by a straightforward extension of the method discussed below. The expansions about $T=0$ used below do not limit the relativistic nature of the interactions or the kinematics of the system. Our results therefore represent temperature corrections to a fully relativistic system.

The finite temperature of the system will be incorporated as boundary conditions on the fermion Green's functions which follow from the effective Lagrangian (3.3). As a result of finite temperatures, fermions will not be restricted to their lowest possible energy states. Instead they will occur in the energy state $p^0$ with probability of occupancy given by

$$n_F(p^0,\beta)=\frac{1}{e^{\beta(p^0-\mu)}+1}. \tag{5.1}$$

The finite-temperature Green's function for fermions is thus given by (2.5) with $n_F(p)$ replaced by (5.1).

This method of introducing boundary conditions is completely equivalent, at least for low temperatures, to the one employed by Bowers and Zimmerman in terms of the elementary Lagrangian (2.1) and the Green's functions. The latter quantities contain suitably defined thermodynamic averages, a specific representation of which has already been discussed. The same thermodynamic averages for fermions are used here. The chemical potential $\mu$ entering through (5.1) is a function of temperature $T=(\beta k)^{-1}$, where $k$ is Boltzmann's constant, and is determined by the number density through

$$n=2\int\frac{d^3p}{(2\pi)^3}n_F(\vec{p},\beta). \tag{5.2}$$

The analysis of Sec. IV is applicable to finite-temperature systems if (5.1) is used. In particular, the Green's function follows from (3.18) if $n_F(p)$ is replaced with (5.1). Note now that $\Sigma_S$ and $\Sigma_V$ will depend on $\beta$. Equation (3.22) now gives

$$n=2\int\frac{d^3p}{(2\pi)^3}(\exp\{\beta[\Sigma_V+\mathcal{E}(\vec{p},\beta)-\mu]\}+1)^{-1}, \tag{5.3}$$

while the proper number density is, from (3.23),

$$\bar{n}(\beta)=2\int\frac{d^3p}{(2\pi)^3}\,\frac{m_e(\beta)}{\mathcal{E}(\vec{p},\beta)}\times(\exp\{\beta[\Sigma_V+\mathcal{E}(\vec{p},\beta)-\mu]\}+1)^{-1}, \tag{5.4}$$

where $m_e(\beta)=m_B-(g_S{}^2/\mu_S{}^2)\bar{n}(\beta)$ and $m_B$ is the baryon physical mass. At $T=0$, $m_e(\infty)$ and $\mathcal{E}(\vec{p},\infty)$ reduce to (3.16) and (3.17). The last equation represents the finite-temperature self-consistency condition and reduces to (3.27) at $T=0$. Equation (5.3) defines $\mu$. Since the two self-energies $\Sigma_V$ and $\Sigma_S$ are each functions of $n$ and $\bar{n}$, we see that the latter are now coupled.

In general integrals over $n_F(\vec{p},\beta)$ must be evaluated numerically. However, we shall assume that $kT\ll\mu$ (semidegeneracy), and expand about the $T=0$ solutions.[14] If we make the change of variable $p=(\mathcal{E}^2-m_e{}^2)^{1/2}$ and define $x\equiv\beta(\Sigma_V+\mathcal{E}-\mu)$ we obtain

$$n=\frac{1}{\beta^3\pi^2}\int_\alpha^\infty(x+\beta\mu-\beta\Sigma_V)\times\frac{[(x+\beta\mu-\beta\Sigma_V)^2-m_e{}^2\beta^2]^{1/2}}{e^x+1}dx, \tag{5.5}$$

where $\alpha\equiv\beta(m_e+\Sigma_V-\mu)$. We shall assume that $\alpha<0$, as is true at $T=0$. As long as this condition is satisfied, $\alpha\to-\infty$ as $T\to0$, and $n$ may be expanded about $T=0$. Retaining lowest-order terms we find that

$$n=\frac{[(\mu-\Sigma_V)^2-m_e{}^2]^{3/2}}{3\pi^2}+\frac{2(\mu-\Sigma_V)^2-m_e{}^2}{6[(\mu-\Sigma_V)^2-m_e{}^2]^{1/2}}(kT)^2. \tag{5.6}$$

Proceeding in similar fashion, the proper number density may be expressed as

$$\bar{n}=\frac{m_e}{\pi^2\beta^2}\int_\alpha^\infty\frac{[(x+\beta\mu-\beta\Sigma_V)^2-\beta^2m_e{}^2]^{1/2}}{e^x+1}dx, \tag{5.7}$$

which is, to lowest order in the temperature,

$$\bar{n}=\frac{m_e(\mu-\Sigma_V)}{2\pi^2}[(\mu-\Sigma_V)^2-m_e{}^2]^{1/2}-\frac{m_e{}^3}{2\pi^2}\ln\left|\frac{\mu-\Sigma_V+[(\mu-\Sigma_V)^2-m_e{}^2]^{1/2}}{m_e}\right|+\frac{m_e(\mu-\Sigma_V)}{6[(\mu-\Sigma_V)^2-m_e{}^2]^{1/2}}(kT)^2. \tag{5.8}$$

Although $m_e$ and $\mu$ are functions of temperature, their $T=0$ values are to be used in the last term of (5.7) and (5.8). The latter equations may be used to show that to lowest order in $T$,

$$\mu(\beta)=\mu(\infty)+A\beta^{-2}, \tag{5.9}$$

$$m_e(\beta)=m_e(\infty)+B\beta^{-2}. \tag{5.10}$$

Substituting (5.9) and (5.10) into (5.6) and retaining lowest-order terms leads to the following relation between $A$ and $B$:

$$A=\frac{m_e}{\mathcal{E}_F}B-\frac{\pi^2(2q_F{}^2+m_e{}^2)}{6q_F{}^2\mathcal{E}_F}. \tag{5.11}$$

We then substitute (5.9) and (5.10) into (5.8), use (5.11) to simplify the result, and find, to lowest order in $T$,

$$B=\frac{(g_S{}^2/6\mu_S{}^2)m_ef(q_F)}{1+(g_S{}^2/2\pi^2\mu_S{}^2)[\mathcal{E}_Fq_F+(2m_e{}^2q_F/\mathcal{E}_F)-3m_e{}^2\ln|(q_F+\mathcal{E}_F)/m_e|]}, \tag{5.12}$$

where

$$f(q_F) \equiv 1 + m_e^2/2q_F^2 . \qquad (5.13)$$

All quantities appearing in (5.11)–(5.13) are evaluated at $T=0$, and are thus known.

The analysis above shows that the vector coupling does not affect the low-temperature corrections to $m_e$ or to $\mu$. We also see that the temperature dependence of $m_e$ results entirely from the scalar coupling, and that $m_e(\beta)=m_B$ if $g_S=0$. When $g_S=0$, $B=0$ and (5.11) equals the coefficient of the lowest-order temperature correction to the chemical potential of a free relativistic gas of fermions.[14] We thus see that the vector coupling has no effect on the chemical potential to lowest order in $T$.

Next consider the high-density limit of $B$ and $A$. Recalling that $m_e(\infty)\to 0$ and $\mu(\infty)\to q_F$, in this limit we find

$$\lim_{q_F/m_B\to\infty} m_e(\beta) = m_e(\infty)\left(1+\frac{\pi^2\beta^{-2}}{3q_F^2}\right), \qquad (5.14)$$

$$\lim_{q_F/m_B\to\infty} \mu(\beta) = \mu(\infty) - \frac{\pi^2\beta^{-2}}{3q_F} . \qquad (5.15)$$

For fixed $q_F$, finite-temperature corrections tend to increase $m_e$, but the coefficient of $\beta^{-2}$ tends to zero with increasing $q_F$. Equation (5.15) shows that the temperature dependence at high density is independent of the interactions; the latter enters only through $\mu(\infty)$.

Once $m_e$ is know at $T=0$, the coefficients $A$ and $B$ may be found at each value of $q_F$.

## VI. DETERMINATION OF COUPLING STRENGTHS

Our model contains four quantities $g_S, g_V, \mu_S, \mu_V$ which have yet to be specified. Inspection of (3.10) shows that these enter as the pairs $g_V^2/\mu_V^2$ and $g_S^2/\mu_S^2$ so that we actually are dealing with only two adjustable parameters. The model has a simple physical interpretation. If $\mu_S < \mu_V$, the scalar attraction will dominate at relatively low densities with the vector repulsion becoming important at higher densities. We fix the two parameters in our theory in at least two ways. Ideally we would fix the two parameters in our theory by requiring that it reproduce observed properties of cold matter at densities $\epsilon \gtrsim 10^{15}$ g/cm$^3$. As a consistency check we could then extrapolate down to the regime of nuclear density and compare our results with those obtained from nonrelativistic phenomenological models of nuclear matter. Unfortunately no terrestial system exists which can be used in this way. The closest candidate is data from high-energy scattering which does not correspond to the relevant density regime (strictly not statistical finite-density systems) or to zero temperature (hydrodynamic models for which $\epsilon \gtrsim 10^{16}$ g/cm$^3$ and $T \gg 0$). We are thus forced to extrapolate down to the region of nuclear density in order to fit the input parameters. One way to accomplish this would be to require that a system of dense matter with equal numbers of protons and neutrons be bound at nuclear densities ($q_{F,n}=q_{F,p}=1.42$ fm$^{-1}$) with binding energy $E_B=-15.75$ MeV per nucleon. The other approach would be to use the observed $\sigma$ and $\omega$ mesons as candidates for our effective meson fields.

For the first case we denote the total energy density by $\epsilon=\epsilon^n+\epsilon^p$, the nucleon number density by $n=n^n+n^p$, and assume a baryon mass $m_B=939.0$ MeV. Then (4.6) and

$$E_B = \frac{\epsilon - n m_B}{n} \qquad (6.1)$$

yield a binding energy of $-15.74$ MeV for $g_S^2 m_B^2/\mu_S^2\pi^2=27.04$ and $g_V^2 m_B^2/\mu_V^2\pi^2=19.83$. In this approach our model would be completely specified by the requirement that it satisfactorily describe nuclear matter. In this context we then realize that the intrinsic limits on this model stem from the uncertainties as to what constitutes nuclear matter.

The energy density (4.6) for nucleons is identical with a result obtained by Walecka through an entirely different approach.[8] We remark that his fit to nuclear matter and ours are the same. A thorough comparison of the model's predicted properties (binding energy, effective mass, symmetry energy, etc.) with those obtained by other methods is given by Chin and Walecka.[15] As stressed there, agreement with phenomenological parameters is quite good.

It may appear surprising that a model as simple as one based on scalar and vector fields should be in such close agreement with data on nuclear matter and nuclei. However, in view of our earlier remarks motivating the expansion in density we see that it should contain the essential physics of very dense matter.

The alternative approach to the evaluation of the coupling strengths depends on the identification of the phenomenological fields with physical mesons as observed or as required to fit nucleon-nucleon scattering. For the density range of interest the candidates for meson-exchange species are the $\pi$, $\eta$, $\rho$, $\omega$, $\delta$, and $\phi$ mesons.[16] A different set of mesons are used for fits to nucleon-nucleon scattering. These are the $\pi$, $\eta$, $\sigma$, $\omega$, and $\phi$. Of these lists of candidates only the $\sigma$ and $\omega$ would couple to the baryon number density or baryon proper number density and thus serve as candidates corresponding to our meson species for the expansion in density. The coupling strengths of these mesons in each set differ considerably. The set of meson parameters fit on nucleon-nucleon scattering would appear to provide a more natural set of candidates for the exchange fields in our effective Lagrangian. The average value of $g_\sigma^2/4\pi$ is 5.84 from a range of 4.7 to 6.97. These correspond to $g_S^2 m_B^2/\mu_S^2\pi^2 = 22.94$ and $g_V^2 m_B^2/\mu_V^2\pi^2 = 21.29$. These values are satisfyingly close to those obtained from the nuclear-matter fit. The effects of any uncertainty in the couplings will be discussed in the subsequent paper. The average value of $g_\omega^2/4\pi$ is 11.6 from a range of 9.05 to 15.3.

As we shall show in the following paper, the equations of state may be obtained in a form which permits evaluation of this sensitivity to variations in the coupling strengths. We consider in a subsequent publication the observable effect of these variations on neutron stars.

## APPENDIX

The pressure and ground-state energy density for a system of baryons may be expressed naturally in terms of Green's functions. The analysis could be carried out starting with (3.1) for $\mathcal{L}(x)$, and the Hartree approximation. The elimination of meson degrees of freedom can be performed at the end of the analysis. This procedure is unduly complicated for our present purposes. Instead consider the effective Lagrangian density

$$\mathcal{L}(x) = \sum_B \bar\psi_B(x)\left\{i\partial\!\!\!/ - m_B + \frac{1}{2}\sum_{B'}\left[\frac{g_S^2}{\mu_S^2}\bar\psi_{B'}(x)\psi_{B'}(x) - \frac{g_V^2}{\mu_V^2}\gamma^0\psi^\dagger_{B'}(x)\psi_{B'}(x)\right]\right\}\psi_B(x). \tag{A1}$$

The equations of motion for $\psi_B(x)$ are given by

$$\frac{\partial\mathcal{L}}{\partial\bar\psi_B} - \partial^\mu\frac{\partial\mathcal{L}}{\partial(\partial^\mu\bar\psi_B)} = 0, \tag{A2}$$

and are

$$(i\partial\!\!\!/ - m_B)\psi_B(x) = -\sum_{B'}\left[\frac{g_S^2}{\mu_S^2}\bar\psi_{B'}(x)\psi_{B'}(x) - \frac{g_V^2}{\mu_V^2}\gamma^0\bar\psi_{B'}(x)\gamma^0\psi_{B'}(x)\right]\psi_B(x) \tag{A3}$$

and its adjoint. The stress-energy density tensor is

$$\mathcal{T}^{\mu\nu} = \sum_B \partial^\mu\psi_B(x)\frac{\partial\mathcal{L}}{\partial[\partial^\nu\psi_B(x)]} - g^{\mu\nu}\mathcal{L}(x). \tag{A4}$$

Using (A1) this becomes

$$\mathcal{T}^{\mu\nu}(x) = \sum_B \bar\psi_B(x) i\gamma^\nu\partial^\mu\psi_B(x) - g^{\mu\nu}\mathcal{L}(x). \tag{A5}$$

The Hamiltonian density $\mathcal{H} \equiv \mathcal{T}^{00}$ follows immediately:

$$\begin{aligned}\mathcal{H}(x) &= \sum_B \bar\psi_B(x)\left\{-i\vec\gamma\cdot\vec\nabla + m_B - \sum_{B'}\left[\frac{g_S^2}{2\mu_S^2}\bar\psi_{B'}(x)\psi_{B'}(x) - \frac{g_V^2}{2\mu_V^2}\gamma^0\bar\psi_{B'}(x)\gamma^0\psi_{B'}(x)\right]\right\}\psi_B(x)\\ &= \sum_B \bar\psi_B(x)\left[-i\vec\gamma\cdot\vec\nabla + m_B - \sum_{B'}\frac{g_S^2}{2\mu_S^2}\bar n^{(B')} + \sum_{B'}\frac{g_V^2}{2\mu_V^2}\gamma^0 n^{(B')}\right]\psi_B(x).\end{aligned} \tag{A6}$$

The last step corresponds to the Hartree approximation, in which $\bar\psi_B(x)\psi_B(x)$ and $\bar\psi_B(x)\gamma^0\psi_B(x)$ are replaced by $\bar n^{(B)}$ and $n^{(B)}$, respectively. The pressure $P$ in a spherically symmetric and homogeneous system may be related to the expectation value of $\mathcal{T}^{\mu\nu}$ as follows:

$$\langle\mathcal{T}^{\mu\nu}\rangle = u^\mu u^\nu(P+\epsilon) - g^{\mu\nu}P. \tag{A7}$$

Here $u^\mu$ is the velocity four-vector of a fluid element, and $\epsilon$ is the ground-state energy density. The thermodynamic average in (A7) is taken over the ground state

$$\begin{aligned}\epsilon &= \langle\Phi_0|\mathcal{T}^{00}|\Phi_0\rangle\\ &= \langle\Phi_0|\mathcal{H}|\Phi_0\rangle.\end{aligned} \tag{A8}$$

It follows from (A7) and (A5) that the total baryon pressure of the ground state is

$$P = \tfrac{1}{3}\sum_{i=1}^{3} \langle \Phi_0 | \mathcal{T}^{ii} | \Phi_0 \rangle , \tag{A9}$$

$$\begin{aligned}\mathcal{T}^{ii} &= \sum_B \bar{\psi}_B(x)\left\{\tfrac{1}{3}(i\gamma^0\partial_0 - m_B) + \sum_{B'}\left[\frac{g_S^2}{2\mu_S^2}\bar{n}^{(B')} - \frac{g_V^2}{2\mu_V^2}\gamma^0 n^{(B')}\right]\right\}\psi_B(x) \\ &= \sum_B \bar{\psi}_B(x)\left\{-\frac{i}{3}\vec{\gamma}\cdot\vec{\nabla} - \sum_{B'}\left[\frac{g_S^2}{2\mu_S^2}\bar{n}^{(B')} - \frac{g_V^2}{2\mu_V^2} n^{(B')}\gamma^0\right]\right\}\psi_B(x).\end{aligned} \tag{A10}$$

The last form follows from the equations of motion.

The definition of the baryon Green's function (2.3) is now used to rewrite the ground-state averages of (A6) and (A9) in terms of $G_F^{(B)}(p)$. Consider the first term in (A6). Take the ground-state average and rewrite it as

$$\begin{aligned}\langle \bar{\psi}_B(x) i\vec{\gamma}\cdot\vec{\nabla}\psi_B(x)\rangle &= \lim_{x'\to x+0} i\vec{\gamma}\cdot\vec{\nabla}\langle\bar{\psi}_B(x')\psi_B(x)\rangle \\ &= -\lim_{x'\to x+0} i\vec{\gamma}\cdot\vec{\nabla}\langle T\psi_B(x)\bar{\psi}_B(x)\rangle \\ &= -i\lim_{x'\to x+0} i\vec{\gamma}\cdot\vec{\nabla}G_F^{(B)}(x-x').\end{aligned} \tag{A11}$$

The notation $x'\to x+0$ signifies that $x_0' = x_0 + \epsilon$, with $\epsilon$ being a positive infinitesimal. The time-ordering operator guarantees that the proper ordering $\bar{\psi}\psi$ will result. The same general procedure allows us to rewrite (A6) in terms of $G_F^{(B)}(x-x')$ to obtain

$$\begin{aligned}\epsilon &= \langle \mathcal{H}\rangle \\ &= -i\lim_{x'\to x+0}\operatorname{tr}\left[\sum_B\left(-i\vec{\gamma}\cdot\vec{\nabla} + m_B - \sum_{B'}\frac{g_S^2}{2\mu_S^2}\bar{n}^{(B')} + \sum_{B'}\frac{g_V^2}{2\mu_V^2}n^{(B')}\gamma^0\right)G_F^{(B)}(x-x')\right].\end{aligned} \tag{A12}$$

The equations of motion may be used to eliminate the interaction terms. If this is done, we obtain

$$\epsilon = -\frac{i}{2}\lim_{x'\to x+0}\operatorname{tr}\left\{\sum_B\left[-i\vec{\gamma}\cdot\vec{\nabla} + m_B + \frac{i}{2}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)\gamma^0\right]\times G_F^{(B)}(x-x')\right\}. \tag{A13}$$

Substituting (3.18) for $G_F^B(x-x')$ into (A13) leads directly to the momentum-space representation (4.5b). In this form all interaction effects enter through $G_F^{(B)}(p)$, and since the integration is over baryon momentum the effective mass may be treated as a constant. We note that (4.5b) and the thermodynamic expression

$$\epsilon^{(B)} = \int \mu^{(B)}\,dn^{(B)} \tag{A14}$$

yield similar integrals (one over $\vec{p}$ and the other over $q_F$). Both give identical results.

Proceeding in similar fashion for $P$ we arrive at (4.3b). The details are straightforward and will not be reproduced here. As a check, the results above may be used to reproduce $P$ and $\epsilon$ for a relativistic gas of noninteracting baryons.

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[1]An excellent review of previous work on this subject may be found in V. Canuto, Annu. Rev. Astron. Astrophys. 12, 169 (1974); 13, 335 (1975).

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