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Relativistic magnetohydrodynamical effects of plasma accreting into a black hole

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By an explicit analytic solution it is shown how, in the accretion of a poloidally magnetized plasma into a Kerr black hole, a torque is exerted on the infalling gas, implying the extraction of rotational energy from the black hole. The torque arises from the twisting of magnetic field lines by the frame-dragging effect. It is also shown how, under suitable conditions, a sizable charge separation can be found in the magnetosphere of accreting black holes and hence an electric charge is expected to be induced on the black hole.

A large number of gravitationally collapsed objects (neutron stars or black holes) are observed to be members of binary systems of which one of the components is a normal star.[1] Due to the frequent occurrence of magnetic fields in stellar atmospheres, it is to be expected that magnetic and electric fields will play a prominent role in the accretion processes. The treatment presented here could therefore be of great relevance in realistic astrophysical processes and at the same time answer two major fundamental issues in black-hole physics: (a) the possibility of extracting rotational or Coulomb energy from a black hole,[2] and (b) the possibility of building a magnetosphere around a black hole with a sizable charge separation.[3] We consider accretion into a Kerr black hole[4] of a plasma embedded in a magnetic field which at infinity approaches a poloidal configuration (that is, an axially symmetric field with zero components about the axis of symmetry). The following simplifying assumptions are made: (a) The plasma is assumed to have negligible pressure; (b) infinite conductivity or $F^{\mu}{}_{\nu}U^{\nu}=0$, U^{ν} being the four-velocity of the plasma stream and $F^{\mu}{}_{\nu}$ the electromagnetic tensor; (c) the mass of the accreting plasma is assumed to be small by comparison to the mass of the black hole; and (d) the magnetic fields are assumed to be too weak to ap- from the need of obtaining simple analytic solutions for the accretion process. More realistic regimes can then be further analyzed by numerical work. The background geometry is assumed to be given by the Kerr metric, which in the Boyer-Lindquist coordinates assumes the form

$$ds^2=\Sigma\Delta^{-1}dr^2+\Sigma d\theta^2+\Sigma^{-1}\sin^2\theta\,[(r^2+a^2)d\varphi-a\,dt]^2 -\Sigma^{-1}\Delta(dt-a\sin^2\theta d\varphi)^2, \tag{1}$$

with $\Delta=r^2-2Mr+a^2$ and $\Sigma=r^2+a^2\cos^2\theta$, M being the mass, and a the angular momentum per unit mass of the black hole.

The geodesics equations can be integrated.[5] It follows that if $U_t=-1$ and $U_\varphi=0$ at infinity then U_θ is a constant of the motion. We then have for the velocity field of the freely falling particles

$$V^r=U^r/U^t=\frac{-\Delta[-\Delta U_\theta^2+2Mr(r^2+a^2)]^{1/2}}{\Sigma(r^2+a^2)+2Mr\,a^2\sin^2\theta}, \tag{2a}$$

$$V^\theta=U^\theta/U^t=\frac{\Delta U_\theta}{\Sigma(r^2+a^2)+2Mr\,a^2\sin^2\theta}, \tag{2b}$$

$$V^\varphi=U^\varphi/U^t=g^{\varphi t}/g^{tt}. \tag{2c}$$

Let us now consider the electromagnetic field associated with accreting plasma. Since we consider an axially symmetric configuration the electromagnetic field is totally described by the component A_φ of the vector potential and by the component $H_\varphi=F_{r\theta}$ of the magnetic field. We have

$$F_{r\theta}=H_\varphi, \tag{3a}$$

$$F_{r\varphi}=A_{\varphi,r}, \tag{3b}$$

$$F_{\theta\varphi}=A_{\varphi,\theta}. \tag{3c}$$

From the θ and r component of the condition of infinite conductivity $F_{\mu\nu}U^\nu=0$ we obtain two more components of the electromagnetic field tensor,

$$F_{tr}=V^\theta H_\varphi+V^\varphi\frac{\partial A_\varphi}{\partial r}, \tag{4a}$$

$$F_{t\theta}=-V^rH_\varphi+V^\varphi\frac{\partial A_\varphi}{\partial\theta}. \tag{4b}$$

The equation for H_φ can be obtained from the Maxwell equation $*F^{\varphi\mu}{}_{;\mu}=0$ and the stationarity condition $(\partial H_\varphi/\partial t=0)$. We then have

$$\frac{\partial}{\partial r}(H_\varphi V^r)+\frac{\partial}{\partial\theta}(H_\varphi V^\theta)+\frac{\partial V^\varphi}{\partial\theta}\frac{\partial A_\varphi}{\partial r}-\frac{\partial V^\varphi}{\partial r}\frac{\partial A_\varphi}{\partial\theta}=0. \tag{5}$$

From the φ component of the equation of infinite conductivity and the condition of stationarity $\partial A_\varphi/\partial t=0$ we obtain

$$V^r\frac{\partial A_\varphi}{\partial r}+V^\theta\frac{\partial A_\varphi}{\partial\theta}=0. \qquad (6)$$

This equation implies that A_φ has to be constant all along a trajectory. This implies that the most general form for A_φ is simply given by (cf. Ref. 6)

$$A_\varphi=A(\theta_\infty), \qquad (7a)$$

with

$$\theta_\infty=\theta-U_\theta\int_r^\infty[-\Delta U_\theta{}^2+2Mr(r^2+a^2)]^{-1/2}dr, \qquad (7b)$$

or equivalently A is an *arbitrary* function of θ. From (6) and from the Maxwell equation ${}^*F^{t\mu}{}_{;\mu}=0$ follows then in complete generality

$$H_\varphi=(V^\varphi/V^r)F_{\theta\varphi}, \qquad (8)$$

which completely determines the electromagnetic field associated with the accreting plasma.

The torque T generated by the black hole on the infalling gas is given by

$$T=\int(J^rF_{r\varphi}+J^\theta F_{\theta\varphi})\sqrt{-g}\,d^3x, \qquad (9)$$

where the integral has to be extended to the entire region outside the horizon. By the Gauss theorem we may transform the volume integral into a surface integral extended on the black-hole horizon. We finally have[6]

$$T=\frac{1}{4\pi}\int_{\rm hor}\Delta\Sigma^{-1}H_\varphi\frac{\partial A_\varphi}{\partial\theta}\sin\theta\,d\theta\,d\varphi$$

$$=A_0{}^2\left[\frac{r_+{}^2+a^2}{a^2r_+}\arctan\left(\frac{a}{r_+}\right)-\frac{1}{a}\right], \qquad (10)$$

$r_+=M+(M^2-a^2)^{1/2}$ being the radial coordinate of the horizon. In evaluating the integral in Eq. (10) we have chosen for the sake of an example $A=A_0(1-|\cos\theta|)$. We also have from Eq. (8) the corresponding function

$$H_\varphi=\frac{-\{2aMrA_0\sin\theta/[2\Delta^2Mr(r^2+a^2)]^{1/2}\}\cos\theta}{|\cos\theta|}.$$

In Fig. 1 the lines of flux for this exact solution are shown in the equatorial plane.[8,9]

Once the function A_φ has been given we can compute from the remaining Maxwell equations the lines of current and the charge distribution in the magnetosphere of the accreting black hole. The total charge is given by the integral

$$Q_{\rm mag}=\int\sqrt{-g}\,J^t d^3x; \qquad (11)$$

which, by use of the Gauss theorem, can be written in terms of an integral extended to the surface of the horizon

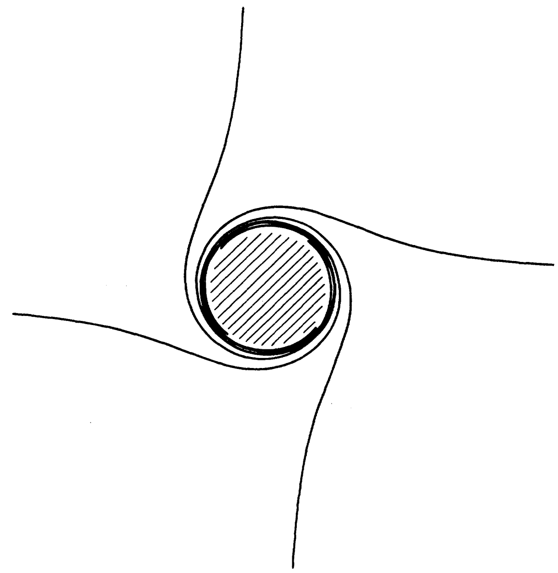

FIG. 1. Magnetic lines of force (see Ref. 9) in the equatorial plane of the black hole. The winding of the lines of force due to the dragging of the inertial frames is most clearly illustrated by this figure. This is the fundamental mechanism by which energy can be extracted from the black hole (see Ref. 8).

$$Q_{\rm hole}=-Q_{\rm mag}$$
$$=+\frac{1}{4\pi}\int_{\rm hor}\sqrt{-g}\,F^{tr}d\theta\,d\varphi$$
$$=\int_{\rm hor}\sqrt{-g}$$
$$\times\left[g^{tt}g^{rr}V^\theta H_\varphi-g^{rr}(g^{tt}V^\varphi-g^{t\varphi})\frac{\partial A_\varphi}{\partial r}\right]d\theta\,d\varphi. \qquad (12)$$

This result clearly implies that if the infalling plasma (a) follows geodesics, (b) is at rest at infinity with zero angular momentum in the φ direction, and (c) has a motion in the θ direction such as $V^\theta(\theta)=V^\theta(\pi-\theta)$ *then* the *net* charge in the magnetosphere and on the surface of the black hole is null. Any plasma accreting into a rotating black hole and breaking these requirements of very high symmetry will generally induce a net charge on the surface of the black hole. As an example we consider in Fig. 2 the lines of flux in a plane $\varphi=$ const with $A=A_0(1-|\cos\theta|)$, $U_\theta(\theta)=-U_\theta(\pi-\theta)=$ const. We then have

$$Q_{\rm hole}=Q_{\rm mag}=\pi aA_0U_\theta/16M^2. \qquad (13)$$

We have therefore shown that under a very large class of regimes of accretion breaking the intrinsic symmetry dictated by the background geometry, a net charge can be induced on the surface of a black

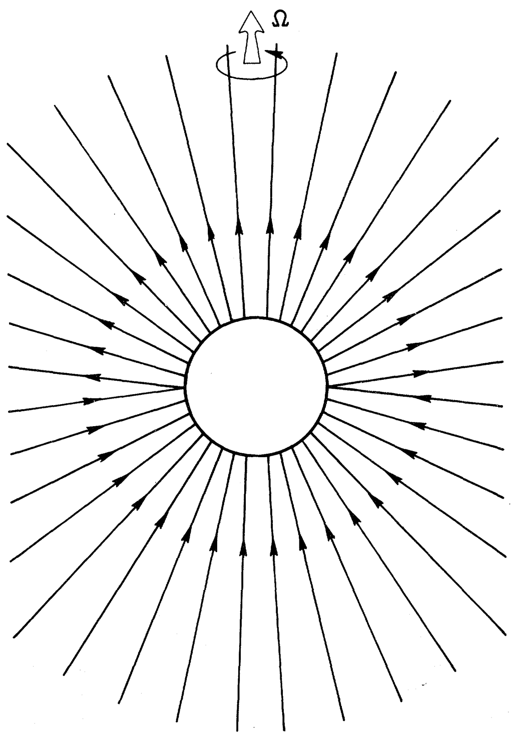

FIG. 2. Magnetic lines of force in a plane φ = const corresponding to $A_\varphi = A_0(1-|\cos\theta|)$ and a $U_\theta(\theta) = -U_\theta(\pi-\theta)$ = const. For the given vector potential and for $V^\theta = 0$ the magnetic lines of force in a plane φ = const would be radially directed, still presenting, of course, the characteristic winding along the φ direction given in Fig. 1.

hole with an equal and opposite charge in its magnetosphere [see Eq. (12)]. We have also shown, through a realistic model, how rotational energy can be extracted from a black hole. It is important to stress that in the analytic solution considered here the angular momentum extracted from the black hole is transmitted to the plasma by the torque of the magnetic field and given back to the black hole by the accreting plasma. In a more realistic situation, however, in which the magnetic field and gas pressure are strong enough to make the particle trajectories depart significantly from a geodesic motion, it has been shown by numerical calculations that a net outward flow of angular momentum is possible.[6,7] In this paper we have mainly considered the magnetic torque exerted by a rotating black hole on infalling material with low values of the angular momentum. The case in which the accreting material is endowed with a large amount of angular momentum would lead to a transfer of angular momentum to the black hole preceding the fall of material into the hole.

It is by now clear that the effects considered here, quite apart from their direct applications to binary x-ray sources, could reveal to be of fundamental importance in the astrophysical processes connected to large extragalactic black holes ($\sim 10^8 M_\odot$). The rotational energy extracted through magnetic fields by the mechanism described in this paper could in fact be essential in explaining some basic features of extragalactic radio sources.

It is a pleasure to thank T. Damour for discussions on these topics.

[1]See, e.g., *X-ray Astronomy,* edited by R. Giacconi and H. Gursky (Reidel, Dordrecht, The Netherlands, 1975); *Neutron Stars, Black Holes and Binary X-ray Sources,* edited by H. Gursky and R. Ruffini (Reidel, Dordrecht, The Netherlands, 1975).

[2]The first gedanken experiment to extract rotational energy from a black hole has been given by R. Penrose and R. M. Floyd, Nat. Phys. Sci. 229, 177 (1971); the one for extracting Coulomb energy by D. Christodoulou and R. Ruffini, Phys. Rev. D 4, 3552 (1971). These processes have been of basic importance in determining the mass formula of a black hole and establishing that up to 50% (29%) of the total mass energy of a black hole can be stored in electromagnetic (rotational) energy and, at least in principle, is extractable. No realistic process of extraction was, however, presented before the one introduced in this paper.

[3]The possibility of having a net charge on the surface of a collapsed object was shown to be expected on pure energetic grounds by R. Ruffini and A. Treves, Astrophys. Lett. 13, 109 (1973), in a classical nonrelativistic model. These considerations have been applied to a relativistic model by R. Wald, Phys. Rev. D 10, 1680 (1974). In both examples, however, it was not shown that this configuration, though energetically favorable, could be reached from a dynamic point of view in accretion of plasma into a black hole and that a configuration could exist in which the magnetosphere could screen the net charge of the black hole. This result is presented here. Finally the relevance of a charge separation on the surface of a collapsed object for coupling between gravitational and electromagnetic radiation has been given by M. Johnston, R. Ruffini, and F. Zerilli, Phys. Lett. 49B, 185 (1974); Phys. Rev. Lett. 31, 1317 (1973).

[4]See, e.g., *Black Holes,* edited by B. DeWitt and C. DeWitt (Gordon and Breach, New York, 1973).

[5]B. Carter, Phys. Rev. 174, 1559 (1968).

[6]T. Damour, R. Ruffini, and J. Wilson (unpublished).

[7]J. Wilson, invited talk at the VII International Texas

meeting on Relativistic Astrophysics (unpublished).

[8]R. Ruffini, invited talk at the VII International Texas meeting on relativistic Astrophysics (unpublished).

[9]For definition of lines of force in a curved background see, e.g., D. Christodoulou and R. Ruffini, in *Black Holes*, edited by B. DeWitt and C. DeWitt (Gordon and Breach, New York, 1973). See also R. Hanni and R. Ruffini, Nuovo Cimento Lett. (to be published).