Mass formulas, strong gravity, and scalar-tensor universality

Kashyap V. Vasavada
Department of Physics, Indiana-Purdue University, Indianapolis, Indiana 46205
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We show that tensor dominance of the stress tensor and a scalar-tensor universality postulate lead to values of the D/F ratio and the coupling constants of the f meson and Pomeron to nucleons in very good agreement with experiments.

In analogy with the vector dominance of the electromagnetic current, tensor dominance of the energy-momentum stress tensor has been considerably explored in recent years.[1] Extensive tensor-field-theoretic treatments[2,3] of the so-called "strong gravity" have been given. Yet, some simple considerations given here have not appeared in the literature. It is known that, if only the f and f' meson contributions to the pole-dominated matrix elements are kept, certain (meson-baryon) universality type relations (e.g., $G_{fNN}/G_{f\pi\pi}=1$) are obtained.[1,4] These are in conflict with experiments by a factor of 2 or 3. In Ref. 4 we introduced an explicit Pomeron contribution to remove this discrepancy.[5] Some interesting bounds were obtained but there were too many unknowns to make a unique comparison with experiments. In the present note we study systematically the symmetry breaking in masses and use a recently proposed scalar-tensor universality postulate.[3] We will be mainly concerned with spin-$\frac{1}{2}$ baryons.

The matrix elements of the energy-momentum tensor between baryon octet states are given by

$$\langle B_i(p')|\theta_{\mu\nu}|B_i(p)\rangle = \bar{u}(p')\left[(\gamma_\mu P_\nu+\gamma_\nu P_\mu)\tfrac{1}{4}F_1(q^2)+\frac{P_\mu P_\nu}{4m_B}F_2(q^2)+\frac{q^2 g_{\mu\nu}-q_\mu q_\nu}{M_B}F_3(q^2)\right]u(p)\,. \quad (1)$$

Here i denotes various SU(3) members. $P=p+p'$, $q=p-p'$, and M_B is the baryon mass. The tensor-meson couplings to the stress tensor and baryon states are defined by

$$\langle T|\theta_{\mu\nu}|0\rangle = M_T{}^3 g_T\epsilon_{\mu\nu}\,, \quad (2)$$

$$\langle B_i(p')|T|B_i(p)\rangle = \epsilon_{\mu\nu}\bar{u}(p')\left[(\gamma_\mu P_\nu+\gamma_\nu P_\mu)\frac{g^{(1)}{}_{TBB}}{4M_T}\left(\frac{\bar{M}}{M_B}\right)+\frac{P_\mu P_\nu}{4M_T M_B}\left(\frac{\bar{M}}{M_B}\right)g^{(2)}_{TBB}\right]u(p)\,. \quad (3)$$

$\bar{M}$, the average mass of the baryon octet, is introduced to make the couplings dimensionless.[6] M_T is the mass of the tensor meson.

As discussed previously,[4] it is necessary to introduce some contribution in addition to that of f and f' mesons. We identify this with the Pomeron term. It is treated as a factorizable SU(3)-singlet Regge pole with slope α'_P and intercept 1. The recently observed logarithmic rise in total cross sections and the considerable difference between πN and KN total cross sections have cast some doubt on this simple picture. On the other hand, these effects and violation of factorization do not seem to be more than 10 or 20%. So these assumptions are still approximately valid. The Pomeron-tensor contribution is taken into account by introducing a spin-2 particle with mass $M_P=1/\sqrt{\alpha'_P}$. This can be regarded as a convenient simple way of parametrizing the Pomeron contribution and the actual existence of such a particle is not really crucial.

The singlet and octet components of f and f' are given by

$$f_1=f\cos\theta-f'\sin\theta\,,\qquad f_8=f\sin\theta+f'\cos\theta\,, \quad (4)$$

The mass formula for a tensor nonet gives the mixing angle $\theta=31°\pm2°$. The so-called ideal mixing angle is given by $\cos\theta=\sqrt{\frac{2}{3}}$ ($\theta=35.3°$). The couplings to the stress tensor are denoted by g_1, g_8, and g_P. The form factor $F_1(q^2)$ gets contributions from spin-2 mesons. Saturating it with P, f_1, and f_8 at $q^2=0$ we get (Ref. 4)

$$M_N=g_P g_{PNN}\bar{M}+g_1 g_{1NN}\bar{M}+g_8 g_{8NN}\bar{M}\,, \quad (5)$$

$$M_\Lambda=g_P g_{PNN}\bar{M}+g_1 g_{1NN}\bar{M}+\frac{2(F-1)}{4F-1}g_8 g_{8NN}\bar{M}\,, \quad (6)$$

$$M_\Sigma=g_P g_{PNN}\bar{M}+g_1 g_{1NN}\bar{M}-\frac{2(F-1)}{4F-1}g_8 g_{8NN}\bar{M}\,, \quad (7)$$

$$M_\Xi=g_P g_{PNN}\bar{M}+g_1 g_{1NN}\bar{M}-\frac{2F+1}{4F-1}g_8 g_{8NN}\bar{M}\,. \quad (8)$$

Here g_{1BB} and g_{8BB} are singlet and octet tensor

(f_1, f_8) coupling constants to the baryons. g_{8BB} are assumed to be SU(3) symmetric with $F + D = 1$.

From Eqs. (5)–(8) it can be readily seen that the baryons obey the Gell-Mann–Okubo mass formula

$$2(M_\Xi + M_N) = 3M_\Lambda + M_\Sigma . \qquad (9)$$

This is not completely unexpected since we allowed only singlet and octet intermediate states, but it does provide insight into the dynamics of symmetry breaking.[7]

Now, taking differences between the masses we obtain

$$F = \frac{M_\Xi - M_N}{2(M_\Xi - M_\Sigma)} \qquad (10)$$

independent of any values of the coupling constants.[8,9] Hence

$$\frac{D}{F} = -\frac{3}{2}\frac{(M_\Sigma - M_\Lambda)}{M_\Xi - M_N} . \qquad (11)$$

This gives $D/F = -0.3$ in excellent agreement with the values (-0.2 to -0.3) determined by Barger, Olsson, and Reeder[10] and Berger and Fox.[11] These authors fitted tensor trajectory contributions to meson-baryon and baryon-baryon reactions. Note that, in contrast, the SU(6) value of $\frac{3}{2}$ is completely off.

With the above value of F, we have only two independent equations:

$$g_p g_{PNN} + g_1 g_{1NN} = 1 , \qquad (12)$$

$$g_8 g_{8NN} = \frac{M_N}{\bar{M}} - 1 . \qquad (13)$$

At this stage we use the scalar-tensor universality postulate recently given by Nath, Arnowitt, and Friedman.[3] This postulate, which would be analogous to the well-known KSRF(Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin) relation, states that the tensor and scalar couplings to the stress tensor are universal. This gives (in our notation)

$$|g_f| m_f = |g_f{}'| m_f{}' = |g_P| M_P = F_\sigma , \qquad (14)$$

where the scalar coupling F_σ is defined by

$$\langle 0|\theta_{\mu\nu}|\sigma, p\rangle = \tfrac{1}{3} F_\sigma (g_{\mu\nu} M_\sigma^2 - p_\mu p_\nu) . \qquad (15)$$

A value of $F_\sigma (\approx F_\pi) = 97$ MeV is found to be consistent with the electron-positron annihilation data.[3] A comparable value is obtained by Carruthers in Ref. 9. With this value of F_σ, we find

$$|g_f| = 0.076, \quad |g_{f'}| = 0.064,$$

and (16)

$$|g_P| = 0.097\sqrt{\alpha_{P'}} .$$

Now it is well established that f' is decoupled from $N\bar{N}$. This implies, from Eq. (4), that

$$g_{fNN} = \frac{g_{1NN}}{\cos\theta} = \frac{g_{8NN}}{\sin\theta} . \qquad (17)$$

If we choose g_{fNN}, $g_{PNN} > 0$, we need the relative signs $g_8 < 0$, $g_1 > 0$, and $g_P < 0$. This can be achieved by taking $g_f > 0$, $g_{f'} < 0$, and $g_P < 0$.

Then Eqs. (12) and (13) give unique values of g_{fNN} and g_{PNN} for a given mixing angle θ. The results are sensitive to the value of θ. We find

$$\theta = 31^\circ \to g_{fNN} = 22.7, \quad g_{PNN} = \frac{9.4}{\sqrt{\alpha_P'}} \qquad (18)$$

and

$$\theta = 35.3^\circ\,(\text{ideal}) \to g_{fNN} = 38.4, \quad g_{PNN} = \frac{21.6}{\sqrt{\alpha_P'}} . \qquad (19)$$

For comparison we note that the meson-baryon tensor universality relations obtained by saturating nucleon and pion matrix elements with the f meson imply[1,4] $g_{fNN} \approx 9.2$, in complete disagreement with the values given above.

There are several dispersion relation estimates[12] for g_{fNN}. The values are: Engels (30.3 ± 2.9); Schaile (20.4); Strauss (32.6 ± 9.7); Goldberg (17.2); Liu and McGee (20.3–24.2). These vary considerably but none of them agrees with the meson-baryon universality value. The experimental value of g_{PNN} can be obtained by comparison with the high-energy proton-proton scattering fits, e.g., those of Barger *et al.*[10] The spin-averaged proton-proton scattering amplitude $A(s,t)$ is normalized such that

$$\sigma_{\text{tot}} = \frac{1}{s}\,\text{Im}A(s, t=0) . \qquad (20)$$

For the spin-2 P exchange we find

$$A(s,t) = -\frac{g^2{}_{PNN}}{M_P{}^2}\left(\frac{\bar{M}}{M_N}\right)^2 \frac{s^2}{t - M_P{}^2} . \qquad (21)$$

Barger *et al.* take

$$A(s,t) = 12\pi \frac{\beta(t)}{\beta(0)} \gamma_{fNN}{}^2 \frac{1 + e^{-i\pi\alpha}}{\sin\pi\alpha}\left(\frac{s}{s_0}\right)^{\alpha_P(t)} , \qquad (22)$$

where the residue factor $\beta(t)/\beta(0)$ has been inserted by us. The data are fitted at $t = 0$.

Extrapolating Eq. (22) to the pole at $t = M_P{}^2$ we obtain

$$g_{PNN} = \frac{M_N}{\bar{M}} \frac{\sqrt{24} M_P}{(\alpha_P')^{1/2} s_0}\left[\frac{\beta(M_P{}^2)}{\beta(0)}\right]^{1/2} \gamma_{PNN} . \qquad (23)$$

Now the Pomeron residues are usually consistent with being structureless. Hence $\beta(M_P{}^2)/\beta(0) \approx 1$. Then from Ref. 10 we have

$$g_{PNN} = 6.3/\alpha_{P'} \ . \qquad (24)$$

It is interesting to note that, even if we take $\beta(t) = 1/\Gamma(\alpha(t))$ given by the Gell-Mann mechanism for ghost elimination or the Veneziano model, $\beta(M_P^2)/\beta(0) = 1$. Equation (24) agrees with our prediction Eq. (18) for $\alpha_P' = 0.44\ \mathrm{GeV}^{-2}$ and with Eq. (19) for $\alpha_P' = 0.08\ \mathrm{GeV}^{-2}$. These values are within the generally accepted range[13]: $0 < \alpha_P' < 0.45$.

Fits of Ref. 10 can also be used to find the experimental value of g_{fNN} by using Eq. (23) with f replacing P (f is identified with P' trajectory). Unlike the Pomeron case, however, $\beta(t)$ could change rapidly here. In particular, a right signature pole at $t \approx -0.5\ \mathrm{GeV}^2$ should be avoided by factors like $\alpha(t)$, $1/\Gamma(\alpha(t))$ etc. The former gives $\beta(M_f^2)/\beta(0) \approx 2$, whereas the latter gives 1.33 for a P' trajectory $\alpha_P'(t) = 0.5 + t$. From the value given in Ref. 10, we find $g_{fNN} = 17.0$ and 11.3 in the two cases. In addition to this, because of the possible structures involved in the residue function, there could be considerable variation in the extrapolation from $t = 0$ to $t = M_f^2$. At any rate, these values are not too far off from the dispersion relation values which are more reliable since, in principle at least, no extrapolation is involved. The value of g_{PNN}, however, essentially comes from the asymptotic value of the total cross section and hence is more reliable.

Thus we have shown that the dominance of the stress tensor by Pomeron, f, and f' mesons gives the Gell-Mann–Okubo mass formula for baryons, the D/F ratio, and PNN and fNN couplings in good agreement with the experiments. The latter results follow from the scalar-tensor universality which is thus seen to be in better agreement with experiments than the meson-baryon universality.

It is a straightforward matter to extend these considerations to other multiplets. Unfortunately the large amount of symmetry breaking in the pseudoscalar meson octet masses does lead to considerable difficulty. For this reason we have decided to consider the baryon case separately. In addition to mass formulas, one can readily find expressions for tensor radii of particles and obtain more relations. These and other matters will be presented elsewhere.

In this note and in Ref. 4 we have treated the Pomeron on a completely phenomenological level, without any restrictions based on the notions of strict two-component duality. Also, unlike the so-called tensor (f-f') dominated Pomeron model,[14] no attempt has been made to relate the Pomeron contribution to the tensor-meson contributions. Only the satisfaction of mass relations has been required. Thus in general different results will be obtained in the two approaches. A drawback of our approach will be that, in order to explain SU(3) breaking of the high-energy total cross sections (which seems to be actually less than 20%), we will have to introduce a small octet component of the Pomeron with free parameters. This is in contrast to the model of Ref. 14 where such parameters are determined. On the other hand, as we have already remarked previously,[4] the strict duality concept does run into various problems. Furthermore, it has not been possible to generate a realistic Pomeron within the framework of dual quark models. Hence, such an alternative approach seems to be worth considering.

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[5]An extra $I = 0$ piece in the trace of the stress tensor has been also previously suggested. See, for example, M. Gell-Mann, in *Proceedings of the Third Hawaii Topical Conference on Particle Physics* (Western Periodicals, North Hollywood, Calif., 1969); P. Carruthers, Phys. Rev. D 2, 2265 (1970).

[6]This definition of the coupling constant g_{TBB} is more convenient for our purpose here than the one used in Ref. 4 (G_{TBB}) which differs by the factor ($\overline{M}/M_B$). Of course, when we assume SU(3) symmetry for these, different results are obtained. Only $g_{TBB}^{(1)}$ is used in the following, and the superscript is dropped.

[7]The fact that we obtained linear mass formulas can be, of course, traced to the definitions of the SU(3) symmetric coupling in Eq. (3). Contrary to the claims which are sometimes made in literature, this cannot be determined *a priori* without appealing to the experiments. We could have obtained quadratic mass formulas in exactly the same manner.

[8]While this work was being completed, we became aware of an unpublished report by H. Pagels [University of

North Carolina technical report (unpublished)] and the work by P. Carruthers quoted in Ref. 9. Pagels obtains our Eqs. (9) and (10). However, he deals with singlet and octet contributions in a general way and does not consider measurable coupling constants as we have done here. The Pomeron is not included there and all the other results of our work are not obtained. I wish to thank Dr. H. Pagels for sending a copy of his unpublished report.

[9]In a scheme with saturation of the trace of stress tensor by scalar mesons, the same relation for the *D/F* ratio would be found. This result has been previously obtained by P. Carruthers, Phys. Rev. D 3, 959 (1971). Of course, the available information on a scalar nonet and its couplings is nowhere near the corresponding case of the tensor nonet.

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