The new resonances observed in the cascade photon decays of ψ(3695)

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The identification of the newly observed resonances in the cascade photon decays of $\psi(3695)$ is studied. The importance of the approximately "ideal" SU(4) structure for the new resonances is pointed out. The masses of the η_c members of various low-lying 16-plets are computed assuming that they satisfy the "ideal" SU(4) structure. The mass and the width of the $\eta_c\,(2^{++})$ are especially discussed without assuming the exact "ideal" SU(4) structure. The relatively narrow width of the $\eta_c\,(2^{++})$ is obtained and the mass is predicted to be around 3.3 GeV, which is close to one of the possible mass values of the new resonance. All our results are obtained in a purely algebraic way, based on asymptotic SU(4), chiral SU(4)⊗SU(4) charge algebra, and the simple mechanism of symmetry breaking. No inter-16-plet SU(4) mixings are considered.

Recently there was an indication of the remarkable cascade photon decay[1] of the $\psi(3695)$, $\psi(3695)\to X+\gamma$, and $X\to\psi(3105)+\gamma$. The mass of the new state X seems to be either around 3.3 or 3.5 GeV. The identification of X is certainly of crucial importance.

In our theoretical framework[2] of asymptotic SU(4), chiral SU(4) ⊗ SU(4) charge algebra, and a simple mechanism of SU(4) and chiral SU(4)⊗SU(4) symmetry breaking, we have recently derived, in a purely *algebraic* way, general SU(4) mass formulas[3] valid for any 16-plet boson multiplet. We denote[4] the members of a 16-plet by π_s, K_s, η_s, η_{cs}, D_s, F_s, and η_s', where the subscript s denotes J^{PC} and other quantum numbers. We have obtained[3] simple intermultiplet *mass-squared* relations,

$$K_s^{\,2}-\pi_s^{\,2}=\text{const}\quad(s\text{ is arbitrary}),\tag{1}$$

$$D_s^{\,2}-\pi_s^{\,2}=\text{const}\quad(s\text{ is arbitrary}),\tag{2}$$

$$F_s^{\,2}-K_s^{\,2}=D_s^{\,2}-\pi_s^{\,2}=\text{const}\quad(s\text{ is arbitrary}),\tag{3}$$

together with the four *mass-squared* constraints involving the SU(4) η_s-η_{cs}-η_s' mixing angles (ϕ_s, θ_s, and ψ_s) valid among the same 16-plet s. These seven independent mass relations completely fix[3] the mixing angles and the masses of K_s, D_s, F_s, and η_{cs} for any 16 plets, once the masses of the π_s, η_s, η_s', $\psi(3105)$ and, for example, the K meson are given.

In our approach, in contrast to the naive quark model, there is an intrinsic dynamical interplay of the masses, SU(4) mixing angles, and the axial-vector matrix elements. For illustration, we read our theoretical constraints obtained using the language of naive quark model. If the η_{cs} takes a pure $c\bar{c}$ configuration, our constraints force the η_s and η_s' to occupy the "ideal"[5] nonet configuration, $\eta_s=s\bar{s}$ and $\eta_s'=(1/\sqrt{2})\,(u\bar{u}+d\bar{d})$, respectively. This ideal SU(4) $q\bar{q}$ configuration corresponds to the ideal values of our η_s-η_{cs}-η_s' mixing angles,

$$\phi^i=30^\circ,\quad \theta^i\simeq 35^\circ(\sin\theta^i=\sqrt{\tfrac{1}{3}}),\quad\text{and }\psi^i=0.\tag{4}$$

In this SU(4) "ideal configuration," our constraints for the masses and couplings take the following simple forms:

(a) Ideal nonet mass relations

$$\pi_s^{\,2}=\eta_s'^{2}$$

and

$$\eta_s^{\,2}-K_s^{\,2}=K_s^{\,2}-\pi_s^{\,2}\quad(s\text{ is ideal})\tag{5}$$

must be satisfied. Furthermore, we possess

$$2D_s^{\,2}=\eta_{cs}^{\,2}+\pi_s^{\,2}\quad(s\text{ is ideal})\tag{6}$$

and the "inter-ideal-multiplet" mass relation

$$\eta_{cs}^{\,2}-\pi_s^{\,2}=\text{const}\quad(s\text{ is ideal})\,.\tag{7}$$

(b) The particular axial-vector matrix elements vanish in our asymptotic limit $\vec{k}\to\infty$, i.e.,

$$\langle\eta_s|A_{\pi^-}|\pi_u^+(\vec{k})\rangle=0\quad\text{and }\langle\eta_{cs}|A_{\pi^-}|\pi_u^+(\vec{k})\rangle=0\,,\tag{8}$$

where s is ideal but u is arbitrary provided $C_sC_u=+1$.

Conversely, if the ideal nonet mass formulas, Eq. (5), for the 16-plet s is imposed or satisfied, the η_s, η_{cs}, and η_s' will take the ideal SU(4) configuration, Eq. (4), and Eqs. (6), (7), and the selection rules, Eqs. (8), follow. Therefore, the important criterion of whether a 16-plet has an ideal SU(4) structure or not is provided by the test of the validity of the ideal nonet mass relation, Eq. (5). The magnitude of the violation of this relation also determines the size of the violation of our remarkable selection rules, Eqs. (8), associated with the ideal SU(4) configuration.

Assuming, for the moment, that the η_{cs} is a *pure* $c\bar{c}$ state (which implies, in our scheme, that the 16-plet s is ideal) as in the naive quark model, we now compute the masses of K_s, η_s, η_s', D_s, F_s, and, in particular, η_{cs} from our ideal mass sum rules Eqs. (5), (6), and (7) for some low-lying 16-plets with $J^{PC}=2^{++}$, 1^{+-}, 1^{++}, 0^{++}, 2^{-+}, 3^{--}, and 4^{++} with the input masses of the π_s and $\psi(3105)$. For this purpose, we rewrite Eq. (7) in the form

$$\eta_{cs}{}^2-\psi^2(3105)=\pi_s{}^2-\rho^2 \quad (s \text{ is ideal}). \tag{9}$$

We have identified the $\psi(3105)$ with the η_c member of the ground state 1^{--} (involving the ρ, ω, K^*, and ϕ) 16-plet which is known to satisfy the ideal constraints well.[2,3] The result is listed in Table I. In Table II, we also list the photon energy associated with the decay $\psi(3695)\to\eta_c(J^{PC})+\gamma$, where $J^{PC}=2^{++}$, 1^{++}, 0^{++}, 2^{-+}, and 4^{++}.

We note from Table I that the predicted masses of the lowest-lying P-wave states are in the range of 3.2–3.3 GeV, while the spectroscopy based on the charmed-quark-antiquark bound states[6,7] put them roughly in the higher range 3.5–3.6 GeV. Therefore, the determination of the mass of X (which may well be one of the lowest-lying P-wave states) will discriminate between these two results. The mass of $\eta_c(2^{++})\simeq 3.28$ GeV (assuming the ideal SU(4) structure for the 2^{++} 16-plet), which has already been obtained in our previous work,[3] is close to one of the possible mass values of the X reported, 3.3 GeV.

In the cascade photon decay of $\psi(3695)$, $\psi(3695)\to\eta_{cs}+\gamma$, and $\eta_{cs}\to\psi(3105)+\gamma$, the intermediate states η_{cs} with a relatively *narrow* width have a better chance to be detected by the observation of the γ ray. Therefore, the estimate of the width of the η_{cs} becomes important.

In almost all the work based on the quark model, each of the η_c members of 16-plets is considered to be a pure $c\bar{c}$ state. As a result their hadronic decay widths are taken to be (or computed to be) *small* unless the decays into $D\bar{D}$, $F\bar{F}$, etc., become energetically possible. For example, in the spectroscopy of η_{cs} of Glashow and co-workers,[6] the widths of the lowest P-wave states of charmonium are predicted to be narrow $\simeq 3$ MeV. However, this may not necessarily be the case in our approach. The smallness of the hadronic width of η_{cs} is highly dependent on the deviation of the 16-plet s from the ideal SU(4) structure. In our scheme, a relatively small deviation of the η_{cs} from a pure $c\bar{c}$ state can produce a sizable *violation* of the ideal nonet configuration of the η_s and η_s', the ideal nonet mass constraints Eq. (5), and the selection rules Eqs. (8). The large phase space available for the η_{cs} decay can then produce a *large* hadronic width of the η_{cs}. The deviations from the SU(4) ideal structure for 16-plet s can be measured and calculated in our scheme in terms of the leakage factors $\eta_s'^2-\pi_s{}^2$ and $\eta_s{}^2-2K_s{}^2+\pi_s{}^2$. The 0^{-+} 16-plet provides a good example. It violates the SU(4) ideal structure significantly. We have recently shown[8] that $\Gamma(\eta_c(0^{-+})\to\text{hadrons})$ is indeed *large* (≥ 200 MeV), in contrast to the estimated width $\simeq 6.5$ MeV based on the colored gauge gluon model,[6] if $\eta'(0^{-+})\equiv E(1420)$. *We therefore stress that even the low-lying η_{cs}' s may not necessarily be narrow resonances. Many of them may have a broad width.*

At present, the only multiplet, among the lowest lying P-wave states with $J^{PC}=0^{++}$, 1^{++}, 2^{++}, and 1^{+-}, which is known to fit *definitely* the pattern of an approximately ideal SU(4) structure is the 2^{++} 16-plet. There is also some evidence[9] for the approximately ideal structure for the 1^{++} mesons involving A_1, K_A, and $D(1285)$.[10] The evidence is based on the possible approximate validity of one of the ideal nonet mass constraints, Eq. (5), i.e., $K_A{}^2-A_1{}^2=D^2-K_A{}^2$. The other constraint, which predicts that the $\eta'(1^{++})$ should be found near the mass of A_1, must also be satisfied.

Therefore, we believe that there is a reasonable chance to observe the $\eta_c(2^{++})$ and possibly the $\eta_c(1^{++})$ [predicted around $\eta_c(1^{++})\simeq 3.2$ GeV] in the search for the photon decay $\psi(3695)\to\eta_c(J^{PC})+\gamma$. At present, we cannot say much about the ideal structures of the 0^{++} and 1^{+-} from Eq. (5). The direct search for the narrow $\eta_c(0^{++})$ (predicted around 3.16 GeV) in the vicinity of the $\eta_c(2^{++})$ and $\eta_c(1^{++})$ may be more informative on this question.

Since we know the nonet masses of the 2^{++} multiplet fairly well, we here estimate the masses of D, F, and η_c members of the 2^{++} and the partial widths of the important decays of the $\eta_c(2^{++})$, using our sum rules *without* assuming the ideal structure. We can compute the first-order deviation from the ideal structure by using Eqs. (25), (26), (27), and (28) of Ref. 3. We then obtain for the SU(4) mixing angles

$$\phi\simeq 31.8^\circ,\quad \theta\simeq 30.2^\circ,\quad \text{and } \psi\simeq 0.20^\circ, \tag{10}$$

with the input masses $f=1.270$, $f'=1.514$, $A_2=1.310$, and $K^{**}=1.420$ GeV. They are close to the ideal values. We then predict for the masses,

$$D(2^{++})=2.483,\quad F(2^{++})=2.543,$$

and (11)

$$\eta_c(2^{++})=3.311 \text{ GeV}.$$

The partial width of some of the important decays of the $\eta_c(2^{++})$ can be computed from our remarkable general sum rule[3,8] (valid for any s and u with $C_sC_u=1$ and at $\vec{k}\to\infty$)

TABLE I. The masses in MeV of the 16-plet, η_s', K_s, η_s, D_s, F_s, and η_{cs}, for various $s = J^{PC}$ from our mass formulas, with the input of the masses of π_s, $K^2 - \pi^2 = 0.227\ \mathrm{GeV}^2$ and $\psi(3105)$. The SU(4) ideal structure is assumed for all the multiplets. The comparison between the observed masses (when available) and the predicted masses suggests the approximately ideal structure for the 2^{++} 16-plet.

J^{PC}	π_s (input)	η_s'	K_s	η_s	D_s	F_s	η_{cs}
2^{++}	$A_2(1310)$	1310 $f(1270)$	1394 $K^{**}(1420)$	1473 $f'(1514)$	2483	2523	3281
1^{+-}	$B(1235)$	1235	1324	1407	2444	2485	3252
1^{++}	$A_1(1100)$	1100	1199	1290 $D(1285)$	2378	2421	3202
0^{++}	$\delta(970)$	970	1081	1181	2321	2365	3161
3^{--}	$g(1680)$	1680	1746	1810	2696	2734	3445
2^{-+}	$A_3(1640)$	1640	1708	1773	2671	2709	3426
4^{++}	$H(2020)$	2020	2075	2129	2920	2955	3627

$$R_c(s,\pi_u^+) \equiv \frac{\langle \eta_{cs} | A_{\pi^-} | \pi_u^+(\vec{k})\rangle}{\langle \eta_s' | A_{\pi^-} | \pi_u^+(\vec{k})\rangle} = -\left(\frac{X_\beta^s}{X_\gamma^s}\right)\left(\frac{\eta_s{}^2 - \eta_s'{}^2}{\eta_s{}^2 - \eta_{cs}{}^2}\right) , \qquad (12)$$

which explicitly demonstrates the dynamical *interplay* of the masses, mixing angles, and couplings. $X_\beta^s \equiv \beta^s - \sqrt{2}\,\beta_c^s$ and $X_\gamma^s \equiv \gamma^s - \sqrt{2}\,\gamma_c^s$ and β, β_c, γ, and γ_c are related to the mixing angles in a simple way.[2,3] $X_\beta^s \to 0$ when the mixing angles $(\phi_s, \theta_s, \psi_s)$ approach to the ideal values, *suppressing* the amplitude for the process $\eta_{cs} \to \pi^- + \pi_u^+$ in the ideal limit. The presence of $\eta_{cs}{}^2$ in the denominator of Eq. (10) further suppresses the above amplitude relative to the amplitude of the usually observed reaction, $\eta_s' \to \pi^- + \pi_u^+$. For $s = 2^{++}$ we obtain from Eq. (10) $R_c(2^{++}, \pi_u^+) \simeq 0.0090$, whereas for $s = 0^{-+}$ we obtain[8] the much larger value $R_c(0^{-+}, \pi_u^+) \simeq 0.160$ (u is arbitrary). If we assume $\langle \eta'(2^{++}) | A_{\pi^-} | \pi_u^+\rangle \simeq \langle \eta'(0^{-+}) | A_{\pi^-} | \pi_u^+\rangle$, i.e., these matrix elements represent the typical orders of magnitude of the usually seen strong hadron interactions, we obtain[8] $\Gamma(\eta_c(2^{++}) \to \text{hadrons}) / \Gamma(\eta_c(0^{-+}) \to \text{hadrons}) \simeq (0.0090/0.160)^2 \simeq 1/300$. Since we have estimated[8] $\Gamma(\eta_c(0^{-+}) \to \text{hadrons}) \geq 200$ MeV, we expect $\Gamma(\eta_c(2^{++}) \to \text{hadrons}) \simeq 1$ MeV. In fact, with $R_c(2^{++}, \pi_u^+) \simeq 0.0090$, PCAC and asymptotic SU(4), we obtain $\Gamma(\eta_c(2^{++}) \to \pi\pi) \simeq 0.20$ MeV and $\Gamma(\eta_c(2^{++}) \to K\bar{K}) \simeq 0.17$ MeV (assuming $f_K \simeq f_\pi$) with the experimental input[10] $\Gamma(f \to \pi\pi) \simeq 140$ MeV. The F-type couplings, $\eta_c(2^{++}) \to K^*\bar{K}$, $K_B\bar{K}$, etc., are generally small,[3,8] since the amplitudes are directly proportional to the small angle ψ. Indeed, we obtain, for example, $\Gamma(\eta_c \to K^*\bar{K}) \simeq 15$ keV. The more important decays may be $\eta_c(2^{++}) \to A_3(2^{-+}) + \pi$, $A_2 + \pi$, $A_1 + \pi$, etc. We find, for example, $\Gamma(\eta_c(2^{++}) \to A_3\pi) \simeq 0.017\,\Gamma(A_3 \to f\pi)$ which may give a few MeV depending on the actual $A_3 \to f\pi$ width. Although we do not compute all the partial widths, the above estimates seem to suggest that the hadronic width of the $\eta_c(2^{++})$ is reasonably *narrow* in contrast to the width[8] of $\eta_c(0^{-+})$. Therefore, the $\eta_c(2^{++})$ should be observed in the decay $\psi(3695) \to \eta_c(2^{++}) + \gamma$. Appelquist *et al.*[6] and Eichten *et al.*[7] have given crude estimates ($\simeq 100$ keV) for the rate.

TABLE II. The photon energy E_γ associated with the decay $\psi(3695) \to \eta_c(J^{PC}) + \gamma$ are listed for $J^{PC} = 2^{++}$, 1^{++}, 0^{++}, 2^{-+}, and 4^{++}. The masses of $\eta_c(J^{PC})$ are obtained from our inter-ideal-multiplet mass relation, $\eta_{cs}{}^2 - \pi_s{}^2 = \psi^2(3105) - \rho^2$. All the particles are assumed to satisfy the ideal "SU(4)" structure.

J^{PC}	π_s (input) (MeV)	η_{cs} (predicted) (MeV)	E_γ (MeV)
2^{++}	$A_2(1310)$	3281	391
1^{++}	$A_1(1100)$	3202	459
0^{++}	$\delta(970)$	3161	496
2^{-+}	$A_3(1640)$	3426	259
4^{++}	$H(2020)$	3627	68

We have computed the *realistic* mass value of the $\eta_c(2^{++})$ around 3.31 GeV (ideal value 3.28 GeV), taking into account the relatively small deviation of the 2^{++} 16-plet from the ideal structure. It is tempting to identify the $\eta_c(2^{++})$ with the X of the mass value $\simeq 3.3$ GeV. We also note (from Table II) that if, for example, the 2^{-+} and 4^{++} are approximately ideal, the $\eta_c(2^{-+})$ and $\eta_c(4^{++})$ predicted around 3.43 and 3.63 GeV, respectively, might also be observed.

If a future experiment finds the $\eta_c(2^{++})$ predicted 3.5–3.6 GeV range instead of the lower range around 3.3 GeV, it tells us the following important point as to the treatment of broken SU(4) in our scheme (see also the *Note added in proof*).

Although we have treated SU(4) η_s-η_{cs}-η_s' mixings in a rigorous manner, we have so far neglected the possible effect of another type of SU(4) mixing—the mixings among the SU(4) multiplets with the J^{PC} or J^P but with *different excitations*. In SU(3) this type of mixing does not seem to be large.[11] However, in SU(4) which is more broken because of the heavy mass of the charmed quark, it is possible that the inter-16-plet SU(4) mixings of the D_s, F_s, and η_{cs} are large and the masses of the D_s, F_s, and η_{cs} receive considerable modifications when we introduce the inter-16-plet SU(4) mixings in our sum rules. We expect that the effect increases[12] the mass of $\eta_c(2^{++})$.

In any case, the confirmation of the mass J^{PC} and width of X is certainly enlightening.

NOTE ADDED IN PROOF

Recently a SLAC group reported[13] evidence for the state χ in the decay $\psi(3684)\to\gamma\chi$, $\chi\to$hadrons. At least two χ states, $\chi(3410)$ and $\chi(3530)$, were observed. The $\chi(3530)$ may or may not be the same as the state reported by the DESY group.[1] It is, however, interesting to speculate that these mesons belong to some low-lying P-wave states, 0^{++}, 1^{++}, and 2^{++}. The predicted mass values of these P-wave states are lower than the observed values but the discrepancies exhibit some regularity. That is, if we scale uniformly (by a factor $3410/3161 \simeq 1.08$) the predicted mass values listed in Table I and also the predicted[3] value 3407 MeV of $\eta_c'(1^{--})$ [which belongs to the 16-plet 1^{--} involving the $\rho'(1600)$] by normalizing the mass of $\eta_c(0^{++})$ to 3410 MeV, we obtain $\eta_c(1^{++}) \simeq 3460$, $\eta_c(2^{++}) \simeq 3540$ MeV, and $\eta_c'(1^{--}) = 3680$ MeV [which is close to the mass of $\psi(3684)$]. Some effect which has not hitherto been considered may produce such more or less uniform scaling.

It is, therefore, tempting to assign the $J^{PC}=0^{++}$ and 2^{++} to the $\chi(3410)$ and $\chi(3530)$, respectively. Indeed, $\chi(3410)$ decays[13] partially into $\pi\pi$ and $K\bar{K}$, which are the expected decay modes of $\eta_c(0^{++})$. As shown in this paper, $\eta_c(2^{++})\to\pi\pi$ or $K\bar{K}$ need not be the dominant decays of $\eta_c(2^{++})$. The $\eta_c(2^{++})\to A_3(2^{-+})\pi$, $A_2\pi$, $A_1\pi$, etc. may be more important, and this is not inconsistent with experiment.[13]

As to the reasons why our predicted masses are more or less uniformly lower ($\simeq$250 MeV) than the observed ones, we have mentioned the possible effect of inter-16-plet SU(4) mixings. Another possible source of uniform error may be the neglect, in our sum rules, of the G-forbidden axial-vector matrix elements $\langle \pi_s | A_{\pi^-} | \pi_u^+ \rangle$ compared with the G-allowed ones, $\langle \eta_s | A_{\pi^-} | \pi_u^+ \rangle$ and $\langle \eta_{cs} | A_{\pi^-} | \pi_u^+ \rangle$ (the allowed ones, however, vanish in the *ideal* limit). This possibility is being studied.

If $\eta_c(0^{-+}) \equiv \chi(3410)$ is established, the 0^{++} 16-plet is preferred to be ideal. We remark that at least one of the ideal mass relations, Eq. (5), is indeed satisfied if $\delta(970)$ and $S^*(993)$ belong to the 0^{++} 16-plet.

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