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Nucleon representation of the algebra of Regge residues and chiral symmetry

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A nucleon saturation scheme for the algebra of Regge residues and chiral symmetry is studied. A representation of this extended algebra of SU(2) × SU(2) × O(5) includes the nucleon $N(940)$, the $\Delta(1236)$, and two other nucleon resonances. The masses for the resonances, the low-energy pionic coupling, and the Regge coupling of the ρ, f, π, and A_1 trajectories to the particles present in the scheme are predicted and the results are compared with experimental numbers.

I. INTRODUCTION

The algebraic approach to chiral symmetry expresses various sum rules of current algebra in a very compact manner.[1,2] Within the framework of chiral saturation schemes having a finite number of particles this algebraic formulation provides an insight into the way the sum rules are built up and leads to predictions for a large number of pionic couplings to particles.[2] For p-wave pions the algebraic formulation yields what may be considered as spectrum-generating algebras.[3] By using the narrow-resonance approximation and the ideas of duality it is possible to give an algebraic formulation also to all finite-energy sum rules for the scattering amplitudes $\pi\alpha \to \pi\beta$, where α, β are any two hadrons belonging to representations of the chiral group.[4] Chiral symmetry predicts the low-energy pionic couplings in reasonable agreement with experiment, and these couplings can be inserted into the algebraic form of finite-energy sum rules to predict a number of Regge couplings for the particles.

Recently this technique has been used to derive the behavior of Regge couplings under chiral transformations and to derive the algebra of Regge residues by writing finite-energy sum rules for $\pi\alpha \to \pi\pi\beta$ and for $\pi\pi\alpha \to \pi\pi\beta$ in the single and double Regge limits.[5,6] This derivation based on duality and current algebra provides a basis for the earlier conjecture of Cabibbo, Horwitz, and Ne'eman[7] that the Regge couplings form an algebra. The extension of the algebra of Regge residues by the chiral group directly gives Regge couplings consistent with low-energy pionic couplings, and this consistency requirement serves to strongly restrict our choices for chiral saturation schemes. At the level of SU(3) symmetry such extensions of the Regge algebra would require the transformation from "constituent" quarks to "current" quarks to be consistent with the Regge algebra, and in this sense we may also talk of "Regge quark" content of the elementary particles.

In the particular case where only the leading trajectories ρ, f, π, and A_1 are taken into consideration the algebra of Regge residues has been shown by Kleinert[6] to be the Lie algebra of O(5). When this is combined with the low-energy theorems of current algebra the extended algebra is SU(2)×SU(2)×O(5). In an earlier paper[5] a detailed derivation of this result has been given together with an application to a meson saturation scheme. The predictions for the pion couplings and for Regge couplings have been shown to agree very well with experimental results.

The purpose of the present paper is to consider a nucleon representation of SU(2)×SU(2)×O(5) and to show that the algebraic picture for Reggeon couplings does lead to good predictions. Since a derivation of the algebra of Regge residues has already been given we present in Sec. II only the final form of the various sum rules. In Sec. III we evaluate the Regge couplings by assigning the nucleons to a suitable representation of the extended algebra of Regge residues and in Sec. IV we compare the results with experiment. We consider a scheme with the $N(940)$, $\Delta(1236)$, and two other nucleon resonances. Using mass sum rules we identify one of the nucleon resonances to be $N(1520)$. The other resonance has mass of 1890 MeV and simulates the contributions of all the high-mass resonances in various sum rules. While this is obviously a rather small set of resonances, all the salient features of our algebraic formulation of scattering amplitudes can be brought out, and what is interesting is the agreement with experimental numbers.

II. ALGEBRAIC FORM OF SUM RULES

We shall consider always the collinear configuration in the scattering of massless pions on the particles α, β. The assumption of partial conservation of the axial-vector current (PCAC) allows us to compare the axial-vector matrix elements with experimental data on pionic couplings because these matrix elements are not expected to vary much when the pion is taken off its mass shell to zero mass. Thus we have

$$\langle\beta p_\beta\lambda'|j_\pi^a|\alpha p_\alpha\lambda\rangle \approx \lim_{q^2\to 0}\frac{m_\pi^2-q_\pi^2}{F_\pi m_\pi^2}\langle\beta|\partial^\mu A_\mu^a|\alpha\rangle\delta_{\lambda\lambda'}$$
$$= -i(|\vec{q}|/F_\pi)\langle\beta|A_a^{0-3}|\alpha\rangle\delta_{\lambda\lambda'}$$
$$\equiv \frac{i\,m_\beta^2-m_\alpha^2}{F_\pi}[X^a(\lambda)]_{\beta\alpha}, \tag{1}$$

where $F_\pi = 0.095$ GeV is the pion decay constant and the helicity λ is conserved. If we define m^2 as the diagonal mass matrix we have

$$\langle\beta|j_\pi^a|\alpha\rangle = (i/F_\pi)[m^2, X_a]_{\beta\alpha} \equiv m_a{}^2/F_\pi \tag{2}$$

with $m_a{}^2$ being an isospin-1 quantity.

The matrix elements of X_a are just the matrix elements of the axial charge in the infinite-momentum frame.[8] The assumption of chiral SU(2)×SU(2) symmetry requires X_a and the isospin operator T_a to satisfy the commutation relations

$$[T_a, T_b] = i\epsilon_{abc}T_c = [X_a, X_b],$$
$$[T_a, X_b] = i\epsilon_{abc}X_c. \tag{3}$$

With these preliminary remarks we present the sum rules we will be discussing:

(i) The superconvergence of the $T=2$ amplitude for $\pi\alpha\to\pi\beta$ leads to the result[1] that the mass matrix m^2 is composed of a chiral scalar $m_0{}^2$ and a term $m_4{}^2$ which together with $m_a{}^2$ belongs to a $(\frac{1}{2},\frac{1}{2})$ representation of the chiral group. This requires the usual assumption that the $\sum$ term (defined below) contains no $T=2$ part. Thus we write

$$m^2 = m_0{}^2 + m_4{}^2. \tag{4}$$

(ii) By using finite-energy sum rules[9] in conjunction with the threshold theorem of current algebra for the $\sum$ term in $\pi^a\alpha\to\pi^b\beta$,

$$\lim_{\nu\to\nu_{\rm th}} T_{\beta\alpha}^{(+)ba}(\nu, t=0) = (i/2F_\pi^2)\{[Q_5^b, \partial A^a] + (a\leftrightarrow b)\}$$
$$\equiv \frac{1}{F_\pi^2}\Sigma_{\beta\alpha}^{b\,a},$$

we can derive the relation[4]

$$\delta_{ab}(m_4{}^2)_{\beta\alpha} = \delta_{ab}\Sigma_{\beta\alpha}^{\pi\pi} + \frac{2F_\pi^2}{\pi\alpha_f(0)}\left(\frac{N}{M^2}\right)^{\alpha_f(0)}(R^f)_{\beta\alpha}(R^f)_{ba}, \tag{5}$$

where the Regge couplings $R^f\times R^f$ are taken to be factorizable and the trajectory intercept is $\alpha_f(0)\approx\frac{1}{2}$. Also, N is the value of the variable $\nu=\frac{1}{2}(s-u)$ beyond which Regge behavior sets in and M^2 is just a scale factor and may be taken to be 1 (GeV)^2.

(iii) Similarly, for the odd amplitude[4] we may write

$$[m_a{}^2, m_b{}^2]_{\beta\alpha} = -\frac{2F_\pi{}^2M^2}{\pi[\alpha_\rho(0)+1]}\left(\frac{N}{M^2}\right)^{\alpha_\rho(0)+1} \times (R^\rho)_{\beta\alpha}(R^\rho)_{ab}. \tag{6}$$

With $m_a{}^2$ and $m_4{}^2$ determined from physical masses and the pion coupling matrices X_a given by chiral saturation schemes we see from Eqs. (5) and (6) that we can *algebraically* estimate the Regge couplings once a judicious choice is made for the energy cutoff N to be a value just above the last resonance beyond which the amplitudes are expected to exhibit smooth Regge behavior.

(iv) We can show[5] that the same kinematics obtains for the reaction "Reggeon" $+\alpha\to\pi_c+\beta$ derived from the reaction $\pi_a\alpha\to\pi_b\pi_c\beta$ in the single-Regge limit. The low-energy behavior of this amplitude is fixed by the Adler zero at $q_c=0$. Using finite-contour dispersion relations we then relate the resonance-saturated amplitude in the single-Regge limit to the double-Regge behavior beyond a certain (energy)^2 value N_π, which leads to the relations

$$[X_a, R_b^{(\rho)}] = i\epsilon_{abc}(\mathcal{G}_\pi)_{\rho A_1}R_c^{(A_1)}\frac{(N_\pi/M^2)^{\alpha_{A_1}-\alpha_\rho}}{\alpha_{A_1}-\alpha_\rho},$$
$$[X_a, R^{(f)}] = i(\mathcal{G}_\pi)_{f\pi}R_a^{(\pi)}\frac{(N_\pi/M^2)^{\alpha_\pi-\alpha_f}}{\alpha_\pi-\alpha_f}, \tag{7}$$

where $(\mathcal{G}_\pi)_{ij}$ are the Reggeon-Reggeon-pion coupling constants and the trajectory intercepts for the π and the A_1 trajectories are $\alpha_\pi\approx\alpha_{A_1}\approx 0$.

(v) In the double-Regge limit the amplitude for the collinear reaction $\pi\pi\alpha\to\pi\pi\beta$ can be considered to be the process "Reggeon" $+\alpha\to$ "Reggeon" $+\beta$. A finite-energy sum rule for this amplitude relates the low-energy resonance-dominated part to the triple-Regge region in this reaction. The algebraic form[6] of these sum rules is

$$[R^i, R^j]_{\beta\alpha} = \frac{-2M^2}{\pi}\frac{(N/M^2)^{\alpha_k+1-\alpha_i-\alpha_j}}{\alpha_k+1-\alpha_i-\alpha_j}g_{ijk}(R^k)_{\beta\alpha}$$
$$(\alpha_k+1>\alpha_i+\alpha_j), \tag{8}$$

with g_{ijk} being the triple-Reggeon coupling.

By defining

$$\tilde\rho_a = -\frac{\pi}{4M^2g}\left(\frac{M^2}{N}\right)^{1/2}R_a^{(\rho)},$$
$$\tilde f = -\frac{\pi}{4M^2g}\left(\frac{M^2}{N}\right)^{1/2}R^{(f)}, \tag{9}$$
$$\tilde\pi_a = -\frac{\sqrt{3}\,\pi}{4Ng}R_a^{(\pi)}, \quad \tilde A_{1a} = -\frac{\sqrt{3}\,\pi}{4Ng}R_a^{(A_1)}$$

we can express the algebra of Regge residues in a compact manner. Also, if we identify $\tilde{\rho}_a, \tilde{f}, \tilde{\pi}_a, \tilde{A}_{1a}$ with the generators L_{bc}, L_{45}, L_{a5}, and L_{a4}, respectively, we have the Lie algebra of O(5):

$$[L_{\mu\nu}, L_{\alpha\beta}] = i(\delta_{\mu\alpha}L_{\nu\beta} + \delta_{\nu\beta}L_{\mu\alpha} - \delta_{\mu\beta}L_{\nu\alpha} - \delta_{\nu\alpha}L_{\mu\beta}), \tag{10}$$

$$\mu, \nu, \alpha, \beta = 1, \ldots, 5 .$$

These definitions together with Eqs. (7) and (10) can be used in Jacobi identities to show[5] that $\tilde{\rho}_a$ and $\tilde{A}_{1a}$ transform according to the $[(1,0) \pm (0,1)]$ representations of the chiral group while the operators $\tilde{\pi}_a, \tilde{f}$ belong to a $(\frac{1}{2}, \frac{1}{2})$ representation:

$$[X_a, \tilde{\rho}_b] = i\epsilon_{abc}\tilde{A}_{1c}\,; \quad [X_a, \tilde{A}_{1b}] = i\epsilon_{abc}\tilde{\rho}_c\,, \tag{11}$$

$$[X_a, \tilde{\pi}_b] = i\,\delta_{ab}\tilde{f}\,; \quad [X_a, \tilde{f}] = -i\tilde{\pi}_a\,.$$

We also obtain the result that

$$|(g_\pi)_{\rho A_1}| = |(g_\pi)_{f\pi}| = \tfrac{1}{2}(3N_\pi/N)^{1/2},$$

where the cutoff energy parameters N_π and N correspond to the $2 \to 3$ and the $3 \to 3$ reactions.

The commutators (10) and (11) now define the algebra of $SU(2) \times SU(2)_R \times O(5)$ since the sets of operators

$$\tfrac{1}{2}(T_a + X_a - \tilde{\rho}_a - \tilde{A}_{1a}),$$

$$\tfrac{1}{2}(T_a - X_a - \tilde{\rho}_a + \tilde{A}_{1a}), \tag{12}$$

$$(\tilde{\rho}_a, \tilde{f}, \tilde{\pi}_a, \tilde{A}_{1a})$$

commute with each other; the subscript R on the $SU(2) \times SU(2)$ subgroup is to distinguish it from the usual chiral group.

III. NUCLEON SATURATION SCHEME

The representations of $O(5)$ are labeled[10] by using two numbers (p, q) and an $O(4)$ classification of the states using the commuting spins $S_a = \tilde{\rho}_a + \tilde{A}_{1a}$ and $J_a = \tilde{\rho}_a - \tilde{A}_{1a}$ is possible. The representations of the group $SU(2) \times SU(2)_R \times O(5)$ are labeled by the numbers $[(i,j)] \times (p,q)$.

Let us consider a saturation scheme with the nucleon $N_1 = N(940)$, the resonance $\Delta(1236)$, and two other nucleon resonances N_2 and N_3 which we can classify as belonging to the $[(\frac{1}{2}, 0)] \times (0,1)$ representation. The $O(4)$ content of the $(0,1)$ representation of $O(5)$ is $(S = \frac{1}{2}, J = \frac{1}{2}) \oplus (S = 0, J = 0)$. The matrix elements of the generators of $O(5)$ are easily evaluated by standard techniques in this representation. Since it is a singlet under $SU(2) \times SU(2)_R$ its chiral content (i.e., its behavior under the usual chiral transformations) is $(\frac{1}{2}, \frac{1}{2}) \oplus (0,0)$. Thus the chiral decomposition of our nucleon representation is

$$(\tfrac{1}{2}, 0) \otimes [(\tfrac{1}{2}, \tfrac{1}{2}) \oplus (0,0)] = (1, \tfrac{1}{2}) \oplus (0, \tfrac{1}{2}) \oplus (\tfrac{1}{2}, 0)$$

$$\Delta, K_1 \qquad K_2 \qquad iK_3\,. \tag{13}$$

We use the Condon-Shortley phase convention together with our choice of imaginary phase for the chiral state $(\frac{1}{2}, 0)$ to make the matrix elements of $\tilde{f}$ real. The physical nucleon states can be expressed as a linear combination of the chiral states K_i in terms of three Euler mixing angles θ, ψ, and ϕ:

$$N_1 = \cos\theta\, K_1 + \sin\theta \sin\psi\, K_2 + \sin\theta \cos\psi\, K_3\,,$$

$$N_2 = \sin\theta \sin\phi\, K_1 + (\cos\psi \cos\phi - \cos\theta \sin\phi \sin\psi) K_2 - (\cos\phi \sin\psi + \cos\theta \sin\phi \cos\psi) K_3\,, \tag{14}$$

$$N_3 = -\sin\theta \cos\phi\, K_1 + (\cos\psi \sin\phi + \cos\theta \cos\phi \sin\psi) K_2 + (\cos\theta \cos\phi \cos\psi - \sin\psi \sin\phi) K_3\,,$$

or in brief

$$N_i = P_{ij} K_j; \quad P^{-1} = P^T.$$

Let us define the reduced matrix elements $G_{\alpha\beta}$ for any isospin $T = 1$ operator, say X_a, as follows (Ref. 4)

$$[X_a]^{1/2,1/2}_{\beta\alpha} = G^{(X)1,1}_{\beta\alpha}\, \chi^{\dagger\prime}(\tfrac{1}{2}\tau_a)\chi\,,$$

$$[X_a]^{3/2,1/2}_{\beta\alpha} = \tfrac{1}{2}\sqrt{3}\, \chi_a^{\prime\dagger}\chi\, G^{(X)3,1}_{\beta\alpha}\,, \tag{15}$$

$$[X_a]^{3/2,3/2}_{\beta\alpha} = \tfrac{3}{2} i\epsilon_{b'aa'}\chi^\dagger_{b'}\chi_{a'}\, G^{(X)3,3}_{\beta\alpha}\,,$$

where the isospin-$\frac{1}{2}$ spinors are denoted by χ and the isospin-$\frac{3}{2}$ spinors are χ_a (with $\tau_a \chi_a = 0$). For the isosinglet operator $\tilde{f}$ we write

$$[\tilde{f}]^{1/2,1/2}_{\beta\alpha} = \chi^{\prime\dagger}\chi\, G^{(f)1,1}_{\beta\alpha}\,,$$

$$[\tilde{f}]^{3/2,3/2}_{\beta\alpha} = \delta_{ab}\chi_b^{\prime\dagger}\chi_a\, G^{(f)3,3}_{\beta\alpha}\,. \tag{16}$$

In the following we shall omit the isospin superscripts since for our calculations α, β are either the Δ or the nucleons.

An explicit construction of the states K_i allows the evaluation of the matrix elements of all the generators. The couplings for the physical states are given by

$$G_{\Delta N_i} = [G_{\Delta K_i} P^T]_i\,,$$

$$G_{N_i N_j} = [P G_{KK} P^T]_{ij}\,. \tag{17}$$

The matrix elements in the chiral basis for all the operators are

$$G^{(X)}_{\Delta\Delta}=\tfrac{1}{3},\quad G^{(X)}_{\Delta K_i}=(\tfrac{4}{3},0,0),$$

$$G^{(X)}_{K_iK_j}=\begin{pmatrix}\tfrac{5}{3}&0&0\\0&-1&0\\0&0&1\end{pmatrix},\tag{18a}$$

(the following couplings have not been arranged in the above matrix form so as to save space)

$$\begin{aligned}
&G^{(\rho)}_{\Delta\Delta}=\tfrac{2}{3},\quad G^{(A_1)}_{\Delta\Delta}=G^{(\pi)}_{\Delta\Delta}=G^{(f)}_{\Delta\Delta}=0,\\
&G^{(\rho)}_{\Delta K_i}=(-\tfrac{1}{3},-1/\sqrt{3},0),\\
&G^{(A)}_{\Delta K_i}=(1,-1/\sqrt{3},0),\\
&G^{(\pi)}_{\Delta K_i}=(0,0,-i2/\sqrt{3}),\\
&G^{(f)}_{\Delta K_i}=0,\\
&3G^{(\rho)}_{K_1K_1}=\sqrt{3}\,G^{(\rho)}_{K_1K_2}=G^{(\rho)}_{K_2K_2}=1,\\
&G^{(A_1)}_{K_1K_1}=\sqrt{3}\,G^{(A)}_{K_1K_2}=-G^{(A)}_{K_2K_2}=1,\\
&\sqrt{3}\,G^{(\pi)}_{K_1K_3}=G^{(\pi)}_{K_2K_3}=-i,\\
&G^{(f)}_{K_1K_3}=-\sqrt{3}\,G^{(f)}_{K_2K_3}=\tfrac{1}{2}\sqrt{3}\,.
\end{aligned}\tag{18b}$$

The "reduced" Regge couplings to nucleons can be obtained by using the transformations (17) on Eqs. (18) after the three mixing angles are determined from experimental data on decay widths. The numbers thus determined are proportional to the physically measured couplings so that we can predict ratios of the couplings. However, using relations (5) and (6) we can also estimate the actual values of the Regge couplings. This requires the masses of the resonances N_2 and N_3 to be known; we shall of course use the masses of $N(940)$ and $\Delta(1236)$ as input.

Instead of directly assuming that N_2, N_3 are two particular resonances in the particle data listings we shall use the fact that $m_4^{\,2}$, which is the chiral symmetry breaking term in the mass matrix, belongs to a $(\frac{1}{2},\frac{1}{2})$ representation to obtain two mass sum rules. Earlier, Pashupathy[11] obtained the same sum rules in another context. Since $(1,\frac{1}{2})\otimes(1,\frac{1}{2})$ does not contain a $(\frac{1}{2},\frac{1}{2})$ in its chiral decomposition we see that

$$\langle\Delta|m^2|\Delta\rangle=\langle K_1|m^2|K_1\rangle\,,$$

and hence

$$m_\Delta^{\,2}=\cos^2\theta\, m_{N_1}^{\,2}+\sin^2\theta(\sin^2\phi\, m_{N_2}^{\,2}+\cos^2\phi\, m_{N_3}^{\,2})\,.\tag{19}$$

Again, because $(1,\frac{1}{2})\otimes(0,\frac{1}{2})$ does not have a $(\frac{1}{2},\frac{1}{2})$, we obtain

$$\langle K_1|m^2|K_2\rangle=\langle K_1|m_4^{\,2}|K_2\rangle=0$$

leading to the mass relation

$$\begin{aligned}
(m_{N_2}^{\,2}-m_{N_3}^{\,2})\cos\psi\sin\phi\cos\phi
&=\cos\theta\sin\psi(m_{N_2}^{\,2}\sin^2\phi+m_{N_3}^{\,2}\cos^2\phi-m_{N_1}^{\,2})\\
&=(\cos\theta\sin\psi/\sin^2\theta)(m_\Delta^{\,2}-m_{N_1}^{\,2})\,.
\end{aligned}\tag{20}$$

IV. COMPARISON WITH EXPERIMENT

With $\cos\theta=0.675$ and

$$|G_A/G_V|=G^{(X)}_{N_1N_1}=\tfrac{5}{3}\cos^2\theta+\sin^2\theta\cos2\psi=1.25\tag{21}$$

as input, we determine $\cos\psi\approx0.975$ while maintaining an adequate width of 98 MeV for the Δ resonance. The third mixing angle is taken to be $\phi=7°$ so that we obtain

$$\begin{aligned}
G^{(X)}_{N_1N_2}&=\sin\theta\cos\theta(\tfrac{5}{3}-\cos2\psi)\sin\phi\\
&\quad-\sin(2\psi)\sin\theta\cos\phi\\
&=-0.273\,,\\
G^{(X)}_{N_1N_3}&=-\sin\theta\cos\theta(\tfrac{5}{3}-\cos2\psi)\cos\phi\\
&\quad-\sin2\psi\sin\theta\sin\phi\\
&=0.418\,.
\end{aligned}\tag{22}$$

If the resonance N_2 is identified as the $N(1688)$ we obtain $\Gamma(N_2\to N\pi)\approx57$ MeV which may be compared with the experimental value[12] of 60–110 MeV. If we let $N_3=N(1520)$ we predict $\Gamma(N_3\to N\pi)\approx130$ MeV which is somewhat higher than the experimental value of 53–75 MeV. With the mixing angles so chosen we give below the predictions of our symmetry scheme:

(a) With $m_{N_1}=940$ MeV and $m_\Delta=1236$ MeV as input in the mass relations (19) and (20) we determine the masses of the resonances N_2 and N_3. We predict

$$m_{N_3}=1440\text{ MeV}\tag{23}$$

so that N_3 may indeed be identified with the $N(1520)$ resonance (with ~5% error), and

$$m_{N_2}=1890\text{ MeV}\tag{24}$$

to be compared with $m_{N_2}=1688$ (with ~12% error). It is of interest to note that if we use the predicted masses instead of the experimental masses the partial widths in the $N\pi$ channel are in close agreement with experiment with decay widths of 107 MeV and 68 MeV, respectively, for N_2 and N_3.

(b) The ρ-Regge couplings of interest are

$$G^{(\rho)}_{N_1N_1}=[(1/\sqrt{3})\cos\theta+\sin\theta\sin\psi]^2\approx 0.308\,, \tag{25}$$

$$G^{(\rho)}_{\Delta N_1}=-(1/\sqrt{3})(\cos\theta/\sqrt{3}+\sin\theta\sin\psi)\approx -0.32\,.$$

The data for $\pi^- p\to\pi^0 n$ and for $\pi^+ p\to\pi^0\Delta^{++}$ of Refs. 13 and 14 lead to a value

$$|G^{(\rho)}_{N_1N_1}/G^{(\rho)}_{\Delta N_1}|=1.06\pm 0.14 \tag{26a}$$

to be compared with the theoretically predicted ratio

$$|G^{(\rho)}_{N_1N_1}/G^{(\rho)}_{\Delta N_1}|_{\text{theory}}=0.96\,. \tag{26b}$$

(c) The f-Regge coupling for the nucleon is

$$G^{(f)}_{N_1N_1}=(\sqrt{3}\cos\theta-\sin\theta\sin\psi)\sin\theta\cos\psi=0.722\,. \tag{27}$$

The experimentally measured ρ and f residues in $\pi N\to\pi N$ have been quoted by Michael[15] to be

$$C(f)^{pp}_{\pi\pi}\equiv R^{(f)}_{N_1N_1}R^{(f)}_{\pi\pi}=-53.6\pm 2 \tag{28}$$

and

$$C(\rho)^{pp}_{\pi\pi}\equiv \tfrac{1}{2}R^{(\rho)}_{N_1N_1}R^{(\rho)}_{\pi\pi}=-9.6\pm 0.6 \tag{29}$$

The errors quoted are probably somewhat low. We may use (28) and (29) together with the exchange degeneracy result $R^{(f)}_{\pi\pi}=R^{(\rho)}_{\pi\pi}$ to obtain

$$G^{(f)}_{N_1N_1}/G^{(\rho)}_{N_1N_1}\simeq 2.8\pm 0.5\,. \tag{30a}$$

From Eqs. (25) and (27) we obtain

$$[G^{(f)}_{N_1N_1}/G^{(\rho)}_{N_1N_1}]_{\text{theory}}=2.35\,, \tag{30b}$$

in excellent agreement with (30a), considering the assumptions that go into the derivation of (30a).

(d) This theory leads to a decoupling of the Reggeized pion from the nucleon. It would require the inclusion of conspiracy considerations[16] in order to have a nonzero value for $G^{(\pi)}_{N_1N_1}$. Within our approximations this null value is not inconsistent. We also have

$$|G^{(\pi)}_{\Delta N_1}|=(2/\sqrt{3})\sin\theta\cos\psi=0.83\,. \tag{31}$$

(e) The couplings to the A_1 Regge trajectory are

$$\begin{aligned}G^{(A)}_{N_1N_1}&=(\cos\theta+\sin\theta\sin\psi/\sqrt{3})^2-\tfrac{4}{3}\sin^2\psi\sin^2\theta\\&\approx 0.557\,,\\G^{(A)}_{\Delta N_1}&=(\cos\theta-\sin\theta\sin\psi/\sqrt{3})\approx 0.58\,,\end{aligned} \tag{32}$$

with

$$|G^{(A)}_{N_1N_1}/G^{(A)}_{\Delta N_1}|\approx 0.96\,.$$

(f) We now make use of the low-energy pionic couplings and the *predicted* masses for N_2 and N_3 in the relations (5) and (6) to give estimates for the actual magnitudes of the Regge couplings. For the ρ-Regge coupling we obtain

$$\begin{aligned}R^{(\rho)}_{N_1N_1}R^{(\rho)}_{\pi\pi}=&-\pi\frac{\alpha_\rho+1}{2F_\pi^{\,2}M^2}\left(\frac{M^2}{N}\right)^{\alpha_\rho+1}\\&\times\Big\{\sum_i[(m_{N_i}^{\,2}-m_{N_1}^{\,2})^2|G^{(X)}_{N_1N_i}|^2]\\&-(m_\Delta^{\,2}-m_{N_1}^{\,2})^2|G^{(X)}_{\Delta N_1}|^2\Big\}.\end{aligned}$$

With $\alpha_\rho\simeq\frac{1}{2}$, $M^2=1$ GeV^2, and $N/M^2=n$ we have

$$R^{(\rho)}_{N_1N_1}R^{(\rho)}_{\pi\pi}\simeq -117n^{-3/2}\,. \tag{33}$$

The value of $N^{1/2}$, the energy above which the Regge approximation to the behavior of the scattering amplitude is expected to hold, may be chosen to be just above the last resonance in the theory. If we let $N^{1/2}=1.95$ GeV, we obtain

$$R^{(\rho)}_{N_1N_1}R^{(\rho)}_{\pi\pi}\approx -16\,, \tag{34}$$

which is in reasonable agreement with the experimental value of -19.2 ± 1.2, considering the approximations inherent in our saturation scheme. We may mention that for $N^{1/2}=1.9$, which is just at the position of the last resonance, the Regge residue is -17.2.

From Eq. (5) we obtain (with $\alpha_f=\frac{1}{2}$):

$$\begin{aligned}R^{(f)}_{N_1N_1}R^{(f)}_{\pi\pi}&=\frac{\pi\alpha_f}{2F_\pi^{\,2}}\left(\frac{M^2}{N}\right)^{\alpha_f}\Big\{\sum_i[-\tfrac{1}{2}(m_{N_i}^{\,2}-m_{N_1}^{\,2})|G^{(X)}_{N_1N_i}|^2]-(m_\Delta^{\,2}-m_{N_1}^{\,2})|G^{(X)}_{N_1\Delta}|^2-\Sigma^{\pi\pi}_{N_1N_1}\Big\}\\&=-87(n)^{-1/2}(0.725+\Sigma^{\pi\pi}_{N_1N_1})\,.\end{aligned} \tag{35}$$

For the same choice $N^{1/2}=1.95$ GeV as before, we obtain (Ref. 17)

$$R^{(f)}_{N_1N_1}R^{(f)}_{\pi\pi}=-32-45\Sigma^{\pi\pi}_{N_1N_1}. \tag{36}$$

The above relation Eq. (36) can be tested if the Σ term is known. However, due to difficulties of extrapolation of the isospin-even amplitude to $m_\pi^2=0$, no reliable estimate is available and values ranging from 20 to 110 MeV are quoted in the literature. Also, inclusion of more resonances will certainly lower the resonance contribution from -32 to a value closer to the experimental value of -53.6 ± 2 for $R^f_{N_1N_1}R^f_{\pi\pi}$, and in view of the generally accepted low values for the Σ term, our small saturation scheme requires modification by the inclusion of additional resonances.

(g) Finally we note that we can obtain an order-of-magnitude estimate for the triple Reggeon coupling constant. The definition (9) can be used to write

$$|g|=(\pi/4M^2)(M^2/N')^{1/2}R^{(\rho)}_{N_1N_1}/G^{(\rho)}_{N_1N_1}. \tag{37}$$

In the absence of a reliable experimental value for the ρ-Regge coupling in $NN\to NN$ we can use our earlier result (Ref. 4)

$$|R^{(\rho)}_{\pi\pi}|\approx\sqrt{30}=5.5$$

based on a meson saturation scheme or, equivalently, the prediction for this number based on the Veneziano model for $\pi\pi\to\pi\pi$ to evaluate $R^{(\rho)}_{N_1N_1}$ using the ρ-Regge residue in $\pi N\to\pi N$. We have

$$R^{(\rho)}_{N_1N_1}=19.2/5.5=3.5$$

so that

$$|g|=(\pi/4M^2)(M^2/N')^{1/2}(3.5/0.308)$$

$$\approx 9(N'/M^2)^{-1/2}\ \text{GeV}^{-2}.$$

We may expect the triple Regge region to be reached for the missing mass variable in the reactions $A+B\to C+$ anything having a value $N'^{1/2}=9-10$ GeV. We then have

$$|g|\approx 1\ \text{GeV}^{-2}. \tag{38}$$

This value for g agrees with our estimate based on a meson saturation scheme[5]; however, the choice of N' is rather arbitrary.

V. CONCLUSIONS

In our saturation scheme we have not demanded that m_4^2 be proportional to the f-Regge couplings; they are taken to be the fourth components of two different chiral four-vectors. They would be proportional to each other for the nucleon representations of $O(5)$ which transform as singlets under $SU(2)\times SU(2)_R$. This in turn would have led to $G^{(\rho)}_{\Delta N_1}=0$, in disagreement with the experimental result that

$$G^{(\rho)}_{N_1N_1}/G^{(\rho)}_{\Delta N_1}\approx 1.$$

It is clear that the extended algebra places constraints on the baryon spectrum when it is based on chiral symmetry considerations.

We have shown that a consistent picture emerges for Regge couplings and low-energy couplings. In order to reemphasize the compactness of the algebraic form of the sum rules we note that the usual Adler-Weisberger relations based on the algebraization of the chiral commutator $[X_a, X_b]=i\epsilon_{abc}T_c$ are given by

$$\begin{aligned}
&\sum_\gamma (G^{(X)11}_{\beta\gamma}G^{(X)11}_{\gamma\alpha}-G^{(X)13}_{\beta\gamma}G^{(X)31}_{\gamma\alpha})=G^{(T)11}_{\beta\alpha}=\delta_{\beta\alpha},\\
&\sum_\gamma (-\tfrac{1}{2}G^{(X)31}_{\beta\gamma}G^{(X)11}_{\gamma\alpha}+(\tfrac{5}{2})G^{(X)33}_{\beta\gamma}G^{(X)31}_{\gamma\alpha})=G^{(T)31}_{\beta\alpha}=0,\\
&\sum_\gamma (\tfrac{1}{2}G^{(X)31}_{\beta\gamma}G^{(X)13}_{\gamma\alpha}+G^{(X)33}_{\beta\gamma}G^{(X)33}_{\gamma\alpha})=G^{(T)33}_{\beta\alpha}=\delta_{\beta\alpha}.
\end{aligned} \tag{39}$$

These relations are readily verified for the coupling matrices in the chiral basis or in the physical nucleon basis. Relations similar to Eq. (39) are derivable for the commutators of the Regge couplings; Such Adler-Weisberger type relations can again be verified in terms of the couplings of Eqs. (18) and they are naturally valid for the nucleon couplings as well since the transformation from the chiral basis to the physical states is an orthogonal one. The relations obtained from the Regge bootstrap algebra are the algebraic versions of finite-energy sum rules based on duality for particle-Reggeon scattering.

The Regge algebra has to be extended to include nonforward directions, helicity changes, and conspiracy effects (to obtain a nonvanishing pion-Regge coupling) and additional trajectories. We have considered only those Regge couplings in this paper for which experimental information is already available, viz. for nucleon matrix elements. The present model, or extensions of it, can be used to anticipate experimental developments and to provide theoretical expectations for the couplings of Reggeons to all particles. Inclusion of more resonances in the saturation scheme and extensions to SU(3) symmetry will be studied by us in the near future as part of the program of algebraization of scattering amplitudes.

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[1]S. Weinberg, Phys. Rev. 177, 2604 (1969); Phys. Rev. Lett. 22, 1023 (1969).

[2]R. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966*, edited by A. Perlmutter, J. Wojtaszck, E. C. G. Sudarshan, and B. Kurşunoğlu (Freeman, San Francisco, Calif., 1966); R. Gatto, L. Maiani, and G. Preparata, Physics (N.Y.) 3, 1 (1967); I. Gerstein and B. W. Lee, Phys. Rev. Lett. 16, 1060 (1966); H. Harari, *ibid.* 16, 964 (1966); F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968); N. Cabibbo and H. Ruegg, Phys. Lett. 22, 85 (1966); F. Buccella *et al.*, Nucl. Phys. B6, 430 (1968); B8, 521 (1968); B22, 651 (1970); Nuovo Cimento 69A, 133 (1970).

[3]S. Weinberg (see Ref. 1); M. Noga and C. Cronstrom, Phys. Rev. D 1, 2414 (1970); L. R. Ram Mohan, *ibid.* 2, 299 (1970); A. McDonald and L. R. Ram Mohan, *ibid.* 3, 3076 (1971).

[4]H. Kleinert, Phys. Lett. 36B, 611 (1972); Fortschr. Phys. 21, 377 (1973); H. Kleinert and L. R. Ram Mohan, Nucl. Phys. B52, 253 (1973).

[5]L. R. Ram Mohan, Nucl. Phys. B72, 201 (1974); R. Carlitz, Nuovo Cimento 12A, 1048 (1972).

[6]H. Kleinert, Lett. Nuovo Cimento 6, 583 (1973); Nucl. Phys. B65, 77 (1973).

[7]N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Lett. 22B, 336 (1966).

[8]S. Fubini and G. Furlan, Lett. Nuovo Cimento 3, 168 (1970); S.-S. Shei, Phys. Rev. 188, 2274 (1969).

[9]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[10]R. T. Sharp and S. C. Pieper, J. Math. Phys. 9, 663 (1968); M. Hamermesh, *Group Theory* (Addison Wesley, Reading, 1962).

[11]J. Pasupathy, Phys. Rev. D 2, 357 (1970).

[12]Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[13]P. Sonderegger *et al.*, Phys. Lett. 20, 75 (1966); J. H. Scharenguival *et al.*, Nucl. Phys. B36, 363 (1972).

[14]G. Hohler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Lett. 20, 79 (1966); R. D. Matthews, Nucl. Phys. B11, 339 (1969).

[15]C. Michael, Springer Tracts Mod. Phys. 55, 174 (1970).

[16]R. J. Phillips, Nucl. Phys. B2, 394 (1967).

[17]Our normalizations are: $S=1-i(2\pi)^4\delta^4(p_f-p_i)T$, and $\langle \vec{p}'|\vec{p}\rangle = 2p_0(2\pi)^3\delta^3(\vec{p}'-\vec{p})$.