Note on mass enhancements and the Goldstone-pair mechanism

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The irreducible components of the $\frac{1}{2}^+$ baryon mass matrix and the 0^- meson mass-squared matrix are calculated and compared with the Li-Pagels relations. The Li-Pagels relations predict correctly orders of magnitude, but one sign is wrong and most of the relations are off by non-negligible factors. An exception is the relation

$$\frac{(m_p-m_n)+2(m_{\Sigma^+}-m_{\Sigma^-})+(m_{\Xi^0}-m_{\Xi^-})}{m_\Xi-m_N}=-\frac{3(m_{K^+}{}^2-m_{K^0}{}^2)+2\sqrt{3}\,(m_\eta{}^2-m_{\pi^0})\sin\beta\cos\beta}{2m_K{}^2+m_\eta{}^2-3m_\pi{}^2}$$

which is satisfied quite well. The angle β is the η-π^0 mixing angle.

I. INTRODUCTION

In the framework of an approximate SU(3) symmetry one can analyze the experimentally observed masses into irreducible components.[1] When such an analysis is carried out for the lowest-lying octet of $\frac{1}{2}^+$ baryons and the lowest-lying octet of 0^- mesons, certain regularities are exposed. The qualitative features of these regularities have been well known for a long time and are referred to as octet mass enhancement.[2] In recent years Pagels and his collaborators[3] have applied the Goldstone-pair mechanism to gain an understanding of the quantitative features of the observed regularities in the mass spectra. The purpose of the present paper is to show that, although their results are encouraging as far as orders of magnitude are concerned, there are nevertheless non-negligible discrepancies between their calculations and experiment.

In Sec. II we update the calculation of the mass expansion coefficients. In Sec. III we review the calculation of the Li-Pagels relations between corresponding expansion coefficients of mesons and baryons. In Sec. IV we compare the Li-Pagels relations with the observed mass expansion coefficients and draw conclusions. Appendix A contains an alternative expansion of the meson mass-squared matrix.

II. MASS EXPANSION COEFFICIENTS

Consider the experimentally observed masses[4] of the ground-state octet of $\frac{1}{2}^+$ baryons as matrix elements of a mass matrix which is diagonal in the subspace spanned by the eight physical states. We may expand this matrix of experimental numbers in a complete set of 8 by 8 matrices $T^{\underline{n}\gamma}_{Y,I,I_3}$ provided by the SU(3) classification scheme. The matrices $T^{\underline{n}\gamma}_{Y,I,I_3}$ are defined as follows[5]:

$$\langle \underline{8}, Y'', I'', I_3''|T^{\underline{n}\gamma}_{Y,I,I_3}|\underline{8}, Y', I', I_3'\rangle =\langle \underline{8}, Y''I''I_3''|\underline{n}, Y, I, I_3; \underline{8}, Y'I', I_3'\rangle_\gamma\sqrt{n}\,, \quad (1)$$

where the factor multiplying $\sqrt{n}$ is an SU(3) Clebsch-Gordan coefficient. The matrices $T^{\underline{n}\gamma}_{Y,I,I_3}$ fulfill the orthonormality relation

$$\mathrm{Tr}\{(T^{\underline{n}\gamma}_{Y,I,I_3})^\dagger\, T^{\underline{n}'\gamma'}_{Y',I',I_3'}\} =8\delta_{\underline{n},\underline{n}'}\,\delta_{\gamma,\gamma'}\,\delta_{Y,Y'}\,\delta_{I,I'}\,\delta_{I_3,I_3'}\,. \quad (2)$$

To utilize conveniently this set of matrices it is desirable to rewrite the mass matrix in a basis in which the isospin I is diagonal. The reason is that the electromagnetic interactions do not conserve isospin, and the states $|\Lambda\rangle$ and $|\Sigma^0\rangle$ which diagonalize the mass matrix are mixtures of an isosinglet state $|s\rangle$ and an $I_3=0$ member of an isotriplet $|t\rangle$. We therefore define

$$\begin{aligned}|\Lambda\rangle &=\cos\alpha\,|s\rangle+\sin\alpha\,|t\rangle\,,\\ |\Sigma^0\rangle &=-\sin\alpha\,|s\rangle+\cos\alpha\,|t\rangle\,,\end{aligned} \quad (3)$$

where α is referred to as the Λ-Σ^0 mixing angle. In a basis in which the isospin I is diagonal the baryon mass matrix $\mathfrak{M}(B)$ is given by

$$\mathfrak{M}(B)=\begin{bmatrix} m_p &&&&&&& \\ & m_n &&&&&& \\ && m_s && m_{st} &&& \\ &&& m_{\Sigma^+} &&&& \\ && m_{st} && m_t &&& \\ &&&&& m_{\Sigma^-} && \\ &&&&&& m_{\Xi^0} & \\ &&&&&&& m_{\Xi^-}\end{bmatrix}, \quad (4)$$

where m_s, m_t, and m_{st} are defined by

$$m_s = m_\Lambda \cos^2\alpha + m_{\Sigma^0}\sin^2\alpha ,$$
$$m_t = m_\Lambda \sin^2\alpha + m_{\Sigma^0}\cos^2\alpha , \qquad (5)$$
$$m_{st} = (m_\Lambda - m_{\Sigma^0})\sin\alpha\cos\alpha .$$

For any given values of the eight masses m_p, m_n, m_Λ, m_{Σ^+}, m_{Σ^0}, m_{Σ^-}, m_{Ξ^0}, m_{Ξ^-} and any given value for the Λ-Σ^0 mixing angle α, the baryon mass matrix $\mathfrak{M}(B)$ can be written as follows:

$$\mathfrak{M}(B) = a^{1} T^{1}_{0,0,0} + a^{S} T^{8S}_{0,0,0} + a^{A} T^{8A}_{0,0,0} + a^{27} T^{27}_{0,0,0} + a_1^{S} T^{8S}_{0,1,0} + a_1^{A} T^{8A}_{0,1,0} + a_+^{10} T_+^{10} + a_1^{27} T^{27}_{0,1,0} + a_2^{27} T^{27}_{0,2,0} , \qquad (6)$$

where T_+^{10} is defined by

$$T_+^{10} = \frac{1}{\sqrt{2}}\,(T^{10}_{0,1,0} + T^{\overline{10}}_{0,1,0}) . \qquad (7)$$

The coefficients a in the expansion Eq. (6) will be referred to as mass expansion coefficients. Before proceeding, we remark that all that expression (6) for the baryon mass matrix does is to trade one set of nine parameters, namely, the mixing angle and the eight masses, for another set of nine parameters, namely, the mass expansion coefficients.

Four out of the nine mass expansion coefficients are actually independent of the Λ-Σ^0 mixing angle α. Indeed, from the experimental values[4] of the masses we have

$$\begin{aligned} a^{1} &= 1.151\,11 \mp 0.000\,08 \text{ GeV},\\ a^{A} &= 0.134\,06 \mp 0.000\,11 \text{ GeV},\\ a_1^{A} &= 0.002\,41 \mp 0.000\,07 \text{ GeV},\\ a_+^{10} &= 0.000\,04 \mp 0.000\,09 \text{ GeV}. \end{aligned} \qquad (8)$$

We see that a_+^{10} is at least four orders of magnitude smaller than a^{1}, and is consistent with zero. Thus, even if the relation

$$a_+^{10} = 0 \qquad (9)$$

is not exact, it must be an excellent approximation. Expressed in terms of masses Eq. (9) reads

$$(m_p - m_n) - (m_{\Sigma^+} - m_{\Sigma^-}) + (m_{\Xi^0} - m_{\Xi^-}) = 0 . \qquad (10)$$

This is the Coleman-Glashow[6] sum rule. To explain the sum rule one assumes that the electromagnetic interactions have the transformation properties of a U-spin scalar member of a SU(3) octet. One also assumes that in the absence of electromagnetic interactions all the coefficients of $T^{n\gamma}_{0,I,0}$ with $I\neq 0$ vanish. From these two assumptions it follows that linear combinations of $T^{n\gamma}_{0,I\neq 0,0}$ which have no U-spin scalar component, will have vanishing coefficients in the expression for $\mathfrak{M}(B)$ Eq. (6). There are two linearly independent combinations with the required property. Up to linear equivalence they are T_+^{10} and $\sqrt{5}\,T^{27}_{0,1,0} - \sqrt{3}\,T^{27}_{0,2,0}$. Therefore, we get two restrictions on the mass expansion coefficients: the above-mentioned Eq. (9),

$$a_+^{10} = 0 ,$$

and

$$\sqrt{5}\,a_1^{27} - \sqrt{3}\,a_2^{27} = 0 . \qquad (11)$$

When reexpressed in terms of the mass matrix elements, Eq. (11) reads

$$(m_p - m_n) - (m_{\Sigma^+} - 2m_t + m_{\Sigma^-}) - (m_{\Xi^0} - m_{\Xi^-}) - 2\sqrt{3}\,m_{st} = 0 \qquad (12)$$

or in terms of the masses and mixing angle

$$(m_p - m_n) - (m_{\Sigma^+} - 2m_{\Sigma^0} + m_{\Sigma^-}) - (m_{\Xi^0} - m_{\Xi^-}) + 2(m_\Lambda - m_{\Sigma^0})(\sin^2\alpha - \sqrt{3}\sin\alpha\cos\alpha) = 0 . \qquad (13)$$

This is the Okubo-Sakita[7] and Dalitz–Von-Hippel[8] sum rule. We follow these authors and define the Λ-Σ^0 mixing angle by the sum rule (13). It is now convenient to define a matrix T_U^{27} by

$$T_U^{27} = (\tfrac{3}{8})^{1/2}\, T^{27}_{0,1,0} + (\tfrac{5}{8})^{1/2}\, T^{27}_{0,2,0} , \qquad (14)$$

and write the expansion of $\mathfrak{M}(B)$ as

$$\mathfrak{M}(B) = a^{1} T^{1}_{0,0,0} + a^{S} T^{8S}_{0,0,0} + a^{A} T^{8A}_{0,0,0} + a^{27} T^{27}_{0,0,0} + a_1^{S} T^{8S}_{0,1,0} + a_1^{A} T^{8A}_{0,2,0} + a_U^{27} T_U^{27} , \qquad (15)$$

where it is understood that the Λ-Σ^0 mixing angle α in expressions (5) is calculated from Eq. (13) and that a_+^{10} vanishes. The orthonormality relations (2) make the calculation of the expansion coefficients immediate.

A completely analogous expansion is possible for the mass-squared matrix of the lowest-lying octet of 0^- mesons if one neglects a possible η-η' mixing by the strong interactions and assumes the state $|\eta\rangle$ to be a pure $I=Y=0$ member of an octet. Assuming $m_{\pi^+}{}^2 = m_{\pi^-}{}^2$, $m_{K^+}{}^2 = m_{K^-}{}^2$, and $m_{K^0}{}^2 = m_{\bar{K}^0}{}^2$ we have

$$\mathfrak{M}^2(P) = b^{1} T^{1}_{0,0,0} + b^{S} T^{8S}_{0,0,0} + b^{27} T^{27}_{0,0,0} + b_1^{S} T^{8S}_{0,1,0} + b_U^{27} T_U^{27} , \qquad (16)$$

where the η-π_0 mixing angle β is defined by

$$(m_{K^+}{}^2 - m_{K^0}{}^2) - (m_{\pi^+}{}^2 - m_{\pi^0}{}^2) + (m_\eta{}^2 - m_{\pi^0}{}^2)(\sin^2\beta - \sqrt{3}\sin\beta\cos\beta) = 0 . \qquad (17)$$

In Table I we exhibit the standard matrices relevant to the mass expansion. Table II and Table III show the values of the expansion coefficients.

TABLE I. Standard matrices relevant to mass matrix expansion.

$$T^{\underline{1}}_{0,0,0}=\begin{bmatrix}1&&&&&&&\\&1&&&&&&\\&&1&&&&&\\&&&1&&&&\\&&&&1&&&\\&&&&&1&&\\&&&&&&1&\\&&&&&&&1\end{bmatrix}\qquad T^{\underline{10}}_{+}=\tfrac{2}{3}\sqrt{3}\begin{bmatrix}1&&&&&&&\\&-1&&&&&&\\&&0&&&&&\\&&&-1&&&&\\&&&&0&&&\\&&&&&1&&\\&&&&&&1&\\&&&&&&&-1\end{bmatrix}$$

$$T^{\underline{8}S}_{0,0,0}=\frac{\sqrt{10}}{5}\begin{bmatrix}-1&&&&&&&\\&-1&&&&&&\\&&-2&&&&&\\&&&2&&&&\\&&&&2&&&\\&&&&&2&&\\&&&&&&-1&\\&&&&&&&-1\end{bmatrix}\qquad T^{\underline{8}S}_{0,1,0}=\frac{\sqrt{30}}{15}\begin{bmatrix}3&&&&&&&\\&-3&&&&&&\\&&0&&2\sqrt{3}&&&\\&&&0&&&&\\&&2\sqrt{3}&&0&&&\\&&&&&0&&\\&&&&&&-3&\\&&&&&&&3\end{bmatrix}$$

$$T^{\underline{8}A}_{0,0,0}=\sqrt{2}\begin{bmatrix}-1&&&&&&&\\&-1&&&&&&\\&&0&&&&&\\&&&0&&&&\\&&&&0&&&\\&&&&&0&&\\&&&&&&1&\\&&&&&&&1\end{bmatrix}\qquad T^{\underline{8}A}_{0,1,0}=\frac{\sqrt{6}}{3}\begin{bmatrix}-1&&&&&&&\\&1&&&&&&\\&&0&&&&&\\&&&-2&&&&\\&&&&0&&&\\&&&&&2&&\\&&&&&&-1&\\&&&&&&&1\end{bmatrix}$$

$$T^{\underline{27}}_{0,0,0}=\frac{\sqrt{15}}{15}\begin{bmatrix}-3&&&&&&&\\&-3&&&&&&\\&&9&&&&&\\&&&1&&&&\\&&&&1&&&\\&&&&&1&&\\&&&&&&-3&\\&&&&&&&-3\end{bmatrix}\qquad T^{\underline{27}}_{U}=\frac{\sqrt{30}}{30}\begin{bmatrix}3&&&&&&&\\&-3&&&&&&\\&&0&&-3\sqrt{3}&&&\\&&&5&&&&\\&&-3\sqrt{3}&&-10&&&\\&&&&&5&&\\&&&&&&-3&\\&&&&&&&3\end{bmatrix}$$

Appendix A contains an alternative expansion for the meson mass-squared matrix.

III. LI-PAGELS RELATIONS

Table II and Table III reveal some well-known features. Roughly speaking Table II shows that $a^{\underline{1}}$ is of order 1 GeV; a^A is of order 0.1 GeV; a^S is of order 0.01 GeV; $a^{\underline{27}}$, a_1^A, and a_1^S are of order 0.001 GeV; and $a_U^{\underline{27}}$ is of order 0.0001 GeV. This ordering provides for various degrees of approximation to the baryonic spectrum. A well-known example is the approximation which neglects mass differences inside isospin multiplets. This amounts to approximating a_U, a_1^S, a_1^A by zero. But since $a^{\underline{27}}$ is about equal to a_1^A one may as well approximate $a^{\underline{27}}$ by zero. The vanishing of $a^{\underline{27}}$ is the Gell-Mann[9]–Okubo[10] sum rule.

Again, roughly speaking, Table III shows that $b^{\underline{1}}$

TABLE II. Mass matrix expansion coefficients for baryons $\underline{8}\ \frac{1}{2}^+$. $\tan\alpha=-0.012\pm0.002$.

$a^{\underline{1}}$	$=1.5111\pm0.00008$ GeV
a^A	$=0.13406\pm0.00011$ GeV
a^S	$=0.03266\mp0.00005$ GeV
$a^{\underline{27}}$	$=0.00250\mp0.00006$ GeV
a_1^A	$=0.00241\mp0.00007$ GeV
a_1^S	$=0.00100\mp0.00014$ GeV
$a_U^{\underline{27}}$	$=0.00033\mp0.00003$ GeV
$a_+^{\underline{10}}$	$=0.00004\mp0.00009$ GeV

TABLE III. Mass-squared matrix expansion coefficients for mesons $\underline{8}\ 0^-$. $\tan\beta=-0.0106\pm0.0003$.

$b^{\underline{1}}$	$=0.16766\pm0.00009\ \text{GeV}^2$
b^S	$=-0.11628\mp0.00011\ \text{GeV}^2$
$b^{\underline{27}}$	$=-0.00585\mp0.00019\ \text{GeV}^2$
b_1^S	$=-0.00203\mp0.00006\ \text{GeV}^2$
$b_U^{\underline{27}}$	$=0.00045\mp0.00001\ \text{GeV}^2$

and b^S are of order 0.1 GeV^2, $b^{\underline{27}}$ and b_1^S are of order 0.005 GeV^2, and $b_U^{\underline{27}}$ is of order 0.0005 GeV^2. Here again if one neglects mass differences inside isospin multiplets, thereby approximating $b_U^{\underline{27}}$ and b_1^S by zero, one may as well approximate $b^{\underline{27}}$ by zero, thereby recovering the Gell-Mann–Okubo relation. However, in the 0^- mesons case the approximation is less relaiable since the magnitude of $b^{\underline{27}}$ is about three times larger than the magnitude of b_1^S. This may be a signal for the necessity of η-η' mixing, which we neglected.

Another well-known feature of the mass expansion coefficient is the smallness of $a_U^{\underline{27}}$ compared to a_1^A and a_1^S which is similar to the smallness of $a^{\underline{27}}$ compared to a^S. This allows an approximate description of the electromagnetic mass splittings by setting $a_U^{\underline{27}}$ equal to zero. The relation obtained

$$3(m_p - m_n) + 5(m_{\Sigma^+} - 2m_{\Sigma^0} + m_{\Sigma^-}) - 3(m_{\Xi^0} - m_{\Xi^-}) - 2(m_\Lambda - m_{\Sigma^0})(5\sin^2\alpha + 3\sqrt{3}\sin\alpha\cos\alpha) = 0 \quad (18)$$

together with the defining relation Eq. (13) and the sum rule Eq. (10) give the approximate description of the electromagnetic mass splittings for the baryons.

For the mesons the smallness of $b_U^{\underline{27}}$ compared to b_1^S is similar to the smallness of $b^{\underline{27}}$ compared to b^S. Here again an approximate description is possible by setting $b_U^{\underline{27}}$ equal to zero.

Finally, Table III shows that if one attempts a very rough approximation of the meson spectrum by neglecting not only mass differences inside isospin multiplets, but also mass differences among different isospin multiplets, one may neglect the meson masses squared altogether. Thus, in an approximation which sets $b_U^{\underline{27}}$, b_1^S, $b^{\underline{27}}$, and b^S all equal to zero, one may set $b^{\perp}$ equal to zero since the magnitude of $b^{\perp}$ is less than $\frac{3}{2}$ times the magnitude of b^S. This is again well known and is the basis for the chiral-symmetry description of the hadronic spectrum.[11]

In a series of papers Li and Pagels[3] address themselves to the task of a detailed quantitative understanding of the entries in Tables II and III. They find interesting relations between corresponding entries in the two tables. The theoretical framework of their analysis is broken chiral[11] SU(3)⊗SU(3) supplemented by the assumption of Goldstone-pair dominance.[3] In this framework the Hamiltonian density is written in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_B\ , \quad (19)$$

where $\mathcal{H}_0$ is SU(3)⊗SU(3) symmetric, while $\mathcal{H}_B$ breaks this symmetry. It is assumed that the SU(3)⊗SU(3) symmetry of $\mathcal{H}_0$ is realized through the Nambu[12]-Goldstone[13] mechanism. This means that the spectrum of $\int \mathcal{H}_0 d^3x$ contains eight massless pseudoscalar mesons, and a vacuum state of $\int \mathcal{H}_0 d^3x$ is invariant only under the diagonal SU(3) subgroup of chiral SU(3)⊗SU(3).

As for the symmetry-breaking Hamiltonian density, its SU(3)⊗SU(3) transformation properties are not important for the Li-Pagels analysis. The only assumption we shall make here is that it is a Lorentz-scalar field.

A concise description of the Li-Pagels method was given by Renner.[14] One considers the baryonic matrix elements of the symmetry-breaking Hamiltonian density $\mathcal{H}_B$ which has the form

$$\langle Bp|\mathcal{H}_B(0)|Bp'\rangle = \bar{u}_B(p)\,u_B(p')\,F(t)\ , \quad (20)$$

where $t = (p - p')^2$. At $t = 0$ one has

$$F(0) = m_B - m_0\ , \quad (21)$$

where m_0 is the same for all members of the baryon octet. Writing an unsubtracted dispersion representation for $F(0)$, one gets

$$F(0) = \frac{1}{\pi}\int_{t_0}^{\infty} dt'\,\frac{\mathrm{disc}F(t')}{t'}\ , \quad (22)$$

and the discontinuity across the cut is given by

$$\mathrm{disc}F = \sum_{|n\rangle} \langle \mathrm{vac}|\mathcal{H}_B(0)|n\rangle\langle n|T^\dagger|\bar{B}B\rangle\ . \quad (23)$$

In the absence of narrow low-lying 0^+ states one assumes that the sum is dominated by states $|PP\rangle$ of two 0^- mesons in the lowest-lying octet. This is the Goldstone-pair mechanism.[3] To actually calculate the Goldstone-pair contribution to disc$F$, one approximates the matrix element $\langle \mathrm{vac}|\mathcal{H}_B(0)|PP\rangle$ by its value at $t = 0$. This is the way the 0^- meson masses squared enter into the expressions below. The matrix element $\langle PP|T^\dagger|\bar{B}B\rangle$ is approximated at threshold by its chirally symmetric value through the Weinberg low-energy theorem.[15] This is the way the f/d value of the baryonic axial-vector couplings enter the expressions below. Finally, the threshold t_0 in the dispersion relation (22) is approximated by zero, and the integral is cut off at $t' = 4\Lambda^2$, presuming dominance of the threshold contributions. It should be noted that the approximations involved in the Li-Pagels scheme go beyond the strict Goldstone-pair dominance hypothesis even if one is willing to assume threshold dominance of the dispersion relation. This may be of some importance when electromagnetic mass differences are concerned, since the state $|\gamma PP\rangle$ of one photon and a Goldstone-pair may give a non-negligible contribution. However, accepting the way Li and Pagels realize the Goldstone-pair dominance hypothesis, one gets the following Li-Pagels relations[3]:

$$a^A = \tfrac{1}{2}\sqrt{5}\, C\Lambda(\alpha^2 - \alpha)\, b^S \,, \tag{24a}$$

$$a^S = \tfrac{1}{4} C\Lambda(2\alpha^2 - 6\alpha + 3)\, b^S \,, \tag{24b}$$

$$a^{\underline{27}} = -\tfrac{1}{6} C\Lambda(2\alpha^2 - 6\alpha + 3)\, b^{\underline{27}} \,, \tag{24c}$$

$$a_1^A = \tfrac{1}{2}\sqrt{5}\, C\Lambda(\alpha^2 - \alpha)\, b_1^S \,, \tag{24d}$$

$$a_1^S = \tfrac{1}{4} C\Lambda(2\alpha^2 - 6\alpha + 3)\, b_1^S \,, \tag{24e}$$

$$a_U^{\underline{27}} = -\tfrac{1}{6} C\Lambda(2\alpha^2 - 6\alpha + 3)\, b_U^{\underline{27}} \,, \tag{24f}$$

$$[(\tfrac{5}{8})^{1/2} a_1^{\underline{27}} - (\tfrac{3}{8})^{1/2} a_2^{\underline{27}}] = -\tfrac{1}{6} C\Lambda(2\alpha^2 - 6\alpha + 3)[(\tfrac{5}{8})^{1/2} b_1^{\underline{27}} - (\tfrac{3}{8})^{1/2} b_2^{\underline{27}}] \,, \tag{24g}$$

$$a_+^{\underline{10}} = 0 \,. \tag{24h}$$

The constant C is given by

$$C = \frac{1}{4\pi}\left(\frac{g_A}{f}\right)^2 \cong 12.85\ (\text{GeV})^{-2} \,, \tag{25}$$

where g_A is the nucleon axial-vector coupling and f is the 0^- mesons decay constant. The quantity $(1-\alpha)/\alpha$ is the f/d ratio of the baryonic axial-vector couplings.

The general form of this relation is

$$a = x\Lambda C\, P(\alpha)\, b \,, \tag{26}$$

where x is a pure number and $P(\alpha)$ is a polynomial in α. Actually there appear only two distinct polynomials in these eight relations. The coefficients a and b correspond to matrices $T_{0,Y,I,0}^{\underline{n}}$ with the same $\underline{n}$ and I. Thus, since for the mesons the combination

$$(m_{K^+}^2 - m_{K^0}^2) - (m_{\pi^+}^2 - m_{\pi^-}^2) + (m_{\bar{K}^0}^2 - m_{K^-}^2) \tag{27}$$

vanishes, it is not surprising that Eq. (24h) predicts that $a_+^{\underline{10}}$ vanishes.[16] Also because of our definition of the Λ-Σ^0 and η-π^0 mixing angles, Eq. (24g) reads $0=0$.

IV. ANALYSIS AND CONCLUSIONS

To compare the Li-Pagels relations with our Tables II and III let us first substitute the experimentally determined values for α:

$$\begin{aligned} &\text{Nieh } et\ al.,^{17} && \alpha = 0.66 \pm 0.02 \,; \\ &\text{Brene } et\ al.,^{18} && \alpha = 0.662 \pm 0.018 \,; \\ &\text{Garcia},^{19} && \alpha = 0.635 \pm 0.010 \,; \end{aligned} \tag{28}$$

and adjust Λ so that Eq. (24a) holds. As noticed by Li and Pagels the polynomial $\alpha^2 - \alpha$ does not change drastically with the above values of α. Indeed, we have

TABLE IV. Comparison of the baryon mass expansion coefficients with the Li-Pagels relations.

	Table II	Li-Pagels
a^A	$0.134\,06 \pm 0.000\,11$	input
a^S	$0.032\,66 \pm 0.000\,05$	input
$a^{\underline{27}}$	$0.002\,50 \pm 0.000\,06$	$-0.001\,10 \pm 0.000\,05$
a_1^A	$0.002\,41 \pm 0.000\,07$	$0.002\,34 \pm 0.000\,06$
a_1^S	$0.001\,00 \pm 0.000\,14$	$0.000\,57 \pm 0.000\,02$
$a_U^{\underline{27}}$	$0.000\,33 \pm 0.000\,03$	$0.000\,08 \pm 0.000\,01$

$$\begin{aligned} &\alpha = 0.66 \pm 0.02 \,, && \alpha^2 - \alpha = -0.22 \pm 0.01 \,; \\ &\alpha = 0.662 \pm 0.018 \,, && \alpha^2 - \alpha = -0.224 \pm 0.006 \,; \\ &\alpha = 0.635 \pm 0.010 \,, && \alpha^2 - \alpha = -0.232 \pm 0.003 \,. \end{aligned} \tag{29}$$

This gives for Λ a value of about 0.35 GeV. However, for the polynomial $2\alpha^2 - 6\alpha + 3$ we have

$$\begin{aligned} &\alpha = 0.66 \pm 0.02 \,, && 2\alpha^2 - 6\alpha + 3 = -0.089 \pm 0.067 \,; \\ &\alpha = 0.662 \pm 0.018 \,, && 2\alpha^2 - 6\alpha + 3 = -0.095 \pm 0.060 \,; \\ &\alpha = 0.635 \pm 0.010 \,, && 2\alpha^2 - 6\alpha + 3 = -0.004 \pm 0.035 \,. \end{aligned} \tag{30}$$

Thus, $2\alpha^2 - 6\alpha + 3$ is not very well determined by the above determinations of α and changes considerably with the above values of α. As stressed by Pagels and his co-workers, this happens because $2\alpha^2 - 6\alpha + 3$ has a zero at $\alpha = \frac{1}{2}(3 - \sqrt{3}) \simeq 0.634$.

One possible way to proceed is to adjust both Λ and α so that (24a) and (24b) are satisfied. The adjusted value of α will then satisfy the relation

$$\frac{a^S}{a^A} = \frac{\sqrt{5}}{10}\,\frac{2\alpha^2 - 6\alpha + 3}{\alpha^2 - \alpha} \,, \tag{31}$$

derived by Li and Pagels.

Let us denote by $R(\alpha)$ the rational function

$$R(\alpha) = \frac{2\alpha^2 - 6\alpha + 3}{\alpha^2 - \alpha} \,. \tag{32}$$

As mentioned above, $R(\alpha)$ vanishes at $\alpha \simeq 0.634$. It increases monotonically to 1 at $\alpha = \frac{1}{2}(5 - \sqrt{13}) \simeq 0.697$, passing through the value $\frac{1}{2}$ at $\alpha = \frac{2}{3}$. Thus, the answer to the question of whether or not $R(\alpha)$ provides a suppression factor is extremely sensitive to the value of α. As a matter of fact, if we calculate $R(\alpha)$ and α from Eq. (31) we get

$$R(\alpha) = 1.089 \pm 0.003 \tag{33}$$

and

$$\alpha = 0.7024 \pm 0.0002 \,. \tag{34}$$

The value of α in Eq. (34) slightly changes the cut-off mass Λ to about 0.37 GeV.

In Table IV we compare the baryon mass expansion coefficients calculated through the relations in Eqs. (24) with the entries of Table II. It is encouraging to note that all calculated coefficients have the correct order of magnitude. However, the coefficients a^{27} and a_1^s calculated according to Li and Pagels are too small by about a factor of 2, and a_U^{27} calculated according to Li and Pagels is too small by about a factor of 4 and a^{27} has the wrong sign. This should be contrasted with the value of a_1^A. Indeed, the relation

$$\frac{a_1^A}{a^A}=\frac{b_1^S}{b^S} \tag{35}$$

which follows from Eqs. (24a) and (24d) holds quite well. We note that all the calculated coefficients which are too small are proportional to the problematic polynomial $2\alpha^2-6\alpha+3$. Writing Eq. (35) in terms of masses we have[20]

$$\frac{(m_p-m_n)+2(m_{\Sigma^+}-m_{\Sigma^-})+(m_{\Xi^0}-m_{\Xi^-})}{m_\Xi-m_N} = -\frac{3(m_{K^+}{}^2-m_{K^0}{}^2)+2\sqrt{3}\,(m_\eta{}^2-m_{\pi^0}{}^2)\sin\beta\cos\beta}{2m_K{}^2+m_\eta{}^2-3m_\pi{}^2}. \tag{36}$$

A possible criticism against this relation having any significance is the neglect of η-η' mixing. In other words, the mesonic sector is different from the baryonic one by the existence of a relatively low-lying η' so that the states $|\eta\rangle$ and $|\eta'\rangle$ may be linear combinations of an SU(3) singlet and an $I=Y=0$ member of an SU(3) octet. This may mean a theoretical uncertainty in the diagonal $I=Y=0$ matrix element of $\mathfrak{M}^2(P)$ in Eq. (15). We overlooked any such uncertainty in calculating the entries in Table III when we neglected η-η' mixing.

To get a feeling for the theoretical uncertainty involved, let us adjust $m_8{}^2$—the diagonal $I=Y=0$ matrix element of $\mathfrak{M}^2(P)$ —so that it fulfills the Gell-Mann–Okubo relation:

$$4m_K{}^2-3m_8{}^2-m_\pi{}^2=0\;. \tag{37}$$

This gives

$$m_8{}^2=0.32128\pm0.00009\;, \tag{38}$$

compared with

$$m_\eta{}^2=0.3012\pm0.0007\;. \tag{39}$$

With the above value for $m_8{}^2$ we get

$$b_1^S=-0.00203\pm0.00006\ \text{GeV}^2\;, \tag{40}$$

$$b^S=-0.1195\pm0.0001\ \text{GeV}^2\;, \tag{41}$$

decreasing the value of b_1^S/b_S from 0.0175 ± 0.0004 to 0.0170 ± 0.0004. Comparing this with the value of a_1^A/a^A which is 0.0180 ± 0.0005, we see that Eq. (31) holds within the experimental accuracy.

TABLE V. Mass-squared matrix expansion coefficients for mesons $\underline{8}\ 0^-$. $\tan\beta=-0.0106\pm0.0002$.

$b_\epsilon=\ 0.20364\pm0.00010\ \text{GeV}^2$
$b_\delta=\ 0.01276\pm0.00010\ \text{GeV}^2$
$b^{27}=-0.00585\pm0.00019\ \text{GeV}^2$
$b_1^S=-0.00209\pm0.00006\ \text{GeV}^2$
$b_U^{27}=\ 0.00045\pm0.00001\ \text{GeV}^2$

Therefore, η-η' mixing does not jeopardize Eq. (36).

We conclude that the Li-Pagels relations give a rough description of the relation between the mesonic and baryonic mass expansion coefficients. What is needed is a way to incorporate the tail of the dispersion integral (22) so that one can judge the validity of the Goldstone-pair dominance hypothesis. Ideas of duality may be of importance in such an evaluation.

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APPENDIX A

To bring out the fact that $m_\pi{}^2$ is a small number, one may alter a little bit the expansion of the meson mass-squared matrix. Define the matrices T_ϵ and T_δ by

$$T_\epsilon=(\tfrac{8}{13})^{1/2}\,T^{\underline{1}}_{0,0,0}-(\tfrac{5}{13})^{1/2}\,T^{\underline{8}S}_{0,0,0}\;, \tag{A1}$$

$$T_\delta=(\tfrac{5}{13})^{1/2}\,T^{\underline{1}}_{0,0,0}+(\tfrac{8}{13})^{1/2}\,T^{\underline{8}S}_{0,0,0}\;. \tag{A2}$$

The matrix T_ϵ describes a spectrum in which $m_\pi{}^2=0$ and $m_K{}^2$ and $m_\eta{}^2$ fulfill the Gell-Mann–Okubo relation

$$4m_K{}^2=3m_\eta{}^2\;. \tag{A3}$$

Expressing $\mathfrak{M}^2(P)$ in terms of T_ϵ and T_δ we write

$$\mathfrak{M}^2(P)=b_\epsilon T_\epsilon+b_\delta T_\delta+b^{27}T^{\underline{27}}_{0,0,0}+b_1^S T^{\underline{8}}_{0,0,0}+b_U^{27}T_U^{27}\,, \tag{A4}$$

and calculate the entries in Table V.

Again we have a familiar pattern where, roughly speaking, b_ϵ is of order 0.1 GeV^2; b_δ is of order 0.01 GeV^2; b^{27} and b_1^S are of order 0.005 GeV^2, and b_U^{27} is of order 0.0005 GeV. Therefore, we may consider a rough approximation to the meson spectrum where b_δ, b^{27}, b_1^S, b_U^{27} are all neglected compared to b_ϵ. This fact is again well known and serves as a basis for the Gell-Mann–Oakes–Renner model of chiral symmetry breaking.[11]

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