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Sum rules for single-pion electroproduction from light-cone commutation relations*

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Light-cone current commutation relations are used to derive fixed-mass sum rules on the amplitudes for the electroproduction of single pions. These sum rules should provide a more direct test of the structure of the commutation relations than the sum rules previously derived.

I. INTRODUCTION

Light-cone current commutation relations are the natural commutators to use to derive fixed-mass sum rules for the structure functions which describe lepton-hadron scattering.[1] The advantage over equal-time commutators is that no infinite-momentum limit (or alternately an assumption about the convergence of a dispersion relation) is necessary.

Many sum rules have been derived[1-4] by using the light-cone commutators which follow from abstracting the free quark[5] or quark-vector-gluon[6] canonical commutators. These sum rules are roughly of two types. The first are sum rules which had previously been derived from equal-time commutation relations and are known to be well satisfied experimentally. These sum rules follow from the ++ light-cone commutator. Examples are the Dashen–Fubini–Gell-Mann sum rule,[1,7] the Drell-Hearn sum rule,[4,8] and an unnamed sum rule derived from equal-time commutators by Goldberg and Gross[9] and by Gerstein[10] and from light-cone commutators by Dicus and Teplitz.[2] This last sum rule has recently been shown by Goldberg[11] to be in excellent agreement with experiment. The second type of sum rules are those which cannot be derived from equal-time commutators or have ambiguities in that type of derivation because of the infinite-momentum limit required and which require a more model-dependent light-cone commutator (usually $+-$). Many examples of these have been given[1-3] but none have been carefully checked experimentally. This is because they usually involve spin-dependent amplitudes, or off-forward direction amplitudes, or both. Unfortunately, this means that we really do not know if the form of the model-dependent light-cone commutation relations has any validity.

This paper is an attempt to find sum rules which can be more readily compared with experiment. We present here sum rules on the amplitudes for electroproduction of pions. These are derived by using the technique Gerstein used to get sum rules on pion-nucleon scattering from equal-time commutation relations.[10] This involves finding sum rules on the invariant amplitudes which represent the commutator of axial-vector currents and then using partially conserved axial-vector current (PCAC) to convert these sum rules to sum rules on the invariant amplitudes for the pion process.

Some of these sum rules follow from the ++ commutators and have been previously derived from equal-time commutation relations. For example, we reproduce the Fubini-Furlan-Rossetti sum rule which relates the nucleon magnetic moments to photoproduction amplitudes.[12] But most of our sum rules follow from the more model-dependent $+-$ commutation relations and, we hope, can be used to check these commutators.

In the next section we define the electroproduction amplitudes and relate them to the invariant amplitudes of the vector-current–axial-vector-current commutator. In Sec. III we derive the sum rules on the current-current invariant amplitudes. Only the sum rules that are used to find sum rules on the electroproduction amplitudes are given in this section; many more sum rules can be derived that are not directly related to electroproduction and these are listed in an appendix. Finally in Sec. IV we give the sum rules on the electroproduction amplitudes and discuss their convergence properties.

II. FORMALISM

The T matrix for the process $eN \to eN\pi$ may be written as

$$\bar{u}(k_2)\gamma_\mu u(k_1)\frac{1}{k^2}\langle N(p_2)\pi^a(q)|\,V_b^\mu(0)\,|N(p_1)\rangle \tag{2.1}$$

in the one-photon approximation. The photon four-momentum is $k = k_1 - k_2$ and we define $P = \frac{1}{2}(p_1 + p_2)$, and $\Delta = p_1 - p_2 = q - k$, and $Q = \frac{1}{2}(k+q)$. a and b are SU(3) indices.

The hadronic matrix element is expressed in terms of invariant scalar functions B_i as

$$\langle N(p_2)\pi^a(q)|\,V_b^\mu(0)|\,N(p_1)\rangle = \bar{u}(p_2)[\tfrac{1}{2}(\gamma^\mu\gamma\cdot k - \gamma\cdot k\gamma^\mu)B_1^{ab} + 2P^\mu B_2^{ab} + 2q^\mu B_3^{ab} + 2k^\mu B_4^{ab} + \gamma^\mu B_5^{ab} - P^\mu\gamma\cdot kB_6^{ab} - k^\mu\gamma\cdot kB_7^{ab} - q^\mu\gamma\cdot kB_8^{ab}]\gamma_5 u(p_1)\,. \tag{2.2}$$

Except for B_1^{ab} these are the amplitudes defined by Ball.[13] Each B_i^{ab} is a function of the variables

$$\nu = P\cdot q = P\cdot k,\, q^2,\, k^2,\, t = \Delta^2\,.$$

Conservation of the vector current gives two conditions on the B_i,

$$2k^2 B_4^{ab} = (t - k^2 - q^2)B_3^{ab} - 2\nu B_2^{ab}\,, \tag{2.3a}$$

$$k^2 B_7^{ab} = B_5^{ab\prime} + \tfrac{1}{2}(t - k^2 - q^2)B_8^{ab}\,, \tag{2.3b}$$

where $B_5^{ab\prime} \equiv B_5^{ab} - \nu B_6^{ab}$.

For most of the calculation we will find it convenient to keep the SU(3) indices general; in the end, however, we will specialize to SU(2) and make the decomposition of the amplitudes

$$B_i^{ab} = B_i^{(+)}\delta_{ab} + B_i^{(-)}\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] + B_i^{(0)}\left\{\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right\}. \tag{2.4}$$

Under the transformation $P \to -P$, $a \leftrightarrow b$ the amplitudes

$$B_1^{(+,0)},\; B_2^{(+,0)},\; B_3^{(-)},\; B_4^{(-)},\; B_5^{(-)},\; B_6^{(+,0)},\; B_7^{(-)},\; B_8^{(-)}$$

are even functions of ν while the rest are odd in ν. This means

$$\mathrm{Im}B_i^{(+,0)}(\nu, t, q^2, k^2) = \eta_i\,\mathrm{Im}B_i^{(+,0)}(-\nu, t, q^2, k^2)\,, \tag{2.5a}$$

$$\mathrm{Im}B_i^{(-)}(\nu, t, q^2, k^2) = -\eta_i\,\mathrm{Im}B_i^{(-)}(-\nu, t, q^2, k^2)\,, \tag{2.5b}$$

with $\eta_i = +1$ for $i = 3, 4, 5, 7, 8$ and $\eta_i = -1$ for $i = 1, 2, 6$.

The absorptive parts of the amplitudes B_i^{ab} can be related to the commutator of an axial-vector current and the vector current $V_b^\mu(0)$ by using PCAC in the usual way. From (2.2)

$$\bar{u}(p_2)[\tfrac{1}{2}(\gamma^\nu\gamma\cdot k - \gamma\cdot k\gamma^\nu)\mathrm{Im}B_1^{ab} + \cdots]\gamma_5 u(p_1) = \frac{m_\pi^{\;2} - q^2}{m_\pi^{\;2} f_\pi} q_\mu \int d^4x\, e^{iq\cdot x}\langle p_2|\,[A_a^\mu(x), V_b^\nu(0)]|\,p_1\rangle\,. \tag{2.6}$$

The Fourier transform of the commutator which appears in (2.6) may be written in terms of invariant functions in a manner similar to (2.2):

$$T_{ab}^{\mu\nu} \equiv \int d^4x\, e^{iq\cdot x}\langle p_2|\,[A_a^\mu(x), V_b^\nu(0)]|\,p_1\rangle \tag{2.7a}$$

$$= \bar{u}(p_2)A_{ab}^{\mu\alpha}\gamma_5 u(p_1)\left(g^\nu_{\;\alpha} - \frac{k_\alpha k^\nu}{k^2}\right). \tag{2.7b}$$

The projection operator on the right of (2.7b) insures that conservation of the vector current is included. There are 32 independent Dirac matrices:

$$\begin{aligned}A_{ab}^{\mu\alpha} = {} & a_1^{ab}P^\mu P^\alpha + \bar{a}_1^{ab}P^\mu P^\alpha\gamma\cdot Q + a_2^{ab}P^\mu q^\alpha + \bar{a}_2^{ab}P^\mu q^\alpha\gamma\cdot Q + a_3^{ab}P^\mu k^\alpha + \bar{a}_3^{ab}P^\mu k^\alpha\gamma\cdot Q + a_4^{ab}q^\mu P^\alpha + \bar{a}_4^{ab}q^\mu P^\alpha\gamma\cdot Q + a_5^{ab}q^\mu q^\alpha \\ & + \bar{a}_5^{ab}q^\mu q^\alpha\gamma\cdot Q + a_6^{ab}q^\mu k^\alpha + \bar{a}_6^{ab}q^\mu k^\alpha\gamma\cdot Q + a_7^{ab}k^\mu P^\alpha + \bar{a}_7^{ab}k^\mu P^\alpha\gamma\cdot Q + a_8^{ab}k^\mu q^\alpha + \bar{a}_8^{ab}k^\mu q^\alpha\gamma\cdot Q + a_9^{ab}k^\mu k^\alpha \\ & + \bar{a}_9^{ab}k^\mu k^\alpha\gamma\cdot Q + a_{10}^{ab}g^{\mu\alpha} + \bar{a}_{10}^{ab}g^{\mu\alpha}\gamma\cdot Q + d_1^{ab}i\sigma^{\mu\alpha} + d_2^{ab}\tfrac{1}{2}(\gamma^\mu\gamma\cdot Q\gamma^\alpha - \gamma^\alpha\gamma\cdot Q\gamma^\mu) + b_1^{ab}P^\mu\gamma^\alpha + b_2^{ab}\gamma^\mu P^\alpha + b_3^{ab}q^\mu\gamma^\alpha \\ & + b_4^{ab}\gamma^\mu q^\alpha + b_5^{ab}k^\mu\gamma^\alpha + b_6^{ab}\gamma^\mu k^\alpha + c_1^{ab}(P^\mu i\sigma^{\alpha\lambda}Q_\lambda + P^\alpha i\sigma^{\mu\lambda}Q_\lambda) + c_2^{ab}(Q^\mu i\sigma^{\alpha\lambda}Q_\lambda + Q^\alpha i\sigma^{\mu\lambda}Q_\lambda) \\ & + c_3^{ab}(\Delta^\mu i\sigma^{\alpha\lambda}Q_\lambda + \Delta^\alpha i\sigma^{\mu\lambda}Q_\lambda) + c_6^{ab}(P^\mu i\sigma^{\alpha\lambda}Q_\lambda - P^\alpha i\sigma^{\mu\lambda}Q_\lambda)\,.\end{aligned} \tag{2.8}$$

Each of the invariant amplitudes $a_i^{ab}, \ldots, d_i^{ab}$ is a function of ν, t, q^2, k^2. The combinations

$$\begin{aligned}&\Delta^\mu i\sigma^{\alpha\lambda}Q_\lambda - \Delta^\alpha i\sigma^{\mu\lambda}Q_\lambda\,,\\ &Q^\mu i\sigma^{\alpha\lambda}Q_\lambda - Q^\alpha i\sigma^{\mu\lambda}Q_\lambda\end{aligned} \tag{2.9}$$

have been eliminated from (2.8) by using the two identities

$$-m\bar{u}(p_2)[\Delta^\mu i\sigma^{\alpha\lambda}Q_\lambda - \Delta^\alpha i\sigma^{\mu\lambda}Q_\lambda]\gamma_5 u(p_1) = \bar{u}(p_2)[\tfrac{1}{4}t(\gamma^\mu\gamma\cdot Q\gamma^\alpha - \gamma^\alpha\gamma\cdot Q\gamma^\mu) + m\Delta\cdot Q i\sigma^{\mu\alpha} - (P^\mu\Delta^\alpha - \Delta^\mu P^\alpha)\gamma\cdot Q - \nu(\Delta^\mu\gamma^\alpha - \Delta^\alpha\gamma^\mu) + Q\cdot\Delta(P^\mu\gamma^\alpha - P^\alpha\gamma^\mu)]\gamma_5 u(p_1)\,, \tag{2.10a}$$

$$\bar{u}(p_2)[(K^\mu i\sigma^{\alpha\lambda}Q_\lambda - K^\alpha i\sigma^{\mu\lambda}Q_\lambda)P^2 + P^2K^2 i\sigma^{\mu\alpha} + \tfrac{1}{2}K^2(P^\mu\Delta^\alpha - P^\alpha\Delta^\mu) - \tfrac{1}{2}\Delta\cdot Q(P^\mu K^\alpha - P^\alpha K^\mu) + \tfrac{1}{2}\nu(\Delta^\mu K^\alpha - \Delta^\alpha K^\mu) + mK^2(P^\mu\gamma^\alpha - P^\alpha\gamma^\mu) - m\nu(K^\mu\gamma^\alpha - K^\alpha\gamma^\mu) - m(P^\mu K^\alpha - P^\alpha K^\mu)\gamma\cdot Q]\gamma_5 u(p_1) = 0, \quad (2.10b)$$

where $K^\mu \equiv Q^\mu - (\nu/P^2)P^\mu - (\Delta\circ Q/\Delta^2)\Delta^\mu$ and $K^2 = Q^2 - \nu^2/P^2 - (\Delta\cdot Q)^2/\Delta^2$. Gerstein[10] has given the expansion (2.8) and the conditions similar to (2.10) for the case of two axial-vector currents. The conditions (2.10) can be derived from his conditions by simply making the transformation $P \leftrightarrow \Delta$.

By using (2.6) the $\mathrm{Im}B_i^{ab}$ may be related to the $a_i^{ab}, \ldots, d_i^{ab}$. The relations are

$$\mathrm{Im}B_1^{ab} = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}(-\nu c_1^{ab} - \nu c_6^{ab} + d_1^{ab} + m d_2^{ab} - q\cdot Q c_2^{ab} - q\cdot\Delta c_3^{ab}), \quad (2.11a)$$

$$\mathrm{Im}B_2^{ab} = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\tfrac{1}{2}(\nu a_1^{ab} - m\nu\bar{a}_1^{ab} + q^2 a_4^{ab} - mq^2\bar{a}_4^{ab} + q\cdot k a_7^{ab} - mq\cdot k\bar{a}_7^{ab} - 2mb_2^{ab} + q\cdot\Delta c_3^{ab} + 2\nu c_6^{ab} + q\cdot Q c_2^{ab} - 2d_1^{ab}), \quad (2.11b)$$

$$\mathrm{Im}B_3^{ab} = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\tfrac{1}{2}(\nu a_2^{ab} - m\nu\bar{a}_2^{ab} + q^2 a_5^{ab} - mq^2\bar{a}_5^{ab} + q\cdot k a_8^{ab} - mq\cdot k\bar{a}_8^{ab} + a_{10}^{ab} - m\bar{a}_{10}^{ab} - 2mb_4^{ab} - \tfrac{1}{2}\nu c_2^{ab} - \nu c_3^{ab}), \quad (2.11c)$$

$$\mathrm{Im}B_4^{ab} = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\frac{1}{2}\Big\{-\frac{\nu}{k^2}[\nu a_1^{ab} - m\nu\bar{a}_1^{ab} + q\cdot k a_2^{ab} - mq\cdot k\bar{a}_2^{ab} + q^2 a_4^{ab} - mq^2\bar{a}_4^{ab} + q\cdot k a_7^{ab} - mq\cdot k\bar{a}_7^{ab} - 2mb_2^{ab} - 2d_1^{ab} + 2\nu c_6^{ab} + \tfrac{1}{2}q^2 c_2^{ab} + q\cdot(q-2k)c_3^{ab}] - \frac{q\cdot k}{k^2}(q^2 a_5^{ab} - mq^2\bar{a}_5^{ab} + q\cdot k a_8^{ab} - mq\cdot k\bar{a}_8^{ab} + a_{10}^{ab} - m\bar{a}_{10}^{ab} - 2mb_4^{ab} - 2mb_5^{ab})\Big\}, \quad (2.11d)$$

$$\mathrm{Im}B_5^{ab} = \frac{m_\pi^2 - q^2}{f_\pi m_\pi^2}(\nu b_1^{ab} + \nu d_2^{ab} + q^2 b_3^{ab} + q\cdot k b_5^{ab}), \quad (2.11e)$$

$$\mathrm{Im}B_6^{ab} = \frac{m_\pi^2 - q^2}{f_\pi m_\pi^2}(-\nu\bar{a}_1^{ab} - b_2^{ab} + d_2^{ab} - q^2\bar{a}_4^{ab} - q\cdot k\bar{a}_7^{ab}), \quad (2.11f)$$

$$\mathrm{Im}B_7^{ab} = \frac{m_\pi^2 - q^2}{m_\pi^2 f_\pi}\left[\frac{\nu}{k^2}(\nu\bar{a}_1^{ab} + q^2\bar{a}_4^{ab} + q\cdot k\bar{a}_7^{ab} + q\cdot k\bar{a}_2^{ab} + b_2^{ab} + b_1^{ab}) + \frac{q\cdot k}{k^2}(q^2\bar{a}_5^{ab} + q\cdot k\bar{a}_8^{ab} + \bar{a}_{10}^{ab} + b_4^{ab} + b_5^{ab}) + \frac{q^2}{k^2}b_3^{ab}\right], \quad (2.11g)$$

$$\mathrm{Im}B_8^{ab} = \frac{(m_\pi^2 - q^2)}{m_\pi^2 f_\pi}(-1)(\nu\bar{a}_2^{ab} + b_4^{ab} + \bar{a}_{10}^{ab} + q^2\bar{a}_5^{ab} + q\cdot k\bar{a}_8^{ab}). \quad (2.11h)$$

The method of deriving sum rules is straightforward: Equation (2.7) is integrated over q^- from $-\infty$ to $+\infty$ while q^+ and k^+ are held fixed at zero. Equation (2.7a) becomes the commutator at $x^+=0$ while (2.7b) becomes integrals over ν of the invariant functions defined in (2.8). (Problems with this method of derivation will be discussed in Sec. IV.) The invariant functions depend on $\nu, q^2 = -\vec{q}_\perp^{\,2}$, $k^2 = -\vec{k}_\perp^{\,2}$, and $t = 2\vec{q}_\perp\cdot\vec{k}_\perp - \vec{q}_\perp^{\,2} - \vec{k}_\perp^{\,2}$. Finally, once sum rules are found for $a_i^{ab}, \ldots, d_i^{ab}$, Eqs. (2.11) may be used to find sum rules on the invariant functions of electroproduction.

Of course we will not be able to find sum rules on all of the $a_i^{ab}, \ldots, d_i^{ab}, \nu a_i^{ab}, \ldots,$ and therefore to simplify (2.11) we will eventually take the soft-pion limit. The amplitudes defined in (2.8) are free of kinematic singularities as $q\to 0$ and so we may drop those terms whose coefficient depends on q (see, however, the discussion in Sec. IV on the Born contribution to such terms). The sum rules we will find in the next section on the a_i^{ab} will allow us to write sum rules on

$$\frac{1}{\nu}\mathrm{Im}B_5^{ab\prime}(\nu, 0, k^2, t=k^2) = \frac{1}{f_\pi}(\nu\bar{a}_1^{ab} + b_1^{ab} + b_2^{ab}), \quad (2.12a)$$

$$\frac{1}{\nu}(m\,\mathrm{Im}B_6^{ab} - \mathrm{Im}B_1^{ab} - \mathrm{Im}B_2^{ab}) = \frac{1}{f_\pi}(-\tfrac{1}{2}a_1^{ab} - \tfrac{1}{2}m\bar{a}_1^{ab} + c_1^{ab}), \quad (2.12b)$$

$$\mathrm{Im}B_5^{ab\prime} = \frac{1}{f_\pi}(\nu^2\bar{a}_1^{ab} + \nu b_1^{ab} + \nu b_2^{ab}), \quad (2.12c)$$

$$\mathrm{Im}B_8^{ab} = -\frac{1}{f_\pi}(\nu\bar{a}_2^{ab} + b_4^{ab} + \bar{a}_{10}^{ab}), \quad (2.12d)$$

$$2\,\mathrm{Im}B_2^{ab} = \frac{1}{f_\pi}(\nu a_1^{ab} - m\nu\bar{a}_1^{ab} - 2mb_2^{ab} + 2\nu c_6^{ab} - 2d_1^{ab}), \quad (2.12e)$$

$$\mathrm{Im}B_1^{ab} = \frac{1}{f_\pi}(-\nu c_1^{ab} - \nu c_6^{ab} + d_1^{ab} + m d_2^{ab}) , \tag{2.12f}$$

where all of the invariant functions depend on $\nu, q^2 = 0,\ k^2,\ t = k^2$ where k^2 is spacelike.

III. SUM RULES

To derive sum rules on the $a_i^{ab}, \ldots,$ we integrate (2.7) over q^-,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dq^- \int d^4x\, e^{iq\cdot x}\langle p_2|[A_a^\mu(x), V_b^\nu(0)]|p_1\rangle_{q^+=0} = \int dx^- d^2x_\perp e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p_2|[A_a^\mu(x), V_b^\nu(0)]|p_1\rangle_{x^+=0} \tag{3.1a}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\nu}{P^+}\bar{u}(p_2)(a_1^{ab}P^\mu P_\alpha + \cdots)\gamma_5 u(p_1)\left(g^{\alpha\nu} - \frac{k^\alpha k^\nu}{k^2}\right), \tag{3.1b}$$

where, in (3.1b), q^+ and k^+ are set equal to zero *before* the integration is performed. To evaluate (3.1a) we need the light-cone commutator. We will use only the commutators which are derived from both a free quark model and a quark model with vector gluons; that is, we will use only the $\mu = +,\ \nu = +$ or the $\mu = \pm,\ \nu = \mp$ commutators. These are

$$[V_a^+(x), A_b^+(0)]_{x^+=0} = if_{abc}A_c^+(x)\delta(x^-)\delta^2(\vec{x}_\perp) , \tag{3.2a}$$

$$\begin{aligned}
[V_a^+(x), A_b^-(0)]_{x^+=0} = {} & if_{abc}A_c^-(0)\delta(x^-)\delta^2(x_\perp) - \tfrac{1}{4}if_{abc}\partial_i^x\{\epsilon(x^-)\delta^2(x_\perp)[\mathfrak{A}_c^i(x|0) - \epsilon^{+-ij}\bar{\mathfrak{V}}_j^c(x|0)]\} \\
& - \tfrac{1}{4}id_{abc}\partial_i^x\{\epsilon(x^-)\delta^2(x_\perp)[\bar{\mathfrak{A}}_c^i(x|0) + \epsilon^{+-ij}\mathfrak{V}_j^c(x|0)]\} \\
& - \tfrac{1}{2}if_{abc}\partial_-^x[\epsilon(x^-)\delta^2(x_\perp)\mathfrak{V}_c^-(x|0)] - \tfrac{1}{2}id_{abc}\partial_-^x[\epsilon(x^-)\delta^2(x_\perp)\bar{\mathfrak{V}}_c^-(x|0)] \\
& + [\tfrac{1}{16}i\epsilon(x^-)\delta^2(x_\perp)\bar{\psi}(x)\gamma^+\gamma^-\gamma_5\Lambda_{ab}^-\psi(0) - \text{H.c.}] .
\end{aligned} \tag{3.2b}$$

The operators which appear on the right-hand side of (3.2b) are the Hermitian combinations of the bilocal generalizations of the vector and axial-vector currents:

$$\mathfrak{V}_a^\mu(x|0) \equiv \tfrac{1}{4}[\bar{\psi}(x)\gamma^\mu\lambda_a\psi(0) + \bar{\psi}(0)\gamma^\mu\lambda_a\psi(x)], \tag{3.3a}$$

$$\bar{\mathfrak{V}}_a^\mu(x|0) \equiv -\tfrac{1}{4}i[\bar{\psi}(x)\gamma^\mu\lambda_a\psi(0) - \bar{\psi}(0)\gamma^\mu\lambda_a\psi(x)], \tag{3.3b}$$

$$\mathfrak{A}_a^\mu(x|0) \equiv \tfrac{1}{4}[\bar{\psi}(x)\gamma^\mu\gamma_5\lambda_a\psi(0) + \bar{\psi}(0)\gamma^\mu\gamma_5\lambda_a\psi(x)], \tag{3.3c}$$

$$\bar{\mathfrak{A}}_a^\mu(x|0) \equiv -\tfrac{1}{4}i[\bar{\psi}(x)\gamma^\mu\gamma_5\lambda_a\psi(0) - \bar{\psi}(0)\gamma^\mu\gamma_5\lambda_a\psi(x)] . \tag{3.3d}$$

Because we have currents which are not conserved the mass has been treated as an SU(3) matrix, M, in deriving (3.2b). The final term in (3.2b) contains the matrix Λ_{ab} which is defined as

$$\Lambda_{ab}^\pm \equiv [M, \lambda_a]_\pm \lambda_b . \tag{3.4}$$

In this paper we will always take $M = \epsilon_0\lambda_0/2 + \epsilon_8\lambda_8/2$.[6]

The commutator

$$[A_a^+(x), V_b^-(0)]_{x^+=0} \tag{3.5}$$

is given by (3.2b), with Λ_{ab}^- replaced by Λ_{ab}^+. We will use both (3.2b) and (3.5) to derive sum rules. No new information can be derived from the $+, i$ commutators that is not contained in the $+, -$.

Eventually we will specialize to SU(2) and continue the pion to zero four-momentum. For SU(2) Λ_{ab}^- is zero while Λ_{ab}^+ is proportional to the pion mass, which, because of our soft-pion approximation, we must also set equal to zero. Therefore, we shall not write out the symmetry-breaking terms when they occur, even when the indices refer to SU(3), but simply note that they are easily determined from (3.2) and (3.4).

Unfortunately, the other terms on the right-hand sides of (3.2) and (3.5) cannot be disposed of so easily and we must expand the matrix elements of the bilocal operators in terms of real form factors. In terms of the nucleon spin, s^μ, given by

$$s^\mu = i\bar{u}(p_2)\gamma^\mu\gamma_5 u(p_1) , \tag{3.6}$$

we have

$$\langle p_2|V_a^\mu(0)|p_1\rangle = P^\mu f_a(t) + \frac{i}{P^2}\epsilon^\mu(P\Delta s)f_s^a(t) , \tag{3.7}$$

$$\begin{aligned}
\langle p_2|\mathfrak{V}_a^\mu(x|0)|p_1\rangle = {} & P^\mu V_1^a + x^\mu V_2^a + i\Delta^\mu V_3^a + i\epsilon^\mu(P\Delta s)V_4^a \\
& + \epsilon^\mu(Pxs)V_5^a + i\epsilon^\mu(\Delta xs)V_6^a \\
& + \Delta\cdot s\epsilon^\mu(P\Delta x)V_7^a + ix\cdot s\epsilon^\mu(P\Delta x)V_8^a ,
\end{aligned} \tag{3.8}$$

$$\langle p_2 | A_a^\mu(0) | p_1\rangle = s^\mu G_A^a(t) + \Delta^\mu s\cdot\Delta G_p^a(t)\,, \qquad (3.9)$$

$$\langle p_2 | \mathcal{A}_a^\mu(x|0) | p_1\rangle = s^\mu A_1^a + P^\mu x\cdot sA_2^a + x^\mu x\cdot sA_3^a + iP^\mu\Delta\cdot sA_4^a + ix^\mu\Delta\cdot sA_5^a + i\Delta^\mu x\cdot sA_6^a + \Delta^\mu\Delta\cdot sA_7^a + i\epsilon^\mu(P\Delta x)A_8^a\,. \qquad (3.10)$$

Each of the V_i^a and A_i^a is a function of x^2, $x\circ P$, $x\cdot\Delta$, and t. We have used the notation $\epsilon^\mu(ABC) = \epsilon^{\mu\alpha\beta\rho}A_\alpha B_\beta C_\rho$. The form factors in (3.9) become the usual axial-vector and induced pseudoscalar coupling constants when $t=0$. Time-reversal invariance requires V_3^a, V_5^a, V_7^a, A_4^a, A_5^a, and A_6^a to be zero when $x\cdot\Delta=0$. Similar decompositions hold for $\overline{\mathcal{V}}$ and $\overline{\mathcal{A}}$ in terms of $\overline{V}_i^a$ and $\overline{A}_i^a$.

Equation (3.1b) is conveniently reduced by using (3.6) and the following:

$$\bar u(p_2)\gamma^\mu u(p_1) = \frac{mP^\mu}{P^2}\bar u u - \frac{1}{2P^2}\epsilon^\mu(P\Delta s)\,, \qquad (3.11)$$

$$\bar u(p_2)\gamma_5 u(p_1) = i\frac{\Delta\cdot s}{2m}\,, \qquad (3.12)$$

$$\bar u(p_2)\gamma^\mu\gamma^\nu u(p_1) = g^{\mu\nu}\bar u u + \frac{u\bar u}{2P^2}[P^\mu\Delta^\nu - P^\nu\Delta^\mu] + \frac{1}{4mP^2}[\Delta^\mu\epsilon^\nu(P\Delta s) - \Delta^\nu\epsilon^\mu(P\Delta s)] + \frac{1}{m}\epsilon^{\mu\nu}(Ps)\,, \qquad (3.13)$$

$$\bar u(p_2)\sigma^{\mu\nu}\gamma_5 u(p_1) = \frac{1}{m}(s^\mu P^\nu - s^\nu P^\mu) + \frac{1}{2P^2}\bar u u\epsilon^{\mu\nu}(\Delta P) + \frac{\Delta\cdot s}{4mP^2}(P^\mu\Delta^\nu - \Delta^\mu P^\nu) + \frac{\Delta^2}{4mP^2}(s^\mu P^\nu - s^\nu P^\mu)\,. \qquad (3.14)$$

The sum rules may now be derived. Using (3.2a) we obtain

$$\int_{-\infty}^{\infty} d\nu\, a_1^{ab}(\nu, q^2, k^2, t) = 0, \quad q^2\le 0\,, \quad k^2\le 0 \qquad (3.15a)$$

$$\int_{-\infty}^{\infty} d\nu\, \bar a_1^{ab} = 0\,, \qquad (3.15b)$$

$$\int_{-\infty}^{\infty} d\nu\, c_1^{ab} = 0\,, \qquad (3.15c)$$

$$\int_{-\infty}^{\infty} d\nu(\nu\bar a_1^{ab} + b_1^{ab} + b_2^{ab}) = -2\pi f_{abc}G_A^c(t)\,. \qquad (3.15d)$$

In deriving these we have treated the "reduced" tensors [that is, the tensors in (2.7b) with $\mu,\nu=+,+$]

$$P^+P^+,\ P^+s^+,\ P^+(s^+P_iQ^i - P^+s_iQ^i),\ P^+\bar u u\epsilon^+(PQ\Delta) \qquad (3.16)$$

as linearly independent.

More sum rules can be derived from (3.2b) and (3.5). To derive these sum rules we take the following set of "reduced" tensors to be linearly independent:

$$1,\ P^+P^-,\ P^+s^-,\ P^-s^+,\ P_iQ^i\,,$$
$$P^+P^-s_iQ^i - P^-s^+P_iQ^i,\ P^+\Delta^-\ (=-P_i\Delta^i)\,,$$
$$\frac{s^+}{P^+},\ s_iQ^i - \frac{s^+}{P^+}P_iQ^i,\ P_iQ^i\left(s_iQ^i - \frac{s^+}{P^+}P_iQ^i\right),$$
$$P^+\Delta^-\left(s_iQ^i - \frac{s^+}{P^+}P_iQ^i\right),\ \frac{s^+}{P^+}P_iQ^i\,, \qquad (3.17)$$
$$\frac{s^+}{P^+}P_i\Delta^i,\ \bar u u\epsilon^{+-}(P\Delta)\,,$$
$$\bar u u\epsilon^{+-}(PQ),\ P^+P^-\epsilon^{+-}(Q\Delta)\,,$$
$$P^+\Delta^-\epsilon^{+-}(Q\Delta),\ \epsilon^{+-}(Q\Delta)\,.$$

All other reduced tensors can be written in terms of these. For example,

$$P_iQ^i\epsilon^{+-}(Q\Delta) = Q^2\bar u u\epsilon^{+-}(P\Delta) - Q\cdot\Delta\bar u u\epsilon^{+-}(PQ)\,.$$

We will only write the sum rules which are useful for finding sum rules on the B_i^{ab} through the relations (2.11). The other sum rules which can be derived from (3.2b) and (3.5) but which are not useful for our purposes are listed in the Appendix. The four sum rules of (3.15) can be derived from these commutators and the following sum rules are also obtained:

$$\frac{m}{P^2}\int_{-\infty}^{\infty} d\nu\, d_2^{ab}(\nu, q^2, k^2, t) = -i\frac{\pi}{2}f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_1^c(\alpha)\,, \qquad (3.18a)$$

$$\int_{-\infty}^{\infty} d\nu[d_1^{ab} - \nu c_6^{ab}] = 0\,, \qquad (3.18b)$$

$$\int_{-\infty}^{\infty} d\nu\,\nu c_1^{ab} = i\frac{\pi}{2}P^2 f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_1^c\,, \qquad (3.18c)$$

$$\int_{-\infty}^{\infty} d\nu\, b_2^{ab} = -2\pi f_{abc}G_A^c(t) + i\frac{\pi}{2}\frac{1}{m}(m^2 - \tfrac{1}{2}q^2 + \tfrac{1}{2}k^2)f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_1^c + \frac{\pi}{4}(q^2 - k^2 - t)f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_4^c\,, \qquad (3.18d)$$

$$\int_{-\infty}^{\infty} d\nu(\nu a_1^{ab} + m b_1^{ab}) = \frac{\pi}{4} m(q^2 - k^2 - t) f_{abc}\left[8G^c(t) + \int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_4^c\right] - i\frac{\pi}{8}(4P^2 + q^2 - k^2 - t) f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_1^c, \tag{3.18e}$$

$$\int_{-\infty}^{\infty} d\nu(\nu\overline{a}_2^{ab} + b_4^{ab}) = i\frac{\pi}{2} d_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)(\overline{A}_1^c + \alpha\overline{A}_2^c), \tag{3.18f}$$

$$\int_{-\infty}^{\infty} d\nu[\nu^2\overline{a}_1^{ab} + \nu(b_1^{ab} + b_2^{ab})] = -i\frac{\pi}{4}(q^2 + k^2 - t) d_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)(\overline{A}_1^c + \alpha\overline{A}_2^c), \tag{3.18g}$$

$$\int_{-\infty}^{\infty} d\nu\, \overline{a}_{10}^{ab} = -i\frac{\pi}{2} d_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)(\overline{A}_1^c + \alpha\overline{A}_2^c), \tag{3.18h}$$

where we have omitted the argument ν, q^2, k^2, t of the amplitudes on the left-hand sides of the sum rules. Similarly on the right-hand side the bilocal form factors are each a function of $x^2 = 0, \alpha, x\cdot\Delta = 0$, and t. q^2 and k^2 are restricted to be spacelike.

The derivation of the final sum rule (3.18h) is more complicated than the others. We derive from (3.1) the sum rules [not listed in (3.18)]

$$\int_{-\infty}^{\infty} d\nu(\nu\overline{a}_4^{ab} + b_3^{ab}) = 0, \tag{3.19a}$$

$$\int_{-\infty}^{\infty} d\nu(\nu\overline{a}_7^{ab} + b_5^{ab}) = i\frac{\pi}{2} d_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)(A_1^c + \alpha A_2^c), \tag{3.19b}$$

$$\int_{-\infty}^{\infty} d\nu[\nu^2\overline{a}_4^{ab} + \nu^2\overline{a}_7^{ab} + \nu(b_3^{ab} + b_5^{ab} + \overline{a}_{10}^{ab})] = \frac{\pi}{4} f_{abc}(q^2 - k^2 - t)\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)(\alpha\overline{V}_6^c + \alpha^2\overline{V}_8^c). \tag{3.19c}$$

The high power of ν in the integrand of (3.19c) insures that we can use the method of Heimann *et al.*[14] to derive a sum rule

$$\int_{-\infty}^{\infty} d\nu(\nu\overline{a}_4^{ab} + \overline{a}_7^{ab} + b_3^{ab} + b_5^{ab} + \overline{a}_{10}^{ab}) = 0. \tag{3.20}$$

This sum rule, together with (3.19a) and (3.19b), gives (3.18h).

IV. RESULTS

It is now fairly simple to use the sum rules (3.15) and (3.18) together with the relations (2.11) to get sum rules on the electroproduction amplitudes B_i. In particular in the soft-pion limit, $q_i \to 0$, the sum rules can be used to eliminate all of the amplitudes $a_1^{ab}, \ldots, d_2^{ab}$ [see (2.12)]. Specializing to SU(2) and using the decomposition (2.4) the sum rules which follow from the ++ commutator through (3.15) are

$$\frac{F_2^{(V,S)}(t)}{2m} = \frac{2m}{g_{\pi N}}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}[\mathrm{Im}B_1^{(+,0)} + \mathrm{Im}B_2^{(+,0)} - m\,\mathrm{Im}B_6^{(+,0)}], \tag{4.1}$$

$$F_1^V(t) - \frac{G_A(t)}{G_A(0)} = \frac{2m}{g_{\pi N}}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\mathrm{Im}B_5'^{(-)}. \tag{4.2}$$

The amplitudes on the right-hand side of (4.1) and (4.2) are functions of ν, $q^2 = 0$, k^2, and $t = k^2$ with k^2 spacelike. $F_1^V, F_2^V, F_1^S, F_2^S$ are the usual vector and scalar, electric and magnetic, form factors. $g_{\pi N}$ is the pion-nucleon coupling constant and the Goldberger-Treiman relation

$$g_{\pi N} f_\pi = -m G_A(0)$$

has been used to simplify the left-hand side of (4.2).

The sum rules (3.18) which follow from the +− commutators give

$$\int_{\nu_0}^{\infty} d\nu\, \mathrm{Im}B_5'^{(+,0)} = 0, \tag{4.3}$$

$$\frac{F_2^{(V,S)}(t)}{2m} = \frac{2}{g_{\pi N}}\int_{\nu_0}^{\infty} d\nu\, \mathrm{Im}B_8^{(+,0)}, \tag{4.4}$$

$$F_1^V(t) - \frac{1}{G_A(0)}[G_A(t) - tG_p(t)] = -\frac{2}{g_{\pi N}}\int_{\nu_0}^{\infty} d\nu\, \mathrm{Im}B_2^{(-)} - \frac{\pi}{mG_A(0)}(\tfrac{1}{8}t - m^2)\int_0^{\infty} d\alpha\, \overline{V}_1^{(-)}, \tag{4.5}$$

$$F_1^V(t) + F_2^V(t) = \frac{2}{g_{\pi N}}\int_{\nu_0}^{\infty} d\nu\, \mathrm{Im}B_1^{(-)} - \frac{\pi}{mG_A(0)}P^2\int_0^{\infty} d\alpha\, \overline{V}_1^{(-)}, \tag{4.6}$$

where $P^2 = \frac{1}{4}(p_1 + p_2)^2 = m^2 - \frac{1}{4}t$.

These nine sum rules constitute our results. The sum rules derived from the ++ commutation relations, (4.1) and (4.2), were derived from equal-time commutation relations many years ago. At $k^2 = 0$ (4.1) is the famous Fubini-Furlan-Rossetti sum rule which relates the nucleon magnetic moments to photoproduction amplitudes.[12] This is not surprising since the ++ light-cone commutator contains the same information as the "good-good" equal-time commutators. The +− light-cone commutator contains new information, however, and we believe the sum rules derived from that commutator are new and constitute an important test of the form of the light-cone commutators.

The Born terms have been separated out of the sum rules by using the device of giving the intermediate nucleon a different mass than the external nucleons and setting the masses equal only after the $q=0$ limit has been taken. This means that terms of the form

$$q\cdot k \int \frac{d\nu}{\nu} a(\nu, q\cdot k)$$

can be set equal to zero because the Born contribution is proportional to

$$\delta(\nu - q\cdot k + m'^2 - m^2),$$

but that terms of the form

$$(m' - m)\int \frac{d\nu}{\nu} a(\nu, q\cdot k)$$

will have nonzero contributions.

There are several things which are questionable about these sum rules and the method of derivation. Unlike many of the sum rules based on equal-time commutation relations these sum rules are not invalidated by Z graphs. Like the sum rules from equal-time commutation relations, however, they are only valid in the absence of Class II singularities. This is discussed in detail in Appendix D of Ref. 1.[15] Also the approach to $q^2=0$ is from the direction of spacelike q^2, not, as one would like, from the physical mass to $q^2 = 0$.

The t-channel helicity amplitudes indicate that the Regge behavior of the electroproduction amplitudes is[16]

$$B_1, B_2, B_5', B_6, B_8 \sim s^{\alpha-1}, \qquad B_3 \sim s^{\alpha}. \tag{4.7}$$

This makes all of the new sum rules (4.3)–(4.6) appear divergent if $\alpha > 0$. Since we are considering electroproduction we can choose t ($=k^2$) sufficiently spacelike that $\alpha(t)$ is less than zero. The $t \approx 0$ case can then be evaluated by analytic continuation if necessary.[17]

This is not a new situation; similar cases of divergent sum rules were found by Cornwall, Corrigan, and Norton[18] and in Ref. 1 and Ref. 2.[19] Cornwall, Corrigan, and Norton subtracted the leading Regge contribution at $t=0$ and then wrote a sum rule for the remainder. We could also do this. In fact the asymptotic behavior may well be better than (4.7) indicates; further improvement could be expected to result from the proper insertion of factorized Regge poles into the t-channel parity-conserving helicity amplitudes. In addition the $t=0$ behavior may be improved by the use of the conspiracy conditions.

APPENDIX

There are many fixed-mass sum rules on the amplitudes (2.8) that are not included in (3.15), (3.18), or (3.19) because they are not directly related to electroproduction amplitudes. These are derived by using (3.2b) and (3.5). Each invariant amplitude is a function of ν, q^2, k^2, and t while the bilocal form factors in the integrands on the right-hand sides are functions of α, $x^2=0$, $x\cdot\Delta=0$, and t:

$$\int_{-\infty}^{\infty} d\nu\, c_2^{ab} = 0, \tag{A1}$$

$$\int_{-\infty}^{\infty} d\nu\, c_3^{ab} = 0, \tag{A2}$$

$$\int_{-\infty}^{\infty} d\nu \left(b_1^{ab} + \frac{2m}{P^2} d_1^{ab}\right) = -2\pi f_{abc} G_A^c(t) - i\frac{\pi}{4}\frac{1}{m}(2m^2 + k^2 - q^2) f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_1^c - \frac{\pi}{4}(q^2 - k^2 + t) f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_4^c, \tag{A3}$$

$$\frac{k^2 + t - q^2}{k^2}\int_{-\infty}^{\infty} d\nu\, \overline{a}_2^{ab} = -i\pi \frac{1}{m} f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_1^c + 2\pi f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_4^c, \tag{A4}$$

$$\int_{-\infty}^{\infty} d\nu\, a_2^{ab} = 4\pi m f_{abc} G_p^c(t) + i\frac{\pi}{4} f_{abc}\int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_1^c - \frac{\pi}{2} m f_{abc}\int_{\infty}^{\infty} d\alpha\, \epsilon(\alpha)\overline{V}_4^c, \tag{A5}$$

$$\int_{-\infty}^{\infty} d\nu(\overline{a}_4^{ab}+\overline{a}_7^{ab})=0\,, \tag{A6}$$

$$\int_{-\infty}^{\infty} d\nu\left(-\overline{a}_4^{ab}+\overline{a}_7^{ab}+\frac{1}{P^2}d_2^{ab}\right)=-\pi f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_4^c\,, \tag{A7}$$

$$\int_{-\infty}^{\infty} d\nu[a_4^{ab}+a_7^{ab}]=0\,, \tag{A8}$$

$$\int_{-\infty}^{\infty} d\nu\left(-a_4^{ab}+a_7^{ab}-\frac{2}{P^2}d_1^{ab}\right)=-8\pi m f_{abc}G_p^c(t)-i\frac{\pi}{2}f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_1^c+\pi m f_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\overline{V}_4^c\,, \tag{A9}$$

$$\int_{-\infty}^{\infty} d\nu[\nu^3\overline{a}_1^{ab}-k^2\nu\overline{a}_{10}^{ab}+\nu^2(b_1^{ab}+b_2^{ab})+\tfrac{1}{2}(q^2-k^2-t)(\nu^2\overline{a}_2^{ab}+\nu b_4^{ab})]=\frac{\pi}{4}k^2(q^2-k^2+t)\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)(\alpha\overline{V}_6^c+\alpha^2\overline{V}_8^c)\,, \tag{A10}$$

$$\int_{-\infty}^{\infty} d\nu\,\nu\left(\overline{a}_4^{ab}+\overline{a}_7^{ab}-\frac{m}{P^2}c_2^{ab}\right)=-i\frac{\pi}{2}d_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\alpha\overline{A}_2^c\,, \tag{A11}$$

$$\int_{-\infty}^{\infty} d\nu\left[\nu a_4^{ab}+\nu a_7^{ab}+a_{10}^{ab}+\frac{\nu}{4P^2}(q^2-k^2)c_2^{ab}\right]=-i\frac{\pi}{2}m\,d_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)[\overline{A}_1^c+(k^2+t-q^2)\overline{A}_7^c]\,, \tag{A12}$$

$$\int_{-\infty}^{\infty} d\nu\left(c_6^{ab}+\frac{\nu}{P^2}c_2^{ab}\right)=-\pi d_{abc}\int_{\infty}^{\infty} d\alpha\,\epsilon(\alpha)\alpha\overline{A}_8^c\,, \tag{A13}$$

$$\begin{aligned}\int_{-\infty}^{\infty} d\nu\,\nu\Big[&\nu\overline{a}_1^{ab}+b_1^{ab}+\tfrac{1}{2}(q^2-k^2-t)\overline{a}_2^{ab}+\frac{t}{4P^2}d_2^{ab}+\frac{m}{P^2}d_1^{ab}\\&+\frac{\nu m}{P^2}(c_1^{ab}-c_6^{ab})+\frac{m}{4P^2}(q^2-k^2-t)c_2^{ab}-\frac{m}{2P^2}(3k^2+t-q^2)c_3^{ab}\Big]=-i\frac{\pi}{2}k^2d_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)\alpha\overline{A}_2^c\,,\end{aligned} \tag{A14}$$

$$\int_{-\infty}^{\infty} d\nu\,\nu[d_1^{ab}+m d_2^{ab}+\nu(c_1^{ab}-c_6^{ab})-\tfrac{1}{2}(3k^2+t-q^2)c_3^{ab}-\tfrac{1}{4}(5k^2+t-q^2)c_2^{ab}]=0\,, \tag{A15}$$

$$\begin{aligned}\int_{-\infty}^{\infty} d\nu\Big[&-\nu^2a_1^{ab}+\tfrac{1}{2}(k^2+t-q^2)\nu a_2^{ab}+a_{10}^{ab}-\nu m b_1^{ab}+\frac{1}{4P^2}(4m^2+k^2-q^2-t)\nu d_1^{ab}+\frac{m}{4P^2}(q^2-k^2)\nu d_2^{ab}\\&-\nu^2(c_1^{ab}+c_6^{ab})-\frac{1}{4P^2}(q^2-k^2)\nu^2(c_1^{ab}-c_6^{ab})+\frac{1}{16P^2}(q^2-k^2)(k^2+t-q^2)\nu c_2^{ab}+\frac{1}{8P^2}(q^2-k^2)(3k^2+t-q^2)\nu c_3^{ab}\Big]\\&=i\frac{\pi}{2}mk^2d_{abc}\int_{-\infty}^{\infty} d\alpha\,\epsilon(\alpha)[\overline{A}_1^c+(q^2+t-k^2)\overline{A}_7^c]\,.\end{aligned} \tag{A16}$$

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