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CP violation in charmed-particle decays*

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The discovery of hadrons bearing new quantum numbers ("charm," collectively) would open up new possibilities to pursue the issue of CP violation. The matter of CP signatures for charm-changing processes is discussed in general, model-independent terms save for the assumption that $\Delta C/\Delta Q = 1$ in semileptonic channels. We note certain qualitative differences in the analysis, as compared to charm-conserving reactions. We then raise the issue of indirect effects of CP violation in charm-changing interactions on the neutral K-meson system. In particular an example is discussed in which CP symmetry breakdown is located in the charm-changing interactions and in which the present CP evidence for charm-conserving decays is qualitatively accommodated. The operational distinctions between this mechanism and another interesting possibility, the presence of a "second" superweak interaction (one with $|\Delta C| = 2$), turn out to be delicately linked to CP tests for exclusive decay processes.

I. INTRODUCTION

A variety of recent experimental developments has led to the indirect, but almost inescapable inference that one or more new kinds of quantum numbers are needed for hadron physics. An indirect, though persuasive, case for novel quark types had emerged even earlier from attempts to incorporate standard phenomenology into models of the weak interactions.[1] Without prejudice to any particular theoretical scheme, we shall denote any new quantum numbers, collectively, as *charm*. Convincing, direct evidence for charmed particles is still lacking at this moment. Nevertheless, there is a general anticipation that experimental proof of the existence of charm-carrying states is not far off.

In this note we take up the question of CP violation for the weak charm-changing interactions. The study of weak decays of charmed particles, if they exist, would open welcome new avenues for the pursuit of CP violation, a phenomenon up till now so narrowly confined to the K^0-$\overline{K}^0$ system. CPT invariance will be assumed throughout; and for ease of discussion we shall suppose that there is only one kind of charm C, additively conserved in strong and electromagnetic interactions. In all cases we shall be concerned with the low-lying charmed-particle states that are stable against strong and electromagnetic decays. In Sec. II A we consider charmed, conjugate pairs of particles with nonvanishing charge and/or baryon number. In Sec. II B we turn to our main topic, pairs of states which form the charmed analogs of K^0 and $\overline{K}^0$; namely, conjugate pairs of charmed states which couple through the weak interactions to common states and hence to each other. We denote such a pair by D^0 and $\overline{D}^0$. They are neutral mesons with nonvanishing and opposite values of charm and zero strangeness.

In the first instance our task will be to discuss general phenomenological signatures of CP violation for $\Delta C \neq 0$ processes. For the $\Delta C = 0$ case these have been analyzed in great detail[2] and the general principles of course carry over for charm-changing decays. However, there are interesting practical differences between these two classes, so that the transition from one to the other is not altogether a story which repeats itself. To us this seems sufficient justification, even at this early stage, for the analysis presented below.

In essence, these practical differences are twofold. First, if there *is* charm even the low-lying states are likely to be fairly massive. This is a promising feature for our purposes, in that there will be many decay channels available, both semileptonic and nonleptonic. Thus we have here many more options as compared, say, to (both charged and neutral) K-meson decays.

The second difference relates specifically to D_1 and D_2, those linear combinations of D^0 and $\overline{D}^0$ which represent objects with definite mass and lifetime, (m_1, λ_1) and (m_2, λ_2), respectively. We shall be concerned not only with these parameters, but also with the presence of CP impurities in the constitution of D_1 and D_2. All these questions are in straight analogy with the K_S, K_L system. For the latter, the rate parameters λ_S and λ_L and the mass difference $m_S - m_L$ could be determined directly, by observation of the time dependence of the decay reactions; and the impurity parameter came from the detection of CP forbidden decays of K_L, from charge asymmetries in semileptonic decays of K_L, and from time-dependent interferences between K_S and K_L. On the other hand, estimates for the lifetimes of both D_1 and D_2 hover in the region of 10^{-12} sec. While this informed guess was made for a specific model,[3] it is most probable, generally, that both lifetimes will be short compared, say, to that of K_S. From this it follows that experimental access to the D_1, D_2 parameters must inevitably be obtained from the analysis of decay processes integrated over time. While this represents a serious loss of information, we shall see in Sec. II B that this time integration has its compensations.

It is clearly impossible to predict reliably what the situation will be regarding CP violations in the charm-changing sector on the basis of evidence from the charm-conserving sector. The CP-violating effects there are still confined exclusively to phenomena associated with neutral K mesons; and on present evidence it is possible to ascribe them to a single cause, whatever its fundamental theoretical origin: the presence of CP impurities in the K_S, K_L states. We may not in advance assume that for charm change the effects are analogously confined. Nor do the powerful limitations imposed by gauge theories enable us, as yet, to relate the CP questions in the two sectors. Even in this framework there remain still many approaches to CP violation, no one of which has so far achieved a convincing and definitive form. Gauge models which contain CP violation in the $|\Delta C|=1$ sector can of course be constructed; but the general possibilities raised here are presented in a more phenomenological spirit.

Within our general analysis we shall introduce one important assumption, namely, that the charge and charm-changing semileptonic interactions obey the rule

$$\Delta C/\Delta Q=1.$$

Such a rule is in accord with all current models, but it is an assumption nevertheless. This rule will play a role in particular in our discussion of the D_1, D_2 system where it implies a vanishing of the amplitudes for $D^0 \to l^- + \bar{\nu} + \cdots$ and $\bar{D}^0 \to l^+ + \nu + \cdots$; but it can be checked elsewhere too, of course, by observation of semileptonic decays of charged charm-carrying mesons for example.

Before we turn to the direct effects of CP violation in weak charm-changing decays in Sec. II, we would like to raise a second issue, less strictly phenomenological, but still quite broad. It concerns the possibility of indirect effects on the neutral K system of CP violation in charm-changing interactions. Indeed, we think it is quite instructive to ask if the CP impurities in that system could arise solely as an indirect effect of CP violation in the charm sector.

These impurities can arise out of the basic CP-violating interactions in several ways: in second-order transitions between K and $\bar{K}$ which pass through intermediate states on or off the mass shell; or from interactions that directly couple K and $\bar{K}$ in first order. Interactions which give rise to on-shell contributions would be expected to produce CP-violation effects also outside the neutral K-meson system, e.g., in $K^{\pm}$ decays. On the other hand, it could be that the effects arise exclusively off shell (virtual transitions to massive intermediate states), so that no effects would show up for physical $K^{\pm}$ decays. This is the kind of situation about which we shall presently elaborate. A third possibility is exemplified by the superweak model of CP violation.[4] Here one postulates a CP-violating $|\Delta S|=2$ interaction which couples K and $\bar{K}$ so weakly that this first-order effect is $\sim 10^{-3}$ smaller than the second-order contribution from ordinary $|\Delta S|=1$ interactions. Outside the neutral K meson the new interaction would be invisible for all practical purposes, e.g., the process $\Xi \to N + \pi$ which it induces would have far too small a rate to be observed.

Present evidence accords with the superweak model, in the sense that CP violation has not been observed outside the neutral K-meson system[5] and that the breakdown observed there appears to be attributable solely to CP impurities in K_S, K_L. It may be that the requisite level of precision has not yet been reached in these tests; but it is important to be reminded that the consequences of the superweak model, so far as present data are concerned, can be reached in a variety of other ways. For example, suppose that the $|\Delta S|=1$ interactions contain terms which transform under isospin like $|\Delta I|=\frac{13}{2}$. Suppose, moreover, that these terms contain substantial CP-violating components. It just happens that there are no experimentally accessible reactions available where such large isospin changes could play a direct, on-shell role. Yet, in the second-order K^0-$\bar{K}^0$ transition these interactions would contribute off shell to the CP impurities of K_S^0 and K_L^0. This whimsical alternative to the superweak model is totally unmotivated by independent evidence. It reminds us, however, that the range of practically accessible weak reactions is very limited, consequently that the range of theoretical possibilities is very broad. In the present context the more interesting possibility arises that the CP impurities observed in the neutral K system are generated by symmetry-violating terms in the charm-changing interactions. We suppose that there are weak $|\Delta C|=1$ interactions containing $\Delta S/\Delta C=1$, -1, and 0 pieces, where S is strangeness. Then the first two can contribute to K-$\bar{K}$ transitions via the sequences $K \rightleftarrows (C=1, S=0) \rightleftarrows \bar{K}$ and $K \rightleftarrows (C=-1, S=0) \rightleftarrows \bar{K}$ (the two sequences contribute identically, according to the CPT theorem).

For neutral K mesons the CP impurity is measured by a complex parameter ϵ which is very small (magnitude of order 10^{-3}). The parameters which measure CP violations in charm-changing interactions might, however, be substantially larger than this: The masses of charmed particles are presumably substantially larger than the K-meson mass; hence charm-changing contributions to the K-$\bar{K}$ transition would be well off mass shell and therefore suppressed by rather large energy denominators. Direct CP-violating effects in charm-changing decays have at least a chance on this view to be fairly sizable. Even within this picture, however, there are several possibilities. It might be that the $\Delta S/\Delta C = 1$ and $\Delta S/\Delta C = -1$ interaction pieces separately contain both CP-violating and CP-conserving terms (similarly, perhaps, for the $\Delta S/\Delta C = 0$ piece). In this case symmetry-violating effects could show up as a widespread phenomenon in charmed-particle decays. There is also the possibility, which suffices to produce CP impurities in the neutral K-meson system, that the $\Delta S/\Delta C = 1$ and $\Delta S/\Delta C = -1$ pieces are separately pure, one of them being odd under CP, the other even, so that the two pieces are maximally out of phase. In this case, if the $\Delta S/\Delta C = 0$ interaction is also pure, symmetry-violating effects would show up only where the $\Delta S/\Delta C = \pm 1$ pieces interfere, namely, in charmed analogs of the K_S, K_L system. On this alternative the CP impurity parameter for the charmed system could well be larger than it is for the K_S, K_L system, even though the basic mechanism is the same for both. The point is that the charmed particles would have open decay channels available near and on the mass shell, so that suppression effects arising from large energy denominators might not be as effective as for the K system.

It is this latter alternative which we shall discuss more fully in Sec. II. It provides an example of a common source for CP violation in the charm-conserving and charm-changing sectors, where, in each sector, the effects are confined solely to the non-self-conjugate neutral-meson systems.

This confinement can of course also arise if one contemplates a quite distinct, alternative mechanism for CP violation, namely, along with the already much discussed superweak $|\Delta S| = 2$ interaction, a "second" superweak $|\Delta C| = 2$ interaction for the charm sector. It will therefore be of interest to ask in what ways the idea, mentioned above, of two interactions with clashing CP properties can be distinguished operationally from the case of two superweak interactions. We shall answer this question in Sec. II B.

The experimental tests to be described below are for the future. Yet the time is ripe, we believe, to anticipate even now the new theoretical options in regard to CP which may emerge with the advent of new hadronic quantum numbers.

II. SIGNATURES OF CP VIOLATION

A. Baryon and charged meson decays

The tests for CP violation that we shall consider rest on a comparison of the decay properties of CPT-conjugate pairs of states, labeled each by charge and baryon number and, as defined by the strong interactions, by strangeness and charm. As prototypes we may consider mesonic examples; e.g., $D^+(C=1, S=0)$ and $D^-(C=-1, S=0)$, or $F^+(C=1, S=1)$ and $F^-(C=-1, S=-1)$. It will be clear how our remarks on these objects extend to charmed baryon pairs.

For the subclass of particles presently under consideration, conjugate pairs do not couple to each other via the weak interactions. Thus we are dealing with objects of definite lifetime; and it is a consequence of CPT invariance that both members of a pair have the same mass and same *net* decay rate. Certain partial rates, summed over subgroups of final channels, must also be equal in pairs. Here one organizes the channels into subgroups which do not couple, one to another, via the strong and electromagnetic interactions (we neglect weak final-state interactions). Then, e.g., the partial rate for D^+ decay, summed over all channels in a given subgroup, must equal that for D^- decay into the corresponding conjugate subgroup.

Let us first consider the case of nonleptonic decay, illustrated on the example of D^+ and D^-. We classify the final states according to *net* strangeness $S = 1, 0, -1$. Then CPT invariance implies the semiinclusive rate equalities

$$\begin{aligned} &\Gamma(D^+ \to X_1) = \Gamma(D^- \to X_{-1}), \\ &\Gamma(D^+ \to X_{-1}) = \Gamma(D^- \to X_1), \\ &\Gamma(D^+ \to X_0) = \Gamma(D^- \to X_0), \end{aligned} \tag{1}$$

where X_S denotes a sum over all states with net strangeness S.

What is of interest for CP tests is the comparison of exclusive pairs of conjugate reactions, e.g., $D^+ \to K^- + \pi^+ + \pi^+$ vs $D^- \to K^+ + \pi^- + \pi^-$. If CP invariance holds, then not only the partial rates but also the detailed decay spectra (angular distributions, energy variations, etc.) must be identical for the two reactions (in going from one reaction to its conjugate for spectral comparisons one, of course, has to make the appropriate CP changes: change particle into antiparticle and reverse all spins and momenta). Any departures would signal a breakdown of CP invariance. The spectral tests are especially interesting, since these are independent of flux normalization. It is clear that semi-inclusive comparisons can also serve as tests for CP breakdown, e.g., comparison of $D^+ \to \pi^+ + X$ and $D^- \to \pi^- + X$, where X in each case denotes a sum over all accompanying hadrons, irrespective of strangeness; or $D^+ \to \pi^- + X$ vs $D^- \to \pi^+ + X$, $D^+ \to K^\pm + X$ vs $D^- \to K^\mp + X$, etc. It should be noted in connection with the various tests discussed here that absolute determination of decay rates is not required. Since D^+ and D^- have the same net lifetimes, it is enough to compare branching ratios, e.g., $B(D^+ \to \pi^- + X)$ vs $B(D^- \to \pi^+ + X)$; or, indeed, ratios of branching ratios, e.g., $B(D^+ \to \pi^- + X)/B(D^+ \to K^+ + X)$ vs $B(D^- \to \pi^- + X)/B(D^- \to K^- + X)$. Even conjugate pairs of two-body decays need not have the same partial rates, e.g., $D^+ \to \pi^+ + \pi^0$ vs $D^- \to \pi^- + \pi^0$. By comparison the equality of rates for two-pion modes $K^+ \to \pi^+ + \pi^0$ and $K^- \to \pi^- + \pi^0$ follows already from CPT invariance (insofar as we neglect photon channels). Similarly the rates for K decays into three pions must be equal when one sums over both the τ and τ' modes. The only opportunity for CP tests here lies in the comparison, separately, of the τ or τ' modes, $K^+ \to 2\pi^+ + \pi^-$ vs $K^- \to 2\pi^- + \pi^+$ or $K^+ \to 2\pi^0 + \pi^+$ vs $K^- \to 2\pi^0 + \pi^-$.

Let us next consider the situations for semileptonic decays of charmed particles. We again classify the final states, this time according to lepton content as well as net strangeness of the accompanying hadrons. From CPT invariance it then follows that

$$\begin{aligned}
\Gamma(D^+ \to l^+ + \nu + X_1) &= \Gamma(D^- \to l^- + \nu + X_{-1}) ,\\
\Gamma(D^+ \to l^+ + \nu + X_{-1}) &= \Gamma(D^- \to l^- + \nu + X_1) , \qquad (2)\\
\Gamma(D^+ \to l^+ + \nu + X_0) &= \Gamma(D^- \to l^- + \nu + X_0) ,
\end{aligned}$$

where again X_S denotes a sum over hadrons with net strangeness S. For ease of writing we have employed the single symbol ν to represent either neutrino or antineutrino, as appropriate. Additional equalities are implied by CPT invariance for the special cases where there are no hadrons or only a single hadron in the final state (we are ignoring, in this, any significant role of photon-containing channels). Thus

$$\Gamma(D^+ \to l^+ + \nu) = \Gamma(D^- \to l^- + \nu) , \qquad (3)$$

and, e.g.,

$$\Gamma(D^+ \to l^+ + \nu + \overline{K}^0) = \Gamma(D^- \to l^- + \nu + K^0) , \qquad (4)$$

What is of interest for CP tests is the comparison of exclusive pairs of conjugate reactions involving two or more hadrons, e.g., $D^+ \to l^+ + \nu + K^- + \pi^+$ and $D^- \to l^- + \nu + K^+ + \pi^-$. Any difference in partial rates or in spectrum shapes would signify a breakdown of CP invariance for charm-changing semileptonic interactions. As with nonleptonic decays it is clear that here too CP can be tested via semi-inclusive comparisons, e.g., $D^+ \to l^+ + \nu + \pi^+ + X$ vs $D^+ \to l^- + \nu + \pi^- + X$; and again it will be clear how much richer are the options in charmed-particle decay, as contrasted, say, with $K^\pm$-meson decays.

B. D_1, D_2 decays

In a parametrization that is familiar from the analysis of K_S and K_L, the objects D_1 and D_2 referred to in Sec. I can be written as

$$D_{1,2} = N[(1+\epsilon_C)D^0 \pm (1-\epsilon_C)\overline{D}^0] ,$$

where $N = [2(1+|\epsilon_C|^2)]^{-1/2}$ and ϵ_C is a complex parameter that measures the impurity of the states. Of the various parameters that characterize the D_1, D_2 system it is the CP impurity parameter ϵ_C that is of chief interest here. In discussing how information on this quantity can be extracted from time-integrated decay data, we will also have to deal with some of the other parameters, namely,

$$x \equiv \frac{m_1 - m_2}{\lambda} , \quad y = \frac{\lambda_1 - \lambda_2}{2\lambda} , \qquad (5)$$

where

$$\lambda \equiv \tfrac{1}{2}(\lambda_1 + \lambda_2) . \qquad (5')$$

Information on the x and y parameters would have its own interest. For the K_S, K_L system we may recall how important a role the x parameter has played in speculations about the weak interactions. The problem is to account for the modest value, magnitude of order unity, that is observed. It would be interesting to learn for the charmed particles whether x is similarly modest—more generally, to distinguish among the qualitative alternatives $|x| \gg 1$, $|x| \ll 1$, and $|x| \approx 1$.

Let us now first consider semileptonic decays of the neutral D's, with due regard to the $\Delta C/\Delta Q = 1$ rule stated earlier. Consider a situation where the state D^0 is produced at the initial time. As time goes on the state evolves into a superposition of $\overline{D}^0$ and D^0, the former contributing to $l^- + \nu + \cdots$ and the latter to $l^+ + \nu + \cdots$ decays. We now consider a conjugate pair of final states, $l^- + \nu + f$ and $l^+ + \nu + \bar{f}$. Here f denotes some particular collection of hadrons, $\bar{f}$ the conjugate collection. We ask for the total number of events in each channel, integrated over time, and define the ratio

$$r(f, \bar{f}) \equiv \frac{N(l^- + \nu + f)}{N(l^+ + \nu + \bar{f})} .$$

Similarly, for the case where the initial state is $\overline{D}^0$ (and $\overline{N}$ the number of events), define

$$\bar{r}(\bar{f}, f) \equiv \frac{\overline{N}(l^+ + \nu + \bar{f})}{\overline{N}(l^- + \nu + f)} .$$

It may happen that the relevant semileptonic interactions are CP-conserving, even though for other reasons the states D_1 and D_2 are impure with respect to CP. In this case the spectra in the $l^- + \nu + f$ and $l^+ + \nu + \bar{f}$ channels will be identical (after due allowance is made for the operation of CP on spins, momenta, and particle-antiparticle interchanges). Moreover, for the count ratios we have in this case

$$r(f,\bar{f}) = \left|\frac{1-\epsilon_C}{1+\epsilon_C}\right|^2 \left(\frac{1-\alpha}{1+\alpha}\right),$$
$$\bar{r}(\bar{f},f) = \left|\frac{1+\epsilon_C}{1-\epsilon_C}\right|^2 \left(\frac{1-\alpha}{1+\alpha}\right), \tag{6}$$
$$\alpha \equiv \frac{1-y^2}{1+x^2}.$$

In addition, one finds

$$B(l^+ + \nu + \bar{f}) = \bar{B}(l^- + \nu + f), \tag{6'}$$

where B and $\bar{B}$ are the branching ratios, respectively, for initial states D and $\bar{D}$. Notice that these results are independent of the channel labels f and $\bar{f}$. This independence, together with the spectral tests mentioned above, can serve as tests of possible CP-violating effects in the semileptonic interactions—effects, that is, beyond those arising from CP impurities in D_1 and D_2. Whether or nor symmetry-violating effects of this latter kind occur, Eqs. (6) survive for the special case where there is only a single hadron in the final state, e.g., for the channel pairs $l^- + \nu + \pi^+$ and $l^+ + \nu + \pi^-$, etc. Equations (6) also hold generally, because of CPT invariance, if one sums over all semileptonic channels (in a more refined version the sums can be restricted to channels with prescribed net strangeness). For this hadron inclusive case, or for the special one-hadron cases, we denote the ratios by r and $\bar{r}$; and from Eqs. (6) we then find

$$\frac{r}{\bar{r}} = \left|\frac{1-\epsilon_C}{1+\epsilon_C}\right|^4, \quad r\bar{r} = \left(\frac{1-\alpha}{1+\alpha}\right)^2. \tag{7}$$

The first of these equations provides information on the CP impurity parameter; the second, on the rate and mass-difference parameters of the D_1, D_2 system. The presence of CP impurities is measured by the departure from unity of $|1-\epsilon_C|/|1+\epsilon_C|$.

We turn now to a discussion of nonleptonic decays of the neutral D mesons. For the analogous $K, \bar{K}$ system there is the simplifying feature that the dominant transition takes place to the 2π $(I=0)$ state. Insofar as electromagnetic effects are neglected this is an eigenstate of the strong interaction S matrix. From CPT invariance it therefore follows that the $K, \bar{K} \to 2\pi$ $(I=0)$ amplitudes are equal in magnitude, even in the presence of CP violation; and equal in phase, if one wishes, by a suitable choice of phase conventions. For the higher-mass $D, \bar{D}$ system, however, with its many open channels, there is a more complicated S matrix. Hence the D and $\bar{D}$ amplitudes to, say, the 2π $(I=0)$ state need no longer be equal in magnitude if the relevant interactions are CP-violating.

Instead of pursuing this issue in general, let us begin to narrow the focus. Suppose it were found that CP violation for charmed-particle decays is confined exclusively to the D_1, D_2 system, just as for charm-conserving processes the symmetry violation appears to be confined to the K_S, K_L system. For the former case this can be tested by study of $D^\pm$ decays, in the manner already discussed. Let us here suppose that these tests have been passed. The situation thus envisaged still leaves open the possibility of symmetry-violating effects in the D_1, D_2 system. These could arise from CP impurities in the D_1, D_2 states, as would be revealed in the manner already discussed via semileptonic measurements. It is the possibility of additional effects, however, that we want now to especially emphasize.

The assumed absence of CP-violating effects in $D^\pm$ decays would imply that the $\Delta S/\Delta C = 1, 0, -1$ pieces of the charm-changing interaction are each, separately, definite under CP (even or odd). Suppose nevertheless that the D_1, D_2 states have CP impurities. There are then two alternatives to be considered. According to one of them, the $\Delta S/\Delta C = 1, 0, -1$ interactions are all CP-conserving, in which case the impurities would have to be attributed to a more remote cause; most simply a second, superweak interaction, CP-violating and with $|\Delta C| = 2$. A second alternative, already discussed in Sec. I, would involve a clash between the CP properties of the $\Delta S/\Delta C = 1$ and $\Delta S/\Delta C = -1$ interactions. For definiteness let us suppose that the $\Delta S/\Delta C = 1$ and 0 terms are even; $\Delta S/\Delta C = -1$, odd. Let us furthermore suppose that the D^0 particle is pseudoscalar. For the remaining discussion we concentrate on the observational distinction between these two alternatives, supposing that the $\Delta S/\Delta C = 0$ interactions are nonvanishing.[6]

Consider first the decay to a self-conjugate channel with $CP = +1$ e.g., $\pi^+\pi^-$, K^+K^-, etc., taking the $\pi^+\pi^-$ example as prototype. On either of the alternatives considered above the $\Delta S/\Delta C = 0$ interaction, which is the only one relevant here, is pure under CP ($CP = +1$ for definiteness). It therefore follows that the $D^0 \to \pi^+\pi^-$ and $\bar{D}^0 \to \pi^+\pi^-$ amplitudes are equal in magnitude, and we now adopt the phase conventions which make them equal also in phase. Let $B(\pi^+\pi^-)$ be the time-integrated branching ratio for the $\pi^+\pi^-$ channel for the case where the initial state is D^0; define $\bar{B}(\pi^+\pi^-)$ in a similar way for initial state $\bar{D}^0$. We then find

$$\frac{B(\pi^+\pi^-)}{\overline{B}(\pi^+\pi^-)}=\rho^2\frac{1-y+|\epsilon_C|^2(1+y)+2\alpha(\mathrm{Re}\epsilon_C-x\,\mathrm{Im}\epsilon_C)}{1-y+|\epsilon_C|^2(1+y)-2\alpha(\mathrm{Re}\epsilon_C-x\,\mathrm{Im}\epsilon_C)},\tag{8}$$

where

$$\rho\equiv\left|\frac{1-\epsilon_C}{1+\epsilon_C}\right|.\tag{8$'$}$$

Phase conventions having been set, the real and imaginary parts of ϵ_C have now been objectively defined. Altogether then, we are dealing with four parameters for the D_1, D_2 system: $\mathrm{Re}\epsilon_C$, $\mathrm{Im}\epsilon_C$, x, and y. Two expressions which relate these to observable quantities are given in Eq. (7), which have a general validity. With the special assumptions presently under consideration we now also have Eq. (8). To complete the set we next consider decay to a self-conjugate channel with $CP=-1$, e.g., $K_S K_S\pi^0$, $3\pi^0$, etc. With the $K_SK_S\pi^0$ channel as prototype we find

$$\frac{B(K_S K_S\pi^0)}{\overline{B}(K_S K_S\pi^0)}=\rho^2\,\frac{1+y+|\epsilon_C|^2(1-y)+2\alpha(\mathrm{Re}\epsilon_C+x\,\mathrm{Im}\epsilon_C)}{1+y+|\epsilon_C|^2(1-y)-2\alpha(\mathrm{Re}\epsilon_C+x\,\mathrm{Im}\epsilon_C)}.\tag{9}$$

Equations (7)–(9) are a complete set for the parameters under discussion. In writing them we have been concerned to show as a matter of principle how the wanted information can be obtained. Of course, it cannot be said, even if charm were found, that the measurements would be easy. Equations (8) and (9) rest on the assumption that the $\Delta S/\Delta C=0$ interaction is CP-conserving, but this can be tested independently in $D^\pm$ decays and also directly, in the sense that Eqs. (8) and (9) are supposed to hold, respectively, for any $CP=1$ or $CP=-1$ channel.

We are concerned with observational differences between the superweak case and CP clash alternatives. The first such distinction may now be noted. It concerns the parameters, x, y, $\mathrm{Re}\epsilon_C$, and $\mathrm{Im}\epsilon_C$. For the superweak case they are related according to

$$-\frac{2x}{y}=\frac{1+|\epsilon_C|^2}{1-|\epsilon_C|^2}\,\frac{\mathrm{Im}\epsilon_C}{\mathrm{Re}\epsilon_C}\tag{10}$$

(for $|\epsilon_C|^2\ll 1$ this reduces to a familiar result). No such relation need hold for the CP clash case.

Further distinctions arise when we turn to decay channels with nonvanishing strangeness. Let us illustrate this on the example of the pair of conjugate channels $K^-\pi^+$ and $K^+\pi^-$. We deal here with four branching ratios: initial state D^0 or $\overline{D}^0$, decay channel $K^-\pi^+$ or $K^+\pi^-$. Let $B(S)$, $S=\pm 1$, be the branching ratio to final states with strangeness S, for initial state D^0; similarly, let $\overline{B}(S)$ refer to the case where the initial state is $\overline{D}^0$. Define

$$\begin{aligned}&\mathrm{Amp}(D^0\to K^-\pi^+)=a,\quad \mathrm{Amp}(\overline{D}^0\to K^+\pi^-)=a',\\&\mathrm{Amp}(\overline{D}^0\to K^-\pi^+)=b,\quad \mathrm{Amp}(D^0\to K^+\pi^-)=b'.\end{aligned}\tag{11}$$

Clearly if the $\Delta S/\Delta C=1$ and -1 interactions have the same, pure CP properties (even, say, under CP), than $a'=a$, $b'=b$. We have called this the superweak alternative. If the two interactions have definite, but clashing properties (say even and odd, respectively), then $a'=a$, $b'=-b$. We have called this the CP clash alternative. On either scheme the basic amplitudes a and b constitute three essential parameters; two magnitudes and a relative phase. Since there are four branching ratios this implies a single relation among them, depending on ϵ_C, x, and y but independent of a and b. The important point is that the relation so obtained for the clash alternative is different from the one that holds for the superweak alternative. Hence the two schemes are distinguishable on the basis of time-integrated branching ratio data. It is enough at the present time to merely call attention to this matter of principle. For general values of the parameter ϵ_C the relations in question are rather complicated. We content ourselves here with the limiting case where the CP impurity parameter ϵ_C is negligible. In this limit, as it happens, a degeneracy develops for the superweak alternative such that one finds two relations among the branching ratios. For the CP clash alternative no such phenomenon occurs, so that even for $\epsilon_C=0$ there is only a single relation. The results are stated below.

(i) Superweak alternative. For $\epsilon_C=0$,

$$\frac{B(+1)}{\overline{B}(-1)}=1,\quad \frac{B(-1)}{\overline{B}(+1)}=1\tag{12}$$

for any pair of conjugate channels with $S=\pm 1$.

(ii) CP clash alternative. Consider, specifically, a pair of two-body channels with $S=\pm 1$ and define

$$\begin{aligned}N_1&=[B(+1)+B(-1)-\overline{B}(+1)-\overline{B}(-1)],\\N_2&=\alpha[B(+1)+\overline{B}(+1)-B(-1)-\overline{B}(-1)],\\N_3&=(1+\alpha)[B(+1)+\overline{B}(-1)]\\&\quad-(1-\alpha)[B(-1)+\overline{B}(+1)],\\N_4&=(1+\alpha)[B(-1)+\overline{B}(+1)]\\&\quad-(1-\alpha)[B(+1)+\overline{B}(-1)].\end{aligned}\tag{13}$$

Then for $\epsilon_C=0$,

$$y^2N_1{}^2+\alpha^2x^2N_2{}^2=x^2y^2N_1N_4,\tag{14}$$

where the parameters x, y, and α have been defined previously in Eqs. (5) and (6). We repeat that Eqs. (12) and (14) refer to zeroth order in ϵ_C, but already in this order they suffice to reveal an observational distinction between the superweak and clash alternatives. Recall also that these alternatives are indistinguishable outside of the D_1, D_2 system.

There is one further remark to be made. We have considered a pair of conjugate, exclusive channels ($K^-\pi^+$ and $K^+\pi^-$) to illustrate and distinguish the two schemes under discussion. We may ask what happens when one treats decays *inclusively*, grouping together all $S=1$ and, separately, all $S=-1$ channels. Let us retain the symbols $B(S)$, $\overline{B}(S)$ and $S=\pm 1$, for these inclusive branching ratios. The analysis here is facilitated by taking, as states to be summed over, the eigenstates of the strong interaction scattering matrix. For the superweak alternative the amplitudes a and b [see Eqs. (11)] are relatively real for each such channel. This is a consequence of the CPT theorem. For the CP clash alternative it similarly follows that a and b are 90° out of phase. For *either* scheme, and for any ϵ_C, one then finds the relation

$$\frac{B(+1)-B(-1)}{\overline{B}(+1)-\overline{B}(-1)}=\rho^2\frac{1-\rho^2+\alpha(1+\rho^2)}{1-\rho^2-\alpha(1+\rho^2)}, \qquad (15)$$

where ρ has been defined in Eq. (8′), and where ρ and α are expressed in terms of semileptonic branching ratios in Eq. (7). Although the *inclusive* measurements cannot distinguish between the superweak and clash alternatives, Eq. (15) *does* test the assumption that the $\Delta S/\Delta C=1$ and -1 interactions are separately pure with respect to CP, whether or not they clash in their CP properties.[7]

After this work was completed, we learned of a publication by Okun', Zakharov, and Pontecorvo[8] which treats some of the issues discussed here.

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[6]In the CP clash case, if the $\Delta S/\Delta C=0$ interaction were absent altogether, there would be no CP-violating effects whatsoever in charmed-particle decays.

[7]One can see by example that CP violation effectively of the superweak type, both for $|\Delta S|=2$ and $|\Delta C|=2$, can be achieved in the framework of gauge theories. On the other hand, the CP clash schemes at least in theories of the $SU(2)\times U(1)$ type, cannot be implemented exactly, although it can be achieved approximately.

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