Pion inclusive momentum distribution at 90° in a hydrodynamical model*

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We show that pion inclusive momentum distributions in pp collisions at 90° from 24 to 1500 GeV lab energies can be accounted for by a hydrodynamical model which has frame-independence symmetry and incorporates the evaporation phenomena. Within our solution, during its space-time development, the matter system only possesses a local thermal equilibrium but not a global equilibrium. The proper-time interval needed to achieve this equilibrium comes out to be comparable (with $c=1$) to the longitudinal dimension estimated previously based on considerations of quantum statistical fluctuation.

I. INTRODUCTION

The observed pion transverse momentum distribution in pp collisions at 90° at present Fermilab and CERN-ISR energies shows interesting trends in both its momentum dependence and its energy dependence. Some typical data[1] ranging from 24 to 1500 GeV incident laboratory energy (E_L) are illustrated in Fig. 1. The distribution has a sharp dropoff in the region of small transverse momenta (p_T). (Both E_L and p_T will be given in units of GeV, with c = 1). As p_T increases, there is a gradual flattening trend. For E_L dependence, one sees that the magnitude of the inclusive cross section at any p_T increases with energy. Furthermore, in the large-p_T region, as the energy increases the distribution becomes flatter and flatter, in reminiscence of the so-called antishrinkage phenomena occurring in some elastic scattering processes.

Several authors[2-5] have already observed that these features may be qualitatively accounted for by the Fermi statistical model,[6] or, more appropriately, its extension—Landau's model.[7] Within the hydrodynamical model, the space-time description of the matter system formed in hadron collisions is, in general, divided into three distinct stages: the formation of a highly condensed hadronic matter system, its expansion, and the subsequent breakup of this system. The initial matter system has a high temperature which is determined by the incident energy. As the system expands it gradually cools off. Local systems break up at the critical temperature $T_c \sim m$, where m is the pion mass.

In the framework of the hydrodynamical model, one considers those pions with small p_T to be predominantly produced at breakup and those with larger p_T to be produced mainly from evaporation during the period of hydrodynamical expansion. Since the initial stage is the hottest, its evaporation will contribute mainly to those very-large-momentum events. If one assumes either Fermi-Dirac or Bose-Einstein momentum distributions for the constituents of the matter system and takes into account pions produced from all three stages, one naturally finds a general flattening feature in the p_T distribution. Also, as the energy increases, the initial temperature becomes higher and higher; in turn, for any given large p_T, more and more pions will be emitted from the matter system. The

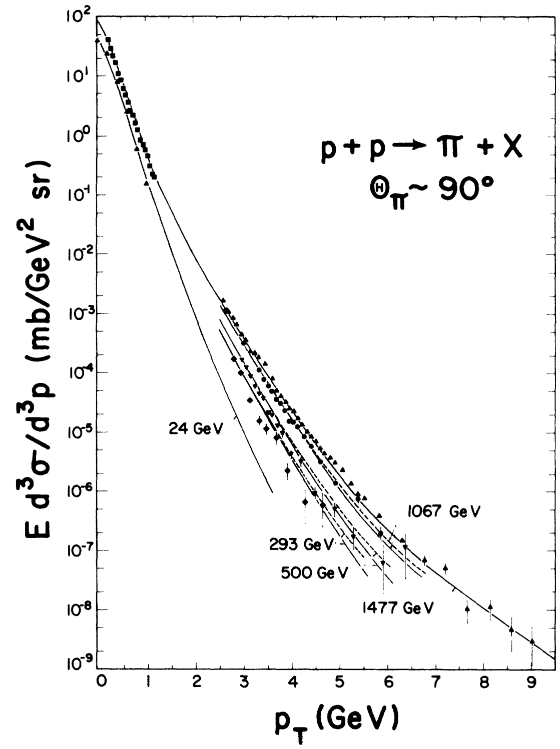

FIG. 1. Pion inclusive transverse-momentum distributions. The solid curve at $E_L = 1477$ GeV is our fit to the data. Those curves at other energies are two sets of theoretical predictions (see text). Data points in the small-p_T region: ■ are at $E_L \sim 1477$ GeV from Banner *et al.* (Ref. 1b); ▲ are at $E_L \sim 24$ GeV from Blobel *et al.* (Ref. 1c). In the large-p_T region, data are from Büsser *et al.* (Ref. 1a). Points with ◆ are at $E_L = 293$ GeV, ▼ at 500 GeV, ● at 1067 GeV, and ▲ at 1477 GeV.

larger is the p_T value, the more pronounced is the effect. This then gives rise to the observed anti-shrinkage phenomena. These points have recently been stressed by Heiko.[4] Also, Gorenshtein *et al.*[5] have made some detailed calculations on the contribution of evaporation based on Landau's model. They found qualitative agreement with the data.

Intrigued by the qualitative success of the hydrodynamical description, we carry out a quantitative calculation on evaporation. The main new feature introduced in our calculation is that we take into account the quantum-mechanical effect in the specification of the initial condition. Also, we base our investigation on a hydrodynamical model with the frame-independence symmetry (FIS). The solution of this hydrodynamical model has been proposed independently in different contexts by various authors.[8-11] Our discussion below will adopt the point of view of Ref. 10.

Our plan for the remainder of the paper is as follows. In Sec. II, we discuss the model considered. In Sec. III we present the phenomenological analysis of the p_T data, and we discuss the implications of our solution in Sec. IV. Several parameters of the FIS solution used are given explicitly in the Appendix A. Some remarks on the frame-independence symmetry model are given in Appendix B.

II. EVAPORATION IN THE HYDRODYNAMICAL MODEL

A. Properties of matter system and possible starting points of thermal equilibrium

As usual we take the point of view that immediately after a pp collision a condensed matter system is formed, which may or may not include the initial proton (s). We assume for definiteness that the constituents within this matter system are massless quarks and antiquarks.[12] After local thermal equilibrium is reached, these constituents are essentially free particles, and they are assumed to satisfy the Fermi-Dirac distributions. In particular, for a local temperature T, the momentum distribution for either quark or antiquark is given by

$$f(p') \equiv \frac{dn}{d^3p'} = \frac{g}{8\pi^3}\,\frac{1}{\exp(p'/T)+1}, \tag{1}$$

where $\vec{p}\,'$ and p' are the momentum and energy of constituents defined in the rest frame of local cells. We consider here only nonstrange quarks and take the statistical weight $g=4$, corresponding to "up" and "down" spin-$\frac{1}{2}$ quarks. For this local system, one has[7]

$$\epsilon = 3p = a_\epsilon T^4 \quad \text{and} \quad n = a_n T^3,$$

with (2)

$$a_\epsilon = 7\pi^2 g/240 \quad \text{and} \quad a_n = 3g\zeta/4\pi^2.$$

Here, ϵ and p are the proper energy density and the proper pressure, respectively, and ζ stands for the zeta function $\zeta=\zeta(3)\approx 1.202$.

We now turn to the following question: At what stage is the thermal equilibrium of the system achieved? Fermi[6] and also Landau[7] assume that this thermal equilibrium is achieved while the system still maintains its initial volume. Taking into account the Lorentz contraction, in the c.m. system the initial volume at the instance when two protons overlap each other is of the order of

$$V_1 \approx \frac{2\pi}{m^2} z_1 \quad \text{with } z_1 = \frac{1}{2m}\left(\frac{2M}{E_L}\right)^{1/2}, \tag{3}$$

where M is the proton mass. With the assumed equation of state, the temperature of the system is given by

$$T_1 \approx (m^3/\pi a_\epsilon)^{1/4} E_L^{1/4}. \tag{4}$$

On the other hand, it has been pointed out[13] that the uncertainty principle does not allow a system in thermal equilibrium to be packed to within the alleged volume of Eq. (3). It is estimated that the thermal equilibrium may be reached when the corresponding volume is[13]

$$V_2 \approx 2\pi z_2/m^2 \quad \text{with } z_2 \approx 0.5\eta(2m^4M)^{-1/6}E_L^{-1/6}, \tag{5}$$

where η is the quantum fluctuation parameter, which measures the accuracy of the quantum statistical approximation.[14]

B. Frame-independence symmetry model with quantum-mechanical initial condition

We mentioned earlier that we based our investigation on the FIS model. We recall that, in contrast to Landau's model, the FIS model predicts[11] the asymptotic behavior of the width of the longitudinal rapidity distribution, $\Delta y \propto \ln E_L$. Within this model, immediately after the collision, there is a short period of violent acceleration along the longitudinal direction not governed by a simple hydrodynamical expansion. Some time after this violent acceleration ceases, local systems reach a state of frame-independence symmetry with thermal equilibrium. Since local cells are now in motion it is more appropriate to use the local proper time than the c.m. time to describe the time evolution of the properties of local systems. Taking the quantum-mechanical effect into account, we assume that the proper time $\tau=\tau_2$ taken for local system to reach the onset of FIS is characterized by z_2, although z_2 was originally estimated for a stationary system. Thus one has

$$\tau=\tau_2\approx 0.5\eta(2m^4M)^{-1/6}E_L{}^{-1/6}, \tag{6}$$

where[15] $\tau=(t^2-z^2)^{1/2}$.

The FIS assumed amounts to specifying the initial condition that at, $\tau=\tau_2$, $v(\tau_2)=z/t$ and $\epsilon(\tau_2)$ = const. From this point on, the time development of the system is governed by the hydrodynamical equation of motion of the ideal fluid:

$$\partial_\mu T^{\mu\nu}=0,$$

with

$$T^{\mu\nu}=(\epsilon+p)u^\mu u^\nu-g^{\mu\nu}p \tag{7}$$

and

$$u^\mu=(t/\tau,0,0,z/\tau)\equiv(\cosh\alpha,0,0,\sinh\alpha),$$

where $g^{00}=-g^{ii}=1$. It turns out that this equation of motion preserves the FIS until breakup. Owing to the conservation of energy,[11] the c.m. rapidities of cells are confined to $|\alpha|<\alpha_M$, where α_M depends on E_L and is given in Appendix A.

C. Evaporation during the hydrodynamical expansion

To ensure the Lorentz invariance, the law of evaporation assumed is most conveniently stated in the rest frame of the cell. We assume that the number of constituents evaporated per unit area per proper-time interval $d\tau'$ within a phase-space volume d^3p' for a cell with c.m. rapidity α is proportional to the differential proper number density of Eq. (1), i.e.,

$$dN_{ev}=2C_2\left\{\frac{2\pi}{m}dz'+\frac{\pi}{m^2}\left[\delta(\alpha-\alpha_M)+\delta(\alpha+\alpha_M)\right]d\alpha\right\}\times d\tau' f(p')d^3\vec{p}\,' , \tag{8}$$

where the factor of 2 is due to the emission of both quarks and antiquarks. We recall that $f(p')$ is the differential number density for either quarks or antiquarks. The first term in the curly brackets corresponds to the area of a ring-shaped surface element, while the second term corresponds to the areas of the two ends. For the latter case the quarks or antiquarks evaporated must have longitudinal rapidity in the c.m. system equal to α_M or $-\alpha_M$. Since we are interested in a near-90° inclusive distribution, we can easily check that the last term gives a negligible contribution. From Eq. (1) it follows that

$$dN_{ev}\approx\frac{C_2 g}{2\pi^2 m}dz'd\tau'\,\frac{d^3\vec{p}\,'}{\exp(p'/T)+1} . \tag{9}$$

We suppose as usual that within the matter system quarks and antiquarks are moving freely as noninteracting particles.[16-17] To describe evaporation we invoke the usual confinement mechanisms for quarks.[17-19] The assumed intuitive picture of interactions among quarks and antiquarks is as follows. As soon as a quark escapes from the matter system, the energy stored in the space between this quark and a certain antiquark within this system increases rapidly, so the field flux between the paired quark and antiquark acts like a rubber band. "Tension" builds up quickly. It eventually forces the antiquark to join the initial escaping quark. If the average over the original isotropic momentum distribution of this antiquark partner is taken, the mean meson momentum of the eventual quark-antiquark pair will be approximately the same as the initial quark momentum. We ignore the smearing due to the folding of the antiquark distribution and identify approximately the quark momentum p' to be the momentum of the eventual pion. Since the evaporation effect of interest is mainly in the region where $p'\gg m$, the energy of this eventual pion is $E'\approx p'$. Hereafter we take Eq. (9) to be the corresponding pion momentum distribution.

We make use of the kinematic relations

$$y=y'+\alpha,$$

$$p_T'=p_T,$$

$$dz'd\tau'=dz\,dt=\tau\,d\alpha\,d\tau,$$

and (10)

$$\frac{d^3p'}{E'}=\frac{d^3p}{E}=dy\,d^2p_T ,$$

where $(\vec{p},E)$ and y are the c.m. four-momentum and c.m. longitudinal rapidity and y' stands for the pion longitudinal rapidity in the rest frame of the cell. The c.m. rapidity of the cell, as before, is α.

We make use of Eq. (A6) to change variable from τ to T. Now Eq. (9) can be rewritten as

$$E\frac{d\sigma}{d^3p}=\sigma\frac{dN_{eV}}{dy\,d^2p_T}$$

$$=C_2\sigma\frac{g}{2\pi^2m}\int_{\tau_2}^{\tau_c}\tau\,d\tau\int_{-\alpha_M}^{\alpha_M}d\alpha\frac{m_T\cosh(y-\alpha)}{\exp[m_T\cosh(y-\alpha)/T]+1}$$

$$=\frac{2C_2\sigma b^2m^3}{3g\xi^2}\left(\frac{E_L^{1/4}}{\alpha_M}\right)^2\int_{T_c}^{T_1}\frac{dT}{T^7}\int_{-\alpha_M}^{\alpha_M}d\alpha\frac{m_T\cosh(y-\alpha)}{\exp[m_T\cosh(y-\alpha)/T]+1}$$

$$=\frac{4C_2\sigma b^2m^3}{3g\xi^2}\left(\frac{E_L^{1/4}}{\alpha_M}\right)^2\int_0^{\alpha_M}\frac{d\alpha}{[m_T\cosh(y-\alpha)]^5}\sum_{n=1}^{\infty}(-1)^{n-1}F_n\,, \tag{11}$$

where

$$m_T=(m^2+p_T^2)^{1/2},\quad x_c=\frac{m_T\cosh(y-\alpha)}{T_c},\quad x_2=\frac{m_T\cosh(y-\alpha)}{T_2},$$

and

$$F_n=\frac{5!}{n^6}\sum_{m=0}^{5}\left[\frac{(x_cn)^m}{m!}\exp(-x_cn)-\frac{(x_2n)^m}{m!}\exp(-x_2n)\right]. \tag{12}$$

σ is the inelastic cross section, which is taken to be independent of E_L.

III. PHENOMENOLOGICAL ANALYSIS OF p_T DISTRIBUTION

The inclusive p_T distribution is contributed by pions from all three stages:

I. production at breakup,

II. evaporation during hydrodynamical expansion, and

III. emission during the violent acceleration stage.

Thus we write

$$E\frac{d^3\sigma}{d^3p}=\frac{d^3\sigma}{dy\,d^2p_T}=\left.\frac{d^3\sigma}{dy\,d^2p_T}\right|_{\rm I}+\left.\frac{d^3\sigma}{dy\,d^2p_T}\right|_{\rm II}+\left.\frac{d^3\sigma}{dy\,d^2p_T}\right|_{\rm III}. \tag{13}$$

The contribution from the first region is given by[10]

$$\left.\frac{d^3\sigma}{dyd^2p_T}\right|_{\rm I}=C_1\sigma\frac{N}{2\alpha_M}\int_{-\alpha_M}^{\alpha_M}d\alpha\frac{m_T\cosh(\alpha-y)}{\exp[m_T\cosh(\alpha-y)/T_c]-1}, \tag{14}$$

where

$N=bE_L^{1/4}$ with $b\approx 3.0$ GeV^{-1}.

Note that at breakup the Bose-Einstein distribution for pions is used. For the present case of interest, we set $y=0$. The contribution of region II is given by Eqs. (11) with (12), again with $y=0$.

In order to compare the relative amount of evaporation between regions II and III, we assign the same law of evaporation to region III as to region II, except for a possible difference in the coefficients of evaporation. Furthermore, we approximate the total contribution in this interval by that of an effective rest system with some mean longitudinal dimension and mean temperature. For this approximate system, the corresponding expression analogous to Eq. (9) is given by

$$\left.\frac{d\sigma}{dy\,d^2p_T}\right|_{\rm III}=C_3\sigma(2\bar z\tau_2)\frac{g}{2\pi^2m}\frac{m_T}{\exp(m_T/\bar T)+1}, \tag{15}$$

where we take for definiteness

$$\bar z=(z_1+z_2)/2 \text{ and } \bar T=(T_1+T_2)/2. \tag{16}$$

It turns out that our fits are not too sensitive to the specific mean values assumed.

Within our approach, $C_1\sigma$, $C_2\sigma$, and $C_3\sigma$ are adjustable parameters which govern the normalization of contributions from the three regions, while the slopes of these contributions are controlled by the temperature $T_c\approx m$, the temperature ranging from T_2 to T_c, and $\bar T$. Through a crude fit to the data at $E_L=1477$ GeV, we fix the normalization coefficients to be

$$C_1\sigma = 145.8 \text{ mb GeV}^{-3},$$
$$C_2\sigma = 0.61 \text{ mb},$$
and
$$C_3\sigma = 1.4\times 10^{-4} \text{ mb} \tag{17}$$

and the quantum fluctuation parameter of Eq. (5), $\eta = 1.2$.

The inclusive p_T distributions at other energies are parameter-free predictions of our model. The comparison between predictions (solid curves) and the data together with the original fit at $E_L = 1477$ GeV are illustrated in Fig. 1. Notice that the gross features of the distributions, both p_T dependence and the antishrinkage phenomena, are reasonably well reproduced. The dashed curves illustrated in Fig. 1 correspond to another solution, where different prescriptions[20] for the mean values $\bar{z}'$ and $\bar{T}'$ are used.

For completeness, we illustrate here the energy dependence of some characteristic temperatures and longitudinal dimensions of the matter system of the present solution. The P_L ($\approx E_L$) dependence of the temperatures T_1, T_2, and $\bar{T}$ is shown in Fig. 2(a), and correspondingly the longitudinal dimensions z_1, z_2, and $\bar{z}$ are shown in Fig. 2(b). In order to see the relative magnitude of z_1 and z_2, we introduce the ratio $R = z_2/z_1$, which is also included in Fig. 2(b). At $P_L \approx E_L = 24$ GeV, $R \approx 1$, while at $E_L = 10^3$ GeV, $R \approx 4$. As a function of energy, R grows like $E_L{}^{1/3}$.

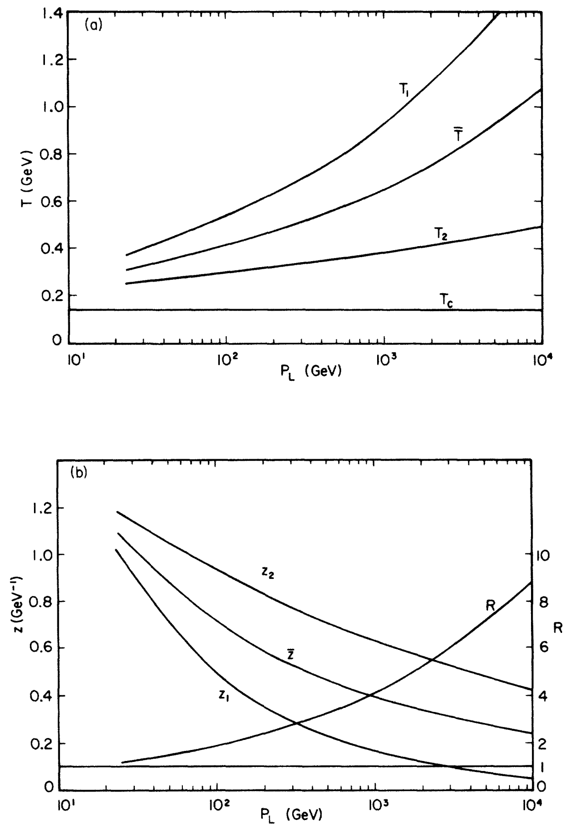

FIG. 2. (a) Momentum dependence of some characteristic temperatures of the matter system. (b) Momentum dependence of some characteristic longitudinal dimensions of the matter system, where $R = z_2/z_1$.

IV. DISCUSSION

The most striking feature in the present solution is the fact that the evaporation in region III is strongly suppressed. This is reflected in the smallness of the ratio

$$C_3/C_2 \approx 2.3\times 10^{-4}. \tag{18}$$

Since in our fit the coefficient C_3 is sensitive only to the large-p_T behavior, the lack of those expected large-p_T events from region III suggests that the corresponding system must not be in thermal equilibrium with the alleged temperature ranging from T_1 to T_2.

Clearly, immediately after the collision the momenta of constituents are mainly directed along the longitudinal direction. It takes some finite time for them to acquire certain transverse momenta and eventually to reach a state of local thermal equilibrium. At this latter stage their momentum distribution is isotropic in the rest frame of local cells.

The extremely small ratio of Eq. (18) suggests that events in region III should thus be associated with the individual large-transverse-momentum collisions of the constituents instead of evaporation of local cells, so our solution gives the picture that local thermal equilibrium begins to set in at the c.m. time t_2. At this point the longitudinal dimension of our matter system is z_2, which especially at high energies is considerably larger than the initial longitudinal dimension z_1 [see Fig. 2(b)]. In fact, within our model the achievement of thermal equilibrium occurs only at the local level, and its onset is governed by the local proper time τ_2, which is comparable to the quantity z_2 given in Eq. (5).

The space-time development of the matter system of our present solution for positive z at $E_L = 1477$ GeV is depicted in Fig. 3. The characteristic dimensions of the initial system are $z_1 \approx 0.13$ GeV^{-1} and the transverse radius $m^{-1} \approx 7.14$ GeV^{-1}. This initial state immediately undergoes a violent acceleration; this period is illustrated schematically as the darkest area in Fig. 3. The mean longitudinal dimensions $\bar{z}$ of Eq. (16) and $\bar{z}'$ given in Ref. 20 for this acceleration period are also indicated in the figure. As this acceleration ceases at the proper time $\tau_2 \approx 0.6$ GeV^{-1}(4×10^{-25} sec), the local frame-independence symmetry sets in. The subsequent hydrodynamical expansion of the system is indicated by the grey area of Fig. 3. At the proper time $\tau = \tau_c \approx 21$ GeV^{-1} (1.4×10^{-23} sec), the breakup occurs. From the point of view of the c.m. frame, this breakup first happens at the center where $t=\tau_c$. Notice that at this very moment when breakup begins, the longitudinal dimension of the matter system in the c.m. frame has $z=21$ GeV^{-1}. This longitudinal dimension is about three times as large as the transverse dimension $\sim m^{-1}$ assumed. As the c.m. time further increases, the matter system is further elongated and the decay occurs for larger and larger values of α. Eventually the decay process ends at the c.m. time $t\approx 703$ GeV^{-1}(4.6×10^{-22} sec). This corresponds to the P in the figure.

Lastly we remark that in the present evaporation calculation we employed a perturbative treatment. We assumed that the evaporation does not disturb the FIS solution in any significant way. One can easily check that as energy increases the relative contribution of evaporation to the total pion production becomes more and more important. It turns out that, for example at $E_L = 1477$ GeV and $p_T = 0$, the contribution of region II is about 17% of the cross section. At $E_L = 10^6$ GeV, for example, region II contributes about 42% of the corresponding cross section. Presumably, beyond this latter energy one must deal explicitly with the nonperturbative treatment.

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APPENDIX A: DETERMINATION OF τ_c, α_M, AND T_2

The FIS solution[8-10] gives the reciprocal law of expansion

$$n = \frac{\text{const}}{\tau} = \frac{n_c \tau_c}{\tau}, \tag{A1}$$

and a plateau for the cell longitudinal rapidity dis-

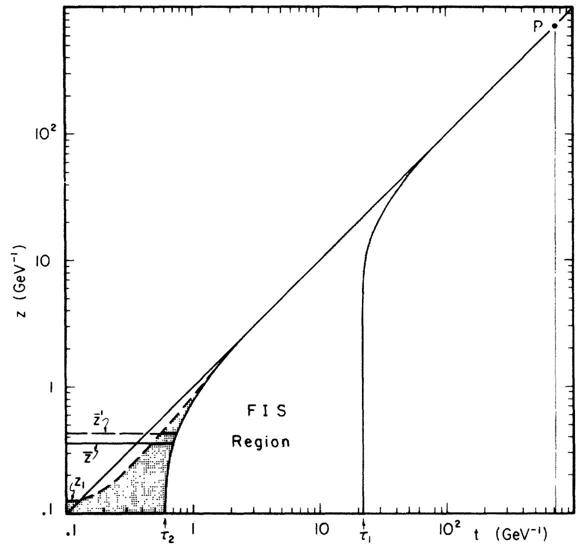

FIG. 3. A schematic illustration of the space-time development of the matter system considered at $E_L = 1477$ GeV, for $z > 0$.

tribution

$$\frac{dN}{d\alpha} = \frac{\pi}{m^2} n_c \tau_c \quad \text{for } |\alpha| \le \alpha_M \text{ and } u_x = u_y = 0,$$
$$= 0 \quad \text{otherwise,} \tag{A2}$$

or

$$N = \frac{2\pi}{m^2} n_c \tau_c \alpha_M. \tag{A3}$$

With $T_c \approx m$, one finds $n_c = (3/4\pi)m^3$. On the other hand, we take from the data

$$N \sim b E_L^{1/4}, \quad \text{with } b = 3.0. \tag{A4}$$

From (A3) and (A4),

$$\tau_c = \frac{m^2 b}{2\pi n_c} \frac{E_L^{1/4}}{\alpha_M}. \tag{A5}$$

In general, we have

$$\tau = \frac{m^2 b}{2\pi a_n} \frac{E_L^{1/4}}{\alpha_M T^3}. \tag{A6}$$

To determine α_M one needs to evaluate the inclusive energy sum rule with pion inclusive momentum distribution based on the step-function longitudinal rapidity distribution of cells [see Eqs. (14) and (A2)]. Unfortunately, even at the highest energy available the central plateau does not dominate the observed inclusive distribution. At the present energy, the energy sum rule only provides the relevant energy dependence up to a multiplicative constant. So we write

$$E_L^{1/2} \propto \int_0^{\alpha_M} \cosh \alpha \frac{dN}{d\alpha} d\alpha = \frac{\pi}{m^2} n_c \tau_c \sinh \alpha_M. \quad (A7)$$

From Eqs. (A3), (A4), and (A7) we arrive at the transcendental equation

$$\frac{\sinh \alpha_M}{\alpha_M} = F E_L^{1/4}, \quad (A8)$$

where F is a parameter. We take the experimental effective value $\alpha_M = 4.2$ at $E_L = 1477$ GeV. This gives $F = 0.13$. For $E_L = 1067$, 500, 293, and 24 GeV, from (A8), we obtain respectively $\alpha_M = 4.1$, 3.9, 3.7, and 2.8. From Eq. (A6) we take $\tau = \tau_2$, as given in Eq. (6), and obtain

$$T_2 = a_2 \alpha_M^{-1/3} E_L^{5/36}$$

with (A9)

$$a_2 = m \left(\frac{b}{\pi a_n \eta}\right)^{1/3} \left(\frac{2M}{m^2}\right)^{1/18}.$$

APPENDIX B: REMARKS ON THE FRAME-INDEPENDENCE SYMMETRY MODEL

The essential difference between the FIS model and Landau's model lies in the difference of initial conditions. Landau assumes that after collision the matter system quickly reaches its state of ideal fluid, while the system is still at rest, so the initial conditions are specified for the global system. In particular, at some c.m. time t_1, they are $\epsilon(z, t_1) = \text{constant}$ and $v(z, t_1) = 0$. These conditions together with the hydrodynamical equations of motion lead to his one-dimensional solution. This solution predicts a Gaussian rapidity distribution with its width $\Delta y \sim (\ln E_L)^{1/2}$. Such a prediction gives reasonable fits to the data, at least up to $E_L \sim 1000$ GeV or so. However, the asymptotic behavior of Δy predicted here differs from that of some other models, such as the multiperipheral model, which has the ultimate behavior $\Delta y \propto \ln E_L$.

On the other hand, the FIS model does predict the asymptotic behavior $\Delta y \propto \ln E_L$. In this model, the matter system formed immediately after the collision is required first to undergo a certain violent acceleration which is not governed by a simple hydrodynamical expansion. We speculate that this acceleration may be related to the quantum-mechanical effect, preventing a highly compressed matter system from standing still. It is only sometime after this violent acceleration has ceased that the state of ideal fluid is reached and followed by simple hydrodynamical expansion. Since the local elements of ideal fluid are in motion, the initial condition for the subsequent expansion should be stated for the local system. We choose the initial conditions to be at some local proper time τ_2 with $\epsilon(\tau_2) = \text{constant}$ and $v(\tau_2) \sim z/t$, as stated later on in the text. To completely specify the FIS model, one further invokes the effect of surface tension at the boundary of the matter system. In the course of hydrodynamical expansion, it turns out there is a continuous transfer of the energy from the constituents onto the surface. The total energy, which is the sum of the energy of the constituents and that of the surface, is conserved.

Lastly, in motivating the initial condition of local equilibrium above, we have adopted the point of view of Landau that the initial matter system first comes to a standstill and subsequentlv expands violently. Actually, the FIS initial condition may alternatively be motivated without ever referring to this standing-still picture. In particular, let us assume that there is a violent interaction between the two overlapping hadrons which takes place only over some small space-time domain. Denote z_0 and t_0 to be the space-time coordinates of a typical cell when leaving this domain. And afterwards, we assume that it has more or less a definite longitudinal velocity v. Later on, the cell reaches a state of local thermal equilibrium at some proper time τ_2. At this instant the longitudinal coordinate of the cell is given by $z \sim z_0 + v(t - t_0) \approx vt$, where in the last step we assume $vt \gg z_0 - vt_0$. At τ_2, this spatial distribution of the velocity together with the condition $\epsilon = \text{constant}$ constitutes the FIS initial condition.

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[1]Pion inclusive p_T distribution data points plotted in Fig. 1:

(a) $pp \to \pi^0 X$: CERN-Columbia-Rockefeller collaboration, F. W. Büsser *et al.*, Phys. Lett. 46B, 471 (1973).
(b) $pp \to \pi^- X$, at $\sqrt{s} = 52.7$ GeV (or $E_L \sim 1477$ GeV): M. Banner *et al.*, Saclay-Strasbourg Collaboration, Phys. Lett. 41B, 547 (1972).
(c) $pp \to \pi^- X$, at $E_L = 24$ GeV: B. V. Blobel *et al.*, Nucl. Phys. B69, 454 (1974).

For Fermilab data and other available data see, for example, the review by S. D. Ellis and R. Thun, CERN Report No. TH 1874, 1974 (unpublished).

[2]E. L. Feinberg, in Proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-356 (1973)].

[3]Meng Ta-Chung, Phys. Rev. D 9, 3062 (1974).

[4]L. Heiko, paper contributed to the XVII International Conference on High Energy Physics, London, 1974 (unpublished). For discussion on large-transverse-momentum distribution in context of hydrodynamical model without invoking evaporation, see S. Sohlo and G. Wilk, Lett. Nuovo Cimento 13, 375 (1975). lished).

[5]M. I. Gorenshtein *et al.*, Kiev Report No. ITP-74-95E, 1974 (unpublished).

[6]E. Fermi, Prog. Theor. Phys. 5, 570 (1950).

[7]L. D. Landau, Isv. Akad. Nauk SSSR 17, 51 (1953); S. V. Belenkij and L. D. Landau, Usp. Fiz. Nauk 56, 309 (1955) [Nuovo Cimento Suppl. 3, 15 (1956)].

[8]R. C. Hwa, Phys. Rev. D 10, 2260 (1974).

[9]F. Cooper, G. Frye, and E. Schonberg, Phys. Rev. D 11, 192 (1975). The scaling solution in their work is essentially the same as the solution of the FIS model discussed in Ref. 10. However, physical motivations are different. These authors regard the scaling solution as an approximation to the exact solution of Landau's model. For discussion on the essential difference between the FIS model and Landau's model, see Appendix B.

[10]C. B. Chiu, E. C. G. Sudarshan, and K.-H. Wang, Phys. Rev. D 12, 902 (1975).

[11]See Appendix B.

[12]C. B. Chiu and K.-H. Wang, Phys. Rev. D 12, 272 (1975). For definiteness, we considered the case where constituents are massless quarks and antiquarks. With the presence of gluons in mind, presumably we should refer to them as the "dressed" quarks and antiquarks. Both the masslessness and the spin-$\frac{1}{2}$ characters assumed are not crucial for our discussions below. The matter system we are concerned with has typically a temperature range of a few pion masses. So long as the effective mass of the dressed quarks is not too large compared to its temperature, the equation of state of the matter system will not deviate significantly from Eq. (2). Also, the "plus-one term" in the Fermi-Dirac distribution of Eq. (1) does not play an important role in the application to evaporation below, since there one deals mainly with the kinematic region $p' \gg T$.

[13]J. Nowakowski and F. Cooper, Phys. Rev. D 9, 771 (1974). For earlier consideration on quantum effect see also S. I. Blokhintsev, Zh. Eksp. Teor. Fiz. 32, 350 (1957) [Sov. Phys.—JETP 5, 286 (1957)].

[14]In the notation of Ref. 13, $\eta \approx j^{4/3}(E/\Delta E)^{1/3}$, where the ratio $\Delta E/E$ measures the energy fluctuation, while j is the number of partitions along the longitudinal direction.

[15]Notice that for $\tau > \tau_2$ the proper time of the local element is approximately given by $\tau = (t^2 - z^2)^{1/2}$, owing to the FIS assumed.

[16]R. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1973).

[17]Within the bag model, see A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[18]In the context of the two-dimensional QED model, see for example A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. 31, 792 (1973); Phys. Rev. D 10, 732 (1974).

[19]In the context of the string model, see for example C. Carlson, L. N. Chang, F. Mansouri, and J. F. Willemsen, Phys. Lett. 49B, 377 (1974).

[20]For the dashed curves illustrated in Fig. 1, the average values of the power of E_L for quantities z_1 and z_2 in Eqs. (3) and (5) and that of T_1 and T_2 in Eqs. (4) and (A9) are used. In particular,

$$\bar{z}' = \frac{1}{m}\left(\frac{2M}{E_L}\right)^{1/2}\left(\frac{E_L}{M}\right)^{1/6}$$

and $\bar{T}' = 0.18\, E_L^{7/36}$.

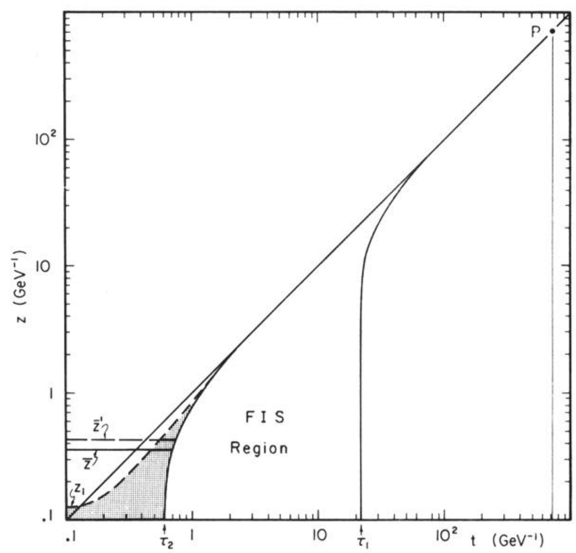

FIG. 3. A schematic illustration of the space-time development of the matter system considered at $E_L = 1477$ GeV, for $z > 0$.