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$^3P_0$ model and helicity structure of photoproduction couplings

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Neglecting the $\Delta L_3 = 2$ term within the possible algebraic structure in a 35-dimensional $SU(6)_W$ representation for the magnetic-dipole operator is shown to result in the $^3P_0$ model for photodecays of nucleon resonances. We then expand the reduced L_3-changing amplitudes into partial waves and impose a simple $SU(6)_W \times O(2)_{L_3}$ constraint on the $W = 1$ and $W = 0$ parts of the $\underline{35}$. This leads to several more relations among helicity amplitudes for these decays. Using vector dominance and the partial-wave parameters from resonance decays into the lowest-mass meson 35-plet, pseudoscalar decays in particular, we get photodecay couplings which are in good agreement with partial-wave analysis.

INTRODUCTION

Following work by Melosh[1] several authors have noted that in both 0^- and radiative decays of hadronic resonances additional freedom in the matrix elements can avoid several previously inescapable conflicts of $SU(6)_W \times O(2)_{L_3}$.[2] In the Melosh construction L_3-changing terms appear naturally within the 35-dimensional representation of $SU(6)_W$ for the matrix elements of the axial charge Q_5 and the dipole operator $\vec{D}_\perp$. Assuming partial conservation of axial-vector current PCAC one can relate the 0^- decay amplitudes to matrix elements of the "good" Q_5 times a "bad" kinematical factor q^-.[3] Radiative decays can be similarly related to matrix elements of $\vec{D}_\perp$.[4,5]

Pseudoscalar decays using PCAC and the algebraic structure of the Melosh construction have been shown[6] to be equivalent to the $^3P_0$ model of Micu.[7,8] For photodecays, however, the $^3P_0$ model[9] is more restrictive.[5] We shall show explicitly the results of the $^3P_0$ model parametrization of the photodecay matrix elements. By expanding in partial waves, several new relations among helicity amplitudes are obtained. Furthermore, using vector dominance and parameters taken from the 0^- decays of baryons we shall see that the model predicts helicity couplings in quite reasonable agreement with partial-wave analysis.

MELOSH RESULTS

The lightlike dipole operator[5,10] is defined in terms of the electromagnetic current $j^\mu(x)$ by

$$\vec{D}_\perp = \int dx\, \delta(x^+)\vec{x}_\perp j^+(x),$$

where

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3), \quad j^\pm = \frac{1}{\sqrt{2}}(j^0 \pm j^3), \quad \vec{x}_\perp = (x^1, x^2), \text{ etc.}$$

For convenience we choose the rest frame of the nucleon resonance (N^*) with $+x^3$ to be the final nucleon (N) direction. The invariant matrix element for $N^* \to N\gamma$ is $\vec{\epsilon}_\perp \cdot \langle N|\vec{j}_\perp(0)|N^*\rangle$, where $\vec{\epsilon}_\perp(\xi)$ is the polarization vector of γ with helicity ξ. By wave-packet arguments in the transverse $\vec{x}_\perp$ plane (sum $i = 1, 2$)

$$\int dx\, \delta(x^+)\partial_i(\vec{x}_\perp j^i) = 0$$

and current conservation ($\partial_i j^i + \partial^+ j^- + \partial^- j^+ = 0$) we get

$$(2\pi)^3\delta_L^3(0)\langle N(p)|\vec{j}_\perp(0)|N^*(p^*)\rangle = iq^-\langle N(p)|\vec{D}_\perp|N^*(p^*)\rangle.$$

In this frame $q^+ = \vec{q}_\perp = 0$. Hence we have our assertion that the relevant matrix elements of the "bad" $\vec{j}_\perp(0)$ have the same algebraic structure as $\vec{D}_\perp$.

The fact that the measure $dx\,\delta(x^+)$ is not rotationally invariant means that those vectors which carry the representations of the "good" lightlike charges cannot have definite spin. For this reason Melosh constructed a unitary (although nonlocal) operator in the free quark model which transforms those vectors into the physical ones, i.e., the *constituent* basis. It is this basis for the $SU(6)_W \times O(3)$ algebra of constituents which contains the nucleons and their resonances.

In the constituent basis the $SU(6)_W$, ΔL_3 properties of $\vec{D}_\perp$ are[1,4,9]

$$\begin{aligned}\vec{D}_\perp \sim & (W=0,\ W_3=0,\ \Delta L_3=\pm1)\\ & +(W=1,\ W_3=\pm1,\ \Delta L_3=0)\\ & +(W=1,\ W_3=0,\ \Delta L_3=\pm1)\\ & +(W=1,\ W_3=\mp1,\ \Delta L_3=\pm2)\end{aligned} \qquad (1)$$

corresponding to the representations

$$(\underline{1}, \underline{8}) + (\underline{8}, \underline{1})_{W_3=0}, \quad (\underline{3}, \underline{\bar{3}})_{W_3=+1},$$
$$(\underline{1}, \underline{8}) - (\underline{8}, \underline{1})_{W_3=0}, \quad (\underline{\bar{3}}, \underline{3})_{W_3=-1},$$

respectively, in SU(3)×SU(3).

The four components of $\vec{D}_\perp$ in (1) are the most freedom any octet operator in a 35-dimensional representation of $SU(6)_W \times O(3)$ of constituents can have, for $\Delta J_3 = \pm 1$. Considering (1) in the usual SU(6) language [not $SU(6)_W$] the electromagnetic $\Delta J_3 = +1$ interactions transform respectively like $S_3L_+, S_+, L_+, S_-(L_+)^2$. The $^3P_0$ model has only the first three terms, i.e., no $\Delta L_3 = 2$. Early forms of the harmonic-oscillator models contain only S_+ (from $\vec{\mu}\cdot\vec{B}$) and L_+ (from $\vec{P}\cdot\vec{A}$). More sophisticated harmonic-oscillator models simply inverting the Dirac Hamiltonian (rather than H^2 as in Feynman, Kislinger, and Ravndal[11] seem to generate all the "spin-orbitlike" terms of (1) but are very difficult to calculate for baryons.[12]

THE $^3P_0$ MODEL

If we impose the constraint that the $\Delta L_3 = 2$ term of (1) does not contribute, the algebraic structure of two-body radiative decays of hadronic resonances into lower-lying hadronic states in the Melosh construction is the same as the $^3P_0$ model.[9] It is conventional in this model to impose the further condition that the relevant reduced matrix elements of the $W=1$, $\Delta L_3 = 1$ and $W=0$, $\Delta L_3 = 1$ terms of (1) are equal. In the final analysis the validity of this assumption is purely empirical. Simple crossing arguments from the quark graphs can be constructed but are outside the context of the Melosh construction. The quark-pair creation model of Le Yaouanc, Oliver, Pène, and Raynal[13] in conjunction with vector dominance will have this property.

Demanding consistency between matrix elements of Q_5 (axial charge) and $\vec{D}_\perp$ in meson decays, insisting on vector dominance assignments of the γ to the $\underline{35}$ meson ($W=1$) multiplet, demands $\Delta L_3 = 2$ does not contribute, but another relationship between $W=1$, $\Delta L_3 = 1$ and $W=0$, $\Delta L_3 = 1$ terms results which depends on whether L is even or odd.[14]

BARYON STRUCTURE

For baryons we have no consistency requirement and the vanishing of the $\Delta L_3 = 2$ is thus speculative. At present levels the data for the $\underline{56}$, $L=2$ resonances are not sufficient to decide. Moreover, parametrizations on the basis of constant (fitted) reduced matrix elements[5,15,16] (multiplied by q^- overall) certainly disagree with expected threshold behavior in the $\underline{70}$, $L=1$ multiplet. That is, the expected $|\vec{p}_\gamma|$ behavior of the reduced L_3-changing amplitudes due to symmetry breaking in the masses is inconsistent with their having constant ratios.

Let us demonstrate the point more precisely. We write the $N^* \to N\gamma$ amplitude as (notation and phases η_W are of Ref. 15)

$$\langle N\gamma|N^*\rangle = \sum_{\nu}\sum_{L_3=-2}^{+2}\sum_{W=0}^{1}(S, \lambda - L_3, L, L_3|J, \lambda)\langle N|\ \underline{35}|N^*\rangle_\nu (N|\gamma|N^*)_\nu \eta_W (1-L_3)(\tfrac{1}{2}, \lambda - 1, W, 1-L_3|S, \lambda - L_3)A_W^{(L_3)}.$$

The initial $J_3 = \lambda$ decays to a spin $J_3^\gamma = 1$ always. In the $\underline{70}$, $L=1$-plet we define partial-wave amplitudes $a^{(l=0)} = S$, $a^{(l=2)} = D$ by (here $L=1$)

$$c_l a^{(l)} = \sum_{L_3}(L, L_3, 1, -L_3|l, 0)A_{W=1}^{(L_3)} \tag{2}$$

with normalizations $c_l = (L, 0, 1, 0|l, 0)$ chosen for convenience (if $A_W^{(L_3\neq 0)} = 0$, then $S = D = SU(6)_W \times O(2)_{L_3}$ amplitudes), and eliminate the parity-violating $l = L$ term. It can be shown that using a general series in two partial waves[8,9] that S, D have the correct partial-wave behavior. This is easily seen to be the case, for example, in the D_{15} which is pure D wave. At threshold, near $q = |\vec{p}_\gamma| \sim 0$ we expect

$$a^{(l)} \sim (\text{const}) \times q^l \tag{3}$$

necessary to cancel the centrifugal-barrier singularity near $r=0$. The familiar Blatt-Weisskopf barrier factors, of course, exhibit this behavior.[17]

Hence the reduced matrix elements

$$A_{W=1}^{(L_3=1)} = \tfrac{1}{3}(D - S), \tag{4a}$$

$$A_{W=1}^{(L_3=0)} = \tfrac{1}{3}(2D + S) \tag{4b}$$

should have rather strong variations in q.

Of course, similar definitions in the case $L=2$ are possible if $A_{W=1}^{(L_3=2)} = 0$. Then we have

$$A_{W=1}^{(L_3=1)} = \frac{\sqrt{3}}{5}(F - P), \tag{5a}$$

$$A_{W=1}^{(L_3=0)} = \frac{1}{5}(3F + 2P). \tag{5b}$$

The P_{31} and F_{37} are thus pure P wave and F wave, respectively.

The additional constraint of the $^3P_0$ model is the relation

$$A_{W=0}^{(L_3=1)} = A_{W=1}^{(L_3=1)}. \tag{6}$$

This leads to the prediction for the $D_{13}(1520) \in {}^28_{3/2}$ ($\underline{70}$, $L=1^-$) that (notation is of Walker, Ref. 18)

$$\frac{A_{3/2}(n)}{A_{3/2}(p)}=-\frac{2}{3} \tag{7}$$

for the neutron vs proton $\lambda=\frac{3}{2}$ excitations. Present data are in agreement with this ratio which is quite independent of (2), having -0.75 ± 0.05,[19] -0.72 ± 0.10,[18] -0.85 ± 0.16.[20]

A similar prediction for the $F_{15}(1688) \in {}^2\underline{8}_{5/2}$ ($\underline{56}$, $L=2^+$)

$$\frac{A_{3/2}(n)}{A_{3/2}(p)}=-\frac{1}{3} \tag{8}$$

is also consistent with the data, but is not compelling.

On the basis of these two assumptions ($A^{(L_3=2)}_{W=1}=0$, and $A^{(L_3=1)}_{W=0}=A^{(L_3=1)}_{W=1}$) we are thus prepared to make a relatively large number of one-partial-wave predictions among helicity amplitudes. (See Table I.) Within the $\underline{70}$, $L=1^-$ multiplet we get for the $D_{13}(1520) \in {}^2\underline{8}_{3/2}$

$$s\text{ wave: } A_{1/2}(n)+\frac{1}{2\sqrt{3}}A_{3/2}(n) = -\tfrac{2}{3}\left[A_{1/2}(p)+\frac{?}{\sqrt{3}}A_{3/2}(p)\right], \tag{9}$$

$$d\text{ wave: } A_{1/2}(n)-\frac{1}{\sqrt{3}}A_{3/2}(n) = \tfrac{1}{3}\left[A_{1/2}(p)-\frac{1}{\sqrt{3}}A_{3/2}(p)\right], \tag{10}$$

TABLE I. Photomatrix elements for nucleon resonances. Amplitudes are for λ helicity excitation of proton (p) or neutron (n) from helicity $+\frac{1}{2}$ nucleon in c.m. frame of resonances: written A_λ(p or n). Partial-wave amplitudes S,P,D,F are related to meson decay amplitudes (Tables III and IV) by $S=(e/\gamma_\rho)S'$, $P=(e/\gamma_\rho)P'$, $D=(e/\gamma_\rho)D'$, $F=(e/\gamma_\rho)F'$ up to mass-breaking dependence on q = c.m. 3-momentum.

$\underline{70}$, $L=1$	$A_{1/2}(p)$	$A_{3/2}(p)$	$A_{1/2}(n)$	$A_{3/2}(n)$
${}^2\underline{8}$ $J=\frac{1}{2}$	$\frac{-\sqrt{2}}{36}(2D+S)$		$\frac{\sqrt{2}}{108}(4D-S)$	
$J=\frac{3}{2}$	$\frac{-1}{18}(2D+S)$	$\frac{1}{6\sqrt{3}}(D-S)$	$\frac{1}{54}(D+2S)$	$-\frac{1}{9\sqrt{3}}(D-S)$
${}^4\underline{8}$ $J=\frac{1}{2}$			$\frac{-\sqrt{2}}{108}(2S-D)$	
$J=\frac{3}{2}$	0	0	$\frac{1}{54\sqrt{10}}(5S-2D)$	$\frac{1}{18\sqrt{30}}(5S+4D)$
$J=\frac{5}{2}$			$\frac{1}{6\sqrt{10}}D$	$\frac{1}{6\sqrt{5}}D$
${}^2\underline{10}$ $J=\frac{1}{2}$	$\frac{\sqrt{2}}{108}(5S-2D)$			
$J=\frac{3}{2}$	$\frac{1}{54}(4D-S)$	$\frac{1}{18\sqrt{3}}(D-S)$		
$\underline{56}$, $L=2$				
${}^2\underline{8}$ $J=\frac{3}{2}$	$\frac{2}{75}(3F+2P)$	$\frac{+2\sqrt{3}}{75}(F-P)$	$\frac{-2}{225}(3F+7P)$	$\frac{-2\sqrt{3}}{225}(F-P)$
$J=\frac{5}{2}$	$\frac{2}{25\sqrt{6}}(3F+2P)$	$\frac{-4\sqrt{3}}{75}(F-P)$	$\frac{-4}{75\sqrt{6}}(4F+P)$	$\frac{4\sqrt{3}}{225}(F-P)$
${}^4\underline{10}$ $J=\frac{1}{2}$	$\frac{2}{45}P$			
$J=\frac{3}{2}$	$\frac{2}{225}(3F+2P)$	$\frac{-2}{45\sqrt{3}}(4P+F)$		
$J=\frac{5}{2}$	$\frac{2}{75\sqrt{21}}(7P-2F)$	$\frac{-4}{45\sqrt{42}}F$		
$J=\frac{7}{2}$	$\frac{4}{15\sqrt{14}}F$	$\frac{-4}{3\sqrt{210}}F$		

the second being the familiar Melosh result (Hey and Weyers,[5] Love and Nanopolous[4]). We get only one for the $D_{15}(1670) \in {}^4\underline{8}_{5/2}$

$$d \text{ wave: } A_{1/2}(n) = \frac{1}{\sqrt{2}} A_{3/2}(n), \tag{11}$$

plus the Moorhouse selection rule $A_\lambda(p) = 0$.[21]

Similarly we get predictions for the $\underline{56}$, $L = 2^+$. If the $P_{13}(1860) \in {}^2\underline{8}_{3/2}$,

$$p \text{ wave: } A_{1/2}(n) + \sqrt{3} A_{3/2}(n) = -\tfrac{2}{3}[A_{1/2}(p) + \sqrt{3} A_{3/2}(p)], \tag{12}$$

$$f \text{ wave: } A_{1/2}(n) - \frac{7}{\sqrt{3}} A_{3/2}(n) = -\tfrac{2}{3}\left[A_{1/2}(p) - \frac{2}{\sqrt{3}} A_{3/2}(p)\right], \tag{13}$$

a linear combination of these two giving

$$A_{3/2}(n) = -\tfrac{1}{3} A_{3/2}(p). \tag{14}$$

And for the $F_{15}(1688) \in {}^2\underline{8}_{5/2}$

$$p \text{ wave: } A_{3/2}(n) + \frac{1}{2\sqrt{2}} A_{1/2}(n) = -\tfrac{1}{2}\left[A_{3/2}(p) + \frac{4}{3\sqrt{2}} A_{1/2}(p)\right], \tag{15}$$

$$f \text{ wave: } A_{3/2}(n) - \sqrt{2} A_{1/2}(n) = -\tfrac{2}{3}[A_{3/2}(p) - \sqrt{2} A_{1/2}(p)], \tag{16}$$

the second being the Melosh result.[5,9] Finally, we get the f-wave relation for the fourth resonance $F_{37}(1950) \in {}^4\underline{10}_{7/2}$:

$$A_{3/2}(p) = \frac{\sqrt{5}}{\sqrt{3}} A_{1/2}(p). \tag{17}$$

This magnetic dipole dominance is an $SU(6)_W \times O(2)_{L_3}$ result.

A further prediction for the $P_{33}(1236) \in {}^4\underline{10}_{3/2}$ in the lowest order $\underline{56}$, $L = 0^+$ is magnetic dipole dominance

$$A_{1/2}(p) = \frac{1}{\sqrt{3}} A_{3/2}(p). \tag{18}$$

Considering the $P_{11}(1470) \in {}^2\underline{8}_{1/2}$ $[\underline{56}, L = 0^+]$ in a "radially excited" $L = 0^+$, we get

$$A_{1/2}(n) = -\tfrac{2}{3} A_{1/2}(p) \tag{19}$$

At present levels it is hard to discern the accuracy of some of these relations, many of which are harmonic-oscillator or $SU(6)_W \times O(2)_{L_3}$ results. However for the D_{13} and F_{15}, (7) and (8), these predictions differ significantly and seem to agree somewhat better. Other analyses[5,15,16] use the D_{13} and F_{15} as input.

NUMERICAL RESULTS

In some cases [notably the $A_{1/2}(p)$ for the D_{13}] the cancellation between S- and D-wave terms is somewhat delicate.[22] In order to incorporate mass breaking we shall use the simple dependence (3), which gives the best χ^2 in the 0^- decays.[16,23] We set

$$\begin{aligned} S &= \frac{e}{\gamma_\rho} \tilde{S}, \\ P &= \frac{e}{\gamma_\rho} \tilde{P}\left(\frac{q}{q_0}\right), \\ D &= \frac{e}{\gamma_\rho} \tilde{D}\left(\frac{q}{q_0}\right)^2, \\ F &= \frac{e}{\gamma_\rho} \tilde{F}\left(\frac{q}{q_0}\right)^3, \end{aligned} \tag{20}$$

where $q_0 = 0.5$ GeV is a convenient scale, to make $\tilde{S}, \tilde{P}, \tilde{D}, \tilde{F}$ dimensionless. We shall use the Orsay determination[24]

$$\frac{e}{\gamma_\rho} = 0.06 \tag{21}$$

and take the $\tilde{S}, \tilde{P}, \tilde{D}, \tilde{F}$ *from the* 0^- *decays*. Hence we fit the photo matrix elements with no fitted parameters from pion photoproduction. (See Table II.)

Tables III and IV give parameter fits to the pseudoscalar decays. The normalization is the familiar one[23] in terms of the "reduced widths"

$$\Gamma = \frac{q M_N}{M} \tilde{\Gamma}, \tag{22}$$

where $\tilde{\Gamma}$ is dimensionless, M_N = nucleon mass, M = resonance mass.

S-wave mixing has been chosen to agree with earlier analysis based on the simple approximations,[8]

$$\begin{aligned} \langle N\eta | S_{11}(1700)\rangle &= \langle N\bar{K} | \Lambda(\text{unseen})\rangle \\ &= \langle N\bar{K} | \Lambda(1670)\rangle \\ &= \langle \Sigma\pi | \Lambda(1670)\rangle \\ &\simeq 0. \end{aligned} \tag{23}$$

Similar fits have been done before (see Ref. 23 for the $\underline{70}$, $L = 1^-$; and also see Ref. 16) with more complicated mixing. It is our intent only to do the simplest thing possible; the monthly variation in these resonant parameters suggests such complications are probably superfluous.

Our l-wave parameters are

$$\begin{aligned} |\tilde{S}| &= 7.6 \pm 0.5, \quad |\tilde{P}| = 2.8 \pm 0.2, \\ |\tilde{D}| &= 4.5 \pm 0.2, \quad |\tilde{F}| = 2.0 \pm 0.1 \end{aligned} \tag{24}$$

in contrast to $|\tilde{S}| = 4.1$, $|\tilde{D}| = 4.9$, $|\tilde{P}| = 2.2$, $|\tilde{F}| = 2.2$

TABLE II. Resonance photoproduction couplings [for normalization, see Eq. (25)]. Signs in parentheses are signs of πN couplings relative to $S(P)$.

Resonance	Ref.[b]	$A^P_{1/2}$	$A^P_{3/2}$	$A^N_{1/2}$	$A^N_{3/2}$
$P_{33}(1236) \in {}^4\underline{10}_{3/2}$	KMORR	-143 ± 2	-257 ± 7		
$(\underline{56},\ L=0^+)_0$	MW	-140 ± 6	-253 ± 20		
	DLR	-134 ± 4	-249 ± 14		
	GK[c]	-110	-190		
	${}^3P_0$ [a,d]	$-110(+)$	-190		
$P_{11}(1470) \in {}^2\underline{8}_{1/2}$	KMORR	-80 ± 4		0 ± 2	
$(\underline{56},\ L=0^+)_2$	MW	-70 ± 23		-43 ± 35	
	DLR	-78 ± 14		62 ± 24	
	${}^3P_0$	$-82(-)$		55	
$S_{11}(1530)$	KMORR	89 ± 21		-52 ± 21	
$(\underline{70},\ L=1^-)$	MW	63 ± 13		-51 ± 21	
	DLR	78 ± 20		-37 ± 23	
${}^2\underline{8}_{1/2}$	${}^3P_0$	$4(+)$		-49	
$\frac{1}{\sqrt{2}}({}^2\underline{8} - {}^4\underline{8})$	${}^3P_0$	$3(+)$		-18	
$S_{11}(1700)$	KMORR	52 ± 11		-55 ± 39	
$(\underline{70},\ L=1^-)$	MW	12 ± 15		-19 ± 22	
	DLR	29 ± 18		-6 ± 31	
${}^4\underline{8}_{1/2}$	${}^3P_0$	$0(+)$		-16	
$\frac{1}{\sqrt{2}}({}^2\underline{8} + {}^4\underline{8})$	${}^3P_0$	$35(+)$		-54	
$D_{13}(1520) \in {}^2\underline{8}_{3/2}$	KMORR	-19 ± 21	170 ± 7	-70 ± 21	-128 ± 7
$(\underline{70},\ L=1^-)$	MW	-6 ± 6	165 ± 11	-66 ± 10	-118 ± 13
	DLR	-8 ± 15	171 ± 12	-89 ± 19	-155 ± 19
	${}^3P_0$	$-3(-)$	173	-32	-115
$S_{31}(1650) \in {}^2\underline{10}_{1/2}$	KMORR	27 ± 18			
$(\underline{70},\ L=1^-)$	MW	105 ± 38			
	DLR	-10 ± 17			
	${}^3P_0$	$78(+)$			
$D_{33}(1670) \in {}^2\underline{10}_{3/2}$	KMORR	79 ± 32	61 ± 39		
$(\underline{70},\ L=1^-)$	MW	0 ± 48	0 ± 41		
	DLR	54 ± 29	72 ± 14		
	${}^3P_0$	$54(-)$	60		
$D_{13}(1700) \in {}^4\underline{8}_{3/2}$	KMORR	22 ± 39	61 ± 32	73 ± 67	51 ± 81
$(\underline{70},\ L=1^-)$	MW	0 ± 34	0 ± 29	0 ± 34	0 ± 44
	DLR	-48 ± 50	-6 ± 14	-21 ± 98	-26 ± 69
	${}^3P_0$	0	0	$44(+)$	9
$D_{15}(1670) \in {}^4\underline{8}_{5/2}$	KMORR	7 ± 24	17 ± 31	-43 ± 24	-90 ± 53
$(\underline{70},\ L=1^-)$	MW	10 ± 13	42 ± 24	4 ± 15	-9 ± 30
	DLR	19 ± 21	14 ± 4	-29 ± 23	-68 ± 20
	${}^3P_0$	0	0	$-44(+)$	-31
$P_{13}(1770) \in {}^2\underline{8}_{3/2}$	KMORR	26 ± 16	-12 ± 12	14 ± 14	-23 ± 23
$(\underline{56},\ L=2^+)$	MW	0 ± 25	0 ± 22	0 ± 50	0 ± 44
	DLR	25 ± 34	-87 ± 57	13 ± 45	-83 ± 90
	${}^3P_0$	$10(-)$	-47	14	-16
$F_{15}(1688) \in {}^2\underline{8}_{5/2}$	KMORR	-25 ± 3	96 ± 4	33 ± 7	-15 ± 15
$(\underline{56},\ L=2^+)$	MW	-8 ± 11	129 ± 16	8 ± 18	0 ± 30
	DLR	27 ± 19	163 ± 11	31 ± 28	-21 ± 28
	${}^3P_0$	$-14(+)$	83	29	-28

TABLE II. *(Continued)*

Resonance	Ref. [b]	$A^P_{1/2}$	$A^P_{3/2}$	$A^N_{1/2}$	$A^N_{3/2}$
$P_{31}(1860) \in {}^4\underline{10}_{1/2}$	KMORR	18 ± 5			
$(\underline{56},\ L=2^+)$	MW	-32 ± 65			
	DLR	0 ± 25			
	$^3P_0$	$22(-)$			
$P_{33}(2000) \in {}^4\underline{10}_{3/2}$	KMORR	-34 ± 8	-9 ± 9		
$(\underline{56},\ L=2^+)$	DLR	-33 ± 37	8 ± 46		
	$^3P_0$	$-30(+)$	-27		
$F_{35}(1870) \in {}^4\underline{10}_{5/2}$	KMORR	25 ± 10	-44 ± 18		
$(\underline{56},\ L=2^+)$	MW	47 ± 67	-28 ± 66		
	DLR	19 ± 27	78 ± 20		
	$^3P_0$	$-29(-)$	-13		
$F_{37}(1950) \in {}^4\underline{10}_{7/2}$	KMORR	-67 ± 7	-80 ± 16		
$(\underline{56},\ L=2^+)$	MW	-59 ± 29	-93 ± 24		
	DLR	-88 ± 25	-80 ± 21		
	$^3P_0$	$-60(-)$	-78		

[a] Parameters used in $^3P_0$ calculations: $\tilde{P}=10.1$ for $P_{33}(1236) \in {}^4\underline{10}_{3/2}$ $(\underline{56},\ L=0^+)_0$ from elastic width of $\Gamma_{\pi N}=114$; $\tilde{P}=2.7$ for $P_{11}(1470) \in {}^2\underline{8}_{1/2}$ $(\underline{56},\ L=0^+)_2$ from elastic width of $\Gamma_{\pi N}=124$; $\tilde{S}=7.6 \pm 0.5$, $\tilde{D}=4.5 \pm 0.2$ from Table III fits to $(\underline{70},\ L=1^-)_1$ baryon 0^- decays; $\tilde{P}=2.8 \pm 0.2$, $\tilde{F}=2.0 \pm 0.1$ from Table IV fits to $(\underline{56},\ L=2^+)_2$ baryon 0^- decays.

[b] Experimental analysis: KMORR is G. Knies, R. G. Moorhouse, H. Oberlack, A. H. Rosenfeld, and A. Rittenberg, LBL Report No. LBL-2673, 1974 (unpublished); MW is W. J. Metcalf and R. L. Walker, Ref. 25; DLR is R. C. E. Devenish, D. H. Lyth, and W. A. Rankin, Ref. 20.

[c] GK refers to the theoretical analysis of Gilman and Karliner (Ref. 15).

[d] Goodness of fits gives χ^2 of

	KMORR	MW	DLR
$\underline{70},\ L=1^-$	49/20 D.F.	61/20 D.F.	22/20 D.F.
$\underline{56},\ L=2^+$	7/15 D.F.	18/15 D.F.	56/15 D.F.

based on 45° mixing of S_{11} states, and *no* fitted parameters.

from Ref. 16.

In order to compare the photocouplings to that of partial-wave analysis we need the normalization of Walker[18]

$$A_\lambda^{P \text{ or } N} = \eta \left(\frac{\pi}{q}\right)^{1/2} A_\lambda(p \text{ or } n), \tag{25}$$

where η is the sign of the πN vertex in pion photoproduction. The sign is determined by comparing the $\gamma N \to N^* \to \pi N$ residue to the one-pion exchange Born term.

SIGNS OF PARAMETERS

Early work involving meson decays suggested $S/D<0$,[7] further reinforced in $\pi N \to \pi\Delta$.[16] In our model, the signs are clearly determined by noting the weakness of the $A_{1/2}(p)$ signal in the second and third resonance regions, namely the ratios

$$\frac{A_{1/2}(p)}{A_{3/2}(p)} = \frac{(1/18)(2D+S)}{(1/6\sqrt{3})(D-S)} \text{ for the } D_{13}(1520) \tag{26}$$

and

$$\frac{A_{1/2}(p)}{A_{3/2}(p)} = \frac{(-\sqrt{6}/75)(3F+2P)}{(4\sqrt{3}/75)(F-P)} \text{ for the } F_{15}(1688) \tag{27}$$

are known to be small. Hence the determinations $S/D<0$, $P/F<0$. While the "anti-SU(6)" solution $S/D<0$ is now generally agreed upon, the second $P/F<0$ is in contradiction to the isobaric analog analysis of $\pi N \to \pi\Delta$ of Cashmore *et al.*[16] Our sign, of course, agrees with the related quark-pair—creation model.[13] Multipole analysis of photocouplings is not completely conclusive, but shows $P/F<0$ slightly more likely.[25] Recent work of Carlitz and Weyers[26] suggesting an expansion of the "lightlike" charges in powers of $1/m$ (quark

TABLE III. Fits to 0^- decay modes of $\underline{70}$, $L=1^-$ baryon resonances to determine $|\tilde{S}|$ and $|\tilde{D}|$ parameters. Determinations give $|\tilde{S}|=7.6\pm0.5$, $|\tilde{D}|=4.5\pm0.2$ with scale in $S'=\tilde{S}$, $D'=\tilde{D}\ (q/q_0)^2$ of $q_0=0.5$ GeV.

State	${}^{2S+1}\underline{R}_J$	Mode	$\tilde{\Gamma}$	(ABG)[a] $\Gamma_{\text{experimental}}$	$\Gamma_{\text{theoretical}}$	χ^2
$S_{01}(1405)$	b	$\Sigma\pi$	$\frac{1}{144}\,\lvert S'\rvert^2$	39 ± 8	38	0.0
$S_{01}(1670)$	b	$\Lambda\eta$	$\frac{1}{144}\,\lvert S'\rvert^2$	12 ± 6	15	0.1
$S_{11}(1530)$	b	$N\pi$	$\frac{1}{432}\,\lvert S'\rvert^2$	33 ± 12	39	0.2
$S_{11}(1530)$	b	$N\eta$	$\frac{1}{108}\,\lvert S'\rvert^2$	61 ± 23	60	0.0
$S_{31}(1650)$	${}^2\underline{10}_{1/2}$	$N\pi$	$\frac{1}{432}\,\lvert S'\rvert^2$	46 ± 10	42	0.2
$D_{13}(1520)$	${}^2\underline{8}_{3/2}$	$N\pi$	$\frac{1}{54}\,\lvert D'\rvert^2$	65 ± 7	73	1.5
$D_{15}(1670)$	${}^4\underline{8}_{5/2}$	$N\pi$	$\frac{1}{360}\,\lvert D'\rvert^2$	61 ± 13	28	6.7
$D_{15}(1670)$	${}^4\underline{8}_{5/2}$	$\Delta\pi$	$\frac{7}{180}\,\lvert D'\rvert^2$	63 ± 11	41	3.6
$D_{33}(1670)$	${}^2\underline{10}_{3/2}$	$N\pi$	$\frac{1}{432}\,\lvert D'\rvert^2$	47 ± 7	23	6.4
$D_{15}(1765)$	${}^4\underline{8}_{5/2}$	$N\bar{K}$	$\frac{1}{135}\,\lvert D'\rvert^2$	43 ± 12	38	0.1
$D_{15}(1765)$	${}^4\underline{8}_{5/2}$	$\Delta\pi$	$\frac{1}{360}\,\lvert D'\rvert^2$	16 ± 5	18	0.1
$D_{15}(1765)$	${}^4\underline{8}_{5/2}$	$Y_1^*\pi$	$\frac{7}{1080}\,\lvert D'\rvert^2$	6 ± 3	3	0.8
$D_{05}(1827)$	${}^4\underline{8}_{5/2}$	$\Sigma\pi$	$\frac{1}{120}\,\lvert D'\rvert^2$	36 ± 22	52	0.6
	${}^4\underline{8}_{5/2}$	$Y_1^*\pi$	$\frac{7}{240}\,\lvert D'\rvert^2$	15 ± 9	30	3.0
				$\chi_{\text{tot}}^2=23.3/12$ D.F.		

[a] ABG refers to experimental determinations taken from A. Barbaro-Galtieri, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 1, p. 159.

[b] S-wave mixing is the same as in Ref. 8, based on the approximations

$$\langle N\bar{K}\,|\,\Lambda(\text{unseen})\rangle = \langle N\bar{K}\,|\,\Lambda(1670)\rangle = \langle\Sigma\pi|\,\Lambda(1670)\rangle \simeq 0$$

for the S_{01} states, and

$$\langle N\eta\,|N^*(1700)\rangle \simeq 0$$

for the S_{11} states. The approximate mixing-matrix relations are

$$\begin{bmatrix} S_{01}(1405) \\ S_{01}(1670) \\ S_{01}(\text{unseen}) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{2/3} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} {}^2\underline{1}_{1/2} \\ {}^2\underline{8}_{1/2} \\ {}^4\underline{8}_{1/2} \end{bmatrix}$$

and

$$\begin{bmatrix} S_{11}(1530) \\ S_{11}(1700) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} {}^2\underline{8}_{1/2} \\ {}^4\underline{8}_{1/2} \end{bmatrix}.$$

TABLE IV. Fits to 0^- decay modes of $\underline{56}$, $L=2^+$ baryon resonances to determine $|\tilde{P}|$ and $|\tilde{F}|$ parameters. Determinations give $|\tilde{P}|=2.8\pm0.2$, $|\tilde{F}|=2.0\pm0.1$ with scale in $P'=\tilde{P}(q/q_0)$, $F'=\tilde{F}(q/q_0)^3$ of $q_0=0.5$ GeV.

State	$^{2S+1}\underline{R}_J$	Mode	$\tilde{\Gamma}$	(ABG) [a] $\Gamma_{\text{experimental}}$	$\Gamma_{\text{theoretical}}$	χ^2
$F_{15}(1968)$	$^{2}\underline{8}_{5/2}$	$N\pi$	$\frac{1}{54}\|F'\|^2$	79 ± 14	54	3.6
$F_{15}(1688)$	$^{2}\underline{8}_{5/2}$	$\Delta\pi$	$\frac{1}{1125}\|P'\|^2+$	14 ± 4	14	0.0
			$\frac{32}{3375}\|F'\|^2$			
$F_{05}(1815)$	$^{2}\underline{8}_{5/2}$	$N\bar{K}$	$\frac{1}{75}\|F'\|^2$	46 ± 13	25	2.8
$F_{05}(1815)$	$^{2}\underline{8}_{5/2}$	$\Sigma\pi$	$\frac{2}{225}\|F'\|^2$	9 ± 2	10	0.4
$F_{05}(1815)$	$^{2}\underline{8}_{5/2}$	$Y_1^*\pi$	$\frac{4}{375}\|P'\|^2+$	9 ± 2	9	0.0
			$\frac{8}{1125}\|F'\|^2$			
$P_{13}(1866)$	$^{2}\underline{8}_{5/2}$	$N\pi$	$\frac{1}{54}\|P'\|^2$	69 ± 43	92	0.3
		ΛK	$\frac{1}{150}\|P'\|^2$	14 ± 9	9	0.3
$F_{35}(1890)$	$^{4}\underline{10}_{5/2}$	$N\pi$	$\frac{8}{4725}\|F'\|^2$	40 ± 19	19	1.7
$P_{31}(1910)$	$^{4}\underline{10}_{1/2}$	$N\pi$	$\frac{8}{675}\|P'\|^2$	66 ± 20	65	0.0
$F_{37}(1950)$	$^{4}\underline{10}_{7/2}$	$N\pi$	$\frac{4}{525}\|F'\|^2$	104 ± 17	117	0.1
		$\Delta\pi$	$\frac{1}{70}\|F'\|^2$	33 ± 7	35	0.5
$F_{17}(2030)$	$^{4}\underline{10}_{7/2}$	$N\bar{K}$	$\frac{4}{1575}\|F'\|^2$	28 ± 4	25	0.5
		$\Lambda\pi$	$\frac{2}{525}\|F'\|^2$	39 ± 6	38	0.1
		$\Sigma\pi$	$\frac{4}{1575}\|F'\|^2$	9 ± 5	15	1.5
					$\chi_{\text{tot}}^2=12.3/12$ D.F.	

[a] See note a of Table III.

effective mass) would have $F/P>0$ at least in $0^-\frac{3}{2}^+$ decays. It may be also that the $\underline{35}$ meson decays are uncorrelated to the photodecays, making vector dominance at this level untenable.

It should be noted, however, considering the simplicity of the present model that the agreement of signs and magnitudes of the photocouplings is quite satisfactory. Furthermore, if we consider mass breaking as independent of the vertex symmetry, the variation in the reduced matrix elements as a function of $|\vec{p}_r|=q$ (i.e., resonance mass) is substantial indeed. In the $\underline{70}$, $L=1^-$ spectral band ($1.530<M<1.730$) the ratio of the reduced matrix elements $A_{W=1}^{(0)}/A_{W=1}^{(1)}$ varies between $0<A^{(0)}/A^{(1)}<0.4$, clearly not a constant. We suggest that in order to study these reduced matrix elements, including $A^{(L_3=2)}$, the mass breaking must be considered. In a scheme like that of Carlitz and Weyers, for example, where the orbital properties of each term in the $1/m$ expansion are obvious, projections like (2) could provide better parametrizations even preserving the independence of the $W=0$ and $W=1$ parts.

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