PHYSICAL REVIEW D VOLUME 12, NUMBER 9 1 NOVEMBER 1975

CP-violating relativistic quark models and the $K^0-\bar{K}^0$ self-energy matrix

D. Flamm*
Institut für Theoretische Physik der Universität Wien, Wien, Austria

P. Kielanowski†
Institute of Theoretical Physics, Warsaw University, Warsaw, Poland

A. Morales, R. Nunez-Lagos, and J. Sanchez Guillen
Departamento de Fisica Nuclear, Universidad de Zaragoza, Zargoza, Spain
(Received 14 April 1975)

With a relativistic quark model which is based on the Bethe-Salpeter equation the K^0-$\bar{K}^0$ self-energy matrix is calculated for the CP-violating models of Glashow, Oakes, and Das. For the first two models the kaon mass difference is in reasonable agreement with experiment. The CP-violating angles are determined from a least-squares fit of the theoretical values of Reϵ, η_{+-}, and η_{00} to the experimental data. While the model of Glashow gives good agreement with experiment, no reasonable fit can be achieved for the models of Oakes and Das.

I. INTRODUCTION

Throughout the years it has remained a challenge to explicitly derive the consequences of various weak Hamiltonians. In particular for CP violation it is necessary to have precise calculations in order to decide between the different models. The relativistic quark model of Böhm, Joos, and Krammer[1] allows us to perform such calculations. It is based on a solution of the Bethe-Salpeter equation with a harmonic-oscillator kernel. If one uses for the spinor structure of the kernel the Fierz-symmetric combination,[1] pseudoscalar plus vector minus scalar, one obtains for the lowest-mass pseudoscalar and scalar mesons the following Bethe-Salpeter (BS) amplitudes[1]:

$$\chi_{0^-} = \frac{4\pi}{\sqrt{3}\beta}\left(1+\frac{\not{P}}{M}\right)\gamma_5 \exp\left(-\frac{r^2}{2\sqrt{\beta}}\right)|q\bar{q}\rangle\,, \tag{1.1}$$

$$\chi_{0^+} = \frac{4\pi\sqrt{2}}{3\beta^{3/4}}\left[\left(1+\frac{\not{P}}{M}\right)\not{r} - \frac{(P\cdot r)\not{P}}{m_\epsilon{}^2} - i\frac{r^2}{M} - \frac{(r\cdot P)}{M} - \frac{i}{M}\frac{(r\cdot P)^2}{m_\epsilon{}^2}\right]\exp\left(-\frac{r^2}{2\sqrt{\beta}}\right)|q\bar{q}\rangle\,. \tag{1.2}$$

Here P is the meson momentum, r the $q-\bar{q}$ relative momentum, m the meson mass, $2\sqrt{\beta}\simeq 1$ GeV^2 is the meson level spacing, and $|q\bar{q}\rangle$ is the SU(3) part of the BS amplitude. The effective quark mass M is determined from the $K\to\mu\nu$ decay constant F_K by[2]

$$M=\frac{4\sqrt{\beta}}{\sqrt{3}\;\pi F_K}\,. \tag{1.3}$$

The advantage of this quark model is that all quark loop diagrams which contain at least one BS amplitude are convergent, and therefore it is possible to calculate any meson transition amplitude explicitly.

In this paper we shall consider[3] the CP violation by phase angles in the charged currents[4] and by neutral currents[5,6] which are extensions of the usual Cabibbo theory. For all these models the Hamiltonian is of current× current type

$$H_W=\frac{G}{2\sqrt{2}}(\{J_c,J_c^\dagger\}+\{J_0,J_0^\dagger\}), \tag{1.4}$$

where J_c is a charged current and J_0 is a neutral current. From these weak Hamiltonians we calculate the CP-violating parameters ϵ, η_{+-}, and η_{00} as well as the kaon mass difference $m_{K_L}-m_{K_S}$. The connection of the K^0-$\bar{K}^0$ self-energy matrix with the mixing parameter ϵ and the K_L-K_S mass difference is exhibited in Sec. II. The numerical analysis for the Glashow model is performed in Sec. III. In Sec. IV the models of Oakes and Das are treated. Finally in Sec. V the results are presented and discussed.

II. THE K^0-$\bar{K}^0$ SELF-ENERGY MATRIX

The K^0-$\bar{K}^0$ self-energy matrix can be written in the form

$$M_{ij}=\begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix}, \tag{2.1}$$

where the indices $(i,j=0,\bar{0})$ refer to K^0 and $\bar{K}^0$, respectively. Due to the instability of the K^0-$\bar{K}^0$ system this matrix is not Hermitian. It can be written in the form

$$M_{ij}=m_{ij}-\frac{i}{2}\Gamma_{ij}\quad (i,j=0,\bar{0}), \tag{2.2}$$

where m_{ij} and Γ_{ij} are Hermitian matrices. *CPT* invariance gives the relation $M_{00}=M_{\bar{0}\bar{0}}$. *CP* invariance would give $M_{\bar{0}0}=M_{0\bar{0}}$ which means that $(M_{0\bar{0}}-M_{\bar{0}0})/M_{0\bar{0}}$ should be of the order 10^{-3}, which is characteristic for the observed *CP*-violating effects. We use the convention $CP|K^0\rangle=|\bar{K}^0\rangle$.

The nonunitary matrix

$$\frac{1}{[2(1+|\epsilon|^2)]^{1/2}}\begin{pmatrix} 1+\epsilon & 1-\epsilon \\ 1+\epsilon & -(1-\epsilon) \end{pmatrix} \tag{2.3}$$

transforms the K^0-$\bar{K}^0$ basis into the K_S-K_L basis in which the self-energy matrix is diagonal and reads

$$\begin{pmatrix} m_{K_S}-\frac{i}{2}\Gamma_{K_S} & 0 \\ 0 & m_{K_L}-\frac{i}{2}\Gamma_{K_L} \end{pmatrix}. \tag{2.4}$$

For ϵ one obtains, neglecting terms of higher order in the *CP* violation,

$$\epsilon=\frac{1}{2}\frac{M_{0\bar{0}}-M_{\bar{0}0}}{M_{0\bar{0}}+M_{\bar{0}0}}. \tag{2.5}$$

The eigenvalues of the self-energy matrix are as follows:

$$m_{K_S}-\frac{i}{2}\Gamma_{K_S}=M_{00}+\frac{1-\epsilon}{1+\epsilon}M_{0\bar{0}} \tag{2.6}$$

and

$$m_{K_L}-\frac{i}{2}\Gamma_{K_L}=M_{00}-\frac{1-\epsilon}{1+\epsilon}M_{0\bar{0}}. \tag{2.7}$$

Up to terms of the order ϵ we get

$$m_{K_L}-m_{K_S}=-2\,\mathrm{Re}M_{0\bar{0}}. \tag{2.8}$$

The off-diagonal elements of the K^0-$\bar{K}^0$ self-energy matrix can be computed from the weak Hamiltonian H_W,

$$m_{ij}=\langle i|H_W|j\rangle+P\sum_n\frac{\langle i|H_W|n\rangle\langle n|H_W|j\rangle}{E_i-E_n} \tag{2.9}$$

and

$$\Gamma_{ij}=2\pi\sum_n\delta(E_i-E_n)\langle i|H_W|n\rangle\langle n|H_W|j\rangle. \tag{2.10}$$

Only virtual states contribute to the sum in Eq. (2.9). We shall in fact restrict the sum to meson pole contributions. The sum in Eq. (2.10) is over real transitions. Here we shall consider only 2π intermediate states.

III. *CP* VIOLATION THROUGH PHASE ANGLES IN THE CHARGED CURRENTS

We now proceed to compute the self-energy matrix elements in the different models of *CP* violation. Let us first consider the class of models with *CP* violation through phase angles in the weak charged current.[4] In these models, the neutral current J_0 vanishes and the charged current is given by

$$J_c=\cos\theta_C(V-e^{i\phi}A)_{1+i2}+\sin\theta_C(V-e^{i\omega}A)_{4+i5}, \tag{3.1}$$

where V and A are vector and axial-vector currents, respectively. In the quark model it takes the form[7]

$$J_c^\mu=\bar{\mathcal{P}}\gamma^\mu(1+e^{i\phi}\gamma_5)\mathfrak{N}\cos\theta_C+\bar{\mathcal{P}}[\gamma^\mu(1+e^{i\omega}\gamma_5)+\delta k^\mu]\lambda\sin\theta_C. \tag{3.2}$$

$\delta=-0.144\ \mathrm{GeV}^{-1}$ is the SU(3)-breaking parameter and k^μ is the relative momentum of the $\mathcal{P}$ and λ quarks.[2] The graphs shown in Fig. 1 determine the matrix element $m_{0\bar{0}}$ for which we find the expression

$$m_{0\bar{0}}=-\frac{1}{2m_K}\left(\frac{G}{\sqrt{2}}\sin\theta_C\cos\theta_C\frac{\beta}{3\pi^2}\right)^2\left\{\frac{D}{4}\left[-16(1-e^{i(\omega-\phi)})+\frac{8m_K^2}{M^2}(1+e^{i(\omega-\phi)})\right]^2+\frac{\sqrt{\beta}}{m_K^2-m_\epsilon^2}\left[\frac{16}{\sqrt{3}}\left(1-\frac{m_K^2}{4m_\epsilon^2}\right)\left(\delta e^{i\phi}+\frac{4}{M}(e^{i\omega}-e^{-i\phi})\right)\right]^2\right\}, \tag{3.3}$$

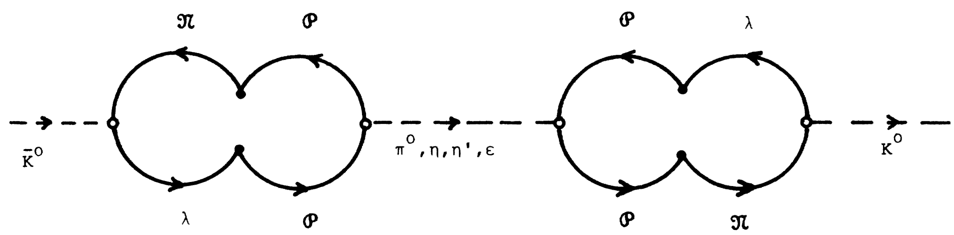

FIG. 1. Diagrams for the $\bar{K}^0 \rightarrow K^0$ transition involving charged currents only. Circles indicate BS amplitudes, black dots stand for the currents, solid lines for quarks, and dashed lines for mesons.

where D is given by

$$D=\frac{1}{m_K{}^2-m_\pi{}^2}+\frac{\sin^2(\theta-\theta_I)}{m_K{}^2-m_\eta{}^2}+\frac{\cos^2(\theta-\theta_I)}{m_K{}^2-m_{\eta'}{}^2}\,. \quad (3.4)$$

In this equation θ is the mixing angle of the pseudoscalar meson octet and $\sin\theta_I=1/\sqrt{3}$.

Taking only the 2π intermediate states into account we obtain for $\Gamma_{0\bar{0}}$ from Eq. (2.10)

$$\Gamma_{0\bar{0}}=\frac{k}{8\pi m_K{}^2}[A(\bar{K}^0\rightarrow\pi^+\pi^-)A^*(K^0\rightarrow\pi^+\pi^-)+A(\bar{K}^0\rightarrow\pi^0\pi^0)A^*(K^0\rightarrow\pi^0\pi^0)]\,. \quad (3.5)$$

The amplitudes $A[K^0(\bar{K}^0)\rightarrow\pi^+\pi^-(\pi^0\pi^0)]$ are calculated from the diagrams in Fig. 2. Including the $\pi\pi$ final-state interactions one obtains[7]

$$\begin{aligned}A_{+-}&=A(K^0\rightarrow\pi^+\pi^-)=z_1e^{-i\phi}+z_2e^{i\omega}\,,\\A_{00}&=A(K^0\rightarrow\pi^0\pi^0)=z_3e^{-i\phi}+z_4e^{i\omega}\,,\\\bar{A}_{+-}&=A(\bar{K}^0\rightarrow\pi^+\pi^-)=z_1e^{i\phi}+z_2e^{-i\omega}\,,\\\bar{A}_{00}&=A(\bar{K}^0\rightarrow\pi^0\pi^0)=z_3e^{i\phi}+z_4e^{-i\omega}\,,\end{aligned} \quad (3.6)$$

where

$$\begin{aligned}z_1&=(4.698+i5.112)\times10^{-6}\text{ GeV}\,,\\z_2&=-(4.506+i4.926)\times10^{-6}\text{ GeV}\,,\\z_3&=-(4.985+i3.615)\times10^{-6}\text{ GeV}\,,\\z_4&=(4.832+i3.483)\times10^{-6}\text{ GeV}\,.\end{aligned} \quad (3.7)$$

It is easily verified that the amplitudes for given

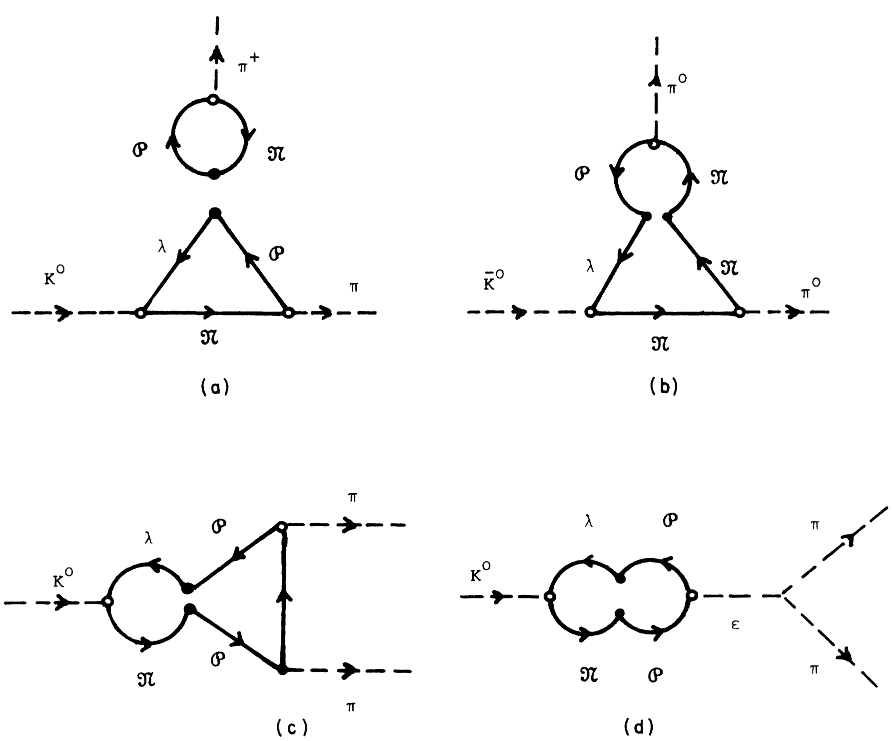

FIG. 2. Quark diagrams for the $K^0\rightarrow\pi\pi$ decay.

isospin in the $\pi\pi$-final state

$$A(K_0 \to 2\pi(I=0)) \equiv A_0 = \frac{1}{\sqrt{3}}(\sqrt{2}\,A_{+-} - A_{00}),$$
$$A(K_0 \to 2\pi(I=2)) \equiv A_2 = \frac{1}{\sqrt{3}}(A_{+-} + \sqrt{2}\,A_{00}) \quad (3.8)$$

obey the usual CPT relations

$$A(\overline{K}_0 \to 2\pi(I=0)) \equiv \overline{A}_0 = A_0^* e^{2i\delta_0},$$
$$A(\overline{K}_0 \to 2\pi(I=2)) \equiv \overline{A}_2 = A_2^* e^{2i\delta_2}, \quad (3.9)$$

where $\delta_0 = 43°$ and $\delta_2 = 0$ (see Ref. 8) are the $I=0$ and $I=2$ $\pi\pi$ phase shifts, respectively.

From Eqs. (3.3), (3.5), (2.2), and (2.5) we can determine the mixing parameter ϵ which is of course a function of the phase angles ϕ and ω. With its help we can calculate the CP-violating parameters for the $K \to \pi\pi$ decays:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \frac{A_{+-} - \overline{A}_{+-} + \epsilon(A_{+-} + \overline{A}_{+-})}{A_{+-} + \overline{A}_{+-} + \epsilon(A_{+-} - \overline{A}_{+-})} \quad (3.10)$$

and

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \frac{A_{00} - \overline{A}_{00} + \epsilon(A_{00} + \overline{A}_{00})}{A_{00} + \overline{A}_{00} + \epsilon(A_{00} - \overline{A}_{00})}. \quad (3.11)$$

Now we determine the two free parameters ϕ and ω of the weak current Eq. (3.1) from a best fit to the experimental values of Reϵ, η_{+-}, and η_{00}.

IV. CP VIOLATION BY NEUTRAL CURRENTS

In the model by Oakes[5] the charged weak current is the usual Cabibbo current. It follows from Eq. (3.1) if we choose $\phi = \omega = 0$. The neutral current J_0 is of $V+A$ type. In terms of the quark fields it reads

$$J_0^\mu = -\{\overline{\mathfrak{N}}\gamma^\mu(1-\gamma_5)\mathfrak{N} - \overline{\lambda}\gamma^\mu(1-\gamma_5)\lambda\}\cos\psi - i\{\overline{\mathfrak{N}}[\gamma^\mu(1-\gamma_5)+\delta k^\mu]\lambda - \overline{\lambda}[\gamma^\mu(1-\gamma_5)+\delta k^\mu]\mathfrak{N}\}\times\sin\psi. \quad (4.1)$$

The transitions $K^0(\overline{K}^0) \to \pi^0, \eta, \eta', \epsilon$ may now in addition to the charged current graph in Fig. 3(a) proceed through the neutral current graphs shown in Figs. 3(b) and 3(c). All together nine graphs arise from these three building blocks for the $K^0 \to \overline{K}^0$ transition. The $\Delta I = \frac{1}{2}$ rule requires parastatistics for the quarks.[9] For the pseudoscalar mesons π, η, η' the two-loop diagrams in Fig. 3(b) and the one-loop diagrams in Fig. 3(c) give for paraquarks contributions which are exactly opposite. As a consequence for neutral currents only give a nonvanishing contribution to the $\overline{K}^0 - K^0$ transition with the orbitally excited scalar meson ϵ as intermediate state. The ϵ contribution does not cancel because $K^0 \to \epsilon$ can only proceed via the single-loop diagrams Figs. 3(a) and 3(c). In addition to Fig. 1 we thus have the diagrams shown in Fig. 4 for the $\overline{K}^0 \to K^0$ transition. The contribution from Fig. 1 is obtained from Eq. (3.3) setting $\phi = \omega = 0$. Altogether we obtain for $m_{0\bar{0}}$ the expression

$$m_{0\bar{0}} = -\frac{1}{2m_K}C^2 D + \frac{1}{2m_K}E^2 \frac{1}{m_K^2 - m_\epsilon^2}\left(1 + i\frac{\sin 2\psi}{\sin 2\theta_C}\right)^2, \quad (4.2)$$

where

$$C = 2G\sin\theta_C\cos\theta_C\left(\frac{4\beta}{3\pi^2}\right)\frac{m_K^2}{M^2}, \quad (4.3)$$

$$E = \frac{G}{\sqrt{2}}\sin\theta_C\cos\theta_C\frac{16\beta^{5/4}}{3\sqrt{3}\,\pi^2}\left(1 - \frac{m_K^2}{4m_\epsilon^2}\right)\delta, \quad (4.4)$$

and D is given in Eq. (3.4). There are also first-order diagrams in this model which cancel because the trace in Fig. 5(a) and the one in Fig. 5(b) have opposite sign for paraquarks.

$\Gamma_{0\bar{0}}$ is again obtained from Eq. (3.5). In addition to the diagrams in Fig. 2 there are now a number of diagrams involving neutral currents which can be found in Ref. 10. The sum of all the diagrams gives for the decay amplitudes the values[10]

$$A_{+-} = u_1 + u_2\sin 2\psi,$$
$$A_{00} = u_3 + u_4\sin 2\psi,$$
$$\overline{A}_{+-} = u_1 - u_2\sin 2\psi,$$
$$\overline{A}_{00} = u_3 - u_4\sin 2\psi, \quad (4.5)$$

where

$$u_1 = (1.918 + i1.867)\times 10^{-7}\text{ GeV},$$
$$u_2 = (-4.666 + i5.185)\times 10^{-7}\text{ GeV},$$
$$u_3 = -(1.535 + i1.320)\times 10^{-7}\text{ GeV},$$
$$u_4 = (3.300 - i3.282)\times 10^{-7}\text{ GeV}. \quad (4.6)$$

These amplitudes fulfill the CPT relation Eq. (3.9). Again we determine the CP violation parameter ψ from a best fit to the experimental values of Reϵ, η_{+-}, and η_{00}.

Next we shall consider the model of Das.[6] As before, the charged current is of Cabibbo type. The neutral current is now given by

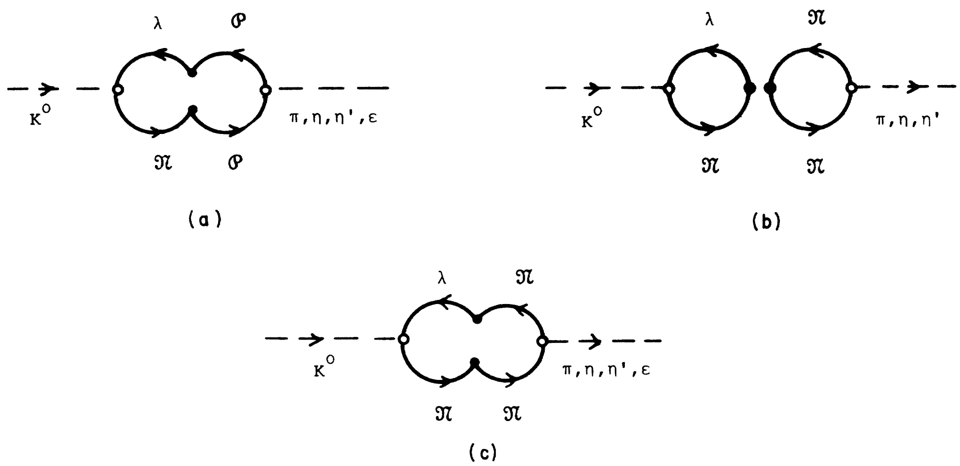

FIG. 3. Quark diagrams for the $K^0 \to \pi$, η, η' and ϵ transitions. While (a) involves only charged currents, (b) and (c) contain the neutral currents Eqs. (4.1) and (4.7), respectively.

$$J_0^\mu = \tfrac{1}{3}[2\bar{\mathcal{P}}\gamma^\mu(1+\gamma_5)\mathcal{P} - \bar{\mathcal{N}}\gamma^\mu(1+\gamma_5)\mathcal{N} - \bar{\lambda}\gamma^\mu(1+\gamma_5)\lambda]\cos\theta_C + \bar{\mathcal{N}}[\gamma^\mu(1+\gamma_5)+\delta k^\mu]\lambda e^{-i\xi}\sin\theta_C. \tag{4.7}$$

The transition $K^0(\bar{K}^0) \to \pi^0, \eta, \eta', \epsilon$ may proceed in addition to the graphs shown in Fig. 3 by the graph in Fig. 6. Out of these four building blocks 16 diagrams for the transition $\bar{K}^0 \to K^0$ result. For the pseudoscalar intermediate states π, η, η' the various combinations of single- and double-loop graphs which contain neutral currents cancel. So finally in addition to the charged current contribution in Fig. 1 there remain six second-order diagrams for the $\bar{K}^0 \to K^0$ transition which are shown in Fig.

7. The first-order diagrams in Fig. 5 cancel as before. For $m_{0\bar{0}}$ we now obtain the expression

$$m_{0\bar{0}} = -\frac{2}{9m_K}C^2D(1.5 - e^{-i\xi})^2 + \frac{1}{18m_K}E^2\frac{1}{m_K{}^2 - m_\epsilon{}^2}(3 - e^{-i\xi})^2, \tag{4.8}$$

where C, D, and E are given in Eqs. (4.3), (3.4), and (4.4), respectively.

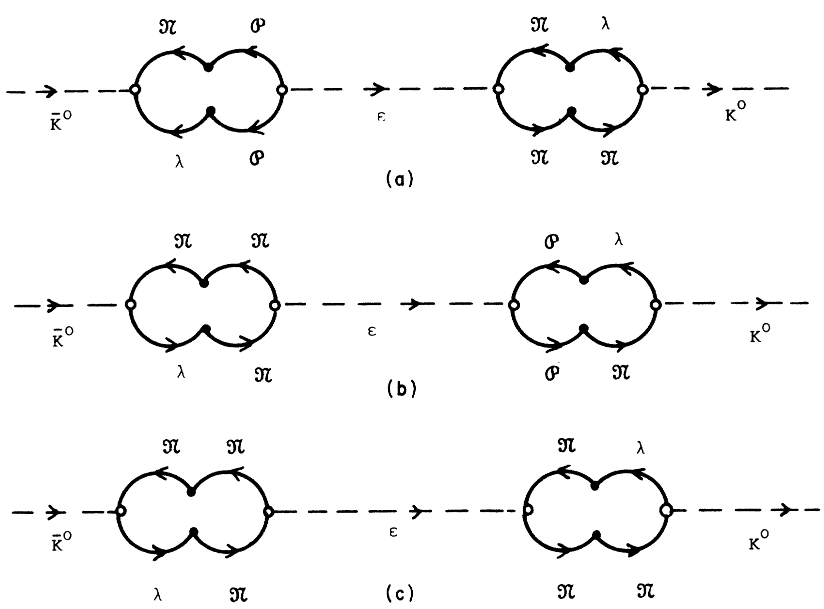

FIG. 4. Additional diagrams for the $\bar{K}^0 \to K^0$ transition in the Oakes model.

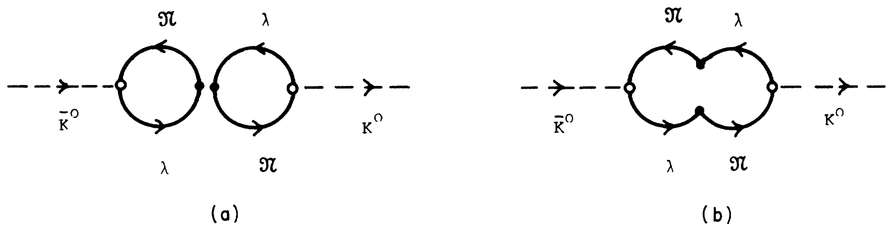

FIG. 5. First-order diagrams for the $\bar{K}^0 \to K^0$ transition in the models of Oakes and Das.

For the kaon decay amplitudes in $\Gamma_{0\bar{0}}$ [Eq. (3.5)] in addition to Fig. 2 there are a large number of neutral current diagrams.[10] Altogether one obtains the expressions[10]

$$\begin{aligned} A_{+-} &= v_1 + v_2 e^{-i\xi}, \\ A_{00} &= v_3 + v_4 e^{-i\xi}, \\ \bar{A}_{+-} &= v_1 + v_2 e^{i\xi}, \\ \bar{A}_{00} &= v_3 + v_4 e^{i\xi}, \end{aligned} \tag{4.9}$$

with

$$\begin{aligned} v_1 &= -(0.3389 + i0.3082)\times 10^{-6}\ \text{GeV}, \\ v_2 &= -(0.1127 + i0.1130)\times 10^{-6}\ \text{GeV}, \\ v_3 &= (0.2217 + i0.2179)\times 10^{-6}\ \text{GeV}, \\ v_4 &= (0.0976 + i0.0799)\times 10^{-6}\ \text{GeV}, \end{aligned} \tag{4.10}$$

which again satisfy the CPT relation Eq. (3.9). The parameter ξ is determined in the same way as in the previous cases.

V. DISCUSSION OF THE RESULTS

Let us first consider those quantities for which CP violation has only a negligible effect. These are the $K_S^0 \to 2\pi$ decay width and the kaon mass difference. While the models of Glashow[4] and Oakes[5] reproduce for these quantities the results of the Cabibbo theory the model of Das gives different values. This is already plausible from an inspection of the currents. While the strangeness-changing parts of the currents in Eqs. (3.2) and (4.1) reduce to the Cabibbo current for $\phi = \omega = 0$ and $\psi = 0$, respectively, the current of Das Eq. (4.7) retains a strangeness-changing neutral part even in the limit $\xi \to 0$. The latter gives rise to the above-mentioned discrepancy. In Table I our theoretical results are compared with experiment. For the models of Glashow and Oakes the theoretical decay rate

$$(K_S \to \pi\pi) = 7.5\times 10^{-12}\ \text{MeV} \tag{5.1}$$

is in very good agreement with the experimental data and the kaon mass difference

$$m_{K_L} - m_{K_S} = 1.9\times 10^{-12}\ \text{MeV} \tag{5.2}$$

is not too far from the experimental value as can be seen from Table I. The Das model, on the other hand, gives a decay rate which is larger than the experimental one but also a larger value for the kaon mass difference. In the first two cases the mass difference is a little bit too small which is presumably due to the fact that we have restricted the sum over intermediate states in Eq. (2.9) to meson pole contributions.

Now let us turn to the CP-violating quantities. For the Glashow model there are two CP-violating angles ϕ and ω [Eq. (3.2)], while for the models of Oakes and of Das there is only one ψ [Eq. (4.1)] and ξ [Eq. (4.7)], respectively. For all three models we determine the CP-violating angles from a best fit of the theoretical values of the CP-violating parameters ϵ, η_{+-}, and η_{00}. More specifically we perform a least-squares fit minimizing the following function:

$$\begin{aligned} F(\text{Re}\epsilon, |\eta_{+-}|, \Phi_{+-}, |\eta_{00}|, \Phi_{00}) &= (\text{Re}\epsilon - \text{Re}\epsilon^{\text{exp}})^2/(\Delta\,\text{Re}\epsilon^{\text{exp}})^2 \\ &+ (|\eta_{+-}| - |\eta_{+-}^{\text{exp}}|)^2/(\Delta|\eta_{+-}^{\text{exp}}|)^2 + (\Phi_{+-} - \Phi_{+-}^{\text{exp}})^2/(\Delta\Phi_{+-}^{\text{exp}})^2 \\ &+ (|\eta_{00}| - |\eta_{00}^{\text{exp}}|)^2/(\Delta|\eta_{00}^{\text{exp}}|)^2 + (\Phi_{00} - \Phi_{00}^{\text{exp}})^2/(\Delta\Phi_{00}^{\text{exp}})^2 . \end{aligned} \tag{5.3}$$

In Eq. (5.3) the label exp means experimental value and Δ indicates the experimental error.

The results of this procedure are listed in Table I. From an inspection of these results one can conclude that the Glashow model is in good agreement with experiment (see Ref. 11), while for the models of Oakes and Das no satisfactory fit can

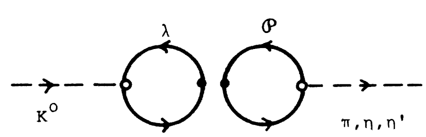

FIG. 6. Additional graph for the $K^0 \to \pi$, η, η' transition arising in the Das model.

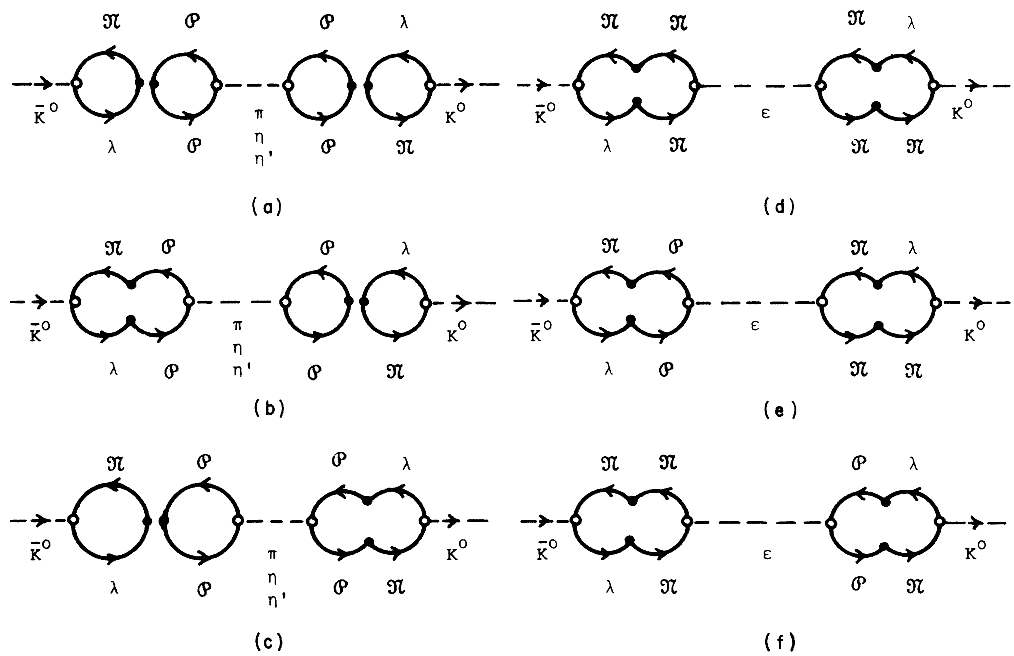

FIG. 7. Quark diagrams for the $\bar{K}^0 \to K^0$ transition which occur in the Das model in addition to Fig. 1.

be achieved. Furthermore, we would like to stress that the CP-violating phase angles ω and ϕ turn out to be of the order 10^{-4} which is one order of magnitude smaller than the original estimate.[4] Such a small value of the CP-violating phase angles may save the model[12] if the experimental upper limit for the neutron electric dipole moment[13] is lowered further.

TABLE I. Comparison of our theoretical results with the experimental data. The CP-violating angles are determined from a least-squares fit of the theoretical predictions for the CP-violating parameters to the experimental data.

	Glashow model	Oakes model	Das model	Experiment (Ref. 11)
$\Gamma(K_S \to \pi\pi)$ (MeV)	7.5×10^{-12}	7.5×10^{-12}	8.2×10^{-12}	$(7.43\pm0.06)\times10^{-12}$
$m_{K_L}-m_{K_S}$ (MeV)	1.9×10^{-12}	1.9×10^{-12}	54×10^{-12}	$(3.56\pm0.02)\times10^{-12}$
CP-violating angles	$\phi=2.94\times10^{-4}$, $\omega=-2.86\times10^{-4}$	$\psi=1.8\times10^{-4}$	$\xi=3.0\times10^{-3}$ [b]	···
Reϵ	1.55×10^{-3} [a]	0.53×10^{-4}	-2.43×10^{-4}	$(1.67\pm0.06)\times10^{-3}$
Imϵ	1.28×10^{-3}	0.91×10^{-3}	-1.48×10^{-3}	···
$\vert\eta_{+-}\vert$	2.34×10^{-3}	1.84×10^{-3}	3.83×10^{-4}	$(2.30\pm0.035)\times10^{-3}$
Φ_{+-}	48.9°	87.2°	−18.6°	49.4 ± 1.7°
$\vert\eta_{00}\vert$	2.38×10^{-3}	1.72×10^{-3}	5.45×10^{-4}	$(2.33\pm0.14)\times10^{-3}$
Φ_{00}	48.9°	−89.6°	82.8°	53.4 ± 13.1°
$\vert\eta_{00}/\eta_{+-}\vert$	1.019	0.938	1.42	1.013 ± 0.046
χ^2 value	7.8	3×10^3	9×10^3	···

[a] This value is in good agreement with the $K_{\mu3}$ average Re$\epsilon=(1.59\pm0.12)\times10^{-3}$ [K. Kleinknecht, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. III-53].

[b] The results for $\vert\xi-\pi\vert\sim10^{-3}$ are equally bad.

Another remarkable feature for the model of Glashow is that the phases of the $\Delta S=0$ and $\Delta S=1$ weak axial currents turn out to be nearly opposite

$$\Phi+\omega=8\times 10^{-6}. \quad (5.4)$$

This numerical result confirms explicitly what had been shown qualitatively before to be a sufficient condition for the equality of η_{+-} and η_{00} (see Ref. 14) and for the approximate $\Delta I=\frac{1}{2}$ rule.[15] In the quark model $\Phi+\omega$ is proportional to $\mathrm{Re}\epsilon-\mathrm{Re}\eta_{+-}$.[7] With the present experimental data $\mathrm{Re}\epsilon$ differs slightly from $\mathrm{Re}\eta_{+-}$ which explains the result (5.4).

---

*Also at Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, Wien, Austria.

†Until August 1974 at Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, Wien, Austria.

[1]M. Böhm, H. Joos, and M. Krammer, Nucl. Phys. B B51, 397 (1973); in *Recent Developments in Mathematical Physics*, proceedings of the XII Schladming Conference on Nuclear Physics, edited by P. Urban (Springer, Berlin, 1973) [Acta Phys. Austriaca Suppl. 11, 3 (1973)].

[2]D. Flamm, P. Kielanowski, and J. Sanchez Guillen, Nuovo Cimento 25A, 16 (1975).

[3]The very successful superweak model of Wolfenstein [L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964)] is not considered here since it cannot be formulated in a simple way in quark language.

[4]S. Glashow, Phys. Rev. Lett. 14, 35 (1965); W. Alles, Phys. Lett. 15, 348 (1965); A. Morales, R. Nunez-Lagos, and M. Soler, Nuovo Cimento 38, 1608 (1965).

[5]R. J. Oakes, Phys. Rev. Lett. 20, 1539 (1968).

[6]T. Das, Phys. Rev. Lett. 21, 409 (1968).

[7]D. Flamm, P. Kielanowski, A. Morales, R. Nunez-Lagos, and J. Sanchez Guillen, University of Zaragoza Report No. FAN.-Z/6/74, 1974 (unpublished).

[8]S. D. Protopopescu *et al.*, Phys. Rev. D 7, 1279 (1973).

[9]D. Flamm, P. Kielanowski, and J. Sanchez Guillen, Acta Phys. Slov. 24, 265 (1974).

[10]D. Flamm, P. Kielanowski, A. Morales, R. Nunez-Lagos, and J. Sanchez Guillen, Acta Phys. Austriaca 42, 289 (1975).

[11]V. Soergel, in proceedings of the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 [J. Phys. (Paris) Suppl. 34, C1-83 (1973)].

[12]L. Wolfenstein, Nucl. Phys. B77, 375 (1974).

[13]W. B. Dress, P. D. Miller, and N. F. Ramsey, Phys. Rev. D 7, 3147 (1973).

[14]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 8, 2317 (1973).

[15]A. Morales and J. Sanchez Guillen, Phys. Rev. D 10, 350 (1974).