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Deep-inelastic electroproduction of pions in a quark-parton model*

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Invariant cross sections for the processes $ep \to e\pi^{\pm} +$ anything are calculated in a quark-parton model. We use a wave-function formalism which we show to be equivalent to the Kuti-Weisskopf quark-parton model of hadrons. Imposing simplifying assumptions valid for a high-momentum final state, we obtain good agreement with existing experimental data.

I. INTRODUCTION

In the past few years refinements of the parton model of hadrons, originally proposed by Feynman,[1] have been successful in predicting some details of high-energy electroproduction. These models[2,3] view hadrons as being composed of point constituents, or partons, which can be taken to have the quantum numbers of quarks. A choice of the parton momentum distribution allows a calculation of the known properties of deep-inelastic lepton-nucleon scattering, and the results have been satisfactory. Since the parton picture does give good results for the deep-inelastic scattering, we want to try the model on a reaction requiring more detailed considerations of the constituents, namely $e+N \to e+\pi^{\pm}+$ anything.

One model for the parton momentum distribution inside a hadron was proposed by Kuti and Weisskopf.[3] They viewed a nucleon as being composed of a core in which the quarks are distributed statistically, and of a valence in which the momentum distribution of the quarks depends on the special dynamics of the valence quarks. The model was used to calculate the invariant structure function $\nu W_2(x)$ for the processes $ep \to e+$ anything and $en \to e+$ anything. Also, $\int_0^1 \nu W_2(x)dx$ and $\nu W_2(\text{neutron})/\nu W_2(\text{proton})$ were calculated. All were found to be in substantial agreement with the available experimental data. Later, McElhaney and Tuan[4] modified the model slightly to obtain good agreement with newer experimental data.

Taking the Kuti-Weisskopf model as a starting point, we have been able to calculate the cross sections for $e+p\to e+\pi^{\pm}+$anything, for pions going backward in the c.m. frame, and obtain tolerable agreement with the data. The calculation is described in the next section (Sec. II), beginning with an explanation of what we do and why we are limited to the backward direction. Our results are compared to the data in Sec. III. Regarding the details of the Kuti-Weisskopf model, we make some changes, mainly notational, so that one can talk of the amplitude rather than the probability for finding a given type of parton at a given momentum. This is discussed in the Appendix.

II. CALCULATION

We choose a frame of reference in which the incoming parton is moving infinitely fast (take $P\to\infty$ below) in a direction perpendicular to the photon's 3-momentum. Explicitly,

$$P^{\mu} = (P+M^2/2P, 0, 0, P), \tag{2.1}$$

$$q^{\mu} = (M\nu/P, \vec{q}_{\perp}, 0). \tag{2.2}$$

M is the proton's mass and $P\cdot q = M\nu$. The proton is replaced by a beam of noninteracting partons and the virtual photon is viewed as interacting with a single free parton, the remaining partons being spectators. The momentum of any given parton will be

$$P_i^{\mu} = (x_i P+\mu^2/2x_i P, \vec{P}_{i\perp}, x_i P),$$

provided $x \gg M/P$ and where μ is the parton's mass. We expect $|\vec{P}_{\perp}| \lesssim 500$ MeV, so that for x not very small and for $\nu \gg M$ we have $x_i = P_i\cdot q/M\nu$.

When the photon interacts with any one of the partons it disturbs the parton configuration, which then no longer forms a proton. The proton "fragments" and the partons, which cannot themselves appear in the final state, recombine into observable physical particles. The details of the recombination are unknown because we do not know the strong parton-parton interactions or the nature of the possible parton fragmentation into hadrons. One may despair of these problems and believe that they limit the parton model to totally inclusive lepton-induced reactions. But any model is likely to be plagued with similar problems, so let us be positive and ask where (i.e., in what special kinematic regions) a simple parton picture *could* give correct quantitative results. We think that partons, or their derivative hadrons, that have an appreciable fraction of the proton momentum (in the infinite-momentum frame) are susceptible to relatively simple calculations.

More specifically, let us hypothesize that after the proton is fragmented, partons with relatively large x escape from the parton blob without having their momentum significantly changed by interaction with the spectators. This means that the large-x partons have the same momentum distribution after the proton fragments and the partons have spread apart as they had before the proton fragmented.

We can attempt to justify this hypothesis heuristically on the grounds that large-x partons are well separated, in rapidity space, from their fellows. Let us first note that there are few large-x partons: Obviously there is not more than one parton with $x > \frac{1}{2}$, not more than two partons (if that many) with $x > \frac{1}{3}$, etc., and most partons will have very small x ("wee partons"). The momentum of one parton is

$$P_1^\mu = (x_1 P + \mu^2/2x_1 P, \vec{P}_{1\perp}, x_1 P), \tag{2.3}$$

so that (neglecting $\vec{P}_\perp$)

$$s_{12} = (P_1 + P_2)^2 = \mu^2(x_1/x_2 + x_2/x_1). \tag{2.4}$$

The rapidity difference, $\ln(s_{12}/M^2)$, determining the decoupling of partons 1 and 2 by well-known arguments,[5] with more than one or two units difference meaning negligible interaction. For x_1 and x_2 very unequal, s_{12} can be a large factor times μ^2. For example, for $x_1 = 0.5$ and $x_2 = 0.05$, the rapidity difference becomes $\ln(10.1\mu^2/M^2)$. If we take $\mu^2 \approx M^2$, this is sufficient to have large-x partons decoupled from wee partons. Of course, one does not know the parton mass. Instead of taking it to be some typical hadron mass, one may take $\mu = 0$ for ease of calculation,[6] argue that $\mu \simeq 300$ MeV, or even that it is infinite.[7] We can only apologize for the uncertainty, adding that it seems to us that a small effective mass is unlikely because one must explain why partons are not seen. They must either be heavy or else be trapped by a dynamical mechanism, from which one can argue that a bound parton is off its energy shell by an amount of $O(1\text{ GeV})$.[6]

So we will take as a working hypothesis the idea that large-x partons do not couple strongly to the other partons and thus we know the parton spectrum for large x. The function of the incoming photon is only to fragment the proton, freeing its constituents (freeing them to become other things, that is). The photon does not have to interact directly with the large-x partons we have been discussing; we do require that it carry enough energy ($\nu \gg M$) to make the parton picture valid.

What happens to the escaping parton? There are two simple models we can think about, which seem different physically and give quantitatively different predictions. In the first model consider the fragmented proton to be a reservoir of partons, with some partons (perhaps only one) having large x and not interacting strongly with the remainder. However, a parton leaving the interaction zone must pick up at least one other parton to make a hadron. Since the interaction is weak, it is likely that only one extra parton will be grabbed,[8] and since this extra parton probably will be "wee", the momentum spectrum of the fast emerging mesons will be that of the fast partons. If the meson has a fraction y of the original proton's momentum in the infinite momentum frame and the distribution of partons is given by $F(x)$, then $d\sigma/dy \propto F(y)$ for large y. (Large y requires that the meson is going backwards in the $\gamma_v p$ c.m. system. Indeed, for the numbers relevant to available experiments any $y \geq 0.06$ is backward going in the c.m. system.) The types of mesons that come out are predicted by considering what types of valence and core quark-partons were in the proton to begin with. (The details are given below.)

Alternately, the large-x parton itself may fragment into hadrons. This has been discussed by Berman, Bjorken, and Kogut[9] in connection with reactions where partons emerge with large transverse momenta. The idea is that the large transverse momentum isolates the parton from its fellows, and we can describe its fragmentation independently of the rest of the partons. We have argued that we have a degree of isolation here also for large-x partons, and so can try to carry the idea over. If the probability of finding a hadron with longitudinal momentum fraction x' of its parent parton is scale invariant, as

$$\frac{dN_h}{dx'} = g_h(x'), \tag{2.5}$$

then the measured hadron distribution should be

$$\frac{d\sigma_h}{dy} \propto \int_y^1 dx\, F_h(x) g_h(y/x). \tag{2.6}$$

It has been suggested[9,10] that for producing pions $g(y/x) \propto (1 - y/x)$ is reasonable. Comparison of this model to the previous model and to experimental data is reserved to Sec. III. The prediction of the data is tolerable with the first model, poor with the second.

We first calculate the matrix element of the hadronic current when one of the partons interacts with the virtual photon. The initial proton state is

$$|P\rangle = \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} \frac{dx_i}{(x_i^2 + \mu^2/P^2)^{1/2}} \delta\left(1 - \sum_{j=1}^{n} x_j\right) Z_P^{1/2} \phi(x_1 \cdots x_n)(n!)^{-1/2} |x_1 \cdots x_n\rangle, \tag{2.7}$$

where $Z_P^{1/2}\phi(x_1\cdots x_n)$, the n-parton wave function, is defined in the Appendix, and the final state is $|z_1\cdots z_n\rangle$, where z_i is the momentum fraction of the ith parton in the final state. Suppose that it is the n^{th} parton that is struck; then

$$\langle z_1\cdots z_n|J^\mu|P\rangle=\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}\frac{dx_i}{(x_i^2+\mu^2/P^2)^{1/2}}\delta\left(1-\sum_{j=1}^{n}x_j\right)Z_P^{1/2}\phi(x_1\cdots x_n)\bar u(z_n)\gamma^\mu u(x_n)\prod_{m=1}^{n-1}\delta(x_m-z_m). \tag{2.8}$$

In the infinite momentum frame $x_n\approx z_n$, and therefore $\bar u(z_n)\gamma^\mu u(x_n)\approx 2P^\mu x_n\delta_{ss'}$, where s, s' are spin indices. Substituting this in Eq. (2.8) and doing the integrations gives

$$\langle z_1\cdots z_n|J^\mu|P\rangle=2P^\mu\delta_{ss'}Z_P^{1/2}\phi(z_1\cdots z_{n-1},\mathfrak{z}_n).$$

But $\mathfrak{z}_n=1-\sum_{j=1}^{n-1}z_j\approx z_n$. Therefore, the hadronic current is

$$\langle z_1\cdots z_n|J^\mu|P\rangle=2P^\mu\delta_{ss'}Z_P^{1/2}\phi(z_1\cdots z_n), \tag{2.9}$$

independent of which parton interacts with the virtual photon.

The spin-averaged cross section is

$$\frac{d\sigma}{d\Omega'dE'}=\frac{E'}{4ME}\int\prod_{i=1}^{n}\frac{d^3P_i'}{(2\pi)^3 2E_i'}2(2\pi)^3\delta\left(q+P-\sum_{j=1}^{n}P_j'\right)\frac{\alpha^2\epsilon^2}{Q^4}\frac{1}{4}\sum_{s,s'}|\bar u(k')\gamma_\mu u(k)\langle z_1\cdots z_n|J^\mu|P\rangle|^2, \tag{2.10}$$

where k' is the final electron momentum, P_i' is the final momentum of the ith parton, and ϵ^2 is the charge squared in units of e of the struck parton. Using the above form for the hadronic current, the differential cross section for observing one parton is

$$\frac{d\sigma}{d\Omega'dE'dy}=\frac{4\alpha^2E'^2}{Q^4}\epsilon^2\frac{M}{y}\sum_{n=0}^{\infty}\int\prod_{i=2}^{n}\frac{dz_i}{(z_i^2+\mu^2/P^2)^{1/2}}\delta_{0,3}{}^2\left(q+P-y-\sum_{j=2}^{n}P_j'\right)Z_P\phi^2(y,z_2\cdots z_n), \tag{2.11}$$

where y is the momentum fraction carried by the parton.

There are three cases to be distinguished, as it makes some small difference which of the three types of quarks ($\mathcal{P}$ valence, $\mathfrak{N}$ valence, core) is the fast parton after the interaction.

Case I: A core quark is the fast parton. The initial proton state is

$$\begin{aligned}|P\rangle=&\sum_{k_i=0,2,4,\dots;\,l=0,1,2,\dots}\int\prod_{i=1}^{n-1}\frac{dx_i}{(x_i^2+\mu^2/P^2)^{1/2}}\frac{dy}{(y^2+\mu^2/P^2)^{1/2}}\delta\left(1-\sum_{j=1}^{n-1}x_j-y\right)\\&\times\left[Z_Px_1^{1-\alpha(0)}x_2^{1-\alpha(0)}(1-x_3)x_3^{1-\alpha(0)}\frac{(\frac{1}{3}g)^{k_1}(\frac{1}{3}g)^{k_2}(\frac{1}{3}g)^{k_3}g'^{l}}{k_1!\;k_2!\;k_3!\;l!}\right]^{1/2}\\&\times(k_1!k_2!k_3!l!)^{-1/2}|x_1\cdots x_{n-1},y\rangle,\end{aligned} \tag{2.12}$$

where y is the momentum fraction of the fast quark. For this form of the proton state, the normalization condition, Eq. (A4), implies that

$$\begin{aligned}1=&\sum_{k_i=0,2,4,\dots;\,l=0,1,2,\dots}\int\prod_{i=1}^{n-1}\frac{dx_i}{(x_i^2+\mu^2/P^2)^{1/2}}\frac{dy}{(y^2+\mu^2/P^2)^{1/2}}\delta\left(1-\sum_{j=1}^{n-1}x_j-y\right)\\&\times Z_Px_1^{1-\alpha(0)}x_2^{1-\alpha(0)}(1-x_3)x_3^{1-\alpha(0)}\frac{(\frac{1}{3}g)^{k_1}(\frac{1}{3}g)^{k_2}(\frac{1}{3}g)^{k_3}g'^{l}}{k_1!\;k_2!\;k_3!\;l!}.\end{aligned} \tag{2.13}$$

This integration can be performed using the techniques described in Appendix B of Ref. 3 to obtain

$$Z_P=a^\gamma\frac{\Gamma(\gamma+3[1-\alpha(0)])}{\Gamma^3(1-\alpha(0))}\left(1-\frac{1-\alpha(0)}{\gamma+3[1-\alpha(0)]}\right)^{-1}, \tag{2.14}$$

where $a^2\equiv\mu^2/P^2$ and $\gamma\equiv g+g'$.

The exact form of the two-dimensional δ function in the differential cross section, Eq. (2.11), depends on which parton interacts with the virtual photon. We shall exclude the case where the virtual photon interacts with the fast quark.

If the struck parton is a $\mathcal{P}$ valence quark, say x_1, then

$$\delta_{Q,3}{}^2\left(q+P-\sum_{j=1}^{n}P_j'\right)=2k\frac{z_1}{M\nu}\delta(z_1-x)\,\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right), \tag{2.15}$$

where the factor of 2 indicates that there are two $\mathcal{P}$ valence quarks in the proton, and the k indicates there are k choices for the core quark which becomes the fast parton. If the struck parton is an $\mathfrak{N}$ valence quark,

$$\delta_{Q,3}{}^2\left(q+P-\sum_{j=1}^{n}P_j'\right)=k\frac{z_3}{M\nu}\delta(z_3-x)\,\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right), \tag{2.16}$$

and if the struck parton is a core quark, but not the fast parton, then

$$\delta_{Q,3}{}^2\left(q+P-\sum_{j=1}^{n}P_j'\right)=k(k-1)\frac{z_4}{M\nu}\delta(z_4-x)\,\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right) \quad . \tag{2.17}$$

Substituting these δ functions into Eq. (2.11), the differential cross section for case I is

$$y\frac{d\sigma_{\rm I}}{d\Omega' dE' dy}=\frac{4\alpha^2E'^2}{Q^4}\frac{1}{\nu}\left(2\tfrac{4}{9}gI_{\mathcal{P}}+\tfrac{1}{9}gI_{\mathfrak{N}}+\tfrac{2}{9}g^2I_c\right), \tag{2.18}$$

where

$$I_{\mathcal{P}}=x^{1-\alpha(0)}B^{-1}(1-\alpha(0),\gamma+2[1-\alpha(0)])\,(1-y-x)^{-1+\gamma+2(1-\alpha(0))}\left(1-\frac{1-\alpha(0)}{\gamma+2[1-\alpha(0)]}(1-y-x)\right)\left(1-\frac{1-\alpha(0)}{\gamma+3[1-\alpha(0)]}\right)^{-1},$$

$$I_{\mathfrak{N}}=(1-x)x^{1-\alpha(0)}B^{-1}(1-\alpha(0),\gamma+2[1-\alpha(0)])\,(1-y-x)^{-1+\gamma+2(1-\alpha(0))}\left(1-\frac{1-\alpha(0)}{\gamma+3[1-\alpha(0)]}\right)^{-1}, \tag{2.19}$$

$$I_c=(1-y-x)^{-1+\gamma+3(1-\alpha(0))}\left(1-\frac{1-\alpha(0)}{\gamma+3[1-\alpha(0)]}(1-y-x)\right)\left(1-\frac{1-\alpha(0)}{\gamma+3[1-\alpha(0)]}\right)^{-1}.$$

The $B(\cdots)$ is a beta function, and $x\equiv Q^2/2M\nu$.

Case II: The $\mathfrak{N}$ valence quark becomes the fast parton. For this case the δ function in Eq. (2.11) is

$$\delta_{Q,3}{}^2\left(q+P-\sum_{j=1}^{n}P_j'\right)=2\frac{z_1}{M\nu}\delta(z_1-x)\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right)+k\frac{z_n}{M\nu}\delta(z_n-x)\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right). \tag{2.20}$$

The differential cross section is

$$y\frac{d\sigma_{\rm II}}{d\Omega' dE' dy}=\frac{4\alpha^2E'^2}{Q^4}\frac{1}{\nu}\left(2\tfrac{4}{9}I_{\mathcal{P}}+\tfrac{2}{9}gI_c\right). \tag{2.21}$$

Case III: A $\mathcal{P}$ valence quark becomes the fast parton. The δ function in Eq. (2.11) is

$$\delta_{Q,3}{}^2\left(q+P-\sum_{j=1}^{n}P_j'\right)=\left(\frac{z_1}{M\nu}\delta(z_1-x)+\frac{z_3}{M\nu}\delta(z_3-x)+k\frac{z_n}{M\nu}\delta(z_n-x)\right)\delta\left(1-y-\sum_{j=1}^{n-1}z_j\right). \tag{2.22}$$

The cross section is

$$y\frac{d\sigma_{\rm III}}{d\Omega' dE' dy}=\frac{4\alpha^2E'^2}{Q^4}\frac{1}{\nu}2\left(\tfrac{4}{9}I_{\mathcal{P}}+\tfrac{1}{9}I_{\mathfrak{N}}+\tfrac{2}{9}gI_c\right). \tag{2.23}$$

To be able to compare our cross sections with the experiments, we must write them in terms of the invariant cross section $E(d^3\sigma/dp^3)$ for the process $\gamma_v p\to\pi+$anything. In the infinite-momentum frame the invariant cross section is

$$E\frac{d^3\sigma}{dp^3}=\frac{y}{\pi}\frac{d\sigma}{dy\,dP_\perp^2}, \tag{2.24}$$

where γ_v is the virtual photon, and, as before, y is the momentum fraction of the outgoing parton. In terms of the cross section we calculated above

$$\frac{d\sigma}{d\Omega' dE' dy}=\Gamma\frac{d\sigma}{dy}, \tag{2.25}$$

where Γ is the flux of virtual photons. Combining Eqs. (2.24) and (2.25), integrating over $P_\perp^2$, and normalizing to the total cross section yields the function

$$F(y)\equiv\frac{1}{\sigma_{\rm tot}\pi\Gamma}\int_0^\infty y\frac{d\sigma}{d\Omega' dE' dy\,dP_\perp^2}dP_\perp^2, \tag{2.26}$$

where σ_{tot} is the total electroproduction cross section. It is the function $F(y)$ that will be compared with experiments in the next section. The total cross section in this model can be written in terms of $W_2(x)$,

$$W_2(x) = \frac{K}{4\pi^2\alpha}\,\frac{Q^2}{Q^2+\nu^2}\,\sigma_{\text{tot}}, \tag{2.27}$$

where

$$K = \nu - Q^2/2M.$$

We use the modified Kuti-Weisskopf form for $\nu W_2(x)$ to calculate σ_{tot}.

III. COMPARISON WITH EXPERIMENT

We have calculated the cross sections for finding high-momentum partons. As mentioned in Sec. II, these cross sections must be multiplied by the probability that a given quark will become a specified final-state particle. By taking the extreme position that the high-momentum parton goes into the valence of the final-state particle and that only one additional parton is picked up, the first model discussed in Sec. II, we are able to make an estimate of these probabilities. For example, there are nine ways of selecting a quark and an antiquark from the proton core, but only one of these, $(\mathcal{P}\bar{\mathfrak{N}})$, can become a π^+. Thus, the probability is $\frac{1}{9}$ that the core will contribute to the π^+. Likewise, the probability is $\frac{1}{9}$ that the core will contribute to the π^-, $\frac{1}{3}$ that the $\mathcal{P}$ valence will contribute to the π^+, and $\frac{1}{3}$ that the $\mathfrak{N}$ valence will contribute to the π^-. Therefore, for $\gamma_v p \to \pi^-$ + anything,

$$F(y) = \frac{1}{\sigma_{\text{tot}}\pi\Gamma}\int_0^\infty y\left(\frac{1}{9}\frac{d\sigma_{\text{I}}}{d\Omega' dE' dy} + \frac{1}{3}\frac{d\sigma_{\text{II}}}{d\Omega' dE' dy}\right) \times A e^{-BP_\perp^2}\,dP_\perp^2, \tag{3.1}$$

where we have assumed an exponential form for the transverse-momentum dependence of the cross section. For $\gamma_v p \to \pi^+$ + anything

$$F(y) = \frac{1}{\sigma_{\text{tot}}\pi\Gamma}\int_0^\infty y\left(\frac{1}{9}\frac{d\sigma_{\text{I}}}{d\Omega' dE' dy} + \frac{1}{3}\frac{d\sigma_{\text{III}}}{d\Omega' dE' dy}\right) \times A e^{-BP_\perp^2}\,dP_\perp^2. \tag{3.2}$$

There are, of course, other possibilities, such as the fast parton picking up more than one additional parton or the fast parton not going into the valence, that can affect the pion cross section. However, we shall assume that the probability of these occurrences is small, and compare the data to the simplest assumption described above.

As discussed in Ref. 3, the choices $\alpha(0) = \frac{1}{2}$ and

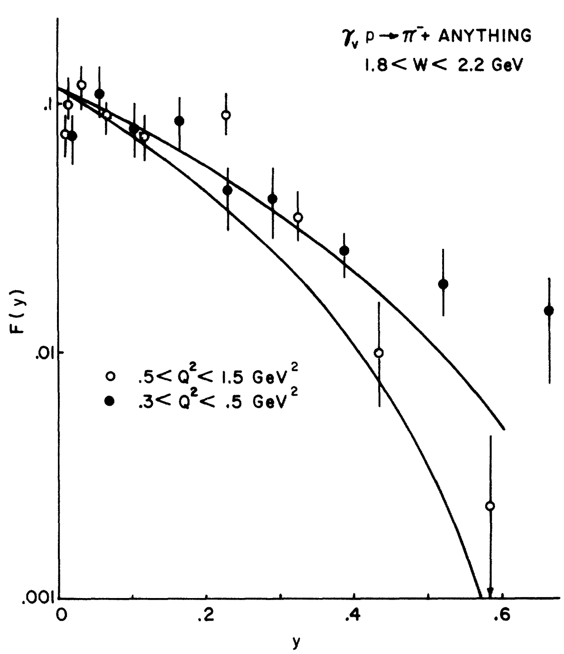

FIG. 1. The invariant function $F(y)$ for inclusive electroproduction of π^-. The top curve is for $W = 2.2$ GeV, $Q^2 = 0.3$ GeV^2, and the bottom curve is for $W = 2.2$, $Q^2 = 1.5$ GeV^2. Both curves have $g = 0.5$.

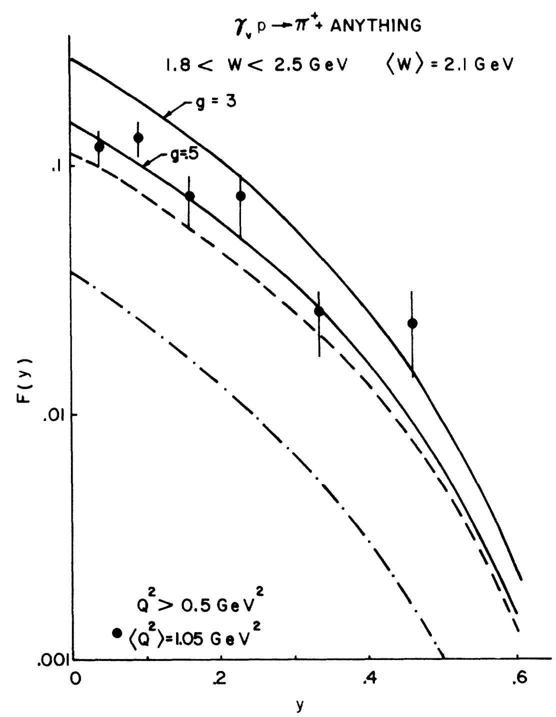

FIG. 2. The invariant function $F(y)$ for inclusive electroproduction of π^+ as a function of g. For all curves shown $W = 2.1$ GeV, $Q^2 = 1.05$ GeV^2. The dashed curve is the valence contribution, and the dash-dot curve is the core contribution to the $g = 0.5$ fit.

$\gamma = 3$ allow a range of possible values for g with the constraint $0 \leq g \leq 3$. McElhaney and Tuan[4] suggest that $g = 0.5$ gives the best agreement with the electroproduction data. We have found that $g = 0.5$ also gives a nice fit for $F(y)$. The experimental data[11,12] for $\gamma_v p \to \pi^-$ + anything and our calculations are plotted vs y in Fig. 1. The plots include only the data for backward-going pions in the c.m.[13] For the backward-going pions ($y \geq 0.06$) we obtain a reasonable fit to the data, but as expected in the region of small y the model predictions are too large [the data for forward-going pions would squeeze in between $y = 0$ and $y = 0.06$ on our plot; experimentally $F(0) \approx 10^{-2}$].

The function $F(y)$ for the process $\gamma_v p \to \pi^+$ + anything is compared to the data[14] in Fig. 2. Again $g = 0.5$ gives a good fit. For comparison, the calculated curve for $g = 3$ is also shown, as are the contributions of the valence quarks and core quarks. The contribution of the core is rather small.

Another interesting quantity is the ratio $\sigma(\gamma_v p \to \pi^+ + \cdots)/\sigma(\gamma_v p \to \pi^- + \cdots)$. Some quark-parton-model considerations of ratios such as these have been made by Dakin and Feldman.[15] Experimental determinations of this ratio have been made for a number of W and Q^2 values, but only in the forward direction in the $\gamma_v p$ c.m. system, which corresponds to $y \leq 0.06$. That is, the experimental points lie in the region where our model cannot be used to make accurate predictions. The experiments show the ratio increasing as γ_v becomes more virtual, and ranging between 1 and 2. We calculated the ratio for the larger values of y, and found that $\sigma(\pi^+)/\sigma(\pi^-) = 1.3$, with little variation as a function of Q^2.

The second model described in Sec. II, the parton fragmentation model, does not work at all well. This model predicts relatively too little π production at high y and too much at low y. The normalization is also too low, if we put in the suggested[6] factor, $g(y) = \frac{2}{3}(1-y)$; even at small y the predicted curve is about half the experimental cross section.

We can perhaps conclude that the high-y region at present experimental energies represents partons in an intermediate kinematic region where "grabbing" a single additional parton is what they are most likely to do. At lower y there will be multiparton interactions, while at higher energies the parton itself shows scaling behavior as it fragments. If this is true, it will be interesting to observe relatively fewer high-y pions produced in higher-energy experiments.

We have thus used a quark-parton model to calculate $ep \to e\pi^+$ + anything. The values of the parameters that we used were taken from the work of Kuti and Weisskopf[3] and McElhaney and Tuan,[4] where they were determined by fitting to the totally inclusive experiment $\gamma_v N \to$ anything. No new parameters are needed for the extension to the one-particle inclusive reaction $\gamma_v N \to \pi^\pm$ + anything, and our calculation of the longitudinal momentum spectrum of the pions has given reasonable agreement with the data in the kinematic region where agreement was expected.

APPENDIX

We will be using a formalism somewhat different from that used by Kuti and Weisskopf.[3] It is easier to extend their model if we take a bound-state picture[16] and describe the partons by writing wave functions rather than probabilities. In this section we explain our formalism and derive $G(x)$, the probability of finding a parton with momentum fraction between x and $x+dx$ in a proton with momentum P. We compare our $G(x)$ with the Kuti-Weisskopf result to determine under what conditions the two formulations are identical.

In its infinite-momentum frame, the proton's momentum vector is

$$P^\mu = (P + M^2/2P, 0, 0, P),$$

where the limit $P \to \infty$ will be taken. In this frame we view the proton as a parallel stream of n noninteracting partons. We define the momentum space wave function of the n parton state to be $\phi(P_1 \cdots P_n)$, where P_i is the momentum of the ith parton. The proton state vector can be written

$$|P\rangle = \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} \frac{d^3P_i}{(2\pi)^3 2E_i} (2\pi)^3 2P\delta\left(P - \sum_{j=1}^{n} P_j\right) \times \phi(P_1 \cdots P_n)(n!)^{-1/2} |P_1 \cdots P_n\rangle, \quad \text{(A1)}$$

where P is the proton momentum in the infinite-momentum frame, and the normalization of the n-particle state vector is $\langle P_1 \cdots P_n | P_1 \cdots P_n\rangle = n!$. To facilitate the calculation we will make the substitution $P_{i\parallel} = x_i P$, and assume that the wave function can be factored as

$$\phi(P_1 \cdots P_n) = \phi(x_1 \cdots x_n) f(P_{1\perp} \cdots P_{n\perp}), \quad \text{(A2)}$$

where $P_\perp$ is a transverse momentum. We will normalize the transverse part of the wave function by itself so that

$$1 = \int \prod_{i=1}^{n} \frac{d^2P_{i\perp}}{2(2\pi)^3} 2(2\pi)^3 \delta\left(\sum_{j=1}^{n} P_{j\perp}\right) \times |f(P_{1\perp} \cdots P_{n\perp})|^2. \quad \text{(A3)}$$

Then normalization of the proton state vector requires that

$$1=\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}\frac{dx_i}{(x_i^2+\mu^2/P^2)^{1/2}}\,\delta\left(1-\sum_{j=1}^{n}x_j\right)\times|\phi(x_1\cdots x_n)|^2. \tag{A4}$$

The probability of finding a parton with momentum fraction between x and $x+dx$ can be obtained from the normalization by not doing the last integration in Eq. (A4) and multiplying by a factor of n:

$$G(x)=\frac{1}{(x^2+\mu^2/P^2)^{1/2}}\sum_{n=0}^{\infty}\int\prod_{i=1}^{n-1}\frac{dx_i}{(x_i^2+\mu^2/P^2)^{1/2}}\times\delta\left(1-\sum_{j=1}^{n-1}x_j-x\right)n\times|\phi(x_1\cdots x_n)|^2. \tag{A5}$$

In the Kuti-Weisskopf model the proton is composed of three valence quarks, and a core of $q\bar{q}$ pairs and neutral gluons. The quarks in the core are distributed statistically,

$$dP_c(x)\propto g\frac{dx}{(x^2+\mu^2/P^2)^{1/2}}, \tag{A6}$$

and the distribution of the valence quarks is based on Regge considerations,

$$dP_v(x)\propto\frac{x^{1-\alpha(0)}}{(x^2+\mu^2/P^2)^{1/2}}\,dx, \tag{A7}$$

where $\alpha(0)$ is the intercept of the nondiffractive trajections. The Kuti-Weisskopf probability distributions of quarks in a proton are given by Eq. (2.7) in Ref. 3. Comparing this with our Eq. (A5) we can make the following identifications:

(i) The wave function for a valence quark is

$$\phi^2(x)\propto x^{1-\alpha(0)}.$$

(ii) The wave function for the k_i $q\bar{q}$ pairs of type i in the core is

$$\phi^2(x_1\cdots x_{ki})\propto\frac{(\frac{1}{3}g)^{k_i}}{k_i!},\quad i=1,2,3.$$

(iii) The wave function for the l neutral gluons is

$$\phi^2(x_1\cdots x_l)\propto\frac{g'^l}{l!}.$$

The modification of McElhaney and Tuan,[4] which brings the Kuti-Weisskopf model into closer agreement with experiments using neutron targets, changes the $\mathfrak{N}$ valence quark distribution to

$$dP_v(x)\propto\frac{(1-x)x^{1-\alpha(0)}}{(x^2+\mu^2/P^2)^{1/2}}\,dx. \tag{A8}$$

Equivalently, the wave function for an $\mathfrak{N}$ valence quark is

$$\phi^2(x)\propto(1-x)x^{1-\alpha(0)}.$$

Thus, the wave function of the n-parton state is

$$\phi^2(x_1\cdots x_n)=Z_p x_1^{1-\alpha(0)}x_2^{1-\alpha(0)}(1-x_3)x_3^{1-\alpha(0)}\times\frac{(\frac{1}{3}g)^{k_1}(\frac{1}{3}g)^{k_2}(\frac{1}{3}g)^{k_3}g'^l}{k_1!\;\;k_2!\;\;k_3!\;\;l!}, \tag{A9}$$

where Z_p is a normalization constant determined by Eq. (A4), and $k_1+k_2+k_3+l+3=n$. This form of the wave function can be used to calculate the $ep\to e+$anything cross section, and the Kuti-Weisskopf form of $\nu W_2(x)$ will be obtained.

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