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Regge model for the reaction $pp \to \pi^+ d$ *

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A calculation is made of the differential cross section and proton polarization for the reaction $pp \to \pi^+ d$ using a Regge-pole model with the exchange of the N_α and N_γ baryon trajectories. The adjustable parameters of the model are varied to obtain a best fit to the cross-section data at incident momenta above 7 GeV/c. The polarization is predicted to have the characteristic feature of vanishing at $t = -0.2$ and $t = -1.2$ (GeV/c)^2.

I. INTRODUCTION

The cross section for the reaction $pp \to \pi^+ d$ has been measured recently at several incident momenta between 7 and 25 GeV/c.[1-3] A few years ago, Barger and Michael,[4] using a Regge formalism similar to that developed by Lee,[5] calculated the differential cross section for $pp \to \pi^+ d$ at incident proton lab momenta above 6 GeV/c. They obtained reasonable agreement with the then existing data. Partly because of the new cross-section data and, more importantly, because of the feasibility of making measurements with polarized proton targets, it is worthwhile to consider this problem again in the Regge formalism. In this paper, we present a Regge calculation of both the differential cross section and the proton polarization.

Six helicity amplitudes are necessary to describe the reaction. Barger and Michael assumed that the two amplitudes in which the deuteron has helicity zero are dominant, and kept only those two amplitudes. In our treatment, we calculate the cross section both with and without the assumption that only these two helicity amplitudes are necessary. We find that we can get fairly good fits to the measured differential cross section with several qualitatively different amplitudes, some containing only two helicity amplitudes and others containing four or six. Thus, we find that the present data are not sufficient to determine the amplitudes uniquely, or to verify the conjecture of Barger and Michael that only two helicity amplitudes are dominant.

In the treatment of Lee and Barger and Michael, the dynamical mechanism causing the reaction is the exchange of baryon Regge trajectories. Barger and Michael's calculation uses only the N_α and N_γ Regge poles and we also confine ourselves to these two poles. We do not include any contribution from Regge cuts. Barger and Michael in their fits to the data used linear Regge trajectories. It is a feature of Lee's formalism that if the trajectories are linear, the polarization is zero. Thus, if we compute the polarization with the amplitudes found by Barger and Michael, we obtain zero. This result comes about because of the restricted nature of the Regge residues given by Lee and Barger and Michael. We therefore relax these restrictions in our own work, but otherwise are guided by the formalism developed by Lee.

In our work, we restrict ourselves to linear Regge trajectories. The polarization then arises from the interference of two trajectories, and is zero whenever the contribution from one of the trajectories vanishes. It is therefore a general feature of this model that the polarization is zero at the wrong-signature nonsense zeros of the N_α and N_γ trajectories. If we allowed the trajectories to be nonlinear, there would also be a contribution to the polarization from a single trajectory. But since the trajectories seem to be approximately linear,[6] this contribution would be expected to be small. It follows that relaxing the linearity condition on the trajectories would not change the qualitative features of the proton polarization. Therefore, a measurement of the proton polarization should provide a good test of this Regge model with two trajectories and wrong-signature nonsense zeros.

II. FORMALISM

We here briefly describe the Regge formalism, which differs from that of Lee[5] in only minor ways. We write the differential cross section $d\sigma/d\Omega$ for the reaction $pp \to \pi^+ d$ and the polarization P in terms of t-channel helicity amplitudes $F_{\lambda_p \lambda_{\bar p} \lambda}$ where λ_p, $\lambda_{\bar p}$, and λ are the helicities of the proton. antiproton, and deuteron, respectively. We have

$$\frac{d\sigma}{d\Omega} = \frac{p_f}{p_i} \frac{1}{16\pi^2 s} \sum |f_{\lambda_p \lambda_{\bar p} \lambda}|^2 , \tag{1}$$

$$P = \frac{-2\,\mathrm{Im}(F_{+++}F^*_{-++} + F_{++0}F^*_{-+0} + F_{++-}F^*_{-+-})}{\sum |F_{\lambda_p \lambda_{\bar p} \lambda}|^2} . \tag{2}$$

Following Lee, we define the parity-conserving amplitudes $f^{\pm}_{\lambda}$ by

$$F_{\pm++} = (1\mp\cos\theta)^{1/2}(f_1^+ \pm f_1^-)\ ,$$

$$F_{\pm+0} = (1\pm\cos\theta)^{1/2}(f_0^- \pm f_0^+)\ , \tag{3}$$

$$F_{\pm+-} = \sin\theta(1\pm\cos\theta)^{1/2}(f_{-1}^- \pm f_{-1}^+)\ ,$$

where θ is the t-channel scattering angle. (Elsewhere in the literature there exist slightly different definitions of the $f^\pm_\lambda$.[7,8]) We now assume that the f^+_λ have the following Regge representations for each trajectory ($\omega=\sqrt{t}$):

$$f^+_\lambda = \frac{(a_\lambda + b_\lambda\omega)\eta K^+_\lambda(\omega)}{\Gamma(\alpha+\frac{1}{2})\cos\pi\alpha}\left(\frac{s}{s_0}\right)^{\alpha-1/2},\quad \lambda=1,0$$

$$f^+_{-1} = \frac{(a_{-1} + b_{-1}\omega)(2\alpha-1)\eta K^+_{-1}(\omega)}{\Gamma(\alpha+\frac{1}{2})\cos\pi\alpha}\left(\frac{s}{s_0}\right)^{\alpha-3/2}\ , \tag{4}$$

where $\alpha(t)$ is the trajectory, the a's and b's and s_0 are adjustable real parameters, η is a signature factor given by

$$\eta=\tfrac{1}{2}(1+\tau e^{-i\pi(\alpha-1/2)}),\quad \tau=\pm1 \tag{5}$$

and the K^+_λ are kinematic factors given by

$$K^+_\lambda(\omega) = (1/\omega)[(\omega+m)^2-\mu^2]^{1/2}[(\omega+m)^2-M^2]^{-1/2}\ ,\quad \lambda=1,0$$

$$K^+_{-1}(\omega) = (p/\omega)[(\omega+m)^2-\mu^2]^{1/2}[(\omega-m)^2-M^2]^{1/2}\ . \tag{6}$$

Here m, μ, and M are the masses of the proton, pion, and deuteron, respectively, and p is the t-channel proton momentum. These kinematic factors are the ones given by Lee. Barger and Michael used different kinematic factors. We assume that the f^-_λ are related to the f^+_λ by MacDowell symmetry[9]

$$f^-_\lambda(\omega) = (-1)^\lambda f^+_\lambda(-\omega)\ . \tag{7}$$

It can be seen from Eqs. (3)–(7) that under the substitutions

$$a_1\to a_0,\quad b_1\to b_0,$$

$$a_0\to a_1,\quad b_0\to b_1 \tag{8}$$

the $\lambda=1,0$ helicity amplitudes transform into each other as follows:

$$F_{\pm++}\to\mp F_{\mp+0},\quad F_{\pm+0}\to\pm F_{\pm+\pm}\ . \tag{9}$$

It then can be seen from Eqs. (1) and (2) that neither the cross section nor the polarization changes under this transformation. We conclude that if the predictions of this model agree with experiment with only helicity-zero amplitudes, the predictions will also agree with experiment with only helicity-one amplitudes.

The factors $a_\lambda + b_\lambda\omega$ appearing in the expressions for the amplitudes [Eq. (4)] are not present in the treatment of Lee or of Barger and Michael. We let the parameters a_λ and b_λ be different for the N_α and N_λ trajectories. This is essential, for without these factors (or if they were the same for N_α and N_γ), we would obtain zero polarization. Because the polarization is sensitive to the form of the parametrization, we do not expect our results to be quantitatively very accurate, even if the Regge model is basically correct. However, we expect that the qualitative features of our calculation will provide a good test of the model.

Since the a's and b's are different in each deuteron helicity state and for each trajectory, they provide 12 adjustable parameters. We take the N_α and N_γ trajectories as fixed

$$\alpha(N_\alpha) = -0.39+1.01t,$$

$$\alpha(N_\gamma) = -0.46+0.88t \tag{10}$$

with t in $(\text{GeV}/c)^2$. We vary s_0 but use the same

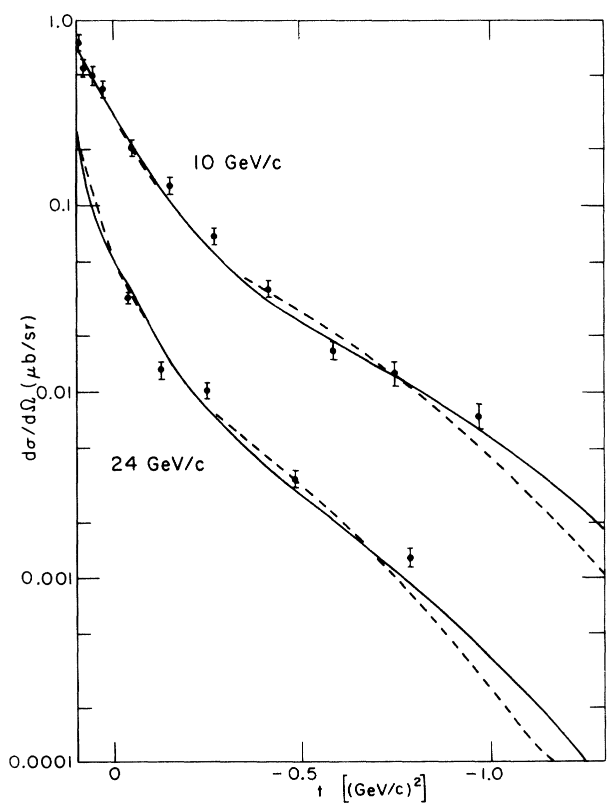

FIG. 1. Fits to the cross-section data of Refs. 1–3 for the reaction $pp\to\pi^+d$ at two momenta. The solid curve is a 13-parameter fit and the dashed curve is a 5-parameter fit.

value of s_0 for both trajectories. Therefore, with the a's and b's we have a total of 13 parameters in the formalism. These parameters are varied to fit the cross-section data, and then the polarization as well as the cross section are calculated at other energies.

III. RESULTS AND DISCUSSION

We fitted 55 cross-section data for $pp \to \pi^+ d$ at several incident laboratory momenta above 7 GeV/c and with $|t| \leq 1.1$ (GeV/c)^2.[1-3] We shall discuss three fits to these data, and present figures showing two of these fits. These illustrate the main features of our solutions. In one of our solutions, we assumed, following the conjecture of Barger and Michael, that the only two amplitudes which contribute are those in which the deuteron helicity is zero. The value of χ^2 was 150. The success of this five-parameter fit to the data does not lend support to the conjecture of Barger and Michael that deuteron helicity-zero amplitudes are dominant. The reason is, as we have noted, that we can obtain an equivalent fit by assuming that only the helicity-one amplitudes contribute. We were not able to obtain a satisfactory fit to the data keeping only the two amplitudes with deuteron helicity minus one.

Our remaining two fits to the data contain contributions from all six helicity amplitudes. In one of these solutions, the deuteron helicity-zero amplitudes are dominant; in the other, all amplitudes contribute comparable amounts. (The solution with dominant helicity-zero amplitudes is, of course, equivalent to a solution with dominant

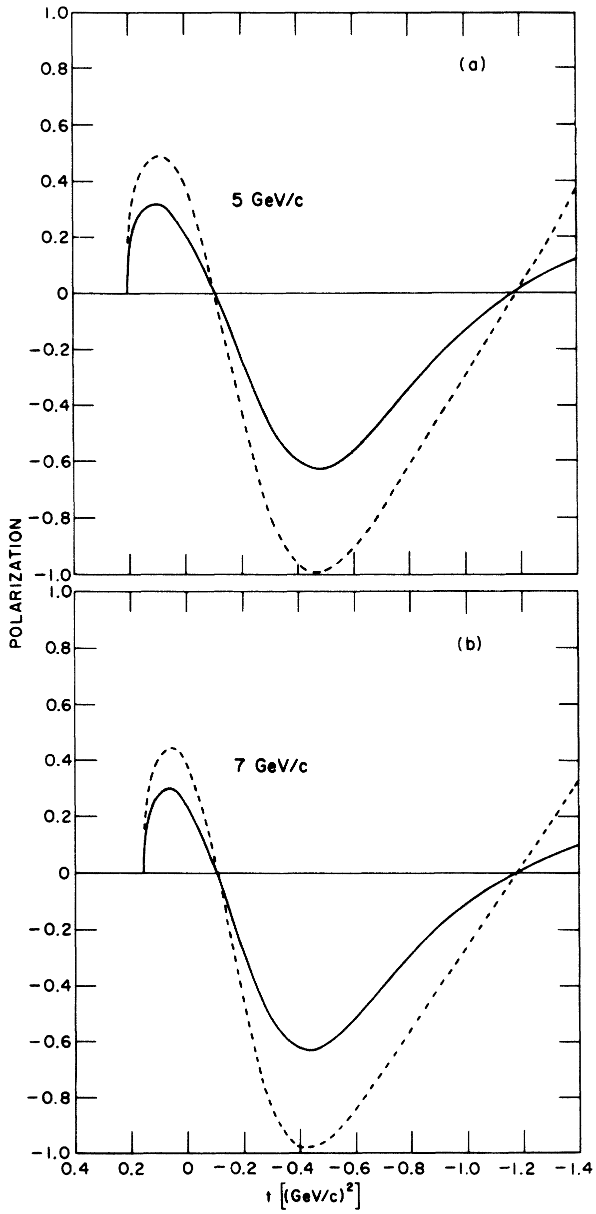

FIG. 2. Predictions for the proton polarization at the momenta (a) 5 GeV/c and (b) 7 GeV/c. The solid curve is a 13-parameter fit and the dashed curve is a 5-parameter fit.

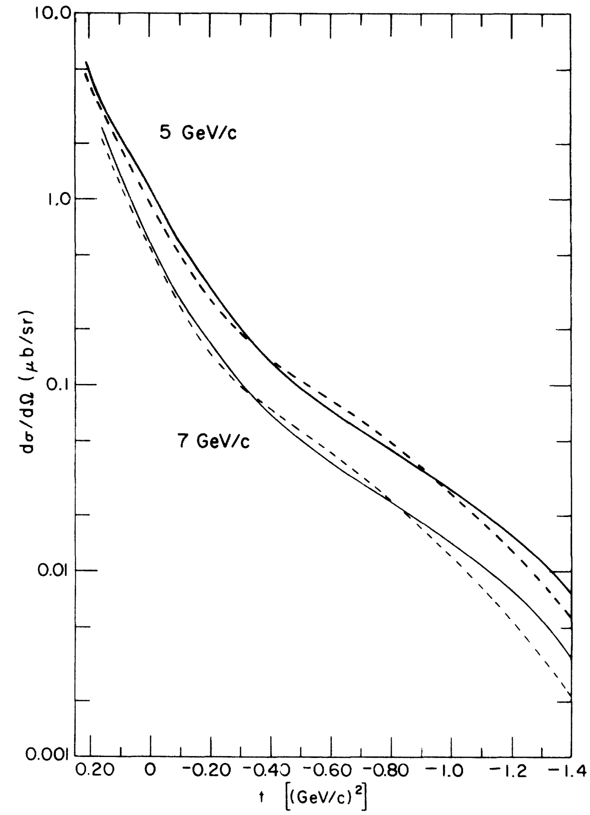

FIG. 3. Predictions for the cross sections at momenta of 5 and 7 GeV/c. The solid curve is a 13-parameter fit and the dashed curve is a 5-parameter fit.

helicity-one amplitudes.) As expected, we find that these fits have a lower χ^2 than the one with only five parameters. The χ^2 values were 126 and 120, respectively.

We show in Fig. 1 two of the fits to the data[1-3] at the highest and lowest momenta. One of these (dashed lines) is the five-parameter fit with only helicity-zero amplitudes contributing and the other (solid line) is a fit with all helicity amplitudes important. On the scale of Fig. 1 the third fit is barely distinguishable from the one shown with the solid line. In Fig. 2 we show predictions for the polarization at two momenta: 5 and 7 GeV/c. These momenta were chosen low enough so that the polarization will not be too difficult to measure using a polarized proton beam or a polarized target, but high enough so that the model should be applicable. The solution with only two helicity amplitudes predicts significantly larger polarization than those with all six amplitudes. Otherwise all solutions yield qualitatively similar predictions. (We have not attempted to predict the over-all sign of the polarization because we do not believe that we can do this reliably with the present model.) In Fig. 3 we show the cross-section predictions at the same two momenta.

This Regge exchange model lends itself readily to an experimental test, because it makes the qualitative prediction that the proton polarization should vanish at $t = -0.2$ and -1.2 $(\text{GeV}/c)^2$ even though the cross section shows no pronounced structure at these values of t. Omitted contributions from other Regge poles and from Regge cuts could change somewhat the position of these zeros. However, if there are not zeros in the polarization near these values this basic Regge model with wrong-signature nonsense zeros would have to be substantially revised.

Note added in proof. Preliminary data from the Minnesota-Rice-Argonne-Indiana collaboration indicate that at 6.0 GeV/c incident proton momentum, the proton polarization is 10 percent or less for $|t| < 0.2$ $(\text{GeV}/c)^2$, and is consistent with zero for $0.2 < |t| < 0.4$ $(\text{GeV}/c)^2$. The model we have discussed can give zero polarization as a special case if the ratio of the parameters a_λ and b_λ is the same for the N_α and N_γ trajectories. We find that we can approximately fit the preliminary polarization data without appreciably changing the predicted differential cross section. However, if the observed polarization turns out to be very nearly zero over the whole range of $|t|$ less than 1.4 $(\text{GeV}/c)^2$, the data do not provide a good test of the qualitative prediction of the model that the polarization should vanish at the characteristic values of $t = -0.2$ and -1.2 $(\text{GeV}/c)^2$. We should like to thank Professor Keith Ruddick for a discussion of the preliminary data prior to publication.

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