Electron-muon mass ratio in a gauge group for leptons*

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We postulate a concept of isospin in purely leptonic space. In particular, we consider a group $G \equiv [SU_L(2)\otimes U(1)][SU_{lL}(2)\otimes SU_{lR}(2)\otimes U_l(1)]$ for leptons. The first group is the group introduced by Salam and Weinberg and generates ordinary weak and electromagnetic interactions. The second group is in leptonic space only such that the leptonic multiplet $L=\frac{1}{2}(1+\gamma_5)\binom{\nu_\mu\ \ \nu_e}{\mu\ \ \ e}$ belongs to a representation $T_{lL}=\frac{1}{2}$, $T_{lR}=0$, $Y_l=1$ of this group and a representation $T_L=\frac{1}{2}$, $Y=-1$ for the group $SU_L(2)\otimes U(1)$. The multiplet $R=\frac{1}{2}(1-\gamma_5)(\mu,e)$ has $T_L=0$, $Y=-2$ for the group $SU_L(2)\otimes U(1)$ and has $T_{lL}=0$, $T_{lR}=\frac{1}{2}$, $Y_l=1$ for the other group. T_{lL} and T_{lR} are left-handed and right-handed isospins, and Y_l is the hypercharge for the leptonic group. The leptonic charge $Q_l=T_{lL3}+T_{lR3}+\frac{1}{2}Y_l$. Thus for the muon $Q_l=1$ and for the electron $Q_l=0$ so that muonic number acquires a group-theoretic meaning. We consider the spontaneously broken gauge symmetry based on the group G such that in zeroth order only the muon acquires a mass and the electron remains massless. We calculate the electron self-mass due to radiative corrections in second order of the gauge coupling constant f of the leptonic group. Our conclusion is that for the electron mass to be calculable, the electron remains massless even in second order so that the electron mass may arise in fourth order of f.

I. INTRODUCTION

It is well known that hadrons exist as isospin multiplets. For example, the proton and the neutron form an isospin multiplet with isospin $I=\frac{1}{2}$. The small mass difference between the proton and the neutron is supposed to arise because of electromagnetic interaction, which violates isospin conservation. In practice, when one attempts to calculate this mass difference due to radiative corrections in second order in electromagnetism, the mass difference comes out to be infinite. This infinity can be removed by renormalization but then the mass difference becomes a free parameter.

In recently formulated unified gauge theories of weak and electromagnetic interactions, the weak interaction basically is of the same order as electromagnetism, and in calculating the mass difference, the weak interaction has to be taken into account. In these theories, the only counterterms necessary for cancellation of all infinities are those allowed by the gauge symmetry. Thus, if a zeroth-order mass difference and the corresponding counterterm are forbidden by the gauge structure, the higher-order contributions to the mass difference must be finite.[1] Such gauge models have been formulated in the last few years. These models[2] give a finite and calculable mass difference between members of a hadron multiplet, but none of these models is experimentally tenable.

The lepton mass spectrum is one of the longstanding mysteries of theoretical physics. We wish to understand the electron-muon mass ratio, which is of order αm_μ, on the same basis as the mass difference between members of a hadron isomultiplet. For this purpose a concept of isospin in purely leptonic space is postulated. This isospin is denoted as T_l. Then in analogy with the Gell-Mann–Nishijima relation $Q=T_3+\frac{1}{2}Y$ we postulate $Q_l=T_{l3}+\frac{1}{2}Y_l$ for leptons such that $Q_l=1$ for muon-type leptons and $Q_l=0$ for electron-type leptons. We associate a chiral group structure with this isospin. In particular, we associate a group $SU_{lL}(2)\otimes SU_{lR}(2)\otimes U_l(1)$ with left-handed and right-handed isospins in lepton space such that $Q_l=T_{lL3}+T_{lR3}+\frac{1}{2}Y_l$. Since this group is in the lepton space, the weak interactions associated with this group structure cannot be extended to hadrons. We call these, interactions which are peculiar to leptons, "anomalous" weak interactions. We believe that ordinary weak interactions, which are universal in the sense that they are applicable to both leptons and hadrons, are adequately described by the group $SU_L(2)\otimes U(1)$ introduced by Salam and Weinberg.[3]

For leptons, we therefore consider a group

$$G \equiv [SU_L(2)\otimes U(1)][SU_{lL}(2)\otimes SU_{lR}(2)\otimes U_l(1)]\,.$$

The first group generates ordinary weak and electromagnetic interactions. The second group is a new group which is introduced in such a way that muonic number appears as muonic charge Q_l. The group $SU_L(2)\otimes U(1)$ is local, and similarly we take $SU_{lL}(2)\otimes SU_{lR}(2)$ as local, but $U_l(1)$ we take as global. The extension to the case when $U_l(1)$ is also local is briefly discussed at the end of Sec. III. Thus we have four vector bosons W_μ, $\overline{W}_\mu$, Z_μ, and A_μ associated with $SU_L(2)\otimes U(1)$. The bosons W_μ, $\overline{W}_\mu$ are coupled to ordinary charged weak currents, and Z_μ is a weak neutral vector boson which is characteristic of the Salam-Weinberg model. A_μ is a photon coupled to the electromagnetic current. The six vector bosons associated with $SU_{lL}(2) \otimes SU_{lR}(2)$ we denote by $G_{L\mu}$ and $G_{R\mu}$. These vector bosons are electrically neutral.

In order to assign leptons to a particular representation of this group, we form the following left-handed and right-handed multiplets:

$$L = \frac{1+\gamma_5}{2}\begin{pmatrix} \nu_\mu & \nu_e \\ \mu & e \end{pmatrix} = \begin{pmatrix} \nu_{\mu L} & \nu_{eL} \\ \mu_L & e_L \end{pmatrix},$$

$$R = \frac{1-\gamma_5}{2}(\mu, e) = (\mu_R, e_R).$$

Thus we see that for the multiplet L we have $T_L = \frac{1}{2}$, $Y = -1$, $T_{lL} = \frac{1}{2}$, $T_{lR} = 0$, $Y = 1$, whereas for R we have $T_L = 0$, $Y = -2$, $T_{lL} = 0$, $T_{lR} = \frac{1}{2}$, $Y_l = 1$. With this assignment, we have correct electric and muonic charges. Note that we take the neutrino to be left-handed because we want neutrinos to remain massless even after radiative corrections.

To start with, all the particles are massless. In order to break the gauge symmetry spontaneously so that all vector mesons except the photon acquire mass, we introduce a set of scalar mesons. The symmetry is broken in such a way that only the muon acquires a mass in zeroth order but the electron remains massless. The hope is that the electron acquires a mass due to radiative corrections by the exchange of those vector bosons G_μ which carry muonic charge, so that a muon in the intermediate state is possible (see Fig. 1). Our conclusions is that the electron mass may arise in the fourth order of radiative corrections

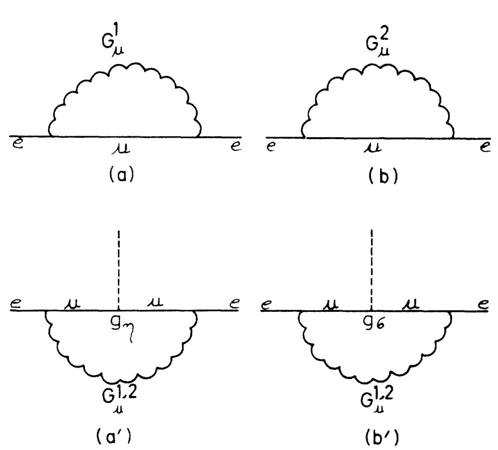

FIG. 1. (a) and (b) Feynman graphs which contribute to electron mass in second order. (a′) and (b′) Feynman graphs which contribute to coupling constants g_η and g_σ in second order.

but not in the second order.

Before we give the plan of this paper, we briefly mention the attempts made by other people to understand the electron-muon mass ratio on gauge models. The first attempt in this respect was made by Weinberg.[4] He considered a chiral gauge group $SU(3) \otimes SU(3)$ for leptons which acts on a triplet

$$\begin{pmatrix} \mu^+ \\ \nu \\ e^- \end{pmatrix}.$$

This model cannot be extended directly to hadrons. Moreover, this model does not give a satisfactory result for the electron-muon mass ratio in second order and has difficulties similar to those of our model. In the model considered by Mohapatra[5] for this purpose, the electron self-mass also arises in fourth order and the model is not simple. Georgi and Glashow[6] have discussed a number of gauge models for calculating the electron-muon mass ratio. They have succeeded in constructing a model in which the electron-muon mass ratio can be calculated in second order, but their model for weak interactions cannot be extended to hadrons. They have to introduce doubly charged vector bosons for this purpose and require 24 vector bosons.

The plan of this paper is as follows. In Sec. II, we write down the gauge-invariant Lagrangian. In this section we also discuss the spontaneous symmetry breaking by introducing three sets of scalars

$$\eta \equiv (\eta_0, \vec{\eta}), \quad \sigma \equiv (\sigma_0, \vec{\sigma}), \text{ and } \epsilon \equiv \begin{pmatrix} \epsilon^+ \\ \epsilon^0 \end{pmatrix}$$

which belong to the representation $(\frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2})$, and $(\frac{1}{2}, 0)$ of the group $SU_{lL}(2) \otimes SU_{lR}(2) \otimes U_l(1)$ but are singlets with respect to the group $SU_L(2) \otimes U(1)$. The vacuum expectation values of scalar mesons are arranged in such a way that only the muon and all the vector bosons $G_{L\mu}$ and $G_{R\mu}$ acquire masses but the electron remains massless in zeroth order. We also discuss the case in which two sets of scalars η and σ are replaced by one set of scalars, so that we get a "natural" zeroth-order gauge symmetry model.

In Sec. III, we investigate the electron self-mass in second order due to exchange of gauge vector bosons $G^{(s)}_\mu$. Our conclusion is that this contribution vanishes and hence the electron does not acquire mass in second order due to exchange of gauge vector bosons. For the second case we show that the electron remains massless even in second order. Thus in either case the electron mass may arise in fourth order, as is the case for most of the models discussed in the literature.

In Sec. IV, we briefly discuss the consequences of "anomalous" weak interactions for lepton decays and lepton scattering. We conclude in this section that none of the known results for these processes are affected by the "anomalous" weak interactions.

II. GAUGE-INVARIANT LAGRANGIAN

For leptons, the Lagrangian invariant under the gauge group

$$G \equiv [SU_L(2) \otimes U(1)][SU_{lL}(2) \otimes SU_{lR}(2)]$$

is given by

$$\mathcal{L}(\text{leptons}) = -\text{Tr}[\bar{L}\gamma_\mu \nabla_\mu L + \bar{R}\gamma_\mu \nabla'_\mu R]\,, \tag{1a}$$

where

$$\nabla_\mu = \partial_\mu - igW_\mu + \frac{i}{2}g'B_\mu - if_L G_{L\mu}\,, \tag{1b}$$

$$\nabla'_\mu = \partial_\mu - ig'B_\mu - if_R G_{R\mu}\,. \tag{1c}$$

Here W_μ and B_μ are gauge vector bosons associated with the group $SU_L(2)\otimes U(1)$ and $G_{L\mu}$ and $G_{R\mu}$ are those associated with the group $SU_{lL}(2) \otimes SU_{lR}(2)$. L and R are given by

$$L = \begin{pmatrix} \nu_{\mu L} & \nu_{eL} \\ \mu_L & e_L \end{pmatrix}\,, \tag{2a}$$

$$R = (\mu_R,\ e_R)\,. \tag{2b}$$

In order to break the gauge symmetry spontaneously we introduce four sets of scalars

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}:\ T_L = \tfrac{1}{2},\quad Y = 1,\quad T_{lL} = 0,\quad T_{lR} = 0\,,$$

$$\eta = (\eta_0, \vec{\eta}):\ T_L = 0,\quad Y = 0,\quad T_{lL} = \tfrac{1}{2},\quad T_{lR} = \tfrac{1}{2}\,,$$

$$\sigma = (\sigma_0, \vec{\sigma}):\ T_L = 0,\quad Y = 0,\quad T_{lL} = \tfrac{1}{2},\quad T_{lR} = \tfrac{1}{2}\,,$$

$$\epsilon = \begin{pmatrix} \epsilon^+ \\ \epsilon^0 \end{pmatrix}:\ T_L = 0,\quad Y = 0,\quad T_{lL} = \tfrac{1}{2},\quad T_{lR} = 0,\quad Y_l = 1\,.$$

Now the complete Lagrangian invariant under the group G is given by

$$\begin{aligned}
\mathcal{L} = &-(\bar{\nu}_{eL}\gamma_\mu\partial_\mu\nu_{eL} + \bar{\nu}_{\mu L}\gamma_\mu\partial_\mu\nu_{\mu L} + \bar{\mu}\gamma_\mu\partial_\mu\mu + \bar{e}\gamma_\mu\partial_\mu e) - \sum_{r=e,\mu}\left[\bar{\xi}_{rL}\gamma_\mu\left(-igW_\mu - \frac{i}{2}g'B_\mu\right)\xi_{rL}\right] \\
&-ig'\left(-\bar{e}\gamma_\mu\frac{1-\gamma_5}{2}e - \bar{\mu}\gamma_\mu\frac{1-\gamma_5}{2}\mu\right)B_\mu - \bar{\zeta}_L\gamma_\mu(-if_L G_{L\mu})\zeta_L - \bar{\zeta}'_L\gamma_\mu(-if_L G_{L\mu})\zeta'_L \\
&-\bar{\zeta}_R\gamma_\mu(-if_R G_{R\mu})\zeta_R - \tfrac{1}{2}[\partial_\mu\bar{\phi} + i\bar{\phi}(gW_\mu - \tfrac{1}{2}g'B_\mu)][\partial_\mu\phi - i(gW_\mu - \tfrac{1}{2}g'B_\mu)\phi] \\
&-\tfrac{1}{2}[\partial_\mu\bar{\epsilon} + if_L\bar{\epsilon}G_{L\mu}][\partial_\mu\epsilon - if_L G_{L\mu}\epsilon] \\
&-\tfrac{1}{2}[\partial_\mu\vec{\eta} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} + f_R\vec{G}_{R\mu})\times\vec{\eta} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\eta_0]\cdot[\partial_\mu\vec{\eta} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} + f_R\vec{G}_{R\mu})\times\vec{\eta} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_{R\mu}\vec{G}_{R\mu})\eta_0] \\
&-\tfrac{1}{2}[\partial_\mu\eta_0 - \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\cdot\vec{\eta}][\partial_\mu\eta_0 - \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\cdot\vec{\eta}] \\
&-\tfrac{1}{2}[\partial_\mu\vec{\sigma} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} + f_R\vec{G}_{R\mu})\times\vec{\sigma} - \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\sigma_0]\cdot[\partial_\mu\vec{\sigma} + \tfrac{1}{2}(f_L\vec{G}_{L\mu} + f_R\vec{G}_{R\mu})\times\vec{\sigma} - \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\sigma_0] \\
&-\tfrac{1}{2}[\partial_\mu\sigma_0 + \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\cdot\vec{\sigma}][\partial_\mu\sigma_0 + \tfrac{1}{2}(f_L\vec{G}_{L\mu} - f_R\vec{G}_{R\mu})\cdot\vec{\sigma}] \\
&-g_\eta\bar{\zeta}(\eta_0 + i\vec{\tau}\cdot\vec{\eta}\gamma_5)\zeta - g_\sigma\bar{\zeta}(i\sigma_0\gamma_5 + \vec{\tau}\cdot\vec{\sigma})\zeta + V(\phi) + V(\eta) + V(\sigma) + V(\epsilon)\,,
\end{aligned} \tag{3a}$$

where

$$\xi_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix},\quad \xi_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix},\quad \zeta = \begin{pmatrix} \mu \\ e \end{pmatrix},\quad \zeta' = \begin{pmatrix} \nu_\mu \\ \nu_e \end{pmatrix}, \tag{3b}$$

$$W_\mu = \tfrac{1}{2}\begin{pmatrix} W_{3\mu} & \sqrt{2}\,\bar{W}_\mu \\ \sqrt{2}\,W_\mu & -W_{3\mu} \end{pmatrix},\quad G_{L,R\mu} = \tfrac{1}{2}\begin{pmatrix} G_{3L,R\mu} & \sqrt{2}\,\bar{G}_{L,R\mu} \\ \sqrt{2}\,G_{L,R\mu} & -G_{3L,R\mu} \end{pmatrix}. \tag{3c}$$

The potentials $V(\phi)$, $V(\eta)$, $V(\sigma)$, and $V(\epsilon)$ will be defined later. From Eq. (3a), we can write the interaction Lagrangians as

$$\begin{aligned}
\mathcal{L}_{\text{int}}^{W,\text{em}} = &-ie(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu)A_\mu + \frac{g}{2\sqrt{2}}[i\bar{\nu}_e\gamma_\mu(1+\gamma_5)e + (e\to\mu)]\bar{W}_\mu + \text{H.c.} \\
&+\frac{(g^2+g'^2)^{1/2}}{4}i[(4\sin^2\theta_W - 1)\bar{e}\gamma_\mu e - \bar{e}\gamma_\mu\gamma_5 e + \bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e + (e\to\mu)]Z_\mu\,,
\end{aligned} \tag{4a}$$

where

$$A_\mu = (g^2+g'^2)^{-1/2}(-g'W_{3\mu}+gB_\mu)\,, \quad Z_\mu=(g^2+g'^2)^{-1/2}(gW_{3\mu}+g'B_\mu)\,, \tag{4b}$$

$$-e = gg'/(g^2+g'^2)^{1/2}\,, \quad g'/g=\tan\theta_W\,. \tag{4c}$$

Equation (4a) gives us the usual Salam-Weinberg interaction Lagrangian for weak and electromagnetic interactions. The interaction Lagrangian for the "anomalous" weak interactions can be written from Eq. (3a) as

$$\begin{aligned}\mathcal{L}^G_{\rm int} &= \frac{f_L}{2\sqrt{2}}[i\bar{\mu}\gamma_\mu(1+\gamma_5)e\overline{G}_{L\mu}+\text{H.c.}]+\frac{f_L}{4}[i\bar{\mu}\gamma_\mu(1+\gamma_5)\mu-i\bar{e}\gamma_\mu(1+\gamma_5)e]G_{3L\mu}\\ &+\frac{f_R}{2\sqrt{2}}[i\bar{\mu}\gamma_\mu(1-\gamma_5)e\overline{G}_{R\mu}+\text{H.c.}]+\frac{f_R}{4}[i\bar{\mu}\gamma_\mu(1-\gamma_5)\mu-i\bar{e}\gamma_\mu(1-\gamma_5)e]G_{3R\mu}\\ &+\frac{f_L}{2\sqrt{2}}[i\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\nu_e\overline{G}_{L\mu}+\text{H.c.}]+\frac{f_L}{4}[i\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\nu_\mu-i\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e]G_{3L\mu}\,.\end{aligned} \tag{5}$$

In order to give masses to all gauge vector bosons except the photon, we break the gauge symmetry spontaneously by requiring that some of the scalar mesons develop nonzero vacuum expectation values. We note that

$$V(\phi) = -\mu_1^2\overline{\phi}\phi - h_\phi(\overline{\phi}\phi)^2\,, \tag{6a}$$

$$V(\eta) = -\tfrac{1}{2}\mu_2^2(\eta_0^2+\vec{\eta}^2)-\tfrac{1}{4}h_\eta(\eta_0^2+\vec{\eta}^2)^2\,, \tag{6b}$$

$$V(\sigma) = -\tfrac{1}{2}\mu_3^2(\sigma_0^2+\vec{\sigma}^2)-\tfrac{1}{4}h_\sigma(\sigma_0^2+\vec{\sigma}^2)^2\,, \tag{6c}$$

and an expression for $V(\epsilon)$ similar to that given in (6a). If μ_1^2, μ_2^2, and $\mu_3^2<0$, then the ϕ, η, and σ must develop a vacuum expectation value so as to make the physical masses of scalar bosons nonnegative. The symmetry is then spontaneously broken.

As is well known, it is possible to select a gauge so that

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \text{ is replaced by } \begin{pmatrix}0\\ \phi+\lambda\end{pmatrix}\,,$$

where $\langle\phi\rangle=0$ and $V(\phi)$ is given by

$$V(\phi) = -\mu_1^2(\lambda+\phi)^2-h_\phi[(\lambda+\phi)^2]^2\,.$$

We now require that in the tree approximation the term linear in ϕ should vanish. Thus, to the lowest order,

$$\lambda^2 = -\mu_1^2/2h_\phi\,.$$

Hence,

$$V(\phi) = -\tfrac{1}{2}(8\lambda^2h_\phi)\phi^2 - h_\phi(\phi^4+4\lambda\phi^3)+h_\phi\lambda^4\,. \tag{7}$$

One can easily see from the Lagrangian (3a) that W_μ and Z_μ acquire the following masses:

$$m_W = \tfrac{1}{2}\lambda g\,, \quad m_Z = \tfrac{1}{2}\lambda(g^2+g'^2)^{1/2}\,.$$

Note that in writing the Lagrangian (3a) we have assumed that the scalar meson is not coupled to leptons, so that leptons are still massless.

We now assume that η_0 and σ_3 have vacuum expectation values

$$\langle\eta_0\rangle = \nu_2\,, \quad \langle\sigma_3\rangle=\nu_3\,.$$

Thus we write $\eta_0=\eta_0'+\nu_2$ and $\sigma_3=\sigma_3'+\nu_3$, so that $\langle\eta_0'\rangle=0$ and $\langle\sigma_3'\rangle=0$. In what follows, we will omit the primes on η_0' and σ_3'. We now require that in the tree approximation the terms linear in η_0 and σ_3 should vanish, so that we get

$$\mu_2^2 = -h_\eta\nu_2^2\,, \quad \mu_3^2=-h_\sigma\nu_3^2\,.$$

Thus we have

$$\begin{aligned}V(\eta) &= -\tfrac{1}{2}(2h_\eta\nu_2^2)\eta_0^2-\tfrac{1}{4}h_\eta[(\eta_0^2+\vec{\eta}^2)^2+4\nu_2\eta_0(\eta_0^2+\vec{\eta}^2)]\\ &\quad+\tfrac{1}{4}h_\eta\nu_2^4\,,\end{aligned} \tag{8a}$$

$$\begin{aligned}V(\sigma) &= -\tfrac{1}{2}(2h_\sigma\nu_3^2)\sigma_3^2-\tfrac{1}{4}h_\sigma[(\sigma_0^2+\vec{\sigma}^2)^2+4\nu_3\sigma_3(\sigma_0^2+\vec{\sigma}^2)]\\ &\quad+\tfrac{1}{4}h_\sigma\nu_3^4\,,\end{aligned} \tag{8b}$$

and

$$m_{\eta_0}^2 = 2h_\eta\nu_2^2\,, \quad m_{\sigma_3}^2 = 2h_\sigma\nu_3^2\,. \tag{9}$$

As is the case for the scalar multiplet ϕ, it is possible to select a gauge such that

$$\epsilon = \begin{pmatrix}\epsilon^+\\ \epsilon^0\end{pmatrix} \text{ is replaced by } \begin{pmatrix}0\\ \epsilon+\nu_1\end{pmatrix}\,,$$

where $\langle\epsilon\rangle=0$ and $V(\epsilon)$ is given by

$$V(\epsilon) = -\tfrac{1}{2}(8\nu_1^2h_\epsilon)\epsilon^2-h_\epsilon(\epsilon^4+4\nu_1\epsilon^3)+h_\epsilon\nu_1^4\,. \tag{10}$$

Now from the term

$$-g_\eta\bar{\zeta}(\eta_0+i\vec{\tau}\cdot\vec{\eta}\gamma_5)\zeta-g_\sigma\bar{\zeta}(i\sigma_0\gamma_5+\vec{\tau}\cdot\vec{\sigma})\zeta$$

in the Lagrangian, we see that the muon and electron masses are given by

$$\begin{aligned}m_\mu &= g_\eta\nu_2+g_\sigma\nu_3\,,\\ m_e &= g_\eta\nu_2-g_\sigma\nu_3\,.\end{aligned} \tag{11}$$

We require the electron mass to be zero in zeroth order. This gives

$$\nu_2/\nu_3 = g_\sigma/g_\eta ,$$

so that

$$m_\mu = 2g_\eta \nu_2 . \quad (12)$$

Also we note that in this case we have

$$g_\eta = m_\mu/2\nu_2 , \quad g_\sigma = m_\mu/2\nu_3 . \quad (13)$$

Now we write down the mass terms for vector bosons $G_{L\mu}$ and $G_{R\mu}$ which arise in the Lagrangian (3a) owing to spontaneous symmetry breaking. These terms are given by

$$\begin{aligned}\mathcal{L}(M)+\mathcal{L}(\text{tadpole}) = &-\tfrac{1}{8}(\nu_1+\epsilon)^2(8f_L{}^2\overline{G}_{L\mu}G_{L\mu}+4f_L{}^2G_{3L\mu}{}^2)-\tfrac{1}{8}(\eta_0+\nu_2)^2(2f_L{}^2\overline{G}_{L\mu}G_{L\mu}+2f_R{}^2\overline{G}_{R\mu}G_{R\mu})\\ &+\tfrac{1}{8}f_Lf_R(\eta_0+\nu_2)^2(\overline{G}_{L\mu}G_{R\mu}+G_{L\mu}\overline{G}_{R\mu})-\tfrac{1}{8}(\eta_0+\nu_2)^2(f_L{}^2G_{3L\mu}{}^2+f_R{}^2G_{3R\mu}{}^2)\\ &+\tfrac{1}{8}2f_Lf_R(\eta_0+\nu_2)^2G_{3L\mu}G_{3R\mu}-\tfrac{1}{8}(\sigma_3+\nu_3)^2(2f_L{}^2\overline{G}_{L\mu}G_{L\mu}+2f_R{}^2\overline{G}_{R\mu}G_{R\mu})\\ &-\tfrac{1}{8}f_Lf_R(\sigma_3+\nu_3)^2(\overline{G}_{L\mu}G_{R\mu}+G_{L\mu}\overline{G}_{R\mu})-\tfrac{1}{8}(\sigma_3+\nu_3)^2(f_L{}^2G_{3L\mu}{}^2+f_R{}^2G_{3R\mu}{}^2)\\ &+\tfrac{1}{8}2f_Lf_R(\sigma_3+\nu_3)^2G_{3L\mu}G_{3R\mu} . \end{aligned} \quad (14)$$

We have written the left-hand side of Eq. (14) as $\mathcal{L}(M)+\mathcal{L}(\text{tadpole})$ because this equation contains both the mass term for vector bosons $G_{L\mu}$ and $G_{R\mu}$ and the terms which contribute to tadpole graphs.

From Eq. (14), we see that the mass matrix for $G_{L\mu}$ and $G_{R\mu}$ is given by

$$M=\tfrac{1}{4}\begin{bmatrix} f_L{}^2(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2) & -f_Lf_R(\nu_2{}^2-\nu_3{}^2)\\ -f_Lf_R(\nu_2{}^2-\nu_3{}^2) & f_R{}^2(\nu_2{}^2+\nu_3{}^2)\end{bmatrix}. \quad (15)$$

The mass matrix for $G_{3L\mu}$ and $G_{3R\mu}$ is given by

$$M_0=\tfrac{1}{4}\begin{bmatrix} f_L{}^2(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2) & -f_Lf_R(\nu_2{}^2+\nu_3{}^2)\\ -f_Lf_R(\nu_2{}^2+\nu_3{}^2) & f_R{}^2(\nu_2{}^2+\nu_3{}^2)\end{bmatrix}. \quad (16)$$

In order to diagonalize the mass matrices M and M_0, we define

$$\begin{aligned} G_\mu^1 &= \sin\alpha\, G_{R\mu} - \cos\alpha\, G_{L\mu} ,\\ G_\mu^2 &= \cos\alpha\, G_{R\mu} + \sin\alpha\, G_{L\mu} , \end{aligned} \quad (17a)$$

$$\begin{aligned} G_{3\mu}^1 &= \sin\beta\, G_{3R\mu} - \cos\beta\, G_{3L\mu} ,\\ G_{3\mu}^2 &= \cos\beta\, G_{3R\mu} + \sin\beta\, G_{3L\mu} . \end{aligned} \quad (17b)$$

We also define an angle θ such that

$$\begin{aligned} f_L &= f\cos\theta ,\\ f_R &= f\sin\theta . \end{aligned} \quad (18)$$

The masses of the vector bosons G_μ^1, G_μ^2, $G_{3\mu}^1$, and $G_{3\mu}^2$ are then given by

$$M_1{}^2=\tfrac{1}{4}f^2[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)\cos^2\alpha+\sin^2\theta(\nu_2{}^2+\nu_3{}^2)\sin^2\alpha+2\cos\theta\sin\theta(\nu_2{}^2-\nu_3{}^2)\cos\alpha\sin\alpha] , \quad (19a)$$

$$M_2{}^2=\tfrac{1}{4}f^2[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)\sin^2\alpha+\sin^2\theta(\nu_2{}^2+\nu_3{}^2)\cos^2\alpha-2\cos\theta\sin\theta(\nu_2{}^2-\nu_3{}^2)\cos\alpha\sin\alpha] , \quad (19b)$$

$$(M_3^1)^2=\tfrac{1}{4}f^2[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)\cos^2\beta+\sin^2\theta(\nu_2{}^2+\nu_3{}^2)\sin^2\beta+2\cos\theta\sin\theta(\nu_2{}^2+\nu_3{}^2)\cos\beta\sin\beta] , \quad (19c)$$

$$(M_3^2)^2=\tfrac{1}{4}f^2[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)\sin^2\beta+\sin^2\theta(\nu_2{}^2+\nu_3{}^2)\cos^2\beta-2\cos\theta\sin\theta(\nu_2{}^2+\nu_3{}^2)\cos\beta\sin\beta] . \quad (19d)$$

We also note that

$$\cot2\alpha\sin2\theta(\nu_2{}^2-\nu_3{}^2)=[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)-\sin^2\theta(\nu_2{}^2+\nu_3{}^2)] , \quad (20a)$$

$$\cot2\beta\sin2\theta(\nu_2{}^2+\nu_3{}^2)=[\cos^2\theta(4\nu_1{}^2+\nu_2{}^2+\nu_3{}^2)-\sin^2\theta(\nu_2{}^2+\nu_3{}^2)] , \quad (20b)$$

$$\frac{\nu_2{}^2-\nu_3{}^2}{\nu_2{}^2+\nu_3{}^2}=\frac{\cot2\beta}{\cot2\alpha} . \quad (20c)$$

In terms of the vector bosons G_μ^1, G_μ^2, $G_{3\mu}^1$, and $G_{3\mu}^2$ of definite mass, the interaction Lagrangian (5) is given by

$$\begin{aligned}\mathcal{L}_{\text{int}}^G = &F_1[i\bar\mu\gamma_\mu(1+r_1\gamma_5)e\overline{G}_\mu^1+\text{H.c.}]+F_2[i\bar\mu\gamma_\mu(1+r_2\gamma_5)e\overline{G}_\mu^2+\text{H.c.}]+F_3^1[i\bar\mu\gamma_\mu(1+r_3^1\gamma_5)\mu-(\mu\to e)]G_{3\mu}^1\\ &+F_3^2[i\bar\mu\gamma_\mu(1+r_3^2\gamma_5)\mu-(\mu\to e)]G_{3\mu}^2+\frac{f_L}{2\sqrt2}\{-\cos\alpha[i\bar\nu_\mu\gamma_\mu(1+\gamma_5)\nu_e]\overline{G}_\mu^1+\sin\alpha[i\bar\nu_\mu\gamma_\mu(1+\gamma_5)\nu_e]\overline{G}_\mu^2+\text{H.c.}\}\\ &+\frac{f_L}{4}\{-\cos\beta[i\bar\nu_\mu\gamma_\mu(1+\gamma_5)\nu_\mu-(\mu\to e)]G_{3\mu}^1+\sin\beta[i\bar\nu_\mu\gamma_\mu(1+\gamma_5)\nu_\mu-(\mu\to e)]G_{3\mu}^2\} ,\end{aligned} \quad (21)$$

where

$$F_1=\frac{f}{2\sqrt{2}}(-\cos\alpha\cos\theta+\sin\alpha\sin\theta)\,,\quad r_1=\frac{\cos\alpha\cos\theta+\sin\alpha\sin\theta}{\cos\alpha\cos\theta-\sin\alpha\sin\theta}\,,$$
$$F_2=\frac{f}{2\sqrt{2}}(\sin\alpha\cos\theta+\cos\alpha\sin\theta)\,,\quad r_2=\frac{\sin\alpha\cos\theta-\cos\alpha\sin\theta}{\sin\alpha\cos\theta+\cos\alpha\sin\theta}\,,$$
$$F_3^1=\frac{f}{4}(-\cos\beta\cos\theta+\sin\beta\sin\theta)\,,\quad r_3^1=\frac{\cos\beta\cos\theta+\sin\beta\sin\theta}{\cos\beta\cos\theta-\sin\beta\sin\theta}\,,\tag{22}$$
$$F_3^2=\frac{f}{4}(\sin\beta\cos\theta+\cos\beta\sin\theta)\,,\quad r_3^2=\frac{\sin\beta\cos\theta-\cos\beta\sin\theta}{\sin\beta\cos\theta+\cos\beta\sin\theta}\,.$$

Now we discuss a particular simple case. This corresponds to using a single scalar multiplet instead of two multiplets η amd σ. This multiplet we write as

$$\Sigma=\begin{bmatrix}\eta_0+\sigma_3+i(\sigma_0+\eta_3) & i\eta^-+\sigma^-\\ i\eta^++\sigma^+ & \eta_0-\sigma_3+i(\sigma_0-\eta_3)\end{bmatrix}\,.$$

Note that this multiplet is electrically neutral. Then interaction of this multiplet with leptons can be written as

$$-g\bar{\xi}_R\Sigma\xi_L+\text{H.c.}=-\tfrac{1}{2}g[2\bar{\mu}\mu(\eta_0+\sigma_3)+2i\bar{\mu}\gamma_5\mu(\sigma_0+\eta_3)+2\bar{\mu}e\sigma^-+2i\bar{\mu}\gamma_5e\eta^-+2\bar{e}\mu\sigma^+$$
$$+2i\bar{e}\gamma_5\mu\eta^++2\bar{e}e(\eta_0-\sigma_3)+2i\bar{e}\gamma_5e(\sigma_0-\eta_3)]\,.$$

Now if $\langle(\eta_0+\sigma_3)\rangle=2\nu$, but $\langle(\eta_0-\sigma_3)\rangle=0$, then we see that

$$m_\mu=2g\nu\,,$$
$$m_e=0\,.$$

This case we call case II to distinguish it from the case I discussed above. In this case we get the results which one gets in case I by taking

$$g_\eta=g_\sigma=g\,,$$
$$\nu_2=\nu_3=\nu\,.$$

In particular, the mass matrix M in Eq. (15) is diagonal, so that $\alpha=0$,

$$G_\mu^1=-G_{L\mu}\,,$$
$$G_\mu^2=G_{R\mu}\,,$$
$$M_1^2=M_L^2=\tfrac{1}{4}f^2\cos^2\theta(4\nu_1^2+2\nu^2)\,,\tag{23}$$
$$M_2^2=M_R^2=\tfrac{1}{4}f^2\sin^2\theta(2\nu^2)\,.$$

III. ELECTRON SELF-MASS

First we discuss the radiative corrections to electron mass in a single-loop approximation due to exchange of vector bosons G_μ. The two graphs which give nonzero contribution to the electron self-energy are shown in Figs. 1(a) and 1(b). The self-energy to order f^2 is finite and is given by

$$\Delta E_1^e=\frac{3}{16\pi^2}m_\mu\frac{\sin2\alpha\sin2\theta}{8}\ln\frac{M_1^2}{M_2^2}\,.\tag{24a}$$

This contribution is of the right order. But, these graphs also give a quadratically divergent contribution of order f^2/M^2 to the electron self-mass which is given by

$$\Delta E_2^e=\frac{m_\mu f^2}{16\pi^2}\sin2\alpha\sin2\theta\left(\frac{1}{M_2^2}-\frac{1}{M_1^2}\right)\times\left(\Lambda^2-m_\mu^2\ln\frac{\Lambda^2}{m_\mu^2}\right)\,.\tag{24b}$$

In Eq. (24b), Λ^2 is the cutoff. In addition, as is clear from Eq. (11), the electron self-mass may also arise from the radiative corrections to Yukawa coupling constants g_η and g_σ due to exchange of vector bosons G_μ. Let us write the radiative corrections to g_η and $-g_\sigma$ as $g_\eta R_\eta$ and $g_\sigma R_\sigma$. Then from Eq. (11), the electron self-mass due to these corrections to g_η and $-g_\sigma$ is given by

$$\delta m_e=\tfrac{1}{2}m_\mu(R_\eta+R_\sigma)\,.\tag{25}$$

Those graphs for which $R_\eta=-R_\sigma$ will not contribute to δm_e. It is then easy to see that as far as radiative corrections due to exchange of vector bosons are concerned, only the graphs shown in Fig. 1(a)′ and 1(b)′ can contribute to δm_e. The contribution of these graphs can easily be calculated and is given by

$$\delta m_e=-\frac{m_\mu}{16\pi^2}f^2\left\{3\frac{\sin2\alpha\sin2\theta}{8}\left[\ln\frac{M_1^2}{M_2^2}+\frac{8}{3}\left(\frac{1}{M_2^2}-\frac{1}{M_1^2}\right)\left(\Lambda^2-m_\mu^2\ln\frac{\Lambda^2}{m_\mu^2}\right)\right]\right\}\,.\tag{26}$$

This exactly cancels the contribution $(\Delta E_1^e + \Delta E_2^e)$ given in Eqs. (24a) and (24b). Thus the electron remains massless in the single-loop approximation owing to exchange of gauge vector bosons.

For case I, the electron self-mass can also arise owing to a shift in the vacuum expectation values ν_2 and ν_3. This shift is due to radiative corrections arising from the one-loop tadpole graphs shown in Figs. 2(a)–2(d). Their contribution can be easily calculated and is given by

$$\delta\nu_2 = \frac{1}{(2\pi)^4}\frac{1}{m_\eta^2}(F^\eta + F_b^\eta + F_c^\eta), \tag{27a}$$

$$\delta\nu_3 = \frac{1}{(2\pi)^4}\frac{1}{m_\sigma^2}(F^\sigma + F_b^\sigma + F_c^\sigma), \tag{27b}$$

where

$$F^\eta = F_a^\eta + F_d^\eta$$

$$= -\pi^2 3\nu_2 f^2\Big[\tfrac{3}{2}\Lambda^2 - \tfrac{1}{2}M_1^2(\cos\alpha\cos\theta + \sin\alpha\sin\theta)^2\ln\frac{\Lambda^2}{M_1^2} - \tfrac{1}{2}M_2^2(\sin\alpha\cos\theta - \cos\alpha\sin\theta)^2\ln\frac{\Lambda^2}{M_2^2}$$
$$-\tfrac{1}{4}(M_3^1)^2(\cos\beta\cos\theta + \sin\beta\sin\theta)^2\ln\frac{\Lambda^2}{(M_3^1)^2} - \tfrac{1}{4}(M_3^2)^2(\sin\beta\cos\theta - \cos\beta\sin\theta)^2\ln\frac{\Lambda^2}{(M_3^2)^2}\Big] \tag{28a}$$

$$F_b^\eta = -\pi^2 3\nu_2 h_\eta\left(\Lambda^2 - m_\eta^2\ln\frac{\Lambda^2}{m_\eta^2}\right), \tag{28b}$$

$$F_c^\eta = -\pi^2 4m_\mu g_\eta^2\left(\Lambda^2 - m_\mu^2\ln\frac{\Lambda^2}{m_\mu^2}\right), \tag{28c}$$

$$F^\sigma = -\pi^2 3\nu_3 f^2\Big[\tfrac{3}{2}\Lambda^2 - \tfrac{1}{2}M_1^2(\cos\alpha\cos\theta - \sin\alpha\sin\theta)^2\ln\frac{\Lambda^2}{M_1^2} - \tfrac{1}{2}M_2^2(\sin\alpha\cos\theta + \cos\alpha\sin\theta)^2\ln\frac{\Lambda^2}{M_2^2}$$
$$-\tfrac{1}{4}(M_3^1)^2(\cos\beta\cos\theta + \sin\beta\sin\theta)^2\ln\frac{\Lambda^2}{(M_3^1)^2} - \tfrac{1}{4}(M_3^2)^2(\sin\beta\cos\theta - \cos\beta\sin\theta)^2\ln\frac{\Lambda^2}{(M_3^2)^2}\Big], \tag{28d}$$

$$F_b^\sigma = -\pi^2 3\nu_3 h_\sigma\left(\Lambda^2 - m_\sigma^2\ln\frac{\Lambda^2}{m_\sigma^2}\right), \tag{28e}$$

$$F_c^\sigma = -\pi^2 4m_\mu g_\sigma^2\left(\Lambda^2 - m_\mu^2\ln\frac{\Lambda^2}{m_\mu^2}\right). \tag{28f}$$

From Eq. (11), the contribution to the electron self-mass due to tadpole graphs is given by

$$\Delta E_T^e = g_\eta\delta\nu_2 - g_\sigma\delta\nu_3$$
$$= \frac{1}{(2\pi)^4}\left[\frac{g_\eta}{m_\eta^2}(F^\eta + F_b^\eta + F_c^\eta) - \frac{g_\sigma}{m_\sigma^2}(F^\sigma + F_b^\sigma + F_c^\sigma)\right]. \tag{29}$$

It is clear from Eqs. (28) that tadpole graphs give both logarithmically and quadratically divergent contributions to the electron self-mass.

Note that in writing Eqs. (28) we have taken into consideration the contribution of "ghost loops" in the tadpole graphs given in Fig. 2(d). The effective interaction of the spin-zero fermion "ghost" field ω is given by

$$f^2\xi^{-1}\{-\tfrac{1}{8}2\nu_2\eta_0[2(\sin\alpha\cos\theta - \cos\alpha\sin\theta)^2\overline{\omega}^2\omega^2 + 2(\cos\alpha\cos\theta + \sin\alpha\sin\theta)^2\overline{\omega}^1\omega^1]$$
$$-\tfrac{1}{8}2\nu_3\sigma_3[2(\sin\alpha\cos\theta + \cos\alpha\sin\theta)^2\overline{\omega}^2\omega^2 + 2(\cos\alpha\cos\theta - \sin\alpha\sin\theta)^2\overline{\omega}^1\omega^1]$$
$$-\tfrac{1}{8}(2\nu_2\eta_0 + 2\nu_3\sigma_3)[(\sin\beta\cos\theta - \cos\beta\sin\theta)^2(\omega_3^2)^2 + (\cos\beta\cos\theta + \sin\beta\sin\theta)^2(\omega_3^1)^2]\}. \tag{30}$$

The propagator of the "ghost" is of the form $\xi/(\xi k^2 - M^2)$. ξ is a gauge-dependent parameter. For the unitary gauge, which we are using, $\xi \to 0$.

We note that both logarithmic and quadratic divergences in Eq. (29) vanish when $M_1^2 = M_2^2$ and $m_\eta^2 = m_\sigma^2$. But in this case the finite contribution also vanishes. In any case, since the contribution of tadpole graphs is equivalent to shifting the vacuum expectation values ν_2 and ν_3 to $\nu_2 + \delta\nu_2$ and $\nu_3 + \delta\nu_3$, this contribution can be absorbed by redefining the polynomials $V(\eta)$ and $V(\sigma)$.

For the case II, since G_μ^1 and G_μ^2 reduce to $G_{L\mu}$ and $G_{R\mu}$ which have $V \mp A$ couplings, it is clear that graphs in Fig. 1(a) and 1(b) give zero contri-

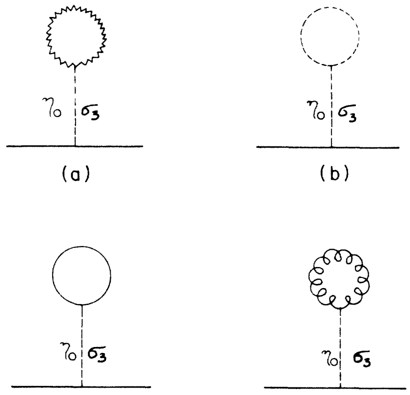

FIG. 2. Tadpole graphs which contribute to electron mass. Here solid lines denote electrons; jagged lines denote vector bosons; dashed lines denote scalar particles; and looped lines denote "ghost loops".

bution to the electron self-mass. Also for this case, there is no tadpole graph which contributes to the electron self-energy in second order. Thus for this case also, the electron remains massless in second order. Our conclusion is that the electron may acquire mass in fourth order, which is not attractive. In this respect our model does not differ from the other models considered in the literature.[4,5,6]

We now briefly discuss the case when the group $U_l(1)$ is also local. We associate a gauge vector boson H_μ with $U_l(1)$. Then we have the additional current which is coupled to H_μ,

$$\frac{i}{2}f'(\bar{\mu}\gamma_\mu\mu+\bar{e}\gamma_\mu e+\bar{\nu}_{\mu L}\gamma_\mu\nu_{\mu L}+\bar{\nu}_{eL}\gamma_\mu\nu_{eL})\,.$$

The current corresponding to muonic charge Q_l is then given by

$$J^l_\mu=(i\bar{\mu}\gamma_\mu\mu+i\bar{\nu}_{\mu L}\gamma_\mu\nu_{\mu L})\,.$$

The gauge vector bosons X_μ coupled to this current are given by

$$X_\mu=e_l\left(\frac{G_{3L\mu}}{f_L}+\frac{G_{3R\mu}}{f_R}+\frac{H_\mu}{f'}\right)\,, \tag{31}$$

where e_l is the coupling strength of this boson with the current J^l_μ and e_l is given by

$$e_l^{\,2}=\left(\frac{1}{f_L^{\,2}}+\frac{1}{f_R^{\,2}}+\frac{1}{f'^2}\right)^{-1}\,. \tag{32}$$

We want the vector boson X_μ to be massive. For this purpose, instead of introducing a scalar multiplet ϵ we introduce a multiplet

$$\epsilon'=\begin{pmatrix}\epsilon'_0\\ \epsilon^-\end{pmatrix}$$

and replace it by

$$\begin{pmatrix}\epsilon'+\nu'_1\\ 0\end{pmatrix},$$

so that instead of the first term in Eq. (14), we get the term

$$-\tfrac{1}{8}(\nu'_1+\epsilon')^2[8f_L^{\,2}\overline{G}_{L\mu}G_{L\mu}+4(f_LG_{3L\mu}+f'H_\mu)^2]\,,$$

and the rest of the terms remain unchanged in Eq. (14). The mass matrix as given in Eq. (16) now becomes

$$M_0=\tfrac{1}{4}\begin{bmatrix} f_L^{\,2}(4\nu_1'^2+\nu_2^{\,2}+\nu_3^{\,2}) & -f_Lf_R(\nu_2^{\,2}+\nu_3^{\,2}) & 4f_Lf'\nu_1'^2\\ -f_Lf_R(\nu_2^{\,2}+\nu_3^{\,2}) & f_R^{\,2}(\nu_2^{\,2}+\nu_3^{\,2}) & 0\\ 4f_Lf'\nu_1'^2 & 0 & 4f'^2\nu_1'^2\end{bmatrix}. \tag{33}$$

Since the contribution to the electron self-energy comes either from the graphs in which the vector bosons G^1_μ and G^2_μ are exchanged or from the tadpole graphs arising from η_0 and σ_3 exchange, therefore none of the results discussed previously are affected by considering $U_l(1)$ as a gauge group.

IV. SOME CONSEQUENCES OF "ANOMALOUS" WEAK INTERACTION

It is clear from the Lagrangian $\mathcal{L}^G_{\rm int}$ given in Eq. (21) that we get a contribution to processes like

$$\nu_e(\bar{\nu}_e)+e\to\nu_e(\bar{\nu}_e)+e\,, \tag{34a}$$

$$\nu_\mu(\bar{\nu}_\mu)+e\to\nu_\mu(\bar{\nu}_\mu)+e\,, \tag{34b}$$

in addition to that given by the Salam-Weinberg theory. The effective Hamiltonian for the process (34a) from Eq. (21) is given by

$$-F^1_3\left(\frac{f\cos\theta\cos\beta}{4(M^1_3)^2}\right)[i\bar{e}\gamma_\mu(1+r^1_3\gamma_5)e]\;[i\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e]$$
$$+F^2_3\left(\frac{f\cos\theta\sin\beta}{4(M^2_3)^2}\right)[i\bar{e}\gamma_\mu(1+r^2_3\gamma_5)e]\;[i\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e]\,.$$

We get a similar expression for the process (34b) by replacing e by μ. It is clear that this contribution is negligible if vector bosons $G^1_{3\mu}$ and $G^2_{3\mu}$ are superheavy, i.e., $(M^1_3)^2$, $(M^2_3)^2\gg M_Z^{\,2}$ and f^2 is of order α. If $f^2/M_3^{\,2}$ is of order G_F, then this effect is similar to that due to a neutral current in the Salam-Weinberg theory.[3]

In the usual theory the effective matrix elements for the decay $\mu^-\to e+\nu_\mu+\bar{\nu}_e$ can be written from Eq. (4a) as

$$\frac{g^2}{8m_W^{\,2}}[i\bar{u}_e\gamma_\mu(1+\gamma_5)v_{\nu_e}][i\bar{u}_{\nu_\mu}\gamma_\mu(1+\gamma_5)u_\mu]\,.$$

The effective matrix elements arising for μ decay from the "anomalous" weak-interaction Lagrangian given in Eq. (21) are given by

$$-F_1\frac{f\cos\theta\cos\alpha}{2\sqrt{2}M_1^{\,2}}[i\bar{u}_e\gamma_\mu(1+r_1\gamma_5)u_\mu]\,[i\bar{u}_{\nu_\mu}\gamma_\mu(1+\gamma_5)v_{\nu_e}]+F_2\frac{f\cos\theta\sin\alpha}{2\sqrt{2}M_2^{\,2}}[i\bar{u}_e\gamma_\mu(1+r_2\gamma_5)u_\mu]\times[i\bar{u}_{\nu_\mu}\gamma_\mu(1+\gamma_5)v_{\nu_e}]\,.$$

Again, if $M_1^{\,2}$, $M_2^{\,2}\gg M^2$ and f^2 is of order α, this contribution is negligible. Even if $f^2/M_1^{\,2}$ or $f^2/M_2^{\,2}$ is of order $g^2/m_W^{\,2}$, we will have negligible effect on the μ-decay parameters, since there are enough parameters available to make the effect of "anomalous" weak interaction negligible.

V. CONCLUSIONS

We have constructed a gauge model for leptons based on a chiral group such that the muon number appears as a muonic charge associated with the group structure. In this model some of the gauge vector bosons carry muonic charge, so that $e-\mu$ transition is possible as in Fig. 1; the result is that the electron self-mass can arise in second order, with the muon in the intermediate state and the electron being massless in zeroth order. But as we have seen for case I in this model, the electron remains massless in second order. For case II of this model, the electron also remains massless in second order. The electron mass may arise in fourth order. In this respect our model gives results similar to the other models[4,5,6] proposed in the literature to calculate the electron-muon mass ratio. None of these models[4,5,6] give satisfactory results in second order. Our model has the advantage that normal weak interactions which are universal in the sense that they are generated in the same way both for leptons and hadrons remain unaffected, and muonic number has a group-theoretic basis.

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