Threshold of pion condensation

G. Bertsch*
Cyclotron Laboratory and Physics Department, Michigan State University, East Lansing, Michigan 48824

Mikkel B. Johnson†
Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544

(Received 23 December 1974)

The threshold density for pion condensation in neutron matter is determined with a model including nuclear interactions and the Δ resonance in π-nucleon scattering. Depending on the magnitude of N-Δ correlations, pion condensation can have a threshold as low as 0.1 neutrons/fm^3 or not occur at all. Applying the model to nuclear matter, we find that in the absence of N-Δ correlations physical pions have a very large effective mass. With N-Δ correlations, these effects are diminished, but pion condensation could still occur at twice normal nuclear matter density.

I. INTRODUCTION

Since the work of Sawyer and Scalapino[1] on the possibility of a phase transition in neutron matter to a phase with π^- in the ground state, there has been considerable interest in the details of the phase transition.[2-8] This interest is due to the complexity of the physics; neutron star structure is probably only mildly affected by the presence or absence of a charged boson condensate.[9] Various effects have been introduced in previous calculations which have an important influence on the existence of the condensation. These effects are (a) nuclear interactions, including correlations,[7] (b) π-nucleon S-wave interactions,[8,4] (c) resonant π-nucleon scattering.[6,4] However, none of the above-referenced calculations have included all of these effects in a reliable way.

In this work we attempt to make a realistic calculation of the location of the threshold, including all of the above effects. In our opinion the most convenient method for examining the threshold for phase transitions is with the pion Green's function for the normal state. Insofar as a pion can couple to the other degrees of freedom of the system, the Green's function contains all the physics necessary to locate the threshold. The various effects mentioned above can be precisely defined and approximately evaluated with diagrammatic perturbation theory.

In the section below, we discuss the structure of the Green's function and threshold condition,[10] since there has been some controversy on this.[11] The following section describes the incorporation of nuclear interactions into the Green's function. The other effects, (b) and (c), are treated in a more primitive way, and are discussed with the results. Finally, we apply the same model to nuclear matter, and essentially confirm the results of Barshay, Brown, and Rho.[12]

II. STRUCTURE OF THE GREEN'S FUNCTION

In Fig. 1 we display the analytic structure of the Green's function for π^- defined in neutron matter

$$G(k,\omega)=\frac{1}{\omega^2-(k^2+\mu^2)-\Pi(k,\omega)}\,, \qquad (1)$$

where $\Pi(k,\omega)$ is a self-energy.

The pole marked A is the π^- pole. The cut on the same side of the real axis marked B is present only if there are physical states having the same quantum numbers as the π^-. There are none such in the pure neutron gas, but in the actual matter of neutron stars there are some protons and we can have a neutron-particle proton-hole excitation.

The cuts and poles above the real axis correspond to states with the quantum numbers of the π^+. We have depicted the π^+ itself as C; the spectrum of proton-particle neutron-hole states as D, and a possible collective pn^{-1} excitation as E.

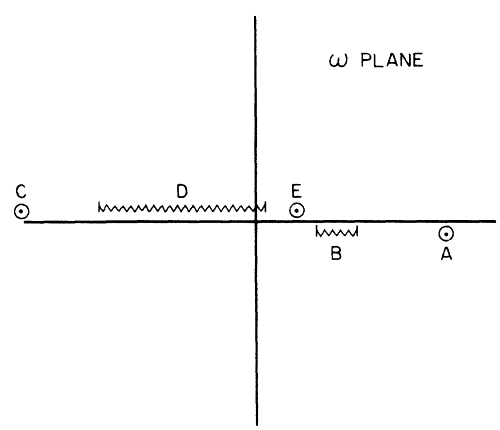


FIG. 1. The complex energy plane with the singularities of the pion Green's function displayed.

Our condition for a phase transition is simply that the upper-plane singularities penetrate leftward of the lower-plane singularities. This simply means that it is energetically favorable to simultaneously create an excitation with the π^+ quantum numbers together with an excitation of the π^- quantum numbers.

The general condition for a phase transition, that the energy gained by creation of a positive-charge excitation just exceed the energy lost for the creation of some negative-charge excitation, may be easily cast in the Green's function language. The minimum energy to create an excitation is given by the leftmost singularity below the real axis of some Green's function, or the rightmost singularity above the real axis. Our condition, that these minimum energies appear in the same Green's function, is clearly sufficient for pion condensation. It is also satisfactory because it guarantees that in a model the same nuclear physics is used for the calculation of the energy of both excitations.

In our calculations below we shall ignore the np^{-1} states. The ratio of neutrons to protons is of the order 10 : 1, so this state cannot have a great effect on the total energy. Also, the Fermi-gas model may not be appropriate for this dilute gas of protons; for example, pairing correlations or clumping into supernuclei (which occurs at the lowest densities at which we find condensation) may completely alter their response characteristics.

The condition for the phase transition which we seek will be that the poles marked A and E meet, or if there is no pole E, that pole A meet the branch cut D.

III. CALCULATION OF Π

The basic graph describing the π-nucleon interaction is given in Fig. 2(a), and gives a contribution to the π self-energy in Eq. (1):

$$\Pi^{(0)}(\vec{q}, \omega) = 2f^2q^2G^{(0)}(\vec{q}, \omega), \tag{2}$$

where $G^{(0)}$ is the Green's function for a proton-particle neutron-hole excitation. The theory of Ref. 1 is obtained by the infinite-mass approximation for $G^{(0)}$

$$G^{(0)}(q, \omega) \approx -\rho/\omega, \tag{3}$$

where ρ is the density of the neutron matter. Nuclear interactions modify $G^{(0)}$ in two ways. First, we use a Hartree-Fock $G^{(0)}$ instead of (3). Second, $G^{(0)}$ is replaced by the random-phase approximation (RPA) Green's function, which allows correlations to be treated. Let us first consider $G^{(0)}$.

Baym and Flowers use a free-particle Green's function similar to the Lindhard function,

(a) (b)

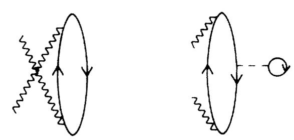


FIG. 2. The elementary nucleon pole contribution to the pion self-energy.

$$G^{(0)}(\vec{q}, \omega) = \sum_p \frac{\theta(p_F - p)}{[p^2 - (\vec{p} - \vec{q})^2]/2M - \omega}. \tag{4}$$

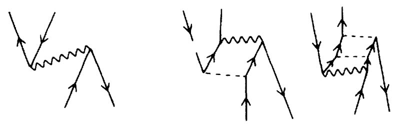
In this work we will use the pn^{-1} Hartree-Fock Green's function, defined by

$$G^{(0)} = \sum_p \frac{\theta(p_F - p)}{\epsilon(p) - \epsilon(\vec{p} - \vec{q}) - \omega}. \tag{5}$$

The self-energies $\epsilon(k)$ contain the kinetic energy and a potential energy, as indicated in Fig. 2(b). The potential is a function of density and the momentum of the nucleon; we shall calculate these in terms of integrals over the effective interaction between nucleons in matter.

This potential can be quite significant for the condensation question. The symmetry potential in ordinary nuclei, if extrapolated to neutron matter, implies that the proton potential is much deeper than the neutron potential. This makes it easier to create a pn^{-1} excitation, and thus lowers the critical density for π condensation. For example, Negele[13] finds for a density of 0.16 fm^{-3} a difference between the average neutron potential and the proton potential of 77 MeV. However, our calculations yield proton and neutron potentials which are nearly equal, if second-order tensor effects are neglected. Our actual calculation of $G^{(0)}$ will be by numerically performing the integration in Eq. (5) with a parameterized potential energy function $U(k, \rho)$.

The nonpionic nuclear interaction is also important because the short-range repulsion will hinder the condensation.[14,15] The same effect in nuclei weakens the π optical potential.[12,16,17] Physically, we understand the effect as follows: The nucleons will stay further apart on the average than in the absence of the potential. Therefore, the pions have to propagate further off their mass shell and the condensation is more difficult to achieve. In our theory, as in Ref. 7, correlations arise from the dynamic effect given by the diagrams in Figs. 3(b) and 3(c) to be added onto the basic π exchange, Fig. 3(a). These graphs represent the exchange of heavy mesons together



(a) (b) (c)

FIG. 3. Correlation effects modifying the basic pion Green's function.

with pions. Instead of calculating these explicitly, let us lump together all interaction effects, except the one-pion exchange, into a t matrix, $\mathcal{V}_{\rm corr}$. Then these graphs can be included in the π self-energy by replacing $G^{(0)}$ by

$$G^{\rm RPA} = G^{(0)}(1 - \mathcal{V}_{\rm corr}G^{(0)})^{-1}. \tag{6}$$

Thus, the π self-energy becomes

$$\Pi^{\rm RPA} = 2f^2k^2G^{\rm RPA}. \tag{7}$$

This RPA Green's function generates the bubble sum of graphs illustrated in Fig. 4. To calculate $\mathcal{V}_{\rm corr}$, we assert that the graphs with multiple meson exchanges are well approximated by the static G-matrix or effective interaction. A calculated G-matrix interaction would include the one-pion exchange (OPE) in the static limit, however. It is only necessary to subtract out this potential from the effective interaction

$$\tilde{\mathcal{V}}_{\rm corr} = \mathcal{V}_{\rm eff} - \mathcal{V}_{\rm OPE}. \tag{8}$$

We now turn to the other contributions to the π self-energy. The π S-wave repulsion we treat in the manner of Ref. 8. Below the condensation point, where π-π interactions are unimportant, this is

$$\Pi_{S\pi} = 2\omega \times 219\ ({\rm MeV})\rho. \tag{9}$$

The resonant π-nucleon scattering is well parameterized by the Chew-Low theory,[18] having an amplitude in the 33 channel

$$\mathcal{F} = \frac{4}{3}\,\frac{f^2k^2}{\omega(1-\omega/\omega_R)}, \tag{10}$$

where $\omega_R \approx 300$ MeV. It may be derived by summing the series of graphs shown in Fig. 5. It contains the nucleon pole $(1/\omega)$ which we already included in the basic theory, Fig. 2.

Thus we write

$$\mathcal{F}_\Delta = \frac{4}{3}\,\frac{f^2k^2}{\omega(1-\omega/\omega_R)} - \frac{4}{3}\,\frac{f^2k^2}{\omega} = \frac{4}{3}\,\frac{f^2k^2}{\omega_R(1-\omega/\omega_R)} \tag{11}$$

and the contribution to the π self-energy from the two graphs in Fig. 6 is

$$\Pi_\Delta = -\frac{8}{3}\,\frac{f^2\rho k^2}{\omega_R}\left(\frac{1}{1-\omega/\omega_R} + \frac{1}{3(1+\omega/\omega_R)}\right). \tag{12}$$

Migdal's expression for Π_Δ (Ref. 4) differs from Eq. (12) only in that the factor $\frac{8}{3}$ is replaced by $\frac{9}{4}$.

If we simply add (12) to the π self-energy, the model neglects ΔN correlations completely. Barshay *et al.*[12] emphasize that the ΔN correlations are as important as the NN correlations in damping the condensation tendency. While the necessary details of the ΔN interaction can only be guessed at, we can make a crude estimate of the possible effects by scaling the ΔN t matrix. to the NN t matrix. In the quark model, the π and magnetic ρ coupling constants are in fact proportional, giving proportional t matrices for the nucleon and isobar. From the Chew-Low theory, Eq. (10), the coupling constant for π in the $N\Delta$ interaction is $\frac{4}{3}$ its value in the NN interaction.

Assuming that all $N\Delta$ t matrices are greater by this factor, we can combine $\Pi_{\rm Nucl}$ and Π_Δ to obtain

$$\Pi_{N\Delta} = 2f^2k^2(G^{(0)} + \tfrac{4}{3}G^\Delta)[1 - \mathcal{V}_{\rm corr}(G^{(0)} + \tfrac{4}{3}G^\Delta)]^{-1}, \tag{13}$$

where $-G^\Delta = 1/(\omega_R - \omega) + \frac{1}{3}/(\omega_R + \omega)$.

IV. NUCLEAR INTERACTION

We now describe the interactions we use to calculate the single-particle potentials U and the par-

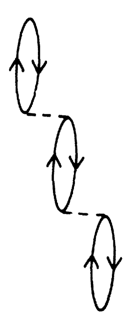


FIG. 4. Bubble graphs of the RPA Green's function.

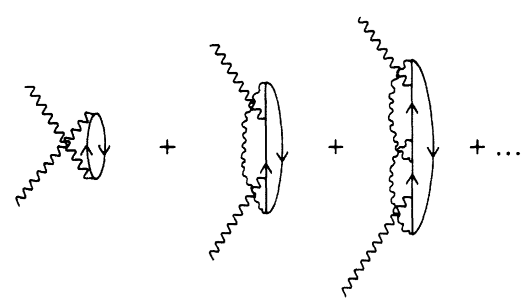


FIG. 5. Sum of graphs giving rise to the Δ resonance in Chew-Low theory.

ticle-hole interaction $\mathcal{V}_{\rm eff}$. We consider two models for the interaction, the first suited for dense matter. Here the two-body interaction $\mathcal{V}$ is taken to be a product of a free nucleon-nucleon potential, and a correlation function calculated from the constrained variational theory of Pandharipande.[19,20] The free nucleon-nucleon potential $v(r)$ is Model III of Ref. 19. This potential has a short-range soft-core repulsion, common to all partial waves. The long-range central part depends on partial waves; we consider four potentials: singlet even $\mathcal{V}_{1+}$, singlet odd $\mathcal{V}_{1-}$, triplet even $\mathcal{V}_{3+}$, and triplet odd $\mathcal{V}_{3-}$. In addition, we consider separately the tensor potential and the singlet-S potential, which is stronger than the singlet D.

Pandharipande calculates the effective interaction as[20]

$$\begin{aligned} \mathcal{V}_<(r) &= \lambda f^2(r), \quad r<d \\ \mathcal{V}_>(r) &= v(r), \quad r>d \end{aligned} \tag{14}$$

where $f(r)$ is his correlation function and λ is the healing constraint. This is approximately equal to[20]

$$\begin{aligned} \mathcal{V}_<(r) &= v(r)f(r), \quad r<d \\ \mathcal{V}_>(r) &= v(r), \quad r>d \end{aligned} \tag{15}$$

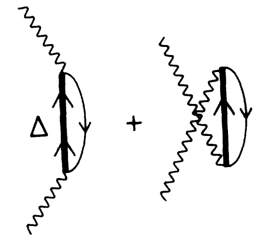


FIG. 6. Δ-resonance contribution to pion self-energy.

We shall use the interaction in the form (15), with correlation function f calculated by Pandharipande's method. The G matrix for the effective interaction is taken as the plane-wave matrix element of this potential.

A second model was considered which would be more realistic at low densities. Borysowicz *et al.*[21] have calculated coordinate-space potentials which closely fit G matrix elements of the Reid[22] potential in finite nuclei. We compute plane-wave matrix elements of these potentials to approximate the G matrix.

The neutron and proton single-particle potentials are given by

$$U_n(k^2,\rho) = 2\int_{<k_{Fn}} \frac{d^3k'}{(2\pi)^3} \sum_l 4\pi(2l+1)\int_0^\infty r^2 dr\, j_l{}^2(|\vec{k}-\vec{k}'|r) \times \begin{cases} \tfrac{3}{2}\mathcal{V}_{3l} & l \text{ odd} \\ \tfrac{1}{2}\mathcal{V}_{1l} & l \text{ even}, \end{cases} \tag{16a}$$

$$U_p(k^2,\rho) = 2\int_{<k_{Fn}} \frac{d^3k'}{(2\pi)^3} \sum_l 4\pi(2l+1)\int_0^\infty r^2 dr\, j_l{}^2(|\vec{k}-\vec{k}'|r)[\tfrac{1}{4}\mathcal{V}_{1l}(r) + \tfrac{3}{4}\mathcal{V}_{3l}(r)]\,. \tag{16b}$$

This was fitted by hand to a power-series expansion in k^2 and ρ; the fit reproduces the potential quite accurately in the range $0.16 \le \rho \le 0.8$ fm^{-3}. The fit with the dense-matter potential is

$$U_p = [-42.2 + 380(\rho - 0.3)^2] + (4.34 + 28.2\rho - 13.95\rho^2)k^2 - (0.55 - 0.13\rho)k^4, \tag{17a}$$

$$U_n = [-56.6 + 295(\rho - 0.46)^2] + 27.8\rho k^2 - (0.16 - 0.416\rho)k^4. \tag{17b}$$

The potential has units MeV, density ρ has units fm^{-3}, and the momentum k has units fm^{-1}. The second-order tensor interaction is not included here. We have estimated this to be a correction to the proton energy of ~-40 MeV, in which case the difference between proton and neutron potentials is 38 MeV at $\rho = 0.16$, roughly half of Negele's result. We will present results both with and without this term, so the prediction of any better models can be estimated. There is a strong k^2 dependence in the proton potential, not found in the neutron potential. This is the well-known effective mass of the Hartree-Fock (HF) potential. It makes U_p actually repulsive at large k^2, inhibiting the condensation.

The calculation of the particle-hole interaction is somewhat complicated because it is off-diagonal. The Green's function carries spin 1 and isospin 1, which determines the contribution of the different spin-isospin component of the force by crossing relations. We express the interaction, in terms of the momentum of the pion k, which is the momentum transfer for the direct part of the interaction, and the momenta p, p' of the neutron holes. We have

$$\tilde{\mathcal{V}}_{\rm corr} = V_c^d(k) - V_c^e(|\vec{p}-\vec{p}'|) + V_S(\vec{p},\vec{p}') + V_T(\vec{p},\vec{p}') - \mathcal{V}_{\rm OPE}, \tag{18}$$

where

$$\mathcal{V}_{\mathrm{OPE}}=\frac{2f^2k^2}{\mu_\pi{}^2+k^2}\ .$$

The first two terms are the direct and exchange parts of the central interaction, given by

$$V_c^d(k)=\tfrac{1}{8}[\mathcal{V}_{3^-}(k)-\mathcal{V}_{3^+}(k)+\mathcal{V}_{1^-}(k)-\mathcal{V}_{1^+}(k)], \qquad (19)$$

$$V_c^e(q)=\tfrac{1}{8}[\mathcal{V}_{3^-}(q)+\mathcal{V}_{3^+}(q)+\mathcal{V}_{1^-}(q)+\mathcal{V}_{1^+}(q)]. \qquad (20)$$

Equation (19) is also given in Ref. 23. The third term in Eq. (18) is the extra singlet S attraction, given by

$$V_S(\vec{p},\vec{p}\,')=-\pi\int_0^\infty r^2dr[\mathcal{V}_{1S}(r)-\mathcal{V}_{1^+}(r)]\times j_0\left(\left|\frac{\vec{p}\,'-\vec{p}-\vec{q}}{2}\right|r\right)j_0\left(\left|\frac{\vec{p}\,'-\vec{p}+\vec{q}}{2}\right|r\right). \qquad (21)$$

The next-to-last term is the tensor interaction, and is given by

$$V_T=\mathcal{V}_T^d(k)+\tfrac{1}{4}[1-3\hat{k}\cdot(\hat{p}-\hat{p}\,')]\mathcal{V}_T^e(|\vec{p}-\vec{p}\,'|) \qquad (22)$$

with

$$\mathcal{V}_T^{d}(q)=-4\pi\int_0^\infty r^2dr\,j_2(qr)[\mathcal{V}_{T^-}(r)\mp\mathcal{V}_{T^+}(r)]\quad\text{for}\begin{cases}\text{direct}\\ \text{exchange}\end{cases}.$$

The tensor interaction is defined in the usual way,

$$V_T(\vec{r}_{12},\vec{\sigma}_1,\vec{\sigma}_2)=S_{12}\mathcal{V}_T(r). \qquad (23)$$

To get an idea of the importance of correlations, we list in Table I some matrix elements. Not only is the effective interaction less attractive than the one-π exchange potential, it actually has the opposite sign for low momenta. As a consequence, G^{RPA} is much reduced from $G^{(0)}$, and there may not exist a collective pn^{-1} state with the π^+ quantum numbers. We also see that the correlation effect is greater in the lower-density model of nuclear interactions, as might be expected.

TABLE I. Interaction potential between nucleons in MeV fm^3 with $k=1.4$ fm^{-1} and $\rho=0.18$.

	$\mathcal{V}_c^d+\mathcal{V}_c^e$	$\mathcal{V}_T$	$\mathcal{V}_{\mathrm{OPE}}$	$\tilde{\mathcal{V}}_{\mathrm{corr}}$
Dense model [a]	160	−330	−650	480
Nuclear model [b]	270	−310	−650	590

[a] See Ref. 19.
[b] See Ref. 21.

V. RESULTS FOR NEUTRON MATTER

We now calculate the threshold neutron densities predicted with the various π self-energy terms. As pointed out by Weise and Brown,[7] the correlation effects on G play an important role. We calculate G by representing the Fermi sea on a discrete mesh in momentum space, computing $\tilde{\mathcal{V}}_{\mathrm{corr}}$ between states of this mesh, and numerically performing the operations in Eq. (5) and Eq. (6). In Table II are quoted sample results for G, computed at a density of 0.18 fm^{-3}, momentum transfer 1.4 fm^{-1}, and energy 100 MeV. The G^{HF} without tensor is close to the model with infinite nucleon mass, $G^{(0)}$. The correlations reduce G by a factor of 2 approximately. This is a greater quenching than what Weise and Brown[7] obtained by a cruder estimation of correlation effects, and also greater than estimated by Migdal.[4]

In Table III we exhibit the threshold conditions for the various different possible models for Π. The simple model has a collective pn^{-1} state which meets the π^- pole. Not all the models with a realistic interaction have a collective state, so the condensation occurs then when the π^- pole meets the pn^{-1} branch cut. Using the realistic nucleon-nucleon Green's function is of great importance, throwing the condensation to a high density. The repulsive S-wave interaction further inhibits it to the point where it no longer occurs at reasonable densities. The contribution of the Δ resonance, when calculated by Eq. (12), brings the condensation point down to less than the density ρ_0 of normal nuclear matter, $\rho_0=0.16$ fm^{-3}. Including ΔN correlations raises the critical density but the extent to which it is raised is very sensitive to the details of the nuclear short-range correlations. For example, using the dense-matter model we find the critical density to be slightly above ρ_0 as shown in Table III, but using the effective interaction of the low-density model we do not find condensation at all. Since the theory depends so strongly on the ΔN correlations, we cannot make any firm conclusions.

In a recent calculation, Bäckman and Weise[24] attempt also to make a realistic assessment of the possibility of condensation including NN and $N\Delta$ correlations and find that condensation will occur. However, these authors do not consider

TABLE II. Particle-hole Green's function in MeV^{-1} fm^{-3} with $k=1.4$ fm^{-1}, $\rho=0.18$, and $\omega=200$ MeV.

$G^{(0)}$	G^{HF}	G^{RPA}
-0.9×10^{-3}	-1.11×10^{-3}	-0.7×10^{-3}

TABLE III. Threshold of π condensation in neutron matter.

Model	ρ_{critical} (fm^{-3})	ω_π (MeV)	k_π (fm^{-1})
Simple ($\Pi^{(0)}$)	0.17	140	1.0
Π^{RPA}	0.3	126	1.3
Π^{RPA} + second-order tensor potential	0.22	152	1.2
$\Pi^{\text{RPA}}+\Pi_{S\pi}$	$\gg$0.8	...	...
$\Pi^{\text{RPA}}+\Pi_{S\pi}+\Pi_\Delta$	0.10	100	1.7
$\Pi^{\text{RPA}}+\Pi_{S\pi}+\Pi_\Delta$ + second-order tensor potential	0.085	130	1.44
$\Pi_{N\Delta}+\Pi_{S\pi}$	>0.225	119	1.50

the S-wave repulsion and use a weaker $\mathcal{V}_{\text{corr}}$ in the NN and the $N\Delta$ channels than we use. [*Note added in proof:* In a subsequent calculation the S-wave repulsion was included; a critical density of twice normal nuclear density was found. See S. O. Bäckman and W. Weise, Phys. Lett. 55B, 1 (1975).]

VI. APPLICATION TO NUCLEAR MATTER

Further guidance can perhaps be otained by studying nuclear matter: An optical potential can be deduced from the dispersion of the poles of the Green's function, and compared with empirical optical potentials. To generalize the theory to nuclear matter, we add a contribution to the self-energy for the protons, $\Pi_N(-\omega)$:

$$\Pi_{\text{nuclear matter}}=\Pi_N(\omega)+\Pi_N(-\omega) \tag{24}$$

since $\Pi(-\omega)$ describes the $\pi^\pm$ system interacting with a proton sea.

In addition in the numerical calculation of Π^{RPA}, we explicitly excluded states below the Fermi surface, although in principle this is unnecessary, since violations of the Pauli principle cancel out of $\Pi(\omega)+\Pi(-\omega)$. The single-particle energies used in G^{RPA} are pure kinetic energies.

TABLE IV. Dispersion of pions in nuclear matter. The coefficient α in Eq. (26) is tabulated for normal and twice-normal nuclear matter density.

Model	$\rho=0.16$ nuc/fm^3	$\rho=0.32$ nuc/fm^3
$\Pi_\Delta+\Pi_N$	0.1	−0.63
$\Pi_{N\Delta}$ (Eq. 13)	0.35	0.035

We calculate numerically the pole of G as a function of momentum. For low-momentum pions, only the Δ contribution is important and the model is very similar to that of Barshay *et al.*[12] To make the identification with their model, it is only necessary to set the energy variable at the pion mass, and choose the Δ coupling constant G^2 as

$$G^2=\frac{4}{3}\frac{f^2}{4\pi}. \tag{25}$$

For momenta below 0.5 fm^{-1}, the dispersion can be expressed quite well as

$$\omega^2=\mu^2+\alpha k^2. \tag{26}$$

The parameter α, which is equal to 1 for free pions, is given in Table IV for the two most realistic models at nuclear matter density, and at twice nuclear matter density. We see that for the case with no $N\Delta$ correlations, the dispersion is almost anomalous at normal nuclear matter density. There is a high density of pionic states which would surely be observable. α is predicted to be negative at supranormal densities, and a condensation would probably occur near twice normal density. However, inclusion of $N\Delta$ correlations changes the picture. The dispersion is normal in nuclear matter, albeit with a large effective mass. At twice normal nuclear density the density of states is again quite high and condensation would probably occur.

ACKNOWLEDGMENT

One of us (M.B.J.) acknowledges discussions with S. O. Bäckman and C. K. Au. G.F.B. thanks G. E. Brown for numerous correspondence and discussions.

*Work supported in part by the National Science Foundation.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]R. F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); D. J. Scalapino, *ibid*. 29, 386 (1972).

[2]J. Kogut and J. T. Manassah, Phys. Lett. 41A, 129 (1972).

[3]R. F. Sawyer and A. C. Yao, Phys. Rev. D 7, 1579 (1973).

[4]A. B. Migdal, Phys. Rev. Lett. 31, 257 (1973); Phys. Lett. 45B, 448 (1973); Nucl. Phys. A210, 421 (1973).

[5]G. Baym, Phys. Rev. Lett. 30, 1340 (1973).

[6]S. Barshay, G. Vagradov, and G. E. Brown, Phys. Lett. 43B, 359 (1973).

[7]W. Weise and G. E. Brown, Phys. Lett. 48B, 297 (1974).

[8]G. Baym and E. Flowers, Nucl. Phys. A222, 29 (1974).

[9]J. B. Hartle, R. F. Sawyer, and D. J. Scalapino, Astrophys. J. 199, 471 (1975); R. C. Malone, M. B. Johnson, and H. A. Bethe, *ibid*. 199, 741 (1975).

[10]G. Bertsch and M. Johnson, Phys. Lett. 48B, 397 (1974).

[11]C. Au, G. Baym, and E. Flowers, Phys. Lett. 51B, 1 (1974).

[12]S. Barshay, G. E. Brown, and M. Rho, Phys. Rev. Lett. 32, 787 (1974).

[13]J. Negele and D. Vautherin, Phys. Rev. C 5, 472 (1972).

[14]R. Rajaraman, Phys. Lett. 48B, 179 (1974).

[15]S. Barshay and G. E. Brown, Phys. Lett. 47B, 107 (1973).

[16]M. Ericson and T. E. O. Ericson, Ann. Phys. (N.Y.) 36, 323 (1966).

[17]J. Eisenberg, J. Hüfner, and E. Moniz, Phys. Lett. 47B, 381 (1973).

[18]G. Chew and F. Low, Phys. Rev. 101, 1570 (1956).

[19]H. Bethe and M. Johnson, Nucl. Phys. A230, 1 (1974).

[20]V. R. Pandharipande, Nucl. Phys. A174, 641 (1971); A178, 123 (1971); A181, 33 (1972).

[21]J. Borysowicz, H. McManus, and G. Bertsch, Michigan State Univ. report (unpublished).

[22]R. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[23]C. Dover, J. Hüfner, and R. Lemmer, Ann. Phys. (N.Y.) 66, 248 (1971).

[24]S. O. Bäckman and W. Weise, report (unpublished).