PHYSICAL REVIEW D VOLUME 12, NUMBER 8 15 OCTOBER 1975

Gravitation field theory in flat space (in the presence of an electomagnetic field) with the most general stress tensor: Equivalence with Einstein's theory

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(Received 18 February 1975; revised manuscript received 23 June 1975)

Gravitational field equations and equations of motion for charged particles are obtained by an iterative method, starting from a Lagrangian density in the "unrenormalized" flat space-time. Assuming (a) that the proper masses and charges of the particles are constant and (b) that the gravitational potential is a tensor (no scalar component) in the linearized version (first order in the coupling constant f) we obtain, by imposing consistency to all orders in f, Einstein's theory which is therefore sounder in the field-theoretic approach than in the usual Riemannian one where there is some arbitrariness (cosmological term, etc). The convergence of our procedure is based on Deser's method but also includes the presence of electromagnetic fields and considers in addition to the energy-momentum tensor $\tilde{T}_{\alpha\beta}$, the most general divergenceless tensor $t_{\alpha\beta}$ depending on four arbitrary parameters which prove to be no longer observable in the "renormalized" space.

I. INTRODUCTION

Einstein postulated his gravitational equations in the Riemannian space, with the criterion of maximum simplicity.

He was, however, doubtful about the possible addition of the cosmological term $\Lambda g_{\alpha\beta}$ whose introduction was strongly claimed by Eddington[1] on the basis of the "natural gauge of the world." Moreover, Eddington[2] again showed that two other alternative equations can be obtained starting from a variational method and using two other different invariants besides the usual completely contracted Riemann tensor R (or, in Eddington's terms, $G = R - 4\Lambda$). In other words, Einstein's theory is not unique even after choosing the observable space to be Riemannian and gravitation to be reduced to purely geometrical terms.

The present paper is mainly aimed at showing that the field-theoretic approach starting from flat, unrenormalized,[3] pseudo-Euclidean space-time leads in a sounder way to the Einstein equations without the cosmological term (i.e., with $\Lambda = 0$). In other words, one is more surely driven toward Einstein's theory with $\Lambda = 0$ in the flat-space approach than in the curved-space approach.

A pure tensor theory (i.e., with a potential represented by a rank-2 tensor) is chosen here on account of experimental evidence.[4]

This fact directly excludes the Jordan-Brans-Dicke theory in Riemannian space; but the flat-space approach in its complete, iterative form, to all orders in f, is able to remove also the cosmological term and the Eddington arbitrarinesses. Apparently, in the introduction of the nonlinearities,[5] arbitrary parameters appear,[6] but it will be shown that they are unobservable.

Moreover, this unique theory coincides, to second order, with Einstein's theory. Finally using the Deser[7] procedure, in one direct step this theory is shown to be equivalent to all orders with Einstein's theory. Actually, with his approach, Deser[7] removed one restriction to the important proof by Wyss.[8]

Indeed, Wyss showed that the field-theoretic approach, starting from flat unrenormalized[3] space, univocally leads to Einstein's theory if gauge invariance (for the tensor potential) is assumed to all orders in the coupling constant f. Then Deser[7] dropped this limitation by introducing the second-order energy-momentum tensor $T_{\alpha\beta}$ obtainable by a variational principle. The present paper shows that such an energy-momentum tensor can be implemented by the most general divergenceless tensor $t_{\alpha\beta}$ (still representing an energy-momentum for free fields and interaction) without changing the theory. In other words, the results are independent of the arbitrary parameters contained in $t_{\alpha\beta}$.

Finally, in addition to the works by Wyss[8] and Deser,[7] the presence of an electromagnetic field (with its coupling to the gravitational field) is considered as well.

II. SECOND-ORDER FIELD EQUATIONS AND EQUATIONS OF MOTION FOR COUPLED GRAVITATIONAL AND ELECTROMAGNETIC FIELDS IN THE FLAT-SPACE APPROACH

The equations of motion are obtained by a "bootstrap" procedure. The inconsistency of first-order field equations is removed by replacing the particle stress tensor $T^{(p)}_{\alpha\beta}$ in (10) of the preceding paper[9] by the total stress tensor $T^{(\mathrm{tot})}_{\alpha\beta}$ including gravitational terms. We get[10]

$$\Box\psi_{\alpha\beta} - \psi_{\lambda(\alpha;\beta)}{}^{\lambda} + \psi_{;\alpha\beta} + a_{\alpha\beta}(\psi_{\lambda\sigma}{}^{;\lambda\sigma} - \Box\psi) = fT^{(\mathrm{tot})}_{\alpha\beta}\,. \tag{1}$$

If the proper masses of our pointlike particles are assumed not to be affected by the gravitational field, the continuity equation is

$$Dm^{(0)}_{(n)}/ds = 0\,, \tag{2}$$

where $m^{(0)}_{(n)}$ is the proper mass of the nth particle.

The stress-energy tensor $T^{(\mathrm{tot})}_{\alpha\beta}$ must be (i) symmetric, and (ii) such that $\int dV_0 T_{\alpha\beta}{}^{;\beta}=0$ gives (2) and the first-order equations of motion as given by (10) of Ref. 9. A particular tensor $\tilde{T}_{\alpha\beta}$ is given by (13) of Ref. 9. But it is not unique, since it can be implemented by the most general divergenceless tensor $t_{\alpha\beta}$ given by Eq. (27) of Ref. 6, containing five arbitrary parameters A, B, C, D, and E: But E must be zero if our theory is Lagrangian.[11] Consequently, we have

$$\begin{aligned}
T^{(\mathrm{tot})}_{\alpha\beta} = {} & \tilde{T}_{\alpha\beta} + A[\psi_{;\alpha\beta}\psi + \psi_{;\alpha}\psi_{;\beta} - a_{\alpha\beta}(\psi_{;\nu}\psi^{;\nu} + \psi\psi_{\sigma\lambda}{}^{;\sigma\lambda})] \\
& + B[-\psi\psi_{;\alpha\beta} + 2\psi_{\alpha\beta;\nu}\psi^{;\nu} + \psi_{\alpha\beta}\psi_{\sigma\lambda}{}^{;\sigma\lambda} - \psi_{\nu(\alpha;\beta)}\psi^{;\nu} - \psi_{;(\alpha}\psi_{\beta)\nu}{}^{;\nu} - \psi_{\nu(\alpha}\psi_{;\beta)}{}^{\nu} + a_{\alpha\beta}(\psi_{\mu\nu}{}^{;\mu\nu}\psi + 2\psi_{\mu\nu}{}^{;\mu}\psi^{;\nu} + \psi_{\mu\nu}\psi^{;\mu\nu})] \\
& + C[\psi_{\gamma\lambda;\alpha\beta}\psi^{\gamma\lambda} + \psi_{\gamma\lambda;\alpha}\psi^{\gamma\lambda}{}_{;\beta} - a_{\alpha\beta}(\psi_{\gamma\lambda;\nu}\psi^{\gamma\lambda;\nu} + 2\psi^{\gamma\lambda}\psi_{\sigma\gamma;\lambda}{}^{\sigma} - \psi^{\gamma\lambda}\psi_{;\gamma\lambda})] \\
& + D[\psi_{\gamma(\alpha}\psi_{\beta)\mu}{}^{;\mu\gamma} - \psi_{\gamma(\alpha}\psi_{;\beta)}{}^{\gamma} + 2\psi_{\alpha\gamma;\nu}\psi_{\beta}{}^{\gamma;\nu} - \psi_{\gamma(\alpha;\beta)\mu}\psi^{\gamma\mu} \\
& \quad - \psi_{\gamma\nu;(\alpha}\psi_{\beta)}{}^{\gamma;\nu} - \psi_{\gamma(\alpha;\beta)}\psi^{\gamma\mu}{}_{;\mu} + a_{\alpha\beta}(2\psi^{\mu}{}_{\gamma;\mu\nu}\psi^{\gamma\nu} + \psi_{\mu\gamma}{}^{;\mu}\psi^{\gamma\nu}{}_{;\nu} + \psi_{\mu\gamma;\nu}\psi^{\gamma\nu;\mu})] \\
& + Bf\psi T^{(p+\mathrm{em})}_{\alpha\beta} - \left(\frac{B}{2} + D\right) f\psi_{\alpha\beta}T^{(p)} + \tfrac{1}{2}(A - B + C) f a_{\alpha\beta}\psi T^{(p)} - Cfa_{\alpha\beta}\psi^{\gamma\lambda}T^{(p+\mathrm{em})}_{\gamma\lambda} + Df\psi^{\gamma}{}_{(\alpha}T^{(p+\mathrm{em})}_{\beta)\gamma}\,,
\end{aligned} \tag{3}$$

where $T^{(p+\mathrm{em})}_{\alpha\beta} = T^{(p)}_{\alpha\beta} + T^{(\mathrm{em})}_{\alpha\beta}$ with $T^{(\mathrm{em})}_{\alpha\beta} = F_{\alpha\gamma}F_{\beta}{}^{\gamma} - \frac{1}{4}a_{\alpha\beta}F_{\gamma\lambda}F^{\gamma\lambda}$, and the particle stress tensor $T^{(p)}_{\alpha\beta} = T^{(p)\gamma}_{\alpha\beta\gamma}$ is given by (12) of Ref. 9.

To obtain the equations of motion as well as the electromagnetic field equations (modified Maxwell equations) to second order in the coupling constant f, the Lagrangian densities for particles and interactions are required with f^2 accuracy. They can be determined by imposing that (1) and (3) are obtained by varying $\psi_{\alpha\beta}$ in the total Lagrangian. With the same procedure as in Ref. 6, we get[12]

$$\begin{aligned}
L^{(p)} + L^{(\mathrm{em})} + L^{(p+g)} + L^{(p+\mathrm{em})} + L^{(\mathrm{em}+g)} = {} & -T^{(p+\mathrm{em})\alpha\beta}a_{\alpha\beta} + f\psi_{\alpha\beta}T^{(p)\alpha\beta} \\
& - (2+D)f^2\psi^{\alpha\beta}\psi^{\rho}_{\alpha}T^{(p+\mathrm{em})}_{\beta\rho} + \left(-\frac{C}{4} + \frac{D}{2} + \frac{1}{2}\right) f^2\psi_{\alpha\beta}\psi^{\alpha\beta}T^{(p)} \\
& + (1-B)f^2\psi\psi^{\lambda\rho}T^{(p+\mathrm{em})}_{\lambda\rho} + \left(-\frac{A}{4} + \frac{B}{2} - \frac{1}{4}\right) f^2\psi\psi T^{(p)} + \tfrac{1}{2}f^2\psi_{\rho\nu}\psi^{\alpha\beta}T^{(p)\rho\nu}{}_{\alpha\beta} \\
& + (-a)^{-1/2}\int ds \sum_{p} e_{(p)}A_{\alpha}\dot{z}^{\alpha}_{(p)}\delta^4(x - z_{(p)}) + f\psi_{\rho\nu}F^{\rho\gamma}F^{\nu}{}_{\gamma} \\
& - \tfrac{1}{4}f\psi F_{\rho\nu}F^{\rho\nu} + 2f^2\psi_{\lambda\sigma}\psi^{\sigma}_{\rho}F^{\rho}{}_{\gamma}F^{\lambda\gamma} + f^2\psi_{\rho\nu}\psi_{\lambda\gamma}F^{\rho\lambda}F^{\nu\gamma} \\
& - \tfrac{1}{4}f^2\psi_{\gamma\lambda}\psi^{\gamma\lambda}F_{\rho\nu}F^{\rho\nu} - f^2\psi\psi^{\lambda\sigma}F_{\lambda\rho}F_{\sigma}{}^{\rho} + \tfrac{1}{8}f^2\psi\psi F_{\rho\nu}F^{\rho\nu}\,,
\end{aligned} \tag{4}$$

where in (4) $L^{(p)}$ refers to particles, $L^{(\mathrm{em})}$ to the em field, $L^{(p+\mathrm{em})}$ to particle interaction with the electromagnetic field, $L^{(p+g)}$ to particle interaction with the gravitational field, and $L^{(\mathrm{em}+g)}$ to the interaction between electromagnetic and gravitational fields. Moreover, as in Ref. 6, $e_{(n)}$, A_{α}, and $F_{\alpha\beta}$ denote the electric charge of the particle and the electromagnetic potential and field, respectively.

By varying the particle coordinates in (4) and integrating over d^4x (as appears in the action integral), the second-order equations of motion for the nth particle are obtained,

$$\frac{D}{ds}\Big[-\dot{z}_\gamma - f\,\psi_{\alpha\beta}\dot{z}^\alpha\dot{z}^\beta\dot{z}_\gamma + 2f\psi_{\gamma\lambda}\dot{z}^\lambda - (2D+4)f^2\psi_{\lambda\gamma}\psi^\lambda_\nu\dot{z}^\nu + (1-B)2f^2\psi\psi_{\lambda\gamma}\dot{z}^\lambda + 2f^2\psi_{\mu\nu}\dot{z}^\mu\dot{z}^\nu\psi_{\lambda\gamma}\dot{z}^\lambda + f^2\left(\frac{D}{2}-\frac{C}{4}+\frac{1}{2}\right)\psi_{\lambda\rho}\psi^{\lambda\rho}\dot{z}_\gamma$$
$$+ f^2\left(\frac{B}{2}-\frac{A}{4}-\frac{1}{4}\right)\psi\psi\dot{z}_\gamma + f^2(D+2)\psi_{\lambda\mu}\psi^\lambda_\rho\dot{z}^\mu\dot{z}^\rho\dot{z}_\gamma - \tfrac{3}{2}f^2(\psi_{\mu\nu}\dot{z}^\mu\dot{z}^\nu)^2\dot{z}_\gamma + f^2(B-1)\psi\psi_{\lambda\rho}\dot{z}^\lambda\dot{z}^\rho\dot{z}_\gamma\Big]$$
$$= -\frac{e}{m}F_{\gamma\beta}\dot{z}^\beta + f\psi_{\alpha\beta;\gamma}\dot{z}^\alpha\dot{z}^\beta - f^2(2D+4)\psi^\lambda_\rho\psi_{\lambda\nu;\gamma}\dot{z}^\rho\dot{z}^\nu + f^2\left(D-\frac{C}{2}+1\right)\psi^{\alpha\beta}\psi_{\alpha\beta;\gamma}$$
$$+ (1-B)f^2(\psi_{;\gamma}\psi_{\lambda\rho}\dot{z}^\lambda\dot{z}^\rho + \psi\psi_{\lambda\rho;\gamma}\dot{z}^\lambda\dot{z}^\rho) + f^2\left(B-\frac{A}{2}-\frac{1}{2}\right)\psi\psi_{;\gamma} + f^2\psi_{\mu\nu;\gamma}\psi_{\lambda\rho}\dot{z}^\mu\dot{z}^\nu\dot{z}^\lambda\dot{z}^\rho\,, \quad (5)$$

where, for simplicity, the subscript (n) is dropped. By varying the em potential A_α in (4), the second-order electromagnetic field equations are obtained,

$$F^{\gamma\alpha}{}_{;\gamma} = J^\alpha + f(F^{\gamma\alpha}\psi - 2F^{\sigma\alpha}\psi^\gamma_\sigma + 2F^{\sigma\gamma}\psi^\alpha_\sigma)_{;\gamma} + f^2[2D\psi^{\sigma\gamma}\psi_{\lambda\sigma}F^{\lambda\alpha} - 2D\psi_{\lambda\sigma}\psi^{\sigma\alpha}F^{\lambda\gamma} - 4\psi^\gamma_\nu\psi^\alpha_\sigma F^{\nu\sigma} - (D+1)\psi_{\sigma\lambda}\psi^{\sigma\lambda}F^{\gamma\alpha}$$
$$- 2(B-2)\psi\psi^\gamma_\sigma F^{\alpha\sigma} + 2(B-2)\psi\psi^\alpha_\sigma F^{\gamma\sigma} - (B-\tfrac{1}{2})\psi\psi F^{\gamma\alpha}]_{;\gamma}\,, \quad (6)$$

where, in the case of point particles, the electric current density is given by $j^\alpha = (-a)^{-1/2}\sum_{(n)}e_{(n)} \times \int ds_{(n)}\delta^4(x-z_{(n)})\dot{z}^\alpha_{(n)}$.

Notice that the motion given by (5) can be described as being due to a pure electromagnetic field in a curved space-time whose metric is given by[6,13]

$$g_{\alpha\beta} = a_{\alpha\beta} - 2f\,\psi_{\alpha\beta} + f^2(2D+4)\psi_{\alpha\gamma}\psi^\gamma_\beta + f^2(2B-2)\psi\psi_{\alpha\beta} + f^2\left(\frac{A}{2}-B+\frac{1}{2}\right)a_{\alpha\beta}\psi\psi + f^2\left(\frac{C}{2}-D-d\right)a_{\alpha\beta}\psi_{\rho\nu}\psi^{\rho\nu}\,. \quad (7)$$

III. EQUIVALENCE OF THE CLASS OF THEORIES OBTAINED, WITH EINSTEIN'S THEORY

Einstein's equations, in the presence of electromagnetic field, are

$$R^*_{\alpha\beta} - \tfrac{1}{2}g^*_{\alpha\beta}R^* = -f^2(T^{(p)*}_{\alpha\beta} + T^{(\mathrm{em})*}_{\alpha\beta})\,, \quad (8)$$

where the asterisk denotes quantities in the Riemannian space. Equations (8) are here translated[14] into flat space-time to second order in f, generalizing the Rosen[15] procedure by the following rules:

$$g^*_{\alpha\beta} = a_{\alpha\beta} - 2f\,\tilde{\psi}_{\alpha\beta} \quad (9)$$

$$z^{*\alpha}_{(n)} = z^\alpha_{(n)}\,,$$
$$m^*_{(n)} = m_{(n)}\,, \quad (10)$$
$$F^*_{\alpha\beta} = F_{\alpha\beta}\,,$$
$$e^*_{(n)} = e_{(n)}\,.$$

We get for the field equations

$$\Box\tilde{\psi}_{\alpha\beta} + \tilde{\psi}_{;\alpha\beta} - \tilde{\psi}_{\gamma(\alpha;\beta)}{}^\gamma + a_{\alpha\beta}(\tilde{\psi}_{\gamma\lambda}{}^{;\gamma\lambda} - \Box\tilde{\psi})$$
$$= -2f\tilde{\psi}^{\rho\gamma}(\tilde{\psi}_{\alpha\beta;\rho\gamma} + \tilde{\psi}_{\rho\gamma;\alpha\beta} - \tilde{\psi}_{\rho(\alpha;\beta)\gamma}) + 2f\,\tilde{\psi}^{\rho\gamma}{}_{;\rho}(\tilde{\psi}_{\gamma(\alpha;\beta)} - \tilde{\psi}_{\alpha\beta;\gamma})$$
$$- f\,\tilde{\psi}^{;\gamma}(\tilde{\psi}_{\gamma(\alpha;\beta)} - \tilde{\psi}_{\alpha\beta;\gamma}) + 2f\,\tilde{\psi}_\alpha{}^{\rho;\gamma}\tilde{\psi}_{\beta\gamma;\rho} - 2f\,\tilde{\psi}_\alpha{}^{\gamma;\rho}\tilde{\psi}_{\beta\gamma;\rho} - f\,\tilde{\psi}^{\rho\gamma}{}_{;\alpha}\tilde{\psi}_{\rho\gamma;\beta}$$
$$+ a_{\alpha\beta}f(-2\tilde{\psi}^{\lambda\gamma}{}_{;\lambda}\tilde{\psi}^\rho{}_{\gamma;\rho} + 2\tilde{\psi}_{;\lambda}\tilde{\psi}^{\lambda\gamma}{}_{;\gamma} - \tfrac{1}{2}\tilde{\psi}^{;\lambda}\tilde{\psi}_{;\lambda} - \tilde{\psi}^{\lambda\rho;\gamma}\tilde{\psi}_{\lambda\gamma;\rho} + \tfrac{3}{2}\tilde{\psi}^{\lambda\gamma;\rho}\tilde{\psi}_{\lambda\gamma;\rho}) + f^2\tilde{\psi}_{\alpha\beta}T^{(p)} + f^2\tilde{\psi}T^{(p)}{}_{\alpha\beta}$$
$$- f^2a_{\alpha\beta}\tilde{\psi}\,T^{(p)} - 2f^2\tilde{\psi}^\rho{}_{(\alpha}T^{(p)}_{\beta)\rho} + 2f^2a_{\alpha\beta}\tilde{\psi}^{\rho\gamma}T^{(p)}_{\rho\gamma} + f^2\tilde{\psi}_{\mu\nu}T^{(p)\mu\nu}_{\alpha\beta}$$
$$+ 2f^2a_{\alpha\beta}\tilde{\psi}^{\rho\nu}T^{(\mathrm{em})}_{\rho\nu} + f\,T^{(p)}_{\alpha\beta} + f\,T^{(\mathrm{em})}_{\alpha\beta} + 2f^2\tilde{\psi}^{\gamma\lambda}F_{\alpha\gamma}F_{\beta\lambda} + \tfrac{1}{2}f^2\tilde{\psi}_{\alpha\beta}F_{\rho\nu}F^{\rho\nu} - f^2F_{\lambda\sigma}F^\sigma_\rho\tilde{\psi}^{\lambda\rho}a_{\alpha\beta}\,. \quad (11)$$

The equations for the electromagnetic field, which in curved Riemannian space are given by $F^{*;\beta}_{\alpha\beta} = \dot{J}^*_\alpha$, become by (9) and (10)

$$F^{\gamma\alpha}{}_{;\gamma} = j^\alpha + f(F^{\gamma\alpha}\tilde{\psi} - 2F^{\sigma\alpha}\tilde{\psi}^\gamma_\sigma + 2F^{\sigma\gamma}\tilde{\psi}^\alpha_\sigma)_{;\gamma}$$
$$+ f^2(-4\tilde{\psi}_{\lambda\sigma}\tilde{\psi}^{\sigma\alpha}F^{\lambda\gamma} + 4\tilde{\psi}_{\lambda\sigma}\tilde{\psi}^{\sigma\gamma}F^{\lambda\alpha} - 4\tilde{\psi}^\gamma_\nu\tilde{\psi}^\alpha_\sigma F^{\nu\sigma} + \tilde{\psi}_{\sigma\lambda}\tilde{\psi}^{\sigma\lambda}F^{\gamma\alpha} + 2\tilde{\psi}\tilde{\psi}^\gamma_\sigma F^{\alpha\sigma} - \tfrac{1}{2}\tilde{\psi}\tilde{\psi}F^{\gamma\alpha})_{;\gamma}\,. \quad (12)$$

The equations of motion for the test particle, obtainable in the curved space by

$$(T^{*(p)}_{\alpha\beta} + T^{*(\mathrm{em})}_{\alpha\beta})^{;\beta} = 0 , \tag{13}$$

are, translated by (9) and (10) into flat space, for the nth particle

$$\frac{D}{ds}[\dot{z}_\alpha(1 + f\tilde{\psi}_{\rho\nu}\dot{z}^\rho\dot{z}^\nu) - 2f\tilde{\psi}_{\alpha\beta}\dot{z}^\beta - 2f^2\tilde{\psi}_{\alpha\beta}\tilde{\psi}_{\rho\nu}\dot{z}^\rho\dot{z}^\nu\dot{z}^\beta + \tfrac{3}{2}f^2\dot{z}_\alpha(\tilde{\psi}_{\rho\nu}\dot{z}^\rho\dot{z}^\nu)^2] = \frac{e}{m}F_{\alpha\beta}\dot{z}^\beta + f\tilde{\psi}_{\rho\nu;\alpha}\dot{z}^\rho\dot{z}^\nu(1 + f\tilde{\psi}_{\lambda\gamma}\dot{z}^\lambda\dot{z}^\gamma) , \tag{14}$$

where, as in (5), for simplicity the subscript (n) was dropped.

Since the tensor potential is not observable, to describe a theory like that of Sec. II, any other function $\tilde{\psi}_{\alpha\beta}$ biunivocally related to $\psi_{\alpha\beta}$ can be chosen.

The expression

$$\psi_{\alpha\beta} = \tilde{\psi}_{\alpha\beta} + f(D+2)\tilde{\psi}_{\alpha\gamma}\tilde{\psi}^\gamma_\beta + f(B-1)\tilde{\psi}\tilde{\psi}_{\alpha\beta} + f\left(\frac{A}{4} - \frac{B}{2} - \frac{1}{4}\right)\tilde{\psi}\tilde{\psi}a_{\alpha\beta} + f\left(\frac{C}{4} - \frac{D}{2} - \frac{1}{2}\right)\psi_{\rho\nu}\psi^{\rho\nu}a_{\alpha\beta} \tag{15}$$

transforms the equations as follows:

Eq. (7) → Eq. (9) ,

Eq. (1) → Eq. (11) ,

Eq. (6) → Eq. (12) ,

Eq. (5) → Eq. (14) .

The last three transformations for the field equations and the equations of motion mean that the class of theories of Sec. II can be described in the same way as Einstein's theory. The transformation Eq. (7) → Eq. (9) says that the new potential, by which the theories of Sec. II are described, is linked to $g^*_{\alpha\beta}$ exactly as $\tilde{\psi}_{\alpha\beta}$ in Einstein's theory. This means that the four arbitrary parameters appearing in the theories of Sec. II are unobservable, and that such theories are just equal to Einstein's theory to second order in f.

To prove that such theories converge to Einstein's theory to any order in f, Deser's very powerful procedure[7] is used directly, including the presence of particles and electromagnetic fields as well. One obtains, to second order, field equations which are a particular case of (1) [with $T_{\alpha\beta}$ given by (3)]. But we showed that the class of the apparently different theories (differentiated by the values of the four arbitrary parameters) corresponds to only one "observable" theory.

Consequently, since one of them converges to general relativity the whole class of theories does the same.

IV. CONCLUSIONS

The class of theories of Sec. II was obtained under the following assumptions: (a) The proper mass and charges of the particles are independent of the gravitational field; (b) the gravitational potential is assumed to be a second-rank tensor (with no scalar component) in the first-order version of the theory (which is unique); (c) field equations and equations of motion are obtainable from a Lagrangian density (Dicke's framework[1]). The various theories studied here are apparently different from one another because of four arbitrary parameters. But, as shown here, they are unobservable and therefore the arbitrariness is irrelevant (to second order in f) and the above gravitational theories coincide with Einstein's theory.[16]

Now the Palatini formulation argument deduces, as shown by Deser,[7] the Einstein result to all orders using the stress-energy tensor to second order only (which is the lowest one containing gravitational terms).

Summarizing, the field-theoretic approach starting from flat, unrenormalized space univocally leads, under assumptions (a), (b), and (c), to general relativity in its simplest form, i.e., without the cosmological term. By contrast, in the curved-space approach, there is no indication either against or in favor of it.

Finally let us also recall, following Thirring,[3] that Einstein's approach does not tell us why the observable space is Riemannian.

On the contrary, the field-theoretic approach (1) predicts that gravitation is attractive (since total energy is positive, in a theory whose potential is an even-rank tensor, only if equal sources attract each other); (2) leads automatically to a Riemannian space-time structure (and thereby defines the standard clocks and rods); (3) constructs the gravitational theory following the pattern of well-understood theories (in particular electrodynamics); and (4) eliminates the cosmological term (which is quite unnatural in it[17]) as well as the Eddington-type ambiguities.[2]

ACKNOWLEDGMENTS

One of the authors (G.S.) is glad to acknowledge that his contribution to the research was performed under the auspices of the C. N. R. Gruppo per la Fisica-Matematica.

[1]A. Eddington, *The Mathematical Theory of Relativity*, second edition (Cambridge, New York, 1954), Sect. 95.

[2]See Sec. 62 of Ref. 1. Moreover, we can start from any function of R or $G_{\alpha\beta}G^{\alpha\beta}$ and so on, as recently done, for example, by G. V. Bicknell, J. Phys. A 7, 1061 (1974).

[3]W. Thirring, Ann. Phys. (N.Y.) 16, 96 (1961). See also R. U. Sexl, Fortschr. Phys. 15, 269 (1967).

[4]I. I. Shapiro, Phys. Rev. Lett. 13, 789 (1964); 20, 1265 (1968); in Proceedings of the Conference on Experimental Tests of Gravitational Theories, California Institute of Technology, Pasadena, 1970 (unpublished); I. I. Shapiro, G. M. Pettengill, M. E. Ash, R. P. Ingalls, D. B. Campbell, and R. B. Dyce, Phys. Rev. Lett. 28, 1594 (1972); for the most recent best data on the radar echoes see P. B. Esposito and J. C. Anderson, Telecommun. J. 41, 299 (1974); for the measurement of solar gravitational deflection of radiowaves see C. C. Counselman III, S. M. Kent, C. A. Knight, I. I. Shapiro, T. A. Clark, H. F. Interegger, A. E. E. Rogers, and A. R. Whitney, Phys. Rev. Lett. 33, 1621 (1974).

[5]A linear theory is inconsistent since it requires that the mass of a (spherical) body does not remain positive-definite for any value of its radius R, and can tend to minus infinity for $R \to 0$ (see the Introduction of Ref. 6). There is also another drawback in a linear theory: Namely a gauge-invariant Lagrangian for the free-field tensor potential is to be assumed to eliminate the scalar components. But then we have a motion by inertia in flat space, as if gravitation were absent. There is the exception to the theory by S. Deser and B. E. Laurent [Ann. Phys. (N.Y.) 50, 76 (1968)], where the source of the field is the divergenceless projection of the matter energy-momentum. The first drawback, however, is still present and, moreover, this theory proved to be contrary to experiments [see C. M. Will, Astrophys. J. 185, 31 (1973)].

[6]G. Cavalleri and G. Spinelli, Nuovo Cimento 21B, 1 (1974).

[7]S. Deser, Gen. Relativ. Gravit. 1, 9 (1970). See also D. G. Boulware and S. Deser, Ann. Phys. (N.Y.) 89, 193 (1975).

[8]W. Wyss, Helv. Phys. Acta 38, 469 (1965); a similar procedure still using gauge invariance was independently used by V. I. Ogievetsky and I. V. Polubarinov, Ann. Phys. (N.Y.) 35, 167 (1975).

[9]G. Cavalleri and G. Spinelli, preceding paper, Phys. Rev. D 12, 2203 (1975).

[10]$g_{\alpha\beta}$ is the fundamental metric tensor in flat space being reduced in orthogonal Cartesian coordinates to $a_{\alpha\beta}=\mathrm{diag}(+1,-1,-1,-1)$. Semicolons denote covariant differentiations and parentheses containing two indices, denote symmetrization, e.g. $\psi_{\alpha(\beta;\gamma)}=\psi_{\alpha\beta;\gamma}+\psi_{\alpha\gamma;\beta}$. Furthermore, $\psi=\psi^{\lambda}_{\lambda}$.

[11]We accept the Dicke framework where a theory is considered to be reasonable if consistent with a Lagrangian formulation, which avoids theoretical drawbacks such as, for example, the self-acceleration of the system mass center. This paradox as shown by G. J. Whithrow and G. B. Morduch [in *Vistas in Astronomy*, edited by A. De Beer (Pergamon, London, 1965), Vol. 6] occurs, for instance in Nordström's theory, whose field equations can be obtained from a Lagrangian formulation [see S. Deser and L. Halpern, Gen. Relativ. Gravit. 1, 131 (1970)] whereas the equations of motion cannot.

[12]The coupling of the charged particles with the electromagnetic field remains as in (8) of Ref. 9, electromagnetism being a linear theory.

[13]G. Cavalleri and G. Spinelli, Nuovo Cimento Lett. 9, 325 (1974).

[14]See G. Cavalleri and G. Spinelli, Nuovo Cimento 21B, 27 (1974), where detailed calculations are given in the absence of em fields.

[15]N. Rosen, Phys. Rev. 57, 147 (1940).

[16]Because of a transcription error two arbitrary parameters appear in the results of Ref. 6. This error has been corrected in G. Cavalleri and G. Spinelli, Nuovo Cimento Lett. 13, 145 (1975).

[17]One could mathematically obtain the cosmological term starting from flat space by adding the term $\int d^4x(-a)^{1/2}\Lambda(\psi-f^{-1})$ to the action integral. Indeed by varying $\psi_{\alpha\beta}$ one obtains the extra term $a_{\alpha\beta}\Lambda/f$ to be added to the right-hand side of the first-order field equations. Then convergence to general relativity with the cosmological term is obtained. But in the field-theoretic approach, the above additional action integral is unacceptable from a physical point of view. Indeed Λ would be a very strange scalar field which couples with the gravitational field but not with matter. Moreover, such a scalar field would be constant and uniform throughout all space either in the presence of or in the absence of matter. Such physical considerations cannot be made directly in Riemannian space. However, some authors [see, e.g., L. Halpern, Ann. Phys. (N.Y.) 25, 387 (1963) and references therein] conceive of Λ as due to a uniform and constant matter density; for example the one arising from quantum vacuum fluctuations: see C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Box. 17.2, part 6h. But in this case Λ should give rise to a further attraction at large distances. This should be in contrast to the original idea of Einstein who introduced Λ in order to have a long-range repulsion.